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A novel hybrid GWO-SCA approach for optimization problems

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ABSTRACT

Recent trend of research is to hybridize two and several number of variants to find out better quality of solution of practical and recent real applications in the field of global optimization problems. In this paper, a new approach hybrid Grey Wolf Optimizer (GWO) – Sine Cosine Algorithm (SCA) is exercised on twenty-two benchmark test, five bio-medical dataset and one sine dataset problems. Hybrid GWOSCA is combination of Grey Wolf Optimizer (GWO) used for exploitation phase and Sine Cosine Algorithm (SCA) for exploration phase in uncertain environment. The movement directions and speed of the grey wolve (alpha) is improved using position update equations of SCA. The numerical and statistical solutions obtained with hybrid GWOSCA approach is compared with other metaheuristics approaches such as Particle Swarm Optimization (PSO), Ant Lion Optimizer (ALO), Whale Optimization Algorithm (WOA), Hybrid Approach GWO (HAGWO), Mean GWO (MGWO), Grey Wolf Optimizer (GWO) and Sine Cosine Algorithm (SCA). The numerical and statistical experimental results prove that the proposed hybrid variant can highly be effective in solving benchmark and real life applications with or without constrained and unknown search areas.

1. Introduction

One of the highly effective techniques in searching the best possible results in benchmark and real life functions is the global optimization technique. In optimization, only a few results are compared to best which are known as the goal. Classical optimization approaches have some deficiencies on finding the global optimal solutions of classical optimization problems. These deficiencies are primarily interdependent on their inherent search systems. These classical algorithms are strongly under effects of choosing proper types of variables, objectives and constraints functions. They also do not grant a universal solution method that can be applied to find global optimal solution of the functions where several types of constrained functions, variables and objective are used [1]. For covering these deficiencies, a new technique with the name of metaheuristics was originated, which is mainly developed from artificial intelligence research that originated by scientists or researchers [2]. Nature inspired techniques researchers are developed for solving the several types of hard global optimization functions without having to full accommodate to each function.

Recently, scientists and researchers have developed several metaheuristics in order to find the best global optimal solution of benchmark and real life applications. The first solution technique for the optimal power flow (OPF) problem was developed by Dommel and Tinney [3] in 1968, and since then several numbers of other nature inspired techniques have been originated, some of them are: Particle Swarm Optimization (PSO) [4], Genetic Algorithm (GA) [5–6], Differential Evolution (DE) [7–8], Ant Colony Optimization (ACO) [9], fuzzy based hybrid particle swarm optimization (fuzzy HPSO) [10], Hybrid Genetic Algorithm (HGA) [11], harmony search algorithm [12], Robust Optimization (RO) [13], Grey Wolf Optimization (GWO) [14], Artificial Neural Network (ANN) [15], Tabu Search (TS) [16], biogeography based optimization algorithm (BBO) [17], Gravitational Search Algorithm (GSA) [18], Ant Lion Optimizer (ALO) [19], adaptive group search optimization (AGSO) [20], krill herd algorithm (KHA) [21], Multi-Verse Optimizer (MVO) [22], Moth Flame Optimizer (MFO) [23], Sine Cosine Algorithm (SCA) [24], Dragonfly Algorithm (DA) [25], Whale Optimization Algorithm (WOA) [26], Grasshopper Optimization Algorithm (GOA) [27], Black-Hole-Based Optimization (BHBO) [28], Cuckoo Search (CS) [29] and In addition, in case of the hybrid convergence, nature inspired algorithm hybridizations using batch modeling are combinations amid evolutionary techniques and techniques of neighbourhood or course.

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Mirjalili et al. [30] Grey Wolf Optimizer is recently developed metaheuristics inspired from the hunting mechanism and leadership hierarchy of grey wolves in nature and has been successfully applied for solving optimizing key values in the cryptography algorithms [31], feature subset selection [32], time forecasting [33], optimal power flow problem [34], economic dispatch problems [35], flow shop scheduling problem [36] and optimal design of double later grids [37]. Several algorithms have also been developed to improve the convergence performance of Grey Wolf Optimizer that includes parallelized GWO [38,39], binary GWO [40], integration of DE with GWO [41], hybrid GWO with Genetic Algorithm (GA) [42], Hybrid DE with GWO [43], Hybrid Grey Wolf Optimizer using Elite Opposition Based Learning Strategy and Simplex Method [44], Modified Grey Wolf Optimizer (mGWO) [45], Mean Grey Wolf Optimizer (MGWO) [46] and Hybrid Particle Swarm Optimization with Grey Wolf Optimizer (HPSOGWO) [47].

Mirjalili [24] was presented a novel population based optimization technique called Sine Cosine Algorithm (SCA) simply based on Sine and Cosine function is applied for exploitation and exploration phases in global optimization functions. The Sine Cosine Algorithm (SCA) creates different initial random agent solutions and requires them to fluctuate outwards or towards the best possible solution using a mathematical model based on sine and cosine functions. The performance of this variant was tested on standard test functions and real life applications including unimodal, multi-modal, composite functions, aircraft's wings and many other biomedical problems.

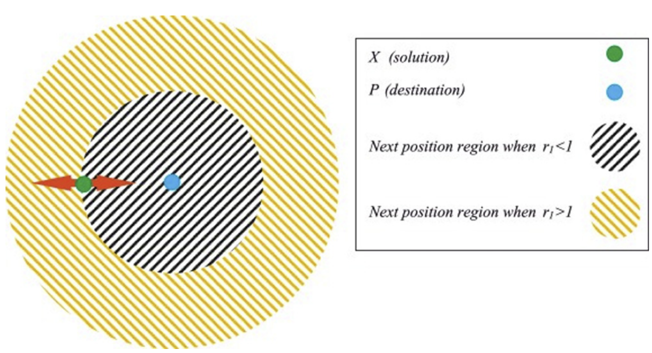


Fig. 1. Illustrating next step towards targeted optimum solution.

After motivated of this metaheuristics the researchers of different areas are developed several new modified and hybrid variants of SCA algorithm to improve the convergence performance of SCA algorithm including SCA integrated with differential evolution (ASCA-DE) [50], Hybrid SCA with multi-orthogonal search strategy [51], Improved SCA based on levy flight [52] and Hybrid Back Tracking Search (BSA) with Sine Cosine Algorithm (SCA) [53].

The help of SCA algorithm the researchers are solved numerious real life problems including a novel Sine Cosine algorithm for the solution of unit commitment problems [48], feature selection [49], Structural Damage Detection [50], the gear train design problem [51], Cantilever beam [51], Welded beam design [51], Pressure vessel design problem [51] and many other biomedical and mechanical engineering problems.

Rodriguez et al. [55] proposed operator is used to the simulation of the hunting process in the variant and has five algorithms are presented. The main purpose of this study the accuracy of the GWO variant when a new hierarchical operator is introduced. On the basis of obtaining solutions authors were presented the quality of the proposed variant.

Rodriguez et al. [56] present a general study of the GWO. This study were divided in two different parts: (i) to determine in the first part which parameters are candidates to be dynamically adjusted and (ii) to determine which are the parameters that have the greatest effect in the variant. Authors also present a solutions of experiments and justification as well as the standard test problems that were applied for the test that are shown.

Rodriguez et al. [57] use of fuzzy logic for dynamic parameter adaptation in the GWO variant. The proposed algorithm of a fuzzy grey wolf optimizer is verified with the traditional standard grey wolf optimizer algorithm with a set of standard test problems. Experimental solutions prove that there is significant advantage of the proposed variant.

In this study, a newly introduced hybrid meta-heuristic optimization technique named Hybrid Grey Wolf Optimizer (GWO)-Sine Cosine Algorithm (SCA) is applied to solve benchmark and real life problems. HGWOSCA comprises of best characteristic of both Grey Wolf Optimizer and Sine Cosine Algorithm. The performance of HGWOSCA algorithm are finding the near best global optimal solution, fast convergence rate due to use of twenty-two classical function, can also handle several real life applications i.e. five bio-medical science dataset and one sine dataset problems.

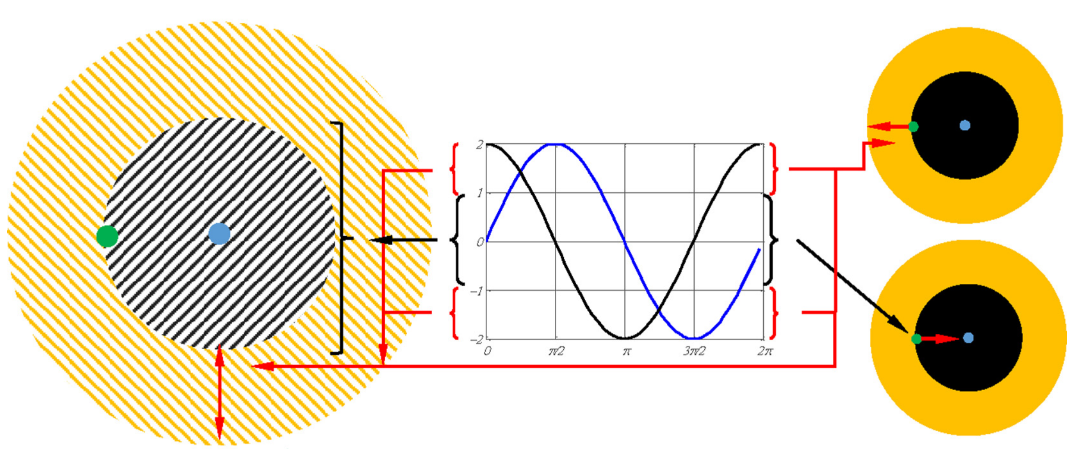


Fig. 2. Basic principle of Sine Cosine Algorithm (SCA).

Table F
Parameter setting.

