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A Linearized OPF Model with Reactive Power and Voltage Magnitude: A Pathway to Improve the MW-Only DC OPF

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***Abstract*— In this study, a linearized and convergence-guaranteed optimal power flow (OPF) model with reactive power (*Q*) and voltage magnitude (*v*) is proposed. Based on a linearized network model, a fully linearly-constrained OPF model is formulated with constraints on *Q* and *v* and limits on the apparent branch flow. Compared with the commonly used DC OPF method, the proposed method narrows the deviation from the AC OPF solution without requiring any additional information of the power grid. The locational marginal price (LMP) of the proposed method is closer to the AC OPF solution than the DC OPF method. The marginal price of the reactive power (Q-LMP) is provided, which offers the opportunity to price the reactive power. Case studies on several IEEE and Polish benchmark systems show that the proposed OPF method substantially enhances the performance of the prevalent DC OPF method. In addition, it is shown that if the accuracy of the linearized network model needs to be further improved, such as during the iterative quasi-optimization process that reconstitutes the AC feasibility, a solution that is notably close to the optimum of the AC OPF model can be obtained by taking only one more iteration.**

***Index Terms*— Linear approximation, locational marginal price (LMP), losses, market clearing, optimal power flow (OPF).**

NOMENCLATURE

Variables and Parameters

c Coefficients for the generator production costs
G_{ij}/B_{ij} Real/imaginary part of Y_{ij} in the admittance matrix
g_{ij}/b_{ij} Conductance/susceptance of branch *(i,j)*
N Number of buses
Offset Offset for the loss linearization
P_i/Q_i Active/reactive power injection at bus *i*
$S_{ij,max}$ Apparent power limitation of branch *(i,j)*
P_{ij}/Q_{ij} Active/reactive power flow from bus *i* to bus *j*
P_{ij}^A/Q_{ij}^A Linear approximation of active/reactive power flow
P_{ij}^L/Q_{ij}^L Active/reactive losses on branch *(i,j)*
$P_{ij,v}^L/Q_{ij,\theta}^L$ Components of loss expression related to v/θ
v_i/θ_i Voltage magnitude/angle at bus *i*
v^s Square of voltage magnitude
α_P/α_Q Scalar determining the fluctuation of the base case

Vectors and Sets

$\mathbf{D}$ Matrix of loss distribution factors
$\mathbf{LF}$ Vector of loss factor
$\mathbf{P_D}$ Vector of active power demands
$\mathbf{P_G}/\mathbf{Q_G}$ Vector of generator active/reactive power production
$\mathbf{P_{loss}}$ Vector of active power losses
$\mathbf{P_f}$ Vector of active power flow
$\mathbf{T}$ Matrix of generator shift distribution factor
$\boldsymbol{\theta}$ Vector of voltage angle
$\mathbf{v^s}$ Square of voltage magnitude
$\mathcal{N}, \mathcal{K}, \mathcal{G}$ Sets of buses, branches, and generators

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I. INTRODUCTION

A. Research Motivation

Optimal power flow (OPF) calculation is crucial to facilitate the secure and economic operation of power grids. System operators perform OPF calculation in day-ahead and real-time operations to determine the most economic generator schedule within the operational limits. Improvements in OPF solvers can potentially save billions of dollars per year for power industries [1].

However, the OPF is a difficult problem to solve [2]. Finding the global optimum of the OPF model is *non-deterministic polynomial-time* (NP) hard [3]. In a power grid with thousands of buses, the computational burden of solving a full OPF model with various operational requirements is considerable [4]. Although numerous solving algorithms have been proposed, methods that achieve the global optimum cannot guarantee the convergence because of the nonlinearity and nonconvexity nature of the OPF problem. For practical system operations, one of the most critical requirements of an OPF solver is the computational robustness [5].

For the OPF problem, to guarantee convergence, accuracy must be sacrificed. There are basically two methods to ensure computational robustness: 1) convex relaxation and 2) network model linearization. For the convex relaxation approach, power flow equations are relaxed into inequalities to obtain a convex

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formulation [6]-[8]. Under certain conditions, a global optimum for the original OPF problem can be obtained from the relaxed model. However, when the conditions do not hold, there may be a large optimality gap, and it is difficult to reconstitute a feasible solution. To our best knowledge, there is still no industry-grade application of the convex relaxation method. By comparison, the network model linearization approach is widely used in power industries. The network model is simplified to guarantee the computational efficiency and robustness. The OPF method based on the DC network model, i.e., the DC OPF method, is one of the representatives. The DC network model becomes linear by ignoring the reactive power and assuming a flat voltage magnitude of all buses [9]. The quasi-linear P-θ relationship in transmission networks is revealed by the DC network model [10]. To distinguish from the DC OPF method, the OPF model with exact AC network model is termed as "AC OPF".

Currently, most system operators use the DC OPF method due to the concerns of computational robustness [5]. However, the DC OPF method oversimplifies the network model, which causes the following major drawbacks: 1) the reactive power (Q) and the voltage magnitude (v) are not modeled, and the voltage/VAR security cannot be considered; 2) the constraints of the active power flow are considered, but the effect of the reactive power flow on line current is ignored. As a result, the DC OPF method may provide uneconomic or insecure solutions, particularly in power networks with strong coupling between active and reactive power [11]. Moreover, using the DC OPF method as the market clearing solver in electricity markets may jeopardize the market efficiency for two reasons: 1) because of the inaccuracy of the DC network model, the solution from the DC OPF method may require manual adjustments in the *energy management system* (EMS) before being applied to the real-time operation; 2) the *locational marginal price* (LMP), which guides the market behavior, may be distorted.

Although the simplified network models may increase operational costs, jeopardize market inefficiency, and even threaten the security, most system operators still use the DC OPF method instead of the numerous AC OPF algorithms [12]. The key problem is the robustness requirement of the practical OPF solver, especially considering the complicated practical operational requirement, such as the N-1 security. The power industries sacrifice accuracy to guarantee the convergence and robustness of the OPF calculation. Besides, system operators prefer linear market clearing models because of the transparency of the obtained solution. In this study, a linear and convergence-guaranteed OPF method is proposed. While maintaining the benefits of the linear DC OPF model, the accuracy of the network model is "sacrificed less", and Q and v are included.

B. Literature Review

The existing solutions of the OPF problem are briefly reviewed.

1) OPF Methods with AC Network Models (AC OPF)

Since the OPF problem was first formulated in the 1960s [13], numerous solution techniques have been proposed, including Newton method, quadratic programming, linear and nonlinear programming, and interior point method [14]-[17]. The *successive linear programming* (SLP) approach is introduced in [11]. In [18], [19], a new linear approximation method that reduces the number of iterations required is proposed. However, the SLP approaches cannot guarantee the convergence. Recently, the convex relaxation method attracts many research interests. Approaches include *semidefinite programming* (SDP) relaxation [3], [6], *quadratic programming* (QP) relaxation [7], and *second-order cone programming* (SOCP) relaxation [8], [20]. These relaxed models are convex and can be solved in a polynomial time. They always provide a lower bound for the OPF problem. However, it is difficult to reconstitute an AC feasible solution from the relaxed model when the duality gap is nonzero. In standard IEEE test systems, the optimality gap can be up to 30% [7].

2) OPF Methods with Linearized Network Models

OPF methods with linearized network models are proposed to reduce the computational burden of the AC OPF problem by simplifying the power flow equations. The commonly used method is the DC OPF, where network losses are ignored and Q and v are not modeled. A quadratic function of bus angles is commonly used to represent losses in the DC OPF method [21], [22]. The nonconvexity is addressed by piecewise linearization [21] or convex relaxation [22]. However, the accuracy of such approximation is reduced when negative LMPs occur [23].

When the system operation approaches its stress condition, active power transfer is frequently constrained by the limits of voltage and reactive power [24]. Incorporating voltage/VAR constraints in the OPF model is becoming increasingly urgent. There have been efforts to include Q and v in the linearized network models. An improved linearized network model based on the Taylor series expansion is used for the OPF problem in [25], [26]. A piecewise linearization method with integer variables is proposed to handle the nonconvexity introduced by the quadratic losses. However, the computational burden is significantly increased because of the "curse of dimensionality".

For existing OPF methods with linearized network models, there is still room to improve the for the accuracy of the modeling of Q and v.

3) Current Practice in Power Industries

The DC OPF method is currently used by most system operators. Losses are handled in different ways. For example, losses can be distributed to the loads based on an estimate of the total system losses [27]. This approach is currently used by system operators in China. In New Zealand and Australia, losses are approximated using a piecewise linear function [28]. However, the power flow balance equations cannot be enforced when there are negative LMP values.

In the major electricity markets in the US, including PJM, MISO, and ISO-NE, losses are estimated using a linearization method based on a base case system operating condition [29], [30]. The sensitivity of the losses with respect to the active power injections is referred to as the "loss factor" (denoted by

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LF). The formulation of losses is as follows:

$$P_{loss} = \mathbf{LF}^T(\mathbf{P_G} - \mathbf{P_D}) + \text{offset}_w \quad (1)$$

A "loss distribution factor" (denoted by **D**) is often selected based on the load pattern and determines the allocation of losses to each network location. The branch flow is computed as follows:

$$\mathbf{P}_f = \mathbf{T}(\mathbf{P_G} - \mathbf{P_D} - \mathbf{D}\mathbf{P}_{loss}) \quad (2)$$

where matrix **T** represents the *generator shift distribution factor* (GSDF). The "loss factor" and the "loss distribution factor" provide the sensitivity of the losses allocated to each bus with respect to the power injections.

In this study, the loss factor-based method is used to handle the losses.

C. Contributions

Although there are many algorithms for the OPF problem, notably few satisfy the requirement of practical operations. There is no robust and convergence-guaranteed OPF method that explicitly considers the constraints of *Q* and *v*.

In this study, a linearly-constrained and convergence-guaranteed OPF method is proposed. The OPF model is formulated based on a new linearized network model with *Q* and *v*. The proposed method improves the performance of the MW-only DC OPF method without requiring any additional information. Compared with the DC OPF method, the notable features of the proposed method are presented as follows:

1) The proposed method explicitly models the reactive power components such as *Q* and *v*, and takes into consideration of their impacts on the dispatch of active power. More importantly, the constraints for voltage magnitude can directly solve the voltage issue that is often encountered by the existing DC OPF based market clearing practice.

2) The proposed method improves the accuracy of the DC OPF solution as well as the corresponding prices for active power, i.e., LMP, as shown in the numerical example. Both enhancements can significantly improve the efficiency of the overall electricity market and induce better pricing, market transparency, and long-term investment.

3) The proposed OPF model is also able to produce the price for reactive power (referred to as Q-LMP) as a side product, and opens the door to designing a better compensation scheme for reactive power devices.

4) The proposed method has desirable computational performance and is suitable for practical implementation of real-world market clearing.

5) If the accuracy of the network model needs to be further improved, such as during the iterative process that reconstitutes the AC feasibility, a solution that is notably close to the AC OPF optimum can be obtained by taking only one more iteration using the proposed method.

The effectiveness of the proposed method is demonstrated in several IEEE and Polish test systems. Compared with the existing DC OPF method, the proposed OPF method improves the overall accuracy of the power flow solution and provides a decent estimate of *Q* and *v*.

To the best of our knowledge, the proposed method presents the first linearly-constrained and convergence-guaranteed OPF model with *Q* and *v*, which creates a great opportunity to enhance the performance of the commonly used DC OPF method.

II. Introducing A New Linearized Network Model

In this section, a network model with *Q* and *v* is introduced. The network model consists of linear components that represent the distribution of power flows and quadratic components that represent the losses.

A. A Linearized Network Model with Q and v

1) Branch Flow Expression

Compared with the DC network model, the proposed network model makes fewer approximations and consequently improves the accuracy. In the derivation, linear components and quadratic components are maintained. The linear terms represent the improved DC power flow equations, whereas the quadratic terms refer to the network losses.

The power flows on branch *(i,j)* are expressed as follows:

$$P_{ij} = (v_i^2 - v_i v_j \cos\theta_{ij})g_{ij} - v_i v_j b_{ij} \sin\theta_{ij} \quad (3)$$

$$Q_{ij} = -(v_i^2 - v_i v_j \cos\theta_{ij})b_{ij} - v_i v_j g_{ij} \sin\theta_{ij} \quad (4)$$

To keep the linear and quadratic terms, the second-order Taylor series expansions of the sine and cosine functions are used. Assuming that the value of θ_{ij} is normally a small number, the following equations are obtained:

$$\sin\theta_{ij} \approx \theta_{ij}, \cos\theta_{ij} \approx 1 - \frac{\theta_{ij}^2}{2} \quad (5)$$

To decouple v and θ, the following approximations are used, assuming that the magnitude of *v* is close to 1.0 *p.u.*:

$$v_i v_j \theta_{ij} \approx \theta_{ij}, v_i v_j \theta_{ij}^2 \approx \theta_{ij}^2 \quad (6)$$

By substituting (5) and (6) into (3) and (4), the following equations are obtained:

$$P_{ij} = g_{ij}(v_i^2 - v_i v_j) - b_{ij}\theta_{ij} + \frac{1}{2} g_{ij}\theta_{ij}^2 \quad (7)$$

$$Q_{ij} = -b_{ij}(v_i^2 - v_i v_j) - g_{ij}\theta_{ij} + \frac{1}{2}(-b_{ij})\theta_{ij}^2 \quad (8)$$

Regarding v^2 as an independent variable, a mathematical transformation for the nonlinear voltage magnitude term is used ($v_{ij} = v_i - v_j$):

$$v_i^2 - v_i v_j = v_i^2 - \left(\frac{v_i^2 + v_j^2}{2} - \frac{v_{ij}^2}{2}\right) = \frac{v_i^2 - v_j^2}{2} + \frac{v_{ij}^2}{2} \quad (9)$$

Without sacrificing the accuracy, the nonlinear term $v_i v_j$ is transformed into a linear term and a quadratic term by (9). By substituting (9) into (7) and (8), the linearized network model with *Q* and *v* is obtained:

$$P_{ij}^A = g_{ij}\frac{v_i^2 - v_j^2}{2} - b_{ij}\theta_{ij} + P_{ij}^L \quad (10)$$

$$Q_{ij}^A = -b_{ij}\frac{v_i^2 - v_j^2}{2} - g_{ij}\theta_{ij} + Q_{ij}^L \quad (11)$$

where

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$$P_{ij}^L = \frac{1}{2} g_{ij} (\theta_{ij}^2 + v_{ij}^2),\ Q_{ij}^L = \frac{1}{2} (-b_{ij})(\theta_{ij}^2 + v_{ij}^2) \quad (12)$$

In (10) and (11), the power flow equations are linear, except for the losses. As shown in (12), the effect of voltage magnitudes on the losses is included.