Parameter	Values
Search Agents	20
Max. number of iterations	40–500
$\vec{a}$	[2, 0]

Additionally, the solutions are compared relying on the meta-heuristics reported in the review of literature. The present research falls into ten sections: Sections 2 and 3 is a devotion for discussing the Grey Wolf Optimizer (Algorithm) and Sine Cosine Algorithm (SCA). The HGWOSCA approach mathematically model and pseudo code is also discussed in Section 4. In Sections 5-7 at presenting the Classical problems, Numerical experiments and parameter settings. While Sections 8-10: aims at discussing the experimental results, bio-medical real life and sine dataset problems. Finally, the conclusion of the work is summarized at end of the text.

This article is organized with different sections and subsections to simplify the presentation of work.

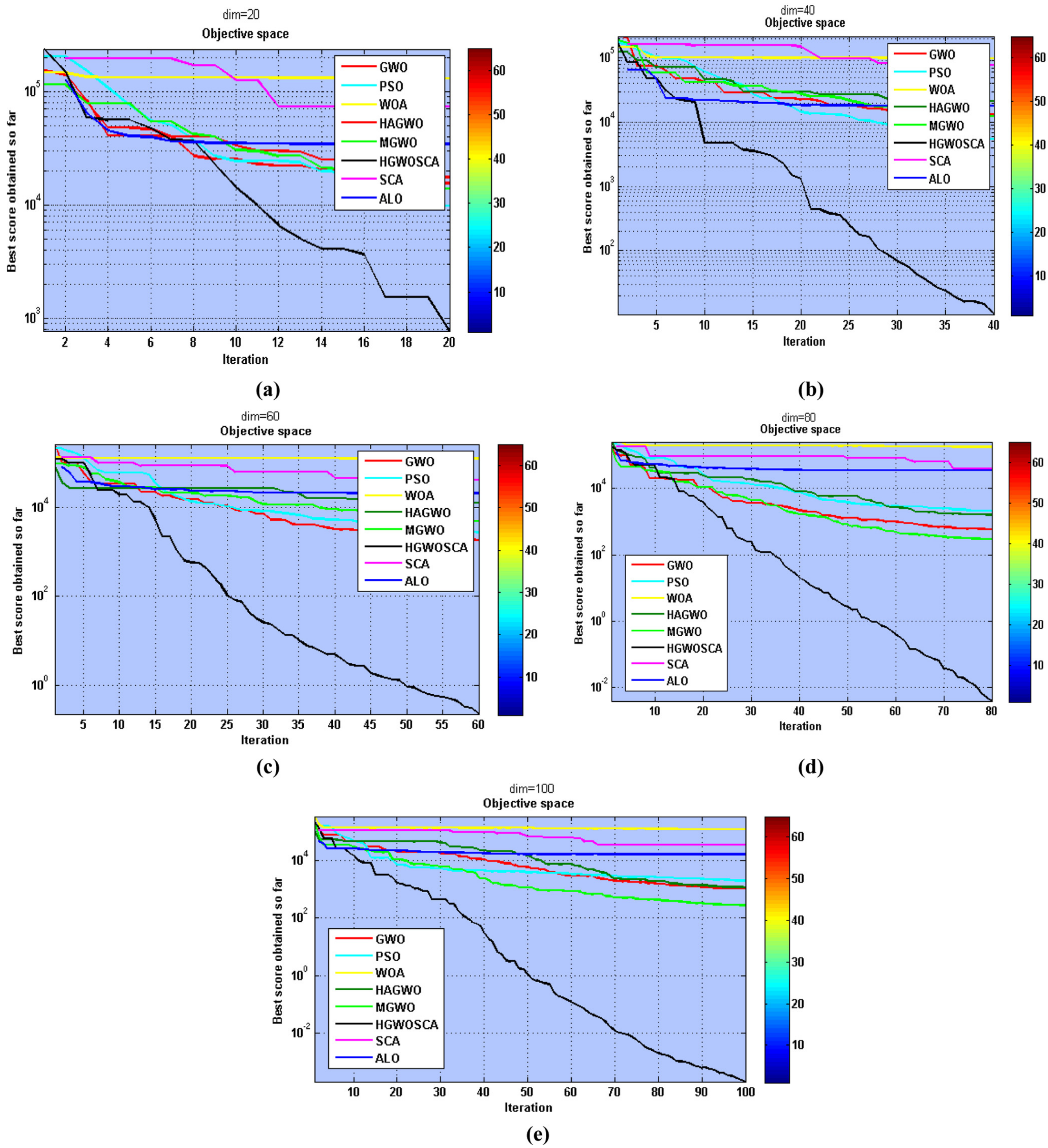


Fig. 3. (a)–(e) The Efficiency of HGOWSCA.

Table 1
Numerical results of unimodal benchmark functions.

Problem	PSO		ALO		WOA		HAGWO	
↓	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{min}	f_{min}	f_{max}
1.	16.1322	6.8723e+04	0.0088	4.8669e+04	7.9586e−10	8.0611e+04	6.3924e−07	7.8880e04
2.	13.9091	7.4148e+14	0.0075	1.2291e+12	3.6985e−4	2.1343e+11	0.0401	5.7106e+12
3.	3.2509e+03	1.0896e05	0.9958	1.2515e+05	1.0825e+05	2.7592e+05	2.4788e+03	1.1168e+05
4.	17.8616	86.6907	0.6523	79.7239	79.6682	86.1808	10.7586	86.5151
5.	888.1723	3.0853e+08	0	1.0752e+08	28.8385	2.5303e+08	27.2331	2.5403e08
6.	90.9179	7.3812e+04	0.9583	4.6168e+04	2.7928	7.5475e+04	3.9647	6.6279e+04
7.	10.8057	130.2038	0.0125	137.5298	0.0027	115.9798	0.0041	128.7699

Table 2
Numerical results of unimodal benchmark functions.

Problem	MGWO		GWO		SCA		HGWOSCA	
↓	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}
1.	0.1614	5.7141e+04	0.0283	4.9634e+04	0.0077	6.2863e+04	0.0053	8.1177e+04
2.	3.7178	7.6983e+13	3.5350	1.0354e+14	0.0666	1.4844e+09	0.0319	8.2182e+08
3.	76.0032	3.5707e+05	1.5468e+03	1.3915e+05	1.0821	1.8592e+05	1.06612e−04	2.8573e+05
4.	16.2001	87.5513	8.4019	86.9014	0.8521	87.7313	0.7785	88.5045
5.	28.6144	3.2644e+08	27.3322	3.0886e+08	0	1.4481e+08	26.5837	3.3996e+08
6.	65.1273	7.2349e+04	67.6751	6.4236e04	0.8825	5.2556e+04	0.0031	7.3972e+04
7.	0.0321	130.7239	0.0246	112.5907	0.2356	129.8274	0.0024	135.8470

Table 3
Statistical results of unimodal benchmark functions.

Problem	PSO		ALO		WOA		HAGWO	
↓	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ
1.	4.9550e+03	1.3474e+04	1.1516e+04	6.4210e+03	4.3237e+03	1.4023e+04	3.7299e+03	1.2422e+04
2.	1.4830e+13	1.0486e+14	2.4583e+10	1.7383e+11	1.8062e+10	5.8526e+10	1.1421e+11	8.0760e+11
3.	1.3074e+04	2.0818e+04	2.4357e+04	1.7511e+04	1.1585e+05	1.8614e+04	2.9885e+04	3.7290e+04
4.	30.8518	19.4085	38.6579	10.5663	79.9653	1.2277	30.7072	20.4292
5.	3.8943e+06	2.7597e07	5.7650e+06	1.2244e+07	1.0583e+07	4.2850e+07	5.1894e+06	2.9108e+07
6.	9.1205e+03	1.7921e+04	1.3042e04	5.9167e+03	5.6896e+03	1.6454e+04	5.3663e+03	1.4436e+04
7.	82.5561	34.6305	11.4101	14.0178	3.6770	16.2741	6.0707	20.8856

Table 4
Statistical results of unimodal benchmark functions.

Problem	MGWO		GWO		SCA		HGWOSCA	
↓	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ
1.	2.9851e+03	9.1449e+03	1.8831e+03	6.7413e+03	2.7451e+04	2.7542e+04	2.5497e+03	1.0281e+04
2.	1.9401e+12	1.3715e+13	2.1773e+12	1.4646e+13	6.4927e+07	2.9503e+08	1.7380e+07	1.1628e+08
3.	1.5908e+04	4.4359e+04	1.6349e+04	2.8998e+04	9.3577e+04	5.6116e+04	1.0717e+04	2.5564e+04
4.	42.6053	25.7953	27.4422	22.9442	83.0504	14.1838	15.2400	24.7636
5.	6.7716e+06	3.8427e+07	4.2861e+06	2.7650e+07	7.2624e+07	6.4987e+07	4.0926e+06	2.9539e+07
6.	6.5470e++03	1.5242e+04	5.8249e+03	1.2912e+04	2.5593e+04	1.9697e+04	3.2264e+03	1.3017e+04
7.	5.3754	19.9518	4.1480	17.7683	71.7481	53.7197	3.2580	16.8661

Table 5
Numerical results of multimodal benchmark functions.

Problem	PSO		ALO		WOA		HAGWO	
↓	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{min}	f_{min}	f_{max}
8.	−5.415e+03	−3.2530e+03	−4.5376e+03	−2.1350e+03	−4.7178e+03	−2.6338e+03	−1.1462e+04	−1.7644e+03
9.	58.0934	404.0174	0	352.1040	0	469.9106	0	416.4556
10.	3.0660	20.8419	0.0029	19.6283	7.0980e−06	20.7132	7.5104e−05	20.5173
11.	1.4987e−05	589.4075	0	309.8401	0	692.6283	0.0244	557.2088
12.	0.8817	5.7002e+08	0.3356	2.9526e+08	0.1484	6.6942e+08	0.1121	5.7990e+08
13.	1.7596	2.6248e+08	0.0256	2.4558e+08	1.0707	1.4122e+09	1.8904	1.2451e+08

Table 6
Numerical results of multimodal benchmark functions.