2) Nodal Power Balance Expression

The nodal active and reactive power injections at bus i can be expressed as follows:

$$P_i = \sum_{j=1}^{N} (G_{ij} v_i v_j \cos\theta_{ij} + B_{ij} v_i v_j \sin\theta_{ij}) \quad (13)$$

$$Q_i = -\sum_{j=1}^{N} (B_{ij} v_i v_j \cos\theta_{ij} - G_{ij} v_i v_j \sin\theta_{ij}) \quad (14)$$

Compared with (3) and (4), it can be inferred that

$$P_i = \sum_{(i,j)\in\mathcal{K}} P_{ij} + (\sum_{j=1}^{N} G_{ij}) v_i^2 \quad (15)$$

$$Q_i = \sum_{(i,j)\in\mathcal{K}} Q_{ij} + (\sum_{j=1}^{N} -B_{ij}) v_i^2 \quad (16)$$

The second terms in (15) and (16), which represent the power flows on the shunt elements, are linear with v^2 as an independent variable.

B. A Fully-Linear Network Model Based on the Loss Factor

In (10) and (11), the only nonlinear terms are the losses, which are naturally quadratic and cannot be linearized using a cold-start in a non-iterative manner. In this paper, the loss factor-based linearization method, which is adopted by most electricity markets in the U.S., is used to facilitate the fully linear formulation of the network model.

Based on a base case system operating condition, the losses are linearized as follows:

$$P_{ij}^L \approx \mathbf{LF}_{P,\theta}^T \boldsymbol{\theta} + \mathbf{LF}_{P,v^s}^T \mathbf{v}^s + \text{offset}_{P,ij} \quad (17)$$

$$Q_{ij}^L \approx \mathbf{LF}_{Q,\theta}^{\mathbf{T}} \boldsymbol{\theta} + \mathbf{LF}_{Q,v^s}^{\mathbf{T}} \mathbf{v}^s + \text{offset}_{Q,ij} \quad (18)$$

where the expressions of the sensitivities are shown in Appendix. A. The coefficients $\mathbf{LF}_{P,\theta}, \mathbf{LF}_{P,v^s}, \mathbf{LF}_{Q,\theta}$, and $\mathbf{LF}_{Q,v^s}$ are the loss factors for the proposed network model. Compared with the commonly used DC OPF method, the estimate of losses is more precise because the change in losses caused by different voltage profiles is represented by $\mathbf{LF}_{P,v^s}$ and $\mathbf{LF}_{Q,v^s}$. The same base case system operating condition used by the commonly adopted DC OPF method can still be used to calculate (17) and (18). Hence, no additional information of the power grid is required for the proposed method.

The fully linear network model is described by (10) and (11) with (17) and (18).

III. Proposed OPF Solution

A. Proposed Linearly-Constrained OPF Model

Based on the proposed linearized network model, a fully linearly-constrained OPF model can be formulated.

A general formulation of the objective function of an OPF model is described as follows:

$$\min_{\mathbf{P_G}, \mathbf{Q_G}, \mathbf{v}^s, \boldsymbol{\theta}} f(\mathbf{P_G}, \mathbf{Q_G}, \mathbf{v}^S, \boldsymbol{\theta}) \quad (19)$$

In this paper, the objective function provided by MATPOWER is used, which is expressed as a polynomial function of active and reactive power production from the generators [31]:

$$\begin{aligned} &\min_{\mathbf{P_G}, \mathbf{Q_G}, \mathbf{v}^s, \boldsymbol{\theta}} f(\mathbf{P_G}, \mathbf{Q_G}, \mathbf{v}^S, \boldsymbol{\theta}) \\ &= \sum_{g\in\mathcal{G}} \left[(c_{g,2}^P P_g^2 + c_{g,1}^P P_g + c_{g,0}^P) + (c_{g,2}^Q Q_g^2 + c_{g,1}^Q Q_g + c_{g,0}^Q) \right] \end{aligned} \quad (20)$$

Because Q and v are explicitly included, other optimization objectives also can be considered, such as minimizing losses, minimizing the deviations from the ideal set points of voltage magnitude, and minimizing the operational costs of FACTS devices [19].

The constraints of the OPF model are presented as follows:

1) Power Flow Equations

$$\text{constraints (10), (11) and (17), (18)}, \quad (i,j)\in\mathcal{K} \quad (21)$$

Using (17) and (18), the losses are expressed by a linear function, which can be positive or negative. However, the actual network losses should always be positive. The losses are expressed as independent functions of θ and v, i.e., $P_{ij,\theta}^L$ and $P_{ij,v}^L$. Because of the quasi-linear P-θ relationship, the values of $P_{ij,\theta}^L$ are usually restricted by the optimization objective of minimizing the operational costs. To avoid the negative values of $P_{ij,v}^L$, the following constraint is added:

$$P_{ij,v}^L + \varepsilon_{ij}^+ \geq 0, \quad (i,j)\in\mathcal{K} \quad (22)$$

where the expression for $P_{ij,v}^L$ can be found in Appendix. A. The penalty factor for ε_{ij}^+ is added to the objective function.

2) Nodal Power Balance Equations

$$\sum_{g\in i} P_g - P_{i,d} = \sum_{(i,j)\in\mathcal{K}} P_{ij}^A + (\sum_{j=1}^{N} G_{ij}) v_i^2 \quad (23)$$

$$\sum_{g\in i} Q_g - Q_{i,d} = \sum_{(i,j)\in\mathcal{K}} Q_{ij}^A + (\sum_{j=1}^{N} -B_{ij}) v_i^2 \quad (24)$$

3) Branch Flow Limits

$$(P_{ij}^A)^2 + (Q_{ij}^A)^2 \leq S_{ij,\max}^2, \ (i,j)\in\mathcal{K} \quad (25)$$

Constraint (25) is quadratic. Because (25) defines a convex region, it can be easily linearized. In this paper, the piecewise linearization method described in [18] is used to facilitate the linear formulation. Constraint (25) can be approximated by a group of linear constraints denoted by Λ:

$$\Lambda(P_{ij}^A, Q_{ij}^A), \ (i,j)\in\mathcal{K} \quad (26)$$

The explanation for the piecewise linearization is provided in the Appendix. B.

4) Operational Constraints

$$P_g^{\min} \leq P_g \leq P_g^{\max}, Q_g^{\min} \leq Q_g \leq Q_g^{\max}, \ g\in\mathcal{G} \quad (27)$$

$$v_{i,\min}^2 \leq v_i^2 \leq v_{i,\max}^2, i\in\mathcal{N} \quad (28)$$

The objective function (19) and constraints (21) -(24) and (26) -(28) define the proposed OPF model. The constraints of the model are fully linear with v^2 as an independent variable.

B. Discussion of the Proposed OPF Model

1) Improvements Compared with the DC OPF Method

In the proposed OPF model, the apparent branch flow limits are included. The congestion of the system can be more precisely reflected compared with the DC OPF method, where only surrogate active power flow limits are enforced.

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Constraints on Q and v are also considered in the proposed model.

In addition, because the effect of the voltage/VAR limits on the LMP is considered, the proposed method provides more precise economic signal than the DC OPF method. The inclusion of reactive power balance equations allows a practical implementation of the reactive power market.

In general, the proposed OPF model improves the solution accuracy and economic efficiency and broadens the range of applications compared with the commonly used DC OPF model without requiring any additional information.

2) Discussion of the Loss Linearization

For the values of losses described in (17) and (18), the component of voltage angles, i.e., $P^L_{ij,\theta}$, is dominant because θ_{ij} is normally larger than v_{ij}. The distance between the voltage angles in the obtained OPF solution and those in the base case system operating condition for calculating the loss factors determines the accuracy of the loss linearization. Because the relationship between the active power flow and the voltage angles is quasi-linear in transmission systems, it can be inferred that the accuracy of the loss linearization is acceptable when the power flow pattern on branches does not dramatically change between the base case system operating condition for the loss linearization and the obtained OPF solution. According to the current experience of power system operations, an acceptable historical base case system operating condition for calculating loss factors is commonly accessible. Case studies demonstrate that even with a large difference between the base case system operating condition for the loss linearization and the obtained OPF solution, the performance of the proposed method remains satisfactory.

IV. Further Improving Accuracy

Similar to the commonly used DC OPF method, the proposed OPF model requires a base case system operating condition to linearize losses. Errors are inevitable. If the accuracy of the network model requires further improvement, such as during the iterative quasi-optimization process that reconstitutes the AC feasibility, this section provides a method to obtain a solution that is notably close the optimum of the AC OPF model by taking only one more iteration.

A. Motivation for Further Improving the Accuracy

Equations (10) and (11) show that the nonlinearity of the power flow equations embodies in the losses, particularly the reactive power losses. The losses are naturally quadratic. Therefore, losses cannot be linearized using a cold start. The accuracy of the loss linearization in Section II. B depends on the distance between the base case system operating condition for calculating the loss factor and the obtained OPF solution. The loss linearization can introduce unpredictable approximation errors in the power flow equations. This is particularly true to the reactive power flow equations considering the large percentage of reactive power losses over the overall reactive power demand.

In certain applications, the accuracy of the linearized network model needs to be further improved. For example, some system operators, such as CAISO, use an iterative quasi-optimization method to constitute the AC feasibility [32]. An AC power flow calculation is performed based on the solution of the DC OPF model. If there is any violation, the DC OPF model is re-optimized. During the DC-AC iteration, the initial point is updated, and a warm-start model can be formulated to further improve the accuracy.

Hence, this subsection investigates how to improve the accuracy of the network model if necessary.

B. Formulation of the Warm-Start OPF Model

Based on the values of (v_1, θ_1) obtained from the solution of the proposed linear OPF model, a warm-start network model can be formulated. The expressions for the coefficients below are shown in Appendix. C.

Approximation (5) is improved as follows:

$$\sin\theta_{ij} \approx s^1_{ij}\theta_{ij} + s^0_{ij}, \cos\theta_{ij} \approx c^1_{ij}\theta_{ij} + c^0_{ij} \tag{29}$$

Using (29), the accuracy of the Taylor series expansions for the sine and cosine functions is improved. The effect of θ^2_{ij} on the network losses is embedded in coefficient c^1_{ij}. An improved approximation for the nonlinear term $v_i v_j \theta_{ij}$ is used:

$$v_i v_j \theta_{ij} \approx v_{i,1} v_{j,1} \theta_{ij} + (v_i v_j - v_{i,1} v_{j,1})\theta_{ij,1} \tag{30}$$

Equation (9) is used to handle the nonlinear term $v_i v_j$. The linearization method described in (36) is used to handle the nonlinearity of v^2_{ij}. By substituting (9), (29), (30) and (36) into (3) and (4), the improved warm-start network model is obtained:

$$P_{ij} = g_{ij} v_i^2 - g^P_{ij} \frac{v_i^2 + v_j^2}{2} - b^P_{ij}(\theta_{ij} - \theta_{ij,1}) + P^L_{ij,v} \tag{31}$$

$$Q_{ij} = -b_{ij} v_i^2 + b^Q_{ij} \frac{v_i^2 + v_j^2}{2} - g^Q_{ij}(\theta_{ij} - \theta_{ij,1}) + Q^L_{ij,v} \tag{32}$$

where the expression of $P^L_{ij,v}$ is given by (36), and $Q^L_{ij,v}$ can be obtained by replacing g_{ij} with $(-b_{ij})$ in (36).

The network model described by (31) and (32) remains linear. By replacing the network model described by (10) and (11) with the warm-start network model (31) and (32), the warm-start OPF model that further improves the modeling accuracy can be obtained.

C. Discussion of the Warm-Start OPF Model

The proposed linearly-constrained OPF model in Section III is already capable of providing satisfactory results. The warm-start OPF solution provides a method to further enhance the performance when necessary, such as during the iterative process that reconstitutes the AC feasibility of the solution.