Problem	MGWO		GWO		SCA		HGWOSCA	
↓	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}
8.	−3.3757e+03	−2.2685e+03	−4.5522e+03	−2.3421e+03	−4.5026e+03	−2.1524e+03	−5.5538e+03	−1.7369e+03
9.	2.4106e−13	485.5179	1.6757e−10	485.1927	0	433.5434	0	486.0063
10.	0.0849	20.6376	0.0789	20.8904	0.0036	20.7229	0.0026	20.9073
11.	0	654.2859	3.3307e−16	598.2472	0	619.4075	0	746.7178
12.	0.5090	6.3560e+08	1.2170	5.7990e+08	0.2215	7.1351e+08	0.0025	7.7386e+08
13.	1.9503	1.3288e+09	2.5835	1.4392e+09	0.2356	1.46683e+09	0.0011	4.6402e+09

Table 7
Statistical results of multimodal benchmark functions.

Problem	PSO		ALO		WOA		HAGWO	
↓	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ
8.	−4.7034e+03	817.5054	−5.4821e+03	437.9171	−9.2452e+03	1.1567e+03	−1.0214e+04	1.6730e+03
9.	188.4820	111.5257	150.4052	76.6545	15.0418	56.7377	15.1999	59.7208
10.	7.7165	4.2107	16.9868	1.8188	3.5705	6.0824	3.1865	6.0335
11.	48.5656	125.6149	23.9759	57.5835	6.0666	49.3150	6.2227	46.1732
12.	1.2001e+07	7.0479e+07	4.9727e+06	3.0094e+07	2.3356e+07	1.0983e+08	2.3018e+07	1.0398e+08
13.	1.6661e+07	8.6318e+07	2.3384e+07	4.4241e+07	5.0588e+07	2.1945e+08	5.8364e+07	1.9388e+08

Table 8
Statistical results of multimodal benchmark functions.

Problem	MGWO		GWO		SCA		HGWOSCA	
↓	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ
8.	−3.2345e+03	281.3776	−3.6515e+03	970.0746	−4.3968e+03	384.9523	−2.7645e+03	319.7960
9.	24.4968	72.7340	25.0832	72.3377	94.4455	165.9993	15.3783	57.2684
10.	3.9455	6.0257	4.0894	6.1931	20.2556	2.0464	2.3481	5.2798
11.	7.0749	48.8287	7.7113	47.1824	115.8053	150.3963	3.8815	35.4040
12.	1.3458e+07	8.0921e+07	1.6550e+07	7.7092e+07	3.6600e+08	3.4946e+08	1.1519e+07	3.1587e+07
13.	3.9456e+07	1.7290e+08	6.4597e+07	2.4396e+08	7.3050e+08	6.3025e+08	2.8421e+07	1.7684e+08

Table 9
Numercial results of fixed-dimension multimodal benchmark functions.

Problem	PSO		ALO		WOA		HAGWO	
↓	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{min}	f_{min}	f_{max}
14.	7.8740	211.0410	1.2563	39.0774	3.9683	42.9403	15.5038	192.9812
15.	7.4144e−04	0.1732	0.0458	0.0253	7.2951e−04	0.2232	3.0802e−04	0.4061
16.	−1.0109	−0.6437	0	−0.6686	−1.0315	1.4164	−1.0315	0.1302
17.	0.3979	1.5016	0.3561	1.9435	0.4056	3.9354	0.3988	2.0514
18.	3	34.5219	0	37.3214	3.0249	87.9213	3.0102	48.6700
19.	−3.8628	−3.2057	−3.8528	0	−3.8587	−2.6925	−3.8549	−3.4726
20.	−3.2022	−1.2967	−3.1480	−1.2563	−2.9195	−1.6207	−2.6640	−0.5264
21.	−9.1520	−0.3532	−9.1532	−0.2816	−4.7144	−0.5816	−9.8512	−0.4161
22.	−10.4007	−0.4086	−5.0877	−0.4036	−2.7209	−0.5224	−5.0847	−0.5396

Table 10
Numercial results of fixed-dimension multimodal benchmark functions.

Problem	MGWO		GWO		SCA		HGWOSCA	
↓	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}	f_{min}	f_{max}
14.	12.6705	457.8792	3.0187	230.4631	1.2356	133.8074	0.9980	326.0632
15.	5.0370e−04	0.1994	6.6952e−04	0.1191	0.0135	0.1098	0.0012	0.4061
16.	−1.0279	−0.6361	−1.0315	0.3933	−1.0314	1.0050	−1.0315	1.5863
17.	0.3980	1.8367	0.3979	1.1224	0.4041	0.5450	0.3979	4.3906
18.	3.0139	112.5218	3.0265	55.5948	0	66.3193	3	136.9078
19.	−3.8549	−2.6168	−3.8606	−2.7591	−3.8428	−3.2586	−3.8625	−2.4271
20.	−3.1392	−1.4655	−3.1036	−2.4001	−1.7394	−1.2563	−3.2761	−0.5221
21.	−9.8681	−0.3681	−2.6028	−0.6386	−1.3354	−0.3325	−10.0419	−0.2658
22.	−10.3661	−0.4723	−10.3551	−0.5396	−3.9777	−0.3526	−10.3882	−0.3345

Table 11
Statistical results of fixed-dimension multimodal benchmark functions.

Problem	PSO		ALO		WOA		HAGWO	
↓	Average μ	S.D. σ	Average μ	S.D. σ	Average $e\mu$	S.D. σ	Average μ	S.D. σ
14.	13.5470	25.7517	7.5965	4.5885	6.5112	4.4488	18.5168	21.2825
15.	0.0040	0.0117	0.0020	0.0028	0.0024	0.0119	0.0021	0.0075
16.	−0.9331	0.1210	−0.6686	0.2945	−0.6751	0.7554	−0.8979	0.3635
17.	0.7627	1.6474	0.4857	0.3764	0.9830	1.2908	0.5126	0.2418
18.	5.5468	6.6712	3.7305	4.8783	6.7592	16.5650	4.0939	6.4381
19.	−3.8441	0.0760	−3.5434	0.6527	−3.7644	0.2527	−3.8432	0.0468
20.	−2.8714	0.4430	−2.9136	0.6190	−2.8238	0.2841	−2.4257	0.2893
21.	−5.3119	3.3247	−8.4484	3.3735	−4.1708	0.8956	−5.5026	2.7400
22.	−5.5657	3.5048	−4.6421	1.2195	−2.3634	0.5113	−4.7298	0.8871

Table 12
Statistical results of fixed-dimension multimodal benchmark functions.

Problem	MGWO		GWO		SCA		HGWOSCA	
↓	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ	Average μ	S.D. σ
14.	18.3599	45.1224	7.8126	24.3280	5.3064	13.4244	1.3887	3.9065
15.	0.0014	0.0105	0.0014	0.0061	0.0035	0.0113	0.0013	0.0069
16.	−0.8377	0.1849	−0.7329	0.5939	−0.6502	0.6641	−0.5511	0.8533
17.	0.5144	0.3267	0.4501	0.1476	0.5171	0.0751	0.5516	0.5820
18.	7.6728	19.5888	5.4840	9.0767	7.2724	11.0238	2.5653	9.1065
19.	−3.8432	0.0867	−3.8502	0.0861	−3.7846	0.2368	−3.8372	0.0358
20.	−3.0345	0.2407	−2.9596	0.1912	−1.4454	0.2897	−3.0890	0.1622
21.	−4.4980	3.0931	−1.7891	0.6176	−0.6456	0.3342	−3.9231	0.1719
22.	−6.0558	3.0424	−5.5217	3.0519	−2.6260	1.1231	−6.4560	3.0491

Table 13
CPU-time consuming results of standard benchmark functions.

Problem No.	PSO CPU time	WOA CPU time	ALO CPU time	SCA CPU time	HAGWO CPU time	GWO CPU time	MGWO CPU time	HGWOSCA CPU time
1.	0.0476203	0.0578204	0.0636401	0.0165214	0.0107219	0.0021020	0.0017014	0.0003716
2.	0.099	0.117	0.581	0.047	0.027	0.011	0.009	0.0001
3.	0.188	0.259	0.311	0.097	0.048	0.010	0.017	0.002
4.	0.01021	0.05097	0.07437	0.00147	0.00120	0.00024	0.00011	0.00011
5.	0.0623713	0.0793459	0.0993007	0.0441057	0.0273717	0.0204114	0.0117079	0.0017419
6.	0.0296307	0.0493407	0.0756104	0.0057471	0.0906301	0.0145004	0.0110017	0.0030041
7.	0.0863427	0.0905327	0.0895007	0.0064404	0.0050007	0.0041170	0.0037410	0.0008873
8.	0.0342537	0.0242709	0.0202708	0.0010004	0.0282304	0.0120101	0.0070102	0.0057901
9.	0.440708	0.804771	0.990727	0.070117	0.043018	0.0011041	0.0016017	0.0010807
10.	0.0418003	0.0397097	0.0708017	0.0011714	0.0010001	0.0007154	0.0120050	0.0038056
11.	0.0072	0.0031	0.0097	0.0018	0.0018	0.0012	0.0011	0.0009
12.	0.0317041	0.0718047	0.0814075	0.0017147	0.0010070	0.0010010	0.001002	0.0010002
13.	0.571309	0.939501	0.441277	0.0281401	0.001001	0.0008940	0.0061016	0.0006087
14.	0.0219	0.0304	0.0118	0.0089	0.067	0.0345021	0.0110104	0.0014101
15.	0.7653770	0.8003070	0.8081401	0.0201201	0.0387040	0.0299711	0.0428091	0.0048098
16.	0.6988789	0.7780703	0.0010741	0.0040110	0.0070971	0.0014134	0.0286867	0.0010074
17.	0.04817	0.06414	0.02417	0.01927	0.00709	0.0006207	0.0006102	0.0001941
18.	0.6177068	0.7008071	0.8188862	0.5178889	0.4007071	0.0186247	0.0090007	0.0087087
19.	0.62443	0.90727	0.08767	0.09079	0.00697	0.0012811	0.0013047	0.041708
20.	0.00144	0.00517	0.00107	0.00099	0.00066	0.00040	0.00033	0.0003301
21.	0.57431	0.01941	0.318411	0.01741	0.01147	0.0011001	0.0040105	0.00088
22.	0.0475816	0.0635010	0.0315210	0.0039010	0.0034909	0.0012100	0.0011002	0.0010509

2. Grey Wolf Optimizer (GWO)

Mirjalili et al. [14] developed a new population based nature inspired algorithm called Grey Wolf Optimization (GWO). This approach mimics the hunting behavior and social leadership of grey wolves in nature. Four types of grey wolves such as alpha, beta, delta, and omega are employed for simulating the leadership hierarchy.