Because of the quasi-linear relationship of P-θ, the solution from the proposed OPF method gives a desirable estimate of the voltage angles. Considering that the losses mainly depend on the voltage angle, the solution from the OPF model provides a new, and more desirable "base case system operating condition" for the loss linearization. Moreover, a better initial point for the Taylor series expansions of the nonlinear terms further improves the accuracy of the network model. In general, the quasi-linear P-θ relationship of the power grids and the efficient linearization method used by the proposed network

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TABLE I COMPARISON OF M_II TO M_IV IN SEVERAL IEEE AND POLISH BENCHMARK SYSTEMS

Test case	Error in objective function			Maximum error in P_{ij} (p.u.)			Maximum error in Q_{ij} (p.u.)		Mean error in LMP ($/MWh)			Solution time (seconds)			
	M_II	M_III	M_IV	M_II	M_III	M_IV	M_III	M_IV	M_II	M_III	M_IV	M_I	M_II	M_III	M_IV[1]
9-bus	0.59%	0.30%	0.062%	0.029	0.28	0.0021	0.027	0.0021	0.15	0.069	0.065	0.11	0.012	0.017	0.014
24-bus	0.91%	0.16%	0.084%	0.24	0.20	0.013	0.026	0.015	3.19	1.20	0.18	0.13	0.015	0.026	0.018
30-bus	1.28%	0.24%	0.054%	0.042	0.014	0.0017	0.0054	0.0015	0.10	0.031	0.018	0.13	0.010	0.028	0.020
57-bus	0.85%	0.056%	0.020%	0.12	0.029	0.0051	0.0084	0.0023	0.46	0.22	0.061	0.15	0.016	0.034	0.021
118-bus	0.29%	0.041%	0.10%	0.28	0.20	0.022	0.040	0.027	0.34	0.077	0.063	0.20	0.036	0.069	0.054
300-bus	1.56%	0.71%	0.057%	2.23	0.98	0.17	0.45	0.22	1.62	0.97	0.58	0.40	0.042	0.14	0.20
2383-wp	0.96%	0.33%	0.048%	0.80	0.82	0.085	0.22	0.024	6.56	6.36	2.19	2.86	0.79	1.34	2.45
2736-sp	1.04%	0.19%	0.082%	1.09	0.32	0.10	0.087	0.022	2.64	1.12	0.58	2.90	0.53	1.14	0.67
2746-wp	1.24%	0.35%	0.12%	1.53	0.45	0.22	0.20	0.037	1.99	1.28	0.81	3.13	0.26	2.29	1.00
3012-wp	2.01%	0.65%	0.27%	5.42	0.78	0.15	0.40	0.13	33.9	5.24	1.64	4.44	1.68	1.63	0.85
3120-sp	1.29%	0.13%	0.10%	5.45	0.42	0.18	0.31	0.28	10.4	3.33	1.33	4.93	0.89	1.44	1.08

1: the solving time for M_IV refers to the time required by solving the warm-start OPF model in the additional iteration.

model together guarantee that the solution from the warm-start OPF model can provide solutions that are notably close to the AC OPF optimum by taking only one more iteration. In addition, compared with the DC OPF method, the proposed OPF method can handle constraint violations more effectively in the iterative process that reconstitutes the AC feasibility. During the traditional DC-AC iteration process, to address the branch flow violations observed in the AC power flow calculation, the branch flow limits in the DC OPF model need to be modified, which changes the feasible region of the system operation [32]. As a result, the modified DC OPF model may lead to sub-optimal or even infeasible solutions. Besides, the violations related to Q and v cannot be directly incorporated. If the proposed OPF method is used to replace the DC OPF method, without modifying the operational limits, the constraint violations are likely to be removed because of the improved accuracy of the warm-start network model. The feasible region of the OPF problem does not have to be modified. Most importantly, violations in Q and v can be directly constrained. Because of these features, the required iterations for the process that reconstitutes the AC feasibility can be notably reduced by replacing the DC OPF method with the proposed OPF method.

Compared with existing SLP approach, the proposed method has distinct advantages. For the SLP method, the step size of voltage angles and magnitudes must be controlled [11]. With the suitable step size that reduces the chances of divergence, the SLP approach requires quite a number of iterations before providing satisfactory solutions. For example, for the IEEE 30-bus system, the recently proposed SLP approach requires 9 iterations before the stop criterion is satisfied [11]. By comparison, as illustrated in the case study, the proposed warm-start OPF method could provide a sufficiently accurate solution by taking only one more iteration.

V. CASE STUDIES

Several IEEE and Polish benchmark systems are tested. The data are obtained from MATPOWER 4.1 [31]. Thorough investigations are provided for the IEEE 30-bus and Polish 2383-bus systems. The following methods are compared: 1) M_I: AC OPF solution, which serves as the benchmark result, 2) M_II: the commonly used DC OPF method, 3) M_III: the proposed OPF method described in Section III, and 4) M_IV: the warm-start OPF method described in Section IV.

For M_II to M_IV, a base case system operating condition is required to calculate the loss factor. To simulate that the base case condition may be quite different from the obtained OPF solution, the OPF solution of the test cases with loads that are randomly varied by at most α is used as the base case condition. To make sure the reproducibility of the case studies, loads of the base case condition are modified by following rules:

$$P'_{i,d} = P_{i,d} \times (1+\alpha_P \times \frac{2i-N}{N}),\ \ Q'_{i,d} = Q_{i,d} \times (1+\alpha_Q \times \frac{2i-N}{N}) \quad (33)$$

where N refers to the number of buses; α_P and α_Q are set to 30% in case studies. To perform a fair comparison, the loss factors for M_II to M_IV are obtained from the identical base case condition created by (33). Gurobi 6.5 is used as the solver for M_II to M_IV. Using the AC OPF solution as the benchmark result, errors in the objective function, approximation errors in P_{ij} and Q_{ij}, and errors in the LMP for M_II to M_IV are compared. The approximation errors of P_{ij} and Q_{ij} are calculated by substituting v and θ solutions from M_II to M_IV into the original branch flow equations (3) and (4) and comparing the obtained value with that in the branch flow solutions of M_II to M_IV. The solution time for M_II to M_IV is also provided. The computer processor is an Intel (R) Core (TM) i7-6700HQ @ 2.60 GHz.

A. The IEEE 30-Bus System

1) Comparison of M_II and M_III

The comparison of M_II and M_III is shown in Fig. 1 and Table I. Comparing M_III with M_II, by replacing the DC network model with the proposed network model, the quality of the OPF solution increases substantially. The error in the objective function is significantly reduced because the proposed OPF model is improved in the following aspects: 1) the branch flow limits are more accurate because the reactive power flow is included; 2) the constraints on Q and v are considered; and 3) the loss linearization is more accurate because the effect of the voltage magnitude is considered. According to Fig. 1, the active power scheduling and LMP obtained from M_III is closer to the AC OPF solution than M_II. Furthermore, M_III provides a result of Q-LMP, which offers an opportunity to price the reactive power. According to Table I, M_III improves the accuracy of the approximation of the active power compared with M_II while providing a decent approximation of the reactive power. The OPF model used by

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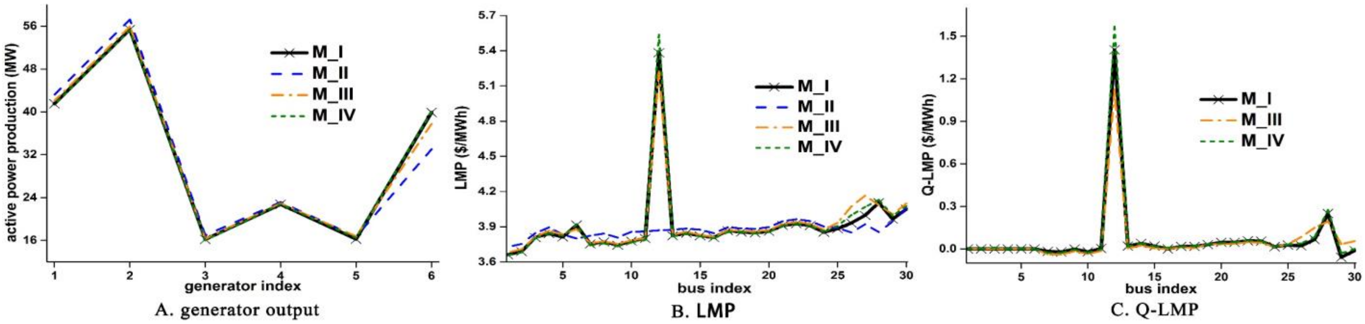

Fig. 1. Comparison of M_I to M_IV in the IEEE 30-bus system

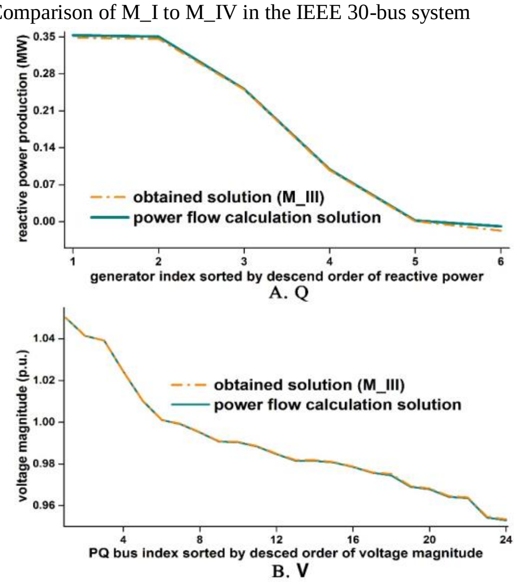

Fig. 2. Comparison of M_III and the power flow calculation result in the IEEE 30-bus system.

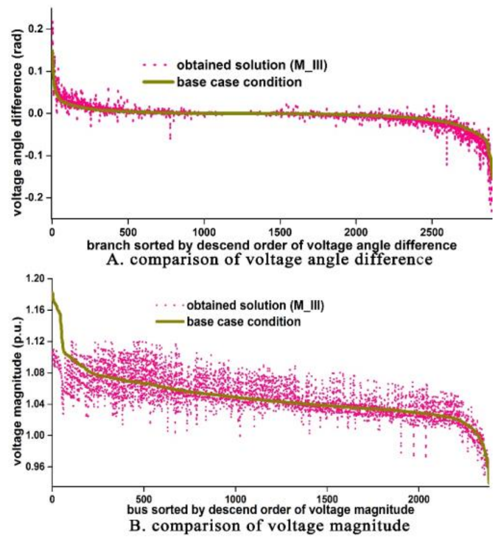

Fig. 3. Comparison of the simulated base case condition for calculating the loss factors and the obtained solution.

M_III has a larger scale than that used by M_II because Q and v are included. Hence, the solution time is larger for M_III. Even so, the computational performance of M_III remains satisfactory. As long as the model is linearly-constrained and can be solved without a convergence problem, the OPF method is preferred by power industries.

To show the accuracy of the network model used in M_III, a power flow calculation is performed by fixing the active and reactive power injections of the PQ buses, active power injections and voltage magnitudes of the PV buses, and voltage angle and magnitude of the swing bus. It can be indicated that the closer the solution from the M_III (or M_IV) and the solution from the power flow calculation is, the more accurate the network model is. It can be observed from Fig. 2 that the network model used in M_III provides a desirable approximation of Q and v.

2) Discussion of M_IV

Compared with M_III, M_IV further narrows the gap from the AC OPF method. According to Fig. 1 and Table I, for M_IV, the errors in the objective function, P_{ij}, Q_{ij}, LMP, and Q-LMP become smaller compared with M_III. The results show that the warm-start OPF model can provide a solution notably close to that obtained from the AC OPF method by taking only one more iteration.

B. The Polish 2383-Bus System

To show the robustness of the proposed method, a realistically sized transmission system with a significant difference between the base case system operating condition for the loss linearization and the obtained OPF solution is investigated

1) Comparison of M_II to M_IV

The simulated base case system operating condition and the OPF solution from M_III are compared in Fig. 3. The base case condition is notably different from the OPF solution. M_III provides an improved result compared with M_II according to Fig. 4 and Table I. The generator outputs provided by M_II to M_IV are similar with M_I for all methods. Compared with M_II, M_III and M_IV produce a much smaller error in the objective function. Hence, the proposed method provides a better scheduling of generators' active power in terms of minimizing generator production costs. The accuracy of the Q and v modeling is satisfactory according to Table I. It can be inferred from Table I and Fig. 4. B that M_III and M_IV provide a better result regarding the LMP compared with M_II. There are several outliers in the values of LMP and Q-LMP for M_III, which are mainly caused by the effects of the loss linearization error on the congestion costs. For M_IV, after improving the accuracy of the network model by using a warm start, the LMP and Q-LMP are notably close to the AC OPF solution, providing a more precise economic signal.

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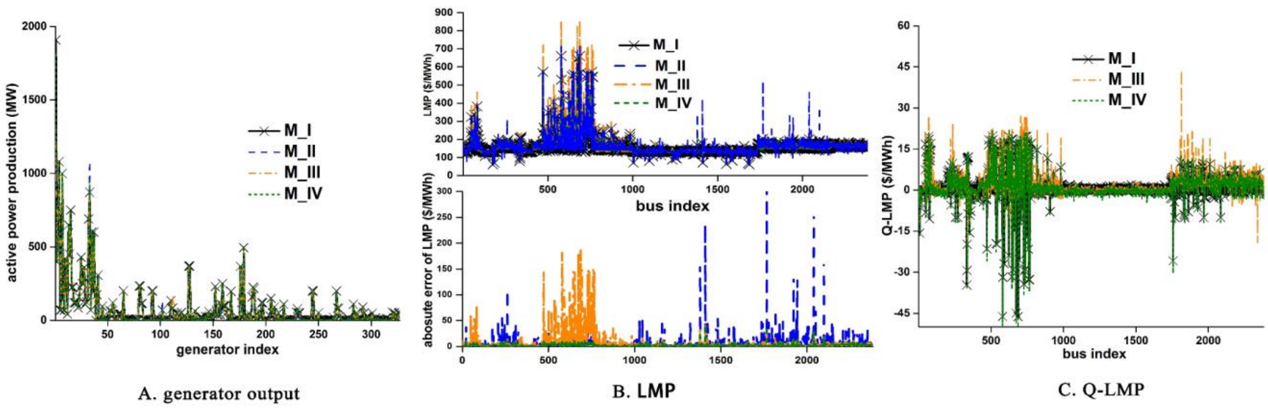

Fig. 4. Comparison of M_I to M_IV in the Polish 2383-bus system. For the LMP, the absolute error of M_II to M_IV compared with M_I is also presented.