The first three best position (fittest) wolves are indicated as α, β and δ who guide the other wolves (ω) of the groups toward promising areas of the search space. The position of each wolf of the group is updated using the following mathematical equations:

The encircling behavior of each agent of the crowd is calculated by the following mathematical equations:

$$\vec{d} = |c.\vec{x}_p^t - \vec{x}^t| \tag{1}$$

$$\vec{x}^{t+1} = \vec{x}_p^t - \vec{a}.\vec{d} \tag{2}$$

The vectors $\vec{a}$ and $\vec{c}$ are formulate as below:

$$\vec{a} = 2l.r_1 \tag{3}$$

$$\vec{c} = 2.r_2 \tag{4}$$

Hunting: In order to mathematically simulate the hunting behavior, it is assumed that the alpha, beta and delta have better knowledge about the potential location of prey. The following equations are developed in this regard.

$$\vec{d}_\alpha = |\vec{c}_1.\vec{x}_\alpha - \vec{x}|, \quad \vec{d}_\beta = |\vec{c}_2.\vec{x}_\beta - \vec{x}|, \quad \vec{d}_\delta = |\vec{c}_3.\vec{x}_\delta - \vec{x}| \tag{5}$$

$$\vec{x}_1 = \vec{x}_\alpha - \vec{a}_1.(\vec{d}_\alpha) \tag{6}$$

$$\vec{x}_2 = \vec{x}_\beta - \vec{a}_2.(\vec{d}_\beta), \quad \vec{x}_3 = \vec{x}_\delta - \vec{a}_3.(\vec{d}_\delta) \tag{7}$$

$$\frac{\vec{x}_1 + \vec{x}_2 + \vec{x}_3}{3} \tag{8}$$

Search for prey and attacking prey: The $\vec{a}$ is random value in the gap $[-2a, 2a]$. When random value $|\vec{a}| < 1$ the wolves are forced to attack the prey. Searching for prey is the exploration ability and attacking the prey is the exploitation ability. The arbitrary values of $\vec{a}$ are utilized to force the search to move away from the prey.

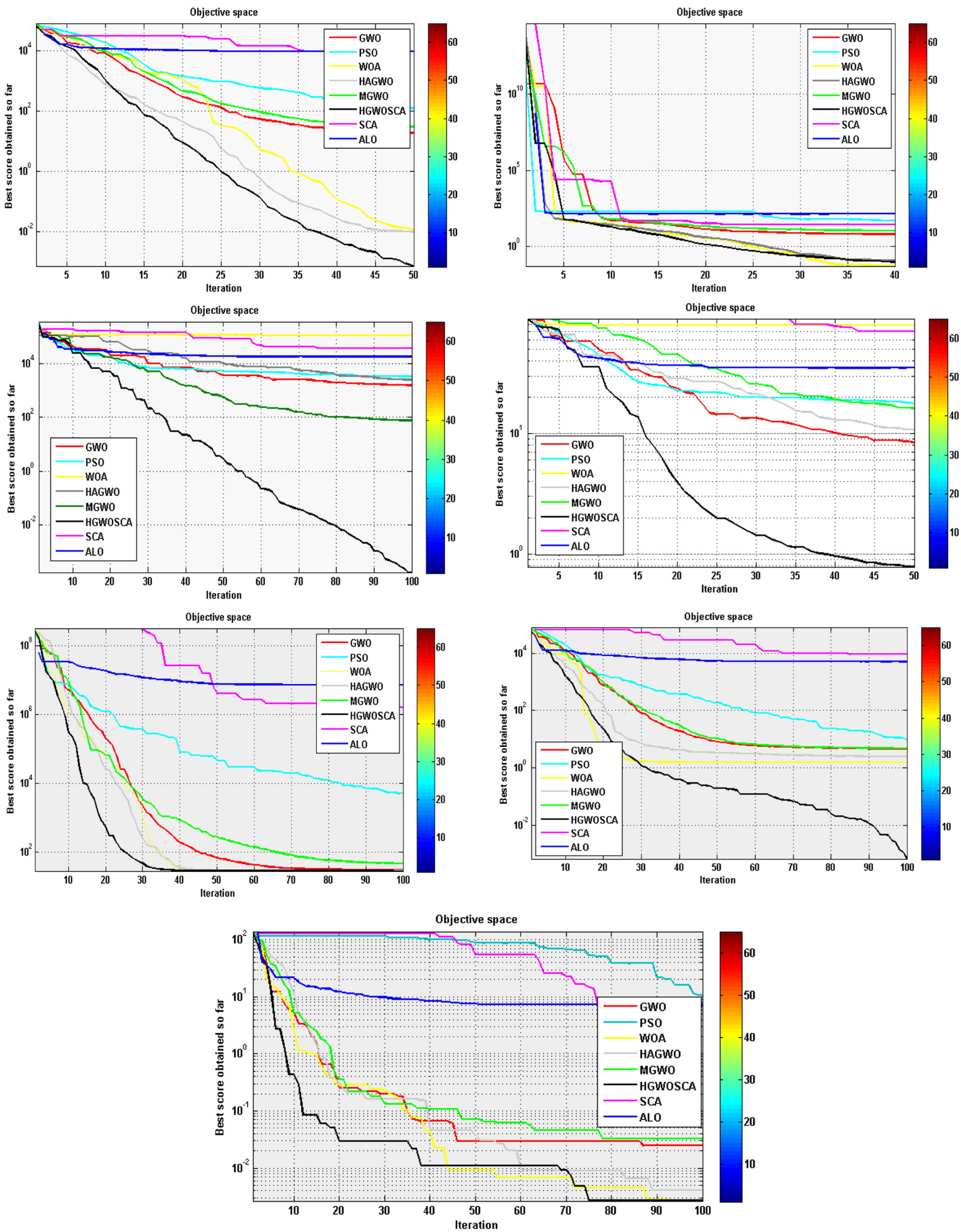
When $|\vec{a}| > 1$, the members of the population are enforced to diverge from the prey.

3. Sine Cosine Algorithm (SCA)

A newly proposed technique by Mirjalili [24] called Sine Cosine Algorithm (SCA) simply based on Sine and Cosine function is applied for exploitation and exploration phases in global optimization functions. The Sine Cosine Algorithm (SCA) creates different initial random agent solutions and requires them to fluctuate outwards or towards the best possible solution using a mathematical model based on sine and cosine functions. Basic principles of Sine Cosine Algorithm (SCA) is represent in Figs. 1 and 2.

Table 14
Experimental results of Bio-Medical real life problems.

	Algorithm	Best Min value	Best Max value	Average	S.D.	Classification Rate
(i)	Iris dataset problem					
	GWO	0.6667	0.8784	0.6727	0.0252	92.21%
	PSO	0.6667	0.8768	0.6908	0.0401	37.80%
	WOA	0.6982	0.8711	0.7311	0.0437	78.7%
	HAGWO	0.6667	0.8621	0.6716	0.0223	93.56%
	MGWO	0.6667	0.8704	0.6709	0.0258	93.79%
	SCA	0.6667	0.8298	0.7431	0.0574	92.69%
	ALO	0.6667	0.8516	0.6683	0.0506	89.10%
	HGWOSCA	**0.6667**	**0.8842**	**0.6703**	**0.0212**	**93.99%**
(ii)	XOR dataset problem					
	Algorithm	Best Min value	Best Max value	Average	S.D.	Classification Rate
	GWO	3.2612e−05	0.2128	0.0065	0.0302	100%
	PSO	2.9305e−198	0.2146	0.0040	0.0292	37.25%
	WOA	0.0723	0.1277	0.0815	0.0152	49.35%
	HAGWO	6.3855e−05	0.1967	0.0036	0.0152	100%
	MGWO	4.5804e−05	0.2305	0.0107	0.0377	100%
	SCA	0.0088	0.1917	0.0189	0.0185	100%
	ALO	0.0045	0.2101	0.0079	0.0290	89.89%
	HGWOSCA	**0.0018**	**0.2595**	**0.0025**	**0.0104**	**100%**
(iii)	Baloon dataset problem					
	Algorithm	Best Min value	Best Max value	Average	S.D.	Classification Rate
	GWO	3.3626e−25	0.1568	0.0017	0.0135	100%
	PSO	4.2536e−25	0.1099	9.3798e−04	0.0093	100%
	WOA	0.0529	0.1673	0.0740	0.0296	100%
	HAGWO	8.2503e−10	0.1461	0.0095	0.0134	100%
	MGWO	6.3038e−21	0.0585	3.9912e−04	0.0093	100%
	SCA	0.0725	0.1154	0.0.0087	0.0205	100%
	ALO	0	0.1391	0.0096	0.0176	100%
	HGWOSCA	**0**	**1625**	**0.0014**	**0.0094**	**100%**
(iv)	Breast cancer dataset problem					
	Algorithm	Best Min value	Best Max value	Average	S.D.	Classification Rate
	GWO	0.0014	0.0467	0.0046	0.0083	99.00%
	PSO	0.0017	0.0451	0.0117	0.0105	25.21%
	WOA	0.0033	0.00439	0.0042	0.0032	59.00%
	HAGWO	0.0013	0.0451	0.0018	0.0032	99.14%
	MGWO	0.0013	0.0454	0.0037	0.0075	99.11
	SCA	0.0014	0.0381	0.0264	0.0071	89.99%
	ALO	0.0012	0.0120	0.0024	0.0036	79.16%
	HGWOSCA	**0.0011**	**0.00508**	**0.0014**	**0.0028**	**99.49%**
(v)	Heart dataset problem					
	Algorithm	Best Min value	Best Max value	Average	S.D.	Classification Rate
	GWO	0.0719	0.2928	0.0905	0.0337	76.00%
	PSO	0.0781	0.2864	0.0899	0.0282	51.25%
	WOA	0.1231	0.2696	0.1284	0.0143	52.66%
	HAGWO	0.0666	0.2371	0.0916	0.0248	76.58%
	MGWO	0.0719	0.2415	0.0933	0.0331	77.00%
	SCA	0	0.2486	0.1948	0.0224	78.48%
	ALO	0	0.2823	0.1103	0.0248	59.98%
	HGWOSCA	**0**	**3048**	**0.0714**	**0.0113**	**78.59%**