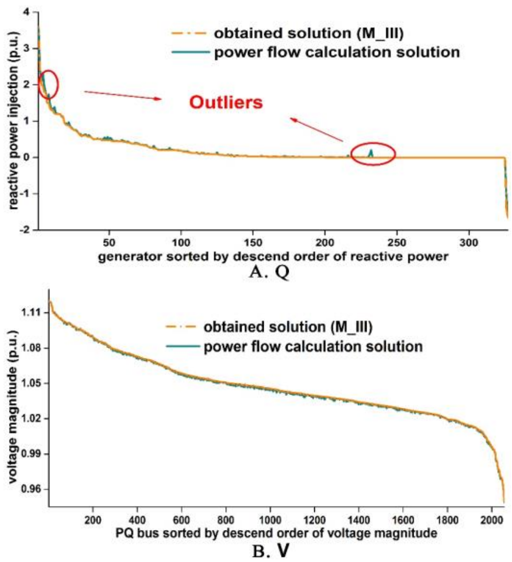

Fig. 5. Comparison of M_III and the power flow calculation result in the Polish 2383-bus system.

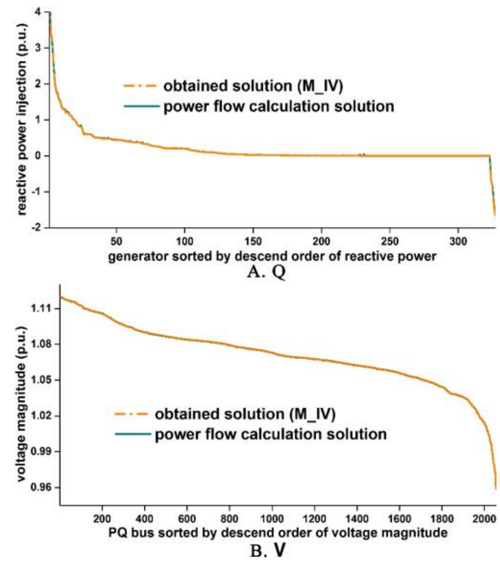

Fig. 6. Comparison of M_IV and the power flow calculation result in the Polish 2383-bus system.

TABLE II RESULTS FOR DIFFERENT SETTINGS OF α

Value of α		Error in objective function			Error in Q_{ij} (p.u.)	
α_P	α_Q	M_II	M_III	M_IV	M_III	M_IV
20%	20%	0.56%	0.47%	0.025%	0.27	0.024
30%	40%	1.21%	0.31%	0.050%	0.22	0.022
40%	20%	0.96%	0.078%	0.046%	0.26	0.025
40%	40%	1.44%	0.10%	0.046%	0.27	0.024
50%	50%	1.77%	0.30%	0.044%	0.26	0.024
60%	60%	2.39%	0.57%	0.045%	0.27	0.028

The solution obtained from M_III and that from the power flow calculation are compared in Fig. 5. The values of v obtained from M_III are close to those from the power flow calculation. There are several outliers for the values of Q. The reason is that the difference between the base case condition and the obtained solution causes inaccuracy in the loss linearization, which affects the accuracy of the Q modeling. However, considering the large deviation between the base case condition and the obtained solution, the modeling accuracy of Q and v for M_III is still impressive.

The comparison of the solutions obtained from M_IV and that from the power flow calculation is shown in Fig. 6. By using a warm-start OPF method, the outliers for the values of Q are eliminated. The modeling accuracy of Q and v is further improved.

TABLE III COMPARISON OF THE ITERATIONS REQUIRED FOR DIFFERENT METHODS

Test cases	M_I	M_II	M_III	M_IV
9-bus	11	6	12	11
24-bus	16	9	15	15
30-bus	15	8	19	20
57-bus	15	9	18	13
118-bus	15	10	21	23
300-bus	26	11	79	89
2383-wp	33	14	68	132
2736-sp	31	346	37	30
2746-wp	34	8	127	42
3012-wp	42	324	58	33
3120-sp	43	14	47	47

2) Analysis for Different Settings of α_P and α_Q

To investigate the effect of choosing different base case conditions for calculating the loss factors on the performance of M_II to M_IV, the comparison is provided in Table II with different settings of α_P and α_Q. When the absolute value of α_P and α_Q increases, the error in the objective function for M_II increases because the error of the loss linearization is larger. By comparison, M_III and M_IV provide improved results. Especially for M_IV, the quality of the solution is stable. The accuracy for the modeling of Q is satisfactory for M_III and M_IV with different settings of α_P and α_Q.

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C. Several Other Benchmark Systems

Several other IEEE and Polish benchmark systems are tested. The results are shown in Table I. For all cases, M_III provides better results than M_II without using any additional information of the power grid.

The performance of M_IV is further improved compared with M_III. In most cases, M_IV provides smaller errors for all the measurements provided in Table I. For the IEEE-118 bus system, although the objective function error is larger for M_IV, the errors of the power flow equations and LMP obtained from M_IV are smaller than those obtained from M_III.

The solution time for M_III and M_IV is satisfactory. It can be observed from Table I that the solution time for M_I is generally longer than M_II to M_IV. The number of inner iterations required by the algorithms for solving M_I to M_IV is shown in Table III. Iterations needed by M_III and M_IV are typically larger than those required by M_II. The reason for this observation is that the models used by M_III and M_IV are larger than the model used by M_II because Q and v are included. It can be observed from Table I and Table III that although the iterations needed by M_III and M_IV are larger than those needed by M_I, M_III and M_IV are actually more computationally efficient because the solution time needed for each iteration is smaller for linearly-constrained models [33].

D. Discussion of the Advantage of Linearized OPF models

From the case studies, it can be observed that the nonlinear optimization solver used by MATPOWER converges in all test cases. However, for OPF methods using linearized models, they have following unique advantages: 1) the convergence is theoretically guaranteed because of the convex formulation; 2) the local optimum corresponds to the global optimum; 3) the convexity of the model provides convenience for pricing the electricity, while the *Karush–Kuhn–Tucker* (KKT) condition for nonconvex problems does not always hold; 4) linearly-constrained optimization models are more computationally robust, especially considering the N-1 security requirement.

To demonstrate the advantage of linearized OPF models, a case study on the IEEE 300-bus system with branch flow limits set to 682 MVA is performed [11]. MATPOWER encounters numerically failure in such a system. By comparison, OPF methods using linearized models, i.e., the DC OPF method and the proposed method, provide a solution. The maximum errors in P_{ij} are 2.48 p.u., 0.74 p.u., and 0.039 p.u. for M_II, M_III, and M_IV, respectively. The maximum errors in Q_{ij} are 0.34 p.u. and 0.15 p.u. for M_III and M_IV, respectively.

It should be noted that the solutions yielded by M_II to M_IV are not strictly AC feasible because linear approximations of power flow equations are used. Based on the solution of M_IV, a solution strictly subjected to power flow equations is obtained by performing a power flow calculation. The AC feasible solution has following violations of branch flow limits: branch *205* (violated by 0.60%), branch *208* (violated by 1.45%), branch *266* (violated by 0.48%), branch *394* (violated by 0.51%), branch *400* (violated by 0.25%). It is difficult to judge whether the IEEE 300-bus system with branch flow limits of 682 MVA is feasible or not. However, while MATPOWER faces numerical difficulty and cannot give any useful information, the proposed method yields a valuable solution, which provides a good starting point to investigate the bottleneck of the branch flow limits in this system.