Figs. 1–7. Convergence Curve of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and HGWOSCA variants on Unimodal functions.

$$\vec{x}_i^{t+1} = \vec{x}_i^t + rand_1 \times \sin(rand_2) \times |rand_3 \times l_i^t - \vec{x}_i^t| \quad (9)$$

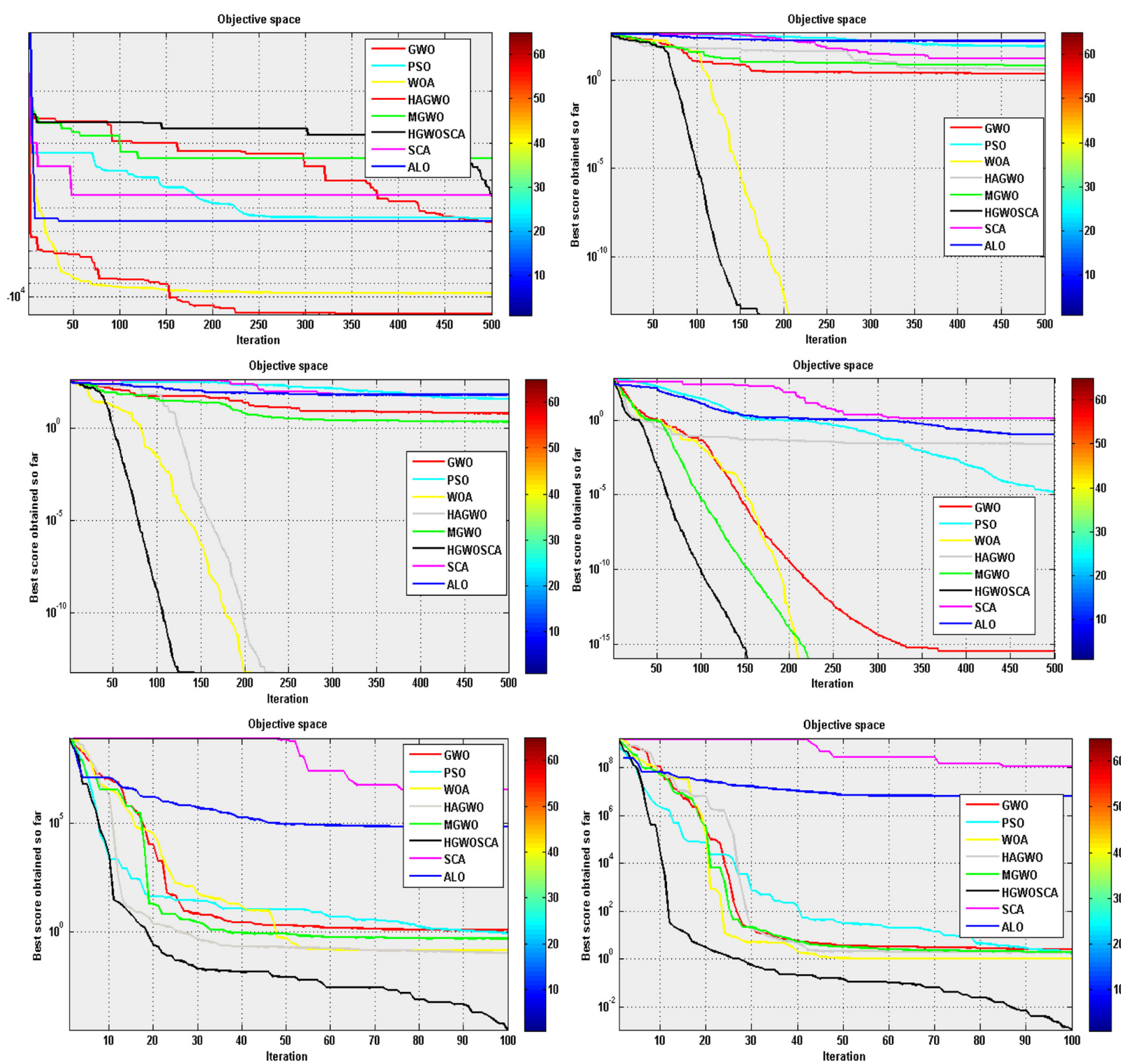
$$\vec{x}_i^{t+1} = \vec{x}_i^t + rand_1 \times \cos(rand_2) \times |rand_3 \times l_i^t - \vec{x}_i^t| \quad (10)$$

where: $\vec{x}_i^t$ current position at tth iteration in ith dimension, $rand_1, rand_2, rand_3 \in [0,1]$ are random numbers and l_i is targeted global optimal solution. The $0.5 \leqslant rand_4 < 0.5$ conditions uses in Eqs. (9) and (10) for exploitation and exploration.

$$\vec{x}_i^{t+1} = \begin{cases} \vec{x}_i^t + rand_1 \times \sin(rand_2) \times |rand_3 \times l_i^t - \vec{x}_i^t|, & rand_4 < 0.5 \\ \vec{x}_i^t + rand_1 \times \cos(rand_2) \times |rand_3 \times l_i^t - \vec{x}_i^t|, & rand_4 \geqslant 0.5 \end{cases} \quad (11)$$

4. Motivation of this work

Although the Grey Wolf Optimizer and Sine Cosine algorithms are able to expose an efficient accuracy in comparison with other well-known swarm intelligence optimization techniques, it is not fitting for highly complex functions and is still may face the difficulty of getting trapped in local optima. To overcome these weakness and to increase its search capability, a newly hybrid variant based on grey wolf optimizer and sine cosine algorithm is proposed to solve recent real life problems. The proposed hybrid variant is called HGWOSCA. In this variant, the movement of alpha agent of the grey wolf algorithm is improve based on sine cosine algorithm. By this methodology, it is intended to improve the global convergence, exploration and exploitation performance by accelerating the search seeking instead of letting the variant running numerious generations without any improvement. The proposed variants have been tested with numerious well-known standard benchmark functions and five bio-medical science dataset and one sine dataset problems. Experimental solutions affirm that the newly hybrid variant is a robust search variant for numerious global optimization functions.



Figs. 8–13. Convergence Curve of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and HGWOSCA variants on Multimodal functions.

5. Hybrid Grey Wolf Optimizer-Sine Cosine Algorithm (HGWOSCA)

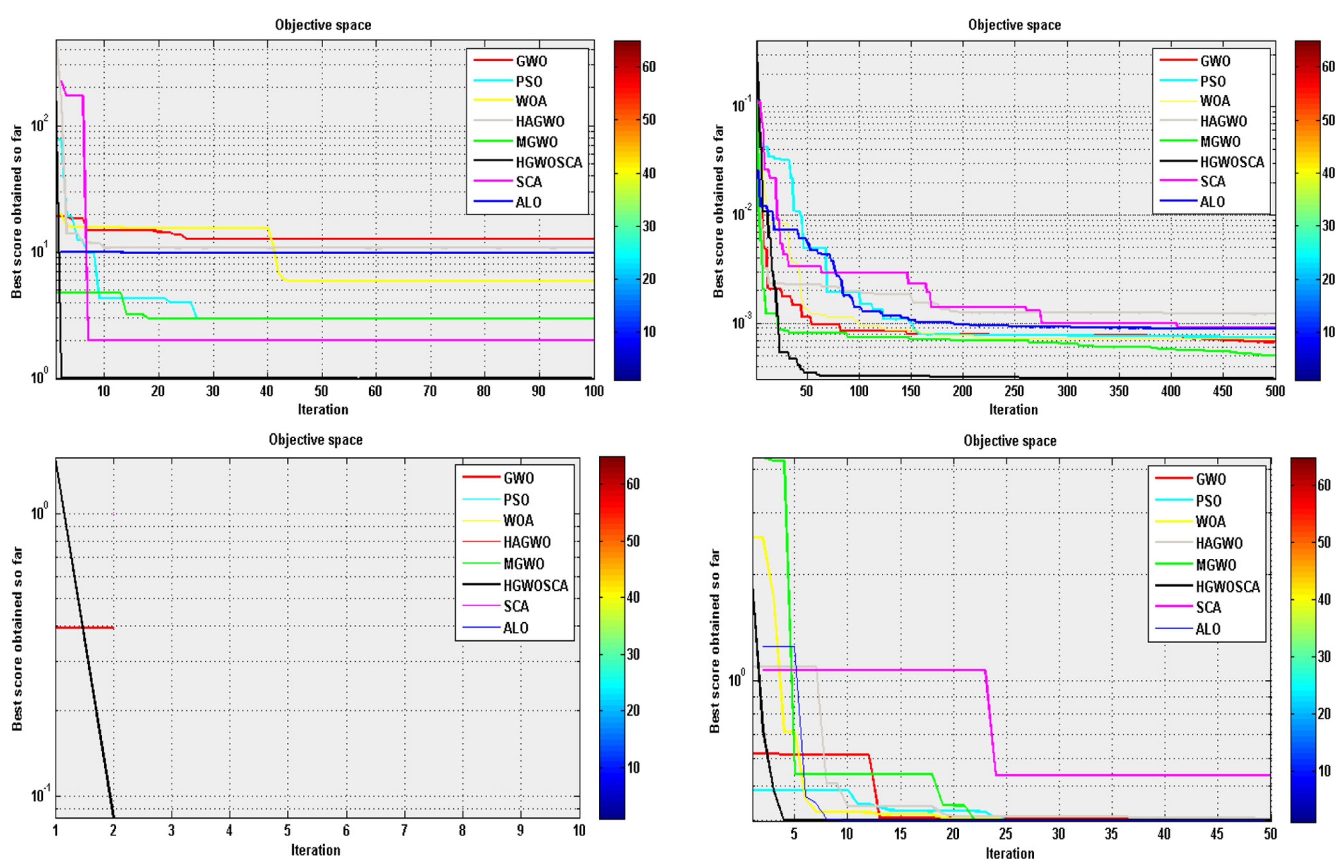
In this section, the details of the newly hybrid algorithm are presented. The basic idea is to use a grey wolf optimizer and sine cosine algorithm then we developed new hybrid variant with the aim to replace the worst result using one to one idea to find new population. The experimental solutions and convergence graphs prove that the combination between the two variants improve the accuracy of the newly hybrid variant. HGWOSCA is based on three procedures, which make it powerful and capable to find the efficient solution of recent real life applications. Now, we start the explanation of the HGWOSCA as follows:

In this variant the position, speed and convergence accuracy of grey wolf (alpha) agent has been improved by applying position update Eq. (11) of SCA for the purpose to balance between the exploration and the exploitation process and extending the convergence performance of grey wolf optimizer algorithm. The rest of the operations of grey wolf optimizer algorithm are same. The following position update equations of grey wolf (alpha) are developed in this regard.

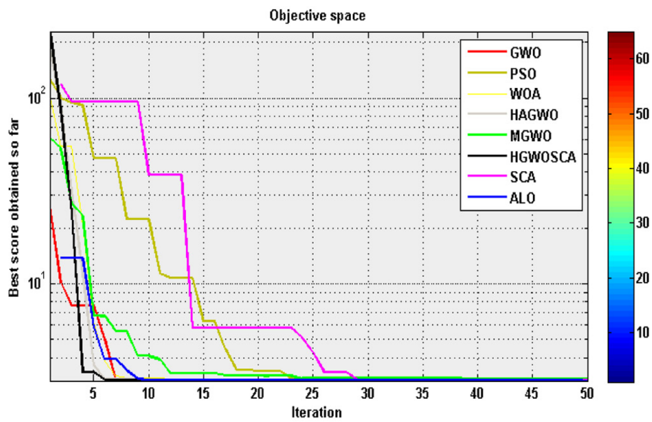
$$\vec{d}_\alpha = \begin{cases} rand() \times \sin(rand()) \times |\vec{c}_1 \times \vec{x}_\alpha - \vec{x}|, & rand() < 0.5 \\ rand() \times \cos(rand()) \times |\vec{c}_1 \times \vec{x}_\alpha - \vec{x}|, & rand() \geqslant 0.5 \end{cases} \tag{12}$$

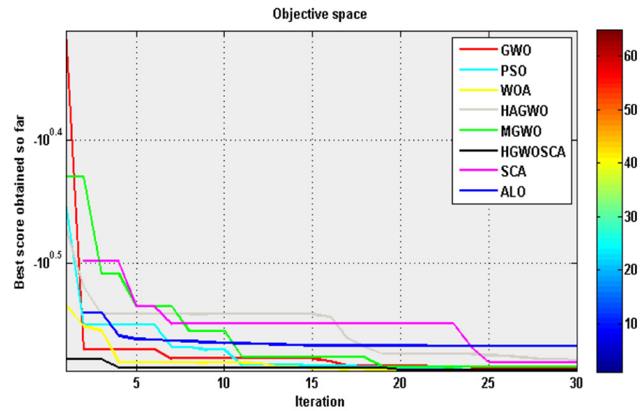
$$\vec{x}_1 = \vec{x}_\alpha - \vec{a}_1.\left(\vec{d}_\alpha\right) \tag{13}$$

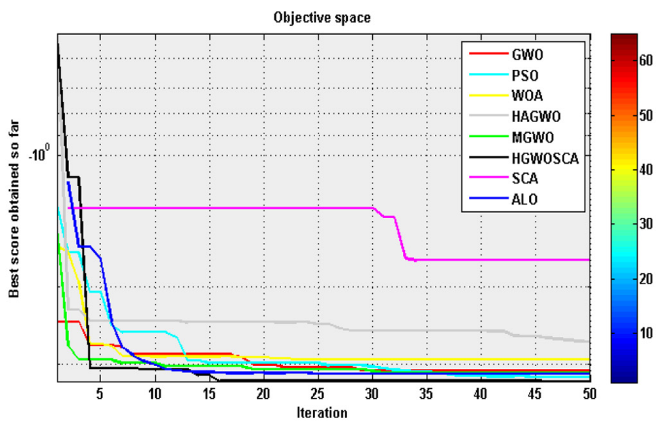
The pseudo code of the HGWOSCA Algorithm
Initialization the population $X_i\ (i = 1, 2, \ldots, n)$
Initialize A, a and C
Find the fitness of each search member
$\vec{x}_\alpha \sim$ *the best search agent*
$\vec{x}_\beta \sim$ *the* 2^{nd} *best search agent*
$\vec{x}_\delta \sim$ *the* 3^{rd} *best search agent*
While (t < Maximum number of iterations)
  for every search member
Update the position of the current search member by Eq. (8)
  end for
  update $\vec{a}$ and $\vec{c}$ by Eqs. (3) and (4)
Calculate the fitness of all search member
Update position of $\vec{x}_\beta, \vec{x}_\delta$ by Eq. (7) and $\vec{x}_\alpha$ as below:
if $rand() < 0.5$
then
  $\vec{d}_\alpha = rand() \times \sin(rand()) \times |\vec{c}_1 \times \vec{x}_\alpha - \vec{x}|$
else
  $\vec{d}_\alpha = rand() \times \cos(rand()) \times |\vec{c}_1 \times \vec{x}_\alpha - \vec{x}|$
   $\vec{x}_1 = \vec{x}_\alpha - \vec{a}_1.(\vec{d}_\alpha)$
end if
end else
  end while
  Return $\vec{x}_\alpha$

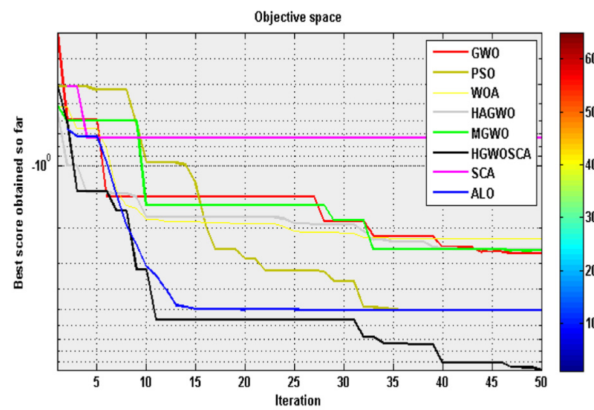


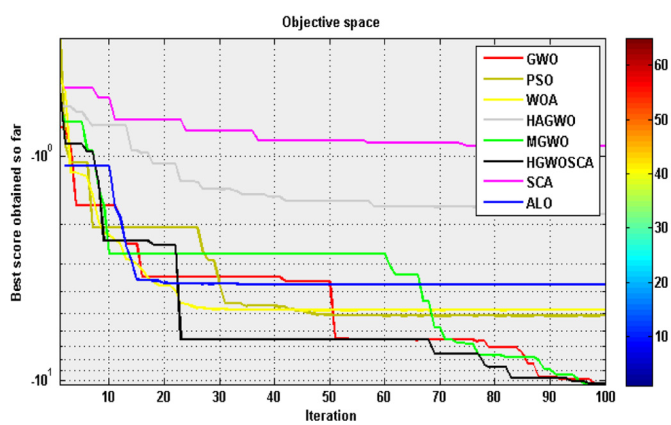
Figs. 14–22. Convergence Curve of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and HGWOSCA variants on Fixed Dimension Multimodal functions.











Figs. 14–22 (*continued*)

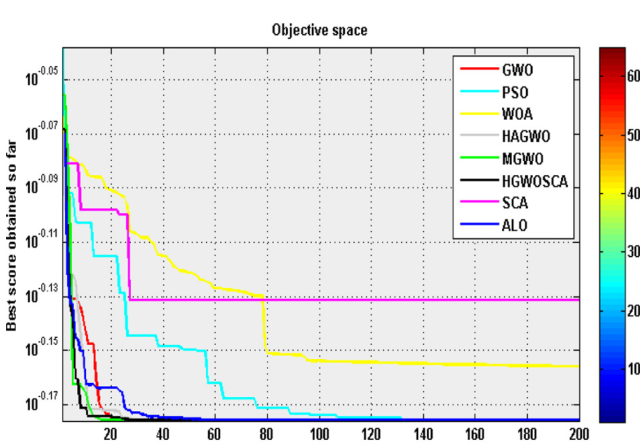


Fig. 23. Convergence graph on Iris dataset problem.