VI. Conclusions

In this study, a linearly-constrained and convergence-guaranteed OPF method with Q and v is proposed. The performance of the commonly used DC OPF method is significantly improved without requiring any additional information of the power grid. In addition, a warm-start OPF method is proposed to narrow the gap from the AC OPF when the accuracy of the network model requires further improvement, such as during the iterative quasi-optimization process that constitutes the AC feasibility. Case studies in several IEEE and Polish benchmark systems validate that the proposed method outperforms the DC OPF method without requiring any additional information of the power grid.

This paper proposes an OPF method that can potentially be applied in practical operations. Considering the increasing demand for solving a full OPF model with Q and v in day-ahead and real-time operations, the proposed method is worthy of further research.

Appendix

A. Expressions of Sensitivities

The active network losses are taken as an example. Suppose in the base case condition, the values of v and θ are (v_0, θ_0). The losses are decomposed into two components: the voltage angle term and the voltage magnitude term:

$$P_{ij}^L = \frac{1}{2} g_{ij}\theta_{ij}^2 + \frac{1}{2} g_{ij} v_{ij}^2 = P_{ij,\theta}^L + P_{ij,v}^L \qquad (34)$$

Based on the first-order Taylor series expansion, the term $P_{ij,\theta}^L$ can be linearized as follows:

$$P_{ij,\theta}^L = \frac{1}{2} g_{ij}\theta_{ij}^2 \approx g_{ij}\theta_{ij,0}\theta_{ij} - \frac{1}{2} g_{ij}\theta_{ij,0}^2 \qquad (35)$$

As shown in (34), $P_{ij,v}^L$ is a function of v. Because v^2 is used as an independent variable, first, $P_{ij,v}^L$ is transformed into a function of v^2; second, the first-order Taylor series expansion is used:

$$\begin{aligned} P_{ij,v}^L &= \frac{g_{ij}}{2} v_{ij}^2 \approx \frac{g_{ij}}{2} \frac{(v_i + v_j)^2}{(v_{i,0} + v_{j,0})^2} v_{ij}^2 = \frac{g_{ij}}{2} \frac{(v_i^2 - v_j^2)^2}{(v_{i,0} + v_{j,0})^2} \\ &= \frac{g_{ij}}{2} \frac{2(v_{i,0}^2 - v_{j,0}^2)}{(v_{i,0} + v_{j,0})^2}(v_i^2 - v_j^2) - \frac{g_{ij}}{2} \frac{(v_{i,0}^2 - v_{j,0}^2)^2}{(v_{i,0} + v_{j,0})^2} \\ &= g_{ij} \frac{v_{i,0} - v_{j,0}}{v_{i,0} + v_{j,0}}(v_i^2 - v_j^2) - \frac{g_{ij}}{2}(v_{i,0} - v_{j,0})^2 \end{aligned} \qquad (36)$$

According to (35) and (36), the expressions for the sensitivities in (17) and (18) are as follows:

$$\mathbf{LF}_{P,\theta}^T = g_{ij}\theta_{ij,0}\mathbf{M}_{ij}, \mathbf{LF}_{P,v^s}^T = g_{ij} \frac{v_{i,0} - v_{j,0}}{v_{i,0} + v_{j,0}} \mathbf{M}_{ij} \qquad (37)$$

$$Offset_{p,ij} = -\frac{1}{2} g_{ij}\left[\theta_{ij,0}^2 + (v_{i,0} - v_{j,0})^2\right] \qquad (38)$$

where $\mathbf{M}_{ij}$ is the incident vector of branch *(i,j)*. In $\mathbf{M}_{ij}$, the i^th^ element is 1, the j^th^ element is -1, and the rest of the elements are zero.

B. Linearization of the Quadratic Branch Flow Limits

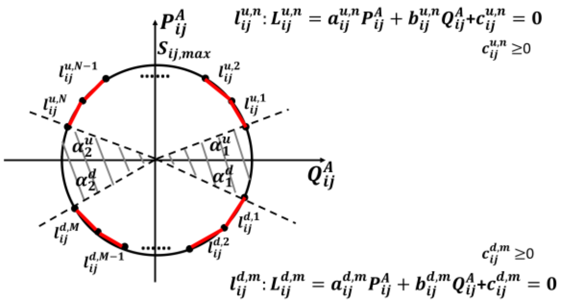

Fig. 7. Linearization of the quadratic branch flow limits.

The feasible region defined by the branch flow limit (25) is represented by the circle in Fig. 7. The circle area can be approximated by the polygon region that is formed by a group of red lines. A general formulation of the red lines can be found in Fig. 7. On the majority of the branches, the active power flow is much greater than the reactive power flow. Hence, the shaded area represents the region where the system operating point rarely locates. The linearization method only focuses on the rest of the circle. In this way, the number of constraints can be reduced without sacrificing the accuracy of the linear approximation. In the test cases, for all branches, $\alpha_1^u = \alpha_2^u = \alpha_1^d = \alpha_2^d = \pi/6$ and *M*=*N*=20.

Hence, the branch flow limit (25) can be denoted by a group of linear constraints as follows:

$$\Lambda(P_{ij}^{A}, Q_{ij}^{A}): \begin{cases} L_{ij}^{u,n} \geq 0, \, n = 1,\dots,N \\ L_{ij}^{d,m} \geq 0, \, m = 1,\dots,M \end{cases}, (i,j) \in \mathcal{K} \quad (39)$$

C. Expression for the Coefficients

The expressions of coefficients in Section IV.B are:

$$s_{ij}^{1} = \cos\theta_{ij,1},\, s_{ij}^{0} = \sin\theta_{ij,1} - \theta_{ij,1}\cos\theta_{ij,1} \quad (40)$$

$$c_{ij}^{1} = -\sin\theta_{ij,1},\, c_{ij}^{0} = \cos\theta_{ij,1} + \theta_{ij,1}\sin\theta_{ij,1} \quad (41)$$

$$g_{ij}^{P} = (g_{ij}c_{ij}^{0} + b_{ij}s_{ij}^{0}) + (g_{ij}c_{ij}^{1} + b_{ij}s_{ij}^{1})\theta_{ij,1} \quad (42)$$

$$b_{ij}^{P} = (g_{ij}c_{ij}^{1} + b_{ij}s_{ij}^{1})v_{i,1}v_{j,1} \quad (43)$$

$$b_{ij}^{Q} = (-g_{ij}s_{ij}^{0} + b_{ij}c_{ij}^{0}) - (g_{ij}s_{ij}^{1} - b_{ij}c_{ij}^{1})\theta_{ij,1} \quad (44)$$

$$g_{ij}^{Q} = (g_{ij}s_{ij}^{1} - b_{ij}c_{ij}^{1})v_{i,1}v_{j,1} \quad (45)$$

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