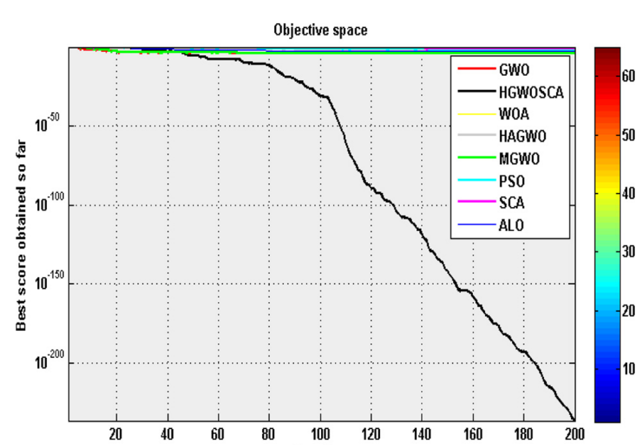


Fig. 24. Convergence graph on XOR dataset problem.

6. Classical problems

The statistical, numerical, convergence and time consuming capability of newly hybrid approach have been tested on twenty-two classical and some real life applications and numerical solutions obtained are compared with recent metaheuristics. These classical functions have been divided into three different parts i.e. Unimodal, Multimodal and fixed dimension multimodal are listed in Appendix-I (Tables A–C).

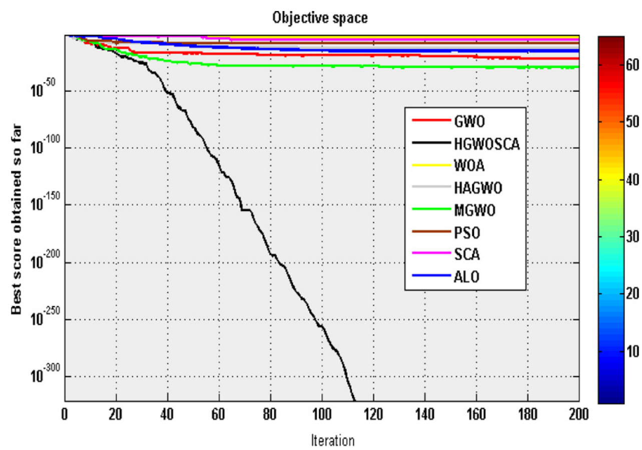


Fig. 25. Convergence on Baloon dataset problem.

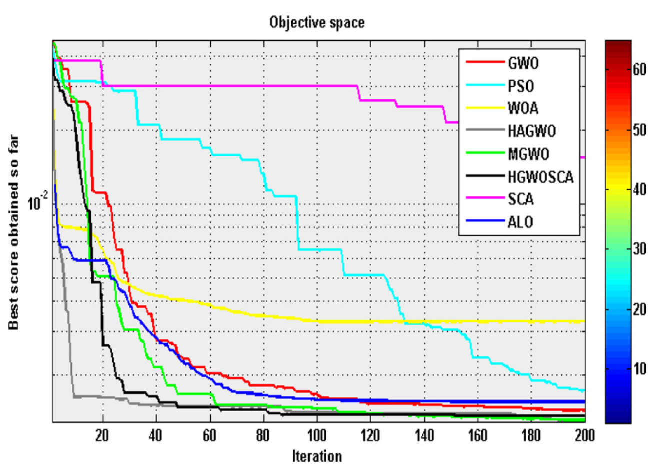


Fig. 26. Convergence on Breast cancer dataset problem.

7. Numerical experiments

We test the accuracy of the proposed hybrid variant on the numerious standard benchmark functions, five bio-medical science dataset and one sine dataset problems on with different number of iterations then we compared it against GWO, PSO, ALO, WOA, HAGWO, MGWO and SCA. We program HGWOSCA, GWO, PSO, ALO, WOA, HAGWO, MGWO and SCA in MATLAB R2013a.

In the following subsections, we report more details the parameter settings of the newly hybrid and all other existing variants (Table F).

8. The efficiency of the proposed hybrid algorithm

In Fig. 3(a)–(e), we verify the general accuracy of the standard metaheuristics with the proposed variant in order to test the efficiency of the proposed variant. We set the same parameter values for the all variants to make fair comparison. We show the solutions in Fig. 3(a)–(e) by plotting the worst optimal values of problem values against the number of generations for simplified model of the molecule with distinct size from 20 to 100 dimension. The figures shows that the standard benchmark problem values quickly decrease as the number of generations increases for proposed hybrid variant solutions than those of the other metaheuristics. In figures A–E, GWO, PSO, ALO, WOA, HAGWO, MGWO and SCA variants suffers from the slow convergence, gets stuck in the partitioning procedure, nevertheless and many local minima and invoking the sine cosine algorithm in the proposed variant avoid trapping in local minima and accelerate the search.

9. Experiment and results

In this section, twenty-two standard benchmark problems have been utilized to demonstrate the performance, strength and efficiency of the proposed hybrid variant, where obtained numerical and statistical results by the newly variant have been verified with the GWO, PSO, ALO, WOA, HAGWO, MGWO and SCA algorithms. The standard problems contain multimodal, unimodal and fixed dimension multimodal examples. The whole results explanation and the convergence graph of the each standard problem are verified in Tables 1–14 and Figs. 1–28, respectively. On other side, five bio-medical science dataset and one sine dataset problems are solved. For these experiments, the variants are coded in MATLAB R2013a, running on a Laptop with an Intel HD Graphics, 15.6″ 16.9 HD LCD, Pentium-Intel Core I, i5 Processor 430 M, 3 GB Mem-

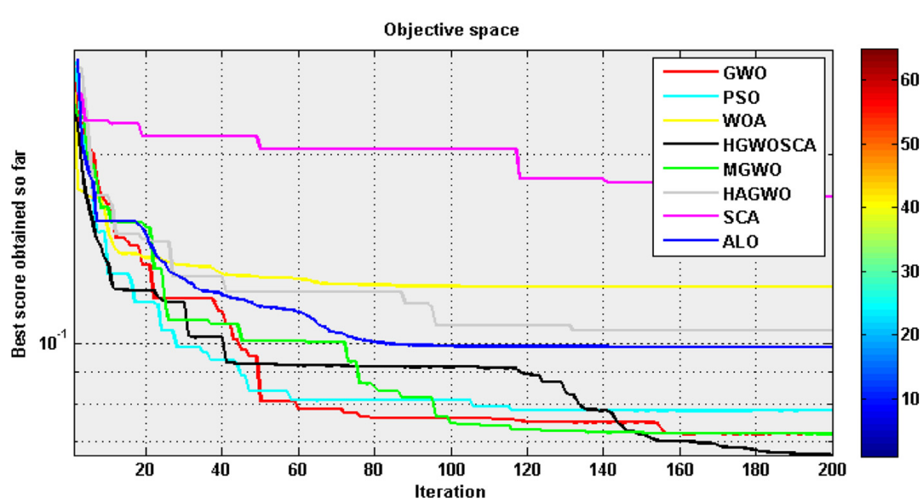


Fig. 27. Convergence on Heart dataset problem.

ory and 320 GB HDD. In addition, to statistically asses the proposed hybrid variant verified with other methodologies, average and standard deviation are introduced.

The capability quality of the proposed variant is evaluated by using a set of standard functions to test the solution quality, solution stability, convergence speed and ability to find the global optimum. The outcome values of the classical functions in the experiments are averaged over 20 runs with different random seeds. The purpose of the optimization is to maximize and minimize the outcome of the classical problems.

The crowd size is set the equal for all the techniques of the proposed hybrid technique and original ones in the experiments. The details of all parameter settings of all testing variants can be found in Section 7. The dimension and initial range of all classical problems are listed in Annexure-I (Tables A–C).

On the basis of Tables 1 and 2 identify the performance quality for the multimodal functions of several recent algorithms of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and the newly developed HGWOSCA. Experimental solutions prove that the newly proposed algorithm superior on all the cases of testing multimodal classical functions as comparison to others and also show that almost increases higher than those obtained from original techniques of GWO and SCA.

A number of criteria has been applied to find out the performance of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and the newly developed HGWOSCA. The mean and standard deviation statistical values are used to evaluate the reliability (Tables 3 and 4). The average computational time of the successful runs and the average number of problem evaluations of successful runs, are applied to estimate the cost of the standard problem.

For Unimodal benchmark functions, the quality of the global optimal solution obtained is considered by the minimize, maximize, mean and standard deviation of the objective function values out of twenty runs. This is shown in Tables 1 and 2 and convergence performance of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and HGWOSCA algorithms are shown in Figs. 1–7.

Further, the performance of the newly developed hybrid approach has been tested on multimodal test functions. The numerical and statistical performance of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and HGWOSCA algorithms are shown in Tables 5–8. We see that newly hybrid approach better performs to other meta-heuristics on all multimodal functions. The obtained results in Tables 5–8 strongly prove that high exploration of HGWOSCA is able to explore the search area extensively and give promising regions of the search area. The convergence performance of all algorithms has been compared with plotting graphs Figs. 8–13.

Fixed-dimension multimodal problems have many local optima with the number growing exponentially with dimension. This makes them fitting for benchmarking the exploration capacity of a technique. As per experimental results of Tables 9–12, newly hybrid approach is competent to find very competitive results on these functions/problems as well. This variant outperforms GWO, PSO, WOA, HAGWO, MGWO, SCA and ALO on the majority of these classical functions. Hence, HGWOSCA approach has merit in terms of exploration.

In Figs. 1–22, the convergence performance of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and HGWOSCA algorithms on all classical functions have been compared, obtaining convergence results show that the HGWOSCA approach is more reliable to search the best global optimal solution in minimum number of generations. Newly hybrid approach HGWOSCA avoids premature convergence of the search process to local optimal point and provides superior exploration of the search course.

Finally, the performance of the proposed variant has been verified using cpu time. These numerical results are provided in Table 13, respectively. It may be seen that the hybrid variant solved most of the standard functions in least time as compared to others.

To sum up, all simulation results assert that the proposed hybrid variant is very helpful in improving the efficiency of the GWO in the terms of result quality as well as computational efforts.

10. Bio-Medical problems

In this section five dataset biomedical real life problems: (i) Iris (ii) XOR (iii) Baloon (iv) Breast Cancer and (v) Heart are employed (Mirjalili [14]). These problems have been solved using newly hybrid approach and compared with GWO, PSO, WOA, HAGWO, MGWO, SCA and ALO algorithms. Different parameter settings have been applied for running code of algorithms and these parameter settings are show in Appendix (Table D) [54]. The performance of the metaheuristics has been compared in terms of minimize and maximize objective function value, average, standard deviation, convergence rate and classification rate (%) of the metaheuristics (Table 14).

The obtaining solutions of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and HGWOSCA metaheuristics on these datasets are illustrate in (Table 14) and convergence capabilities of the techniques are illustrate through Figs. 24–27.

The results of Table 14, prove that newly hybrid metaheuristic gives the better quality of solutions of the real life biomedical problems as comparison to other recent techniques. The experimental

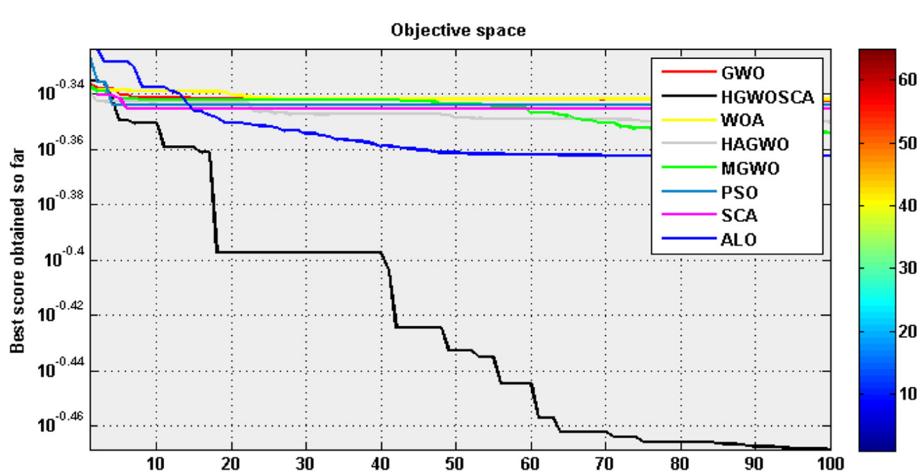


Fig. 28. Convergence Curve of GWO, PSO, WOA, HAGWO, MGWO, SCA, ALO and HGWOSCA variants on Sine dataset function.

Table 15
Experimental results for the Sine dataset Function.

Algorithm	Best Min value	Best Max value	Average	S.D.	Test Error
GWO	0.4547	0.4609	0.4553	9.8223e−04	42.684
PSO	0.3400	0.4623	0.4590	0.0381	126.77
WOA	0.4551	0.4595	0.4558	0.0025	98.11
HAGWO	0.4466	0.4580	0.4494	0.0028	86.56
MGWO	0.4427	0.4602	0.4507	0.0052	85.44
SCA	0.0028	0.4567	0.4469	0.0452	81.29
ALO	0.0019	0.4716	0.4364	0.0452	106.99
HGWOSCA	**0.0021**	**0.4719**	**0.4331**	**0.0023**	**41.400**

solutions of HGWOSCA metaheuristics show that it has the highest capability to avoid the local optima and is considerably superior than other metaheuristics i.e. GWO, PSO, WOA, HAGWO, MGWO, SCA and ALO.

Further the capabilities of the newly hybrid technique have been compared in terms of average, standard deviation, classification rate (in Table 14) and convergence rate (in Figs. 23–27). The low standard deviation and average proves the better local optima avoidance of the metaheuristic. Basis of experimental results, we have concluded that newly hybrid HGWOSCA technique provide highly competitive solutions as compared to other techniques and convergence performance prove that newly hybrid variant provides superiors results rather than GWO, PSO, WOA, HAGWO, MGWO, SCA and ALO algorithms.

11. Sine dataset problem

The sine dataset function is the most difficult function. The parameter setting of this function shown in Appendix (Table E). This dataset problem has four peaks that make it very challenging to be approximated. The experimental solutions of metaheuristics on this function are consistent with those of other two function-approximation datasets. It is worth mentioning that the newly hybrid approach gives most accurate solutions on this dataset function as can be inferred from test errors in Table 15 and convergence performance of the metaheuristics plotted by Fig. 28.

12. Conclusion

A novel proposed newly hybrid approach for the classical and real life applications was presented in the article with the combination of GWO and SCA, namely HGWOSCA. In this proposed algorithm, the position of grey wolf (alpha) in GWO algorithm updated by position update equation of SCA. A newly hybrid approach is developed to explore and exploit the diversity of the algorithm. Twenty-two classical, five bio-medical dataset and one sine dataset functions are applied to verify the accuracy, convergent behavior, best global optimal solution and speed of the newly developed approach. Simulated solutions reveal that the newly hybrid approach increases the accuracy more than the GWO and SCA algorithms and provide highly competitive solutions as compared to other techniques i.e. GWO, PSO, WOA, HAGWO, MGWO, SCA and ALO algorithms.

The future work will be concentrated on two parts: (i) Solving optimal design of double later grids, composite functions, aircraft's wings, unit commitment problems, feature selection, Structural Damage Detection, the gear train design problem, Cantilever beam, Welded beam design, Pressure vessel design problem, built train problem, bionic car problem, multi-objective design problems and many other biomedical and mechanical engineering problems (ii) Developing new metaheuristics for these tasks. To end with, we expectation that this research work will encourage other scientists and young researchers who are working on new population based techniques and optimization concepts.

Appendix A.

Tables A–E.

Table A
Unimodal benchmark functions.

Function	Dim	Range	$f_{\min}$
$F_1(x) = \sum_{i=1}^{n} x_i^2$	30	[−100, 100]	0
$F_2(x) = \sum_{i=1}^{n} \lvert x_i\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	30	[−10, 10]	0
$F_3(x) = \sum_{i=1}^{n} \left(\sum_{j-1}^{i} x_j\right)^2$	30	[−100, 100]	0
$F_4(x) = \max_i\{\lvert x_i\rvert, 1 \leqslant i \leqslant n\}$	30	[−100, 100]	0
$F_5(x) = \sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$	30	[−30, 30]	0
$F_6(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	30	[−100, 100]	0
$F_7(x) = \sum_{i=1}^{n} i x_i^4 + rand[0, 1)$	30	[−1.28, 1.28]	0

Table B
(Multimodal benchmark functions).

Function	Dim	Range	$f_{\min}$
$F_8(x) = \sum_{i=1}^{n} - x_i \sin(\sqrt{\lvert x_i\rvert})$	30	[−500, 500]	−418.9829 × 5
$F_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$	30	[−5.12, 5.12]	0
$F_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	30	[−32, 32]	0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	[−600, 600]	0
$F_{12}(x) = \frac{\pi}{n}\left\{10\sin(\pi y_i) + \sum_{i=1}^{n-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1}) + (y_{n-1})^2\right]\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	30	[−50, 50]	0
$y_i = 1 + \frac{x_i+1}{4}$			
$u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$			
$F_{13}(x) = 0.1\left\{\sin^2(3\pi x_i) + \sum_{i=1}^{n}(x_i - 1)^2[1 + \sin^2(3\pi x_i + 1)] + (x_n - 1)^2\left[1 + \sin^2(2\pi x_n)\right]\right\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	30	[−50, 50]	0

Table C
(Fixed-dimension multimodal benchmark functions).

Function	Dim	Range	$f_{\min}$
$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6}\right)^{-1}$	2	[−65, 65]	1
$F_{15}(x) = \sum_{i=1}^{11}\left[a_i - \frac{x_1(b_i^2+b_ix_2)}{b_i^2+b_ix_i+x_4}\right]^2$	4	[−5, 5]	0.00030
$F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	[−5, 5]	−1.0316
$F_{17}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1-\frac{1}{8\pi}\right)\cos x_1 + 10$	2	[−5, 5]	0.398
$F_{18}(x) = \left[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)\right] \times \left[30+(2x_1-3x_2)^2 \times (18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)\right]$	2	[−2, 2]	3
$F_{19}(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} a_{ij}(x_j-p_{ij})^2\right)$	3	[1, 3]	−3.86
$F_{20}(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} a_{ij}(x_j-p_{ij})^2\right)$	6	[0, 1]	−3.32
$F_{21}(x) = -\sum_{i=1}^{5}\left[(X-a_i)(X-a_i)^T + c_i\right]^{-1}$	4	[0, 10]	−10.1532
$F_{22}(x) = -\sum_{i=1}^{7}\left[(X-a_i)(X-a_i)^T + c_i\right]^{-1}$	4	[0, 10]	−10.4028
$F_{23}(x) = -\sum_{i=1}^{10}\left[(X-a_i)(X-a_i)^T + c_i\right]^{-1}$	4	[0, 10]	−10.5363

Table D
Bio-Medical Classification datasets (Mirjalili et al. (2014)) [54].

Classification datasets	Number of attributes	Number of training samples	Number of test samples	Number of classes
3-bits XOR	3	8	8 as training samples	2
Balloon	4	16	16 as training samples	2
Iris	4	150	150 as training samples	3
Breast cancer	9	599	100	2
Heart	22	80	187	2

Table E
Classification datasets (Mirjalili et al. (2014)) [54].

Function-approximation datasets	Training samples	Test samples
Sine : $y = \sin(2x)$	126 : x *in* $[-2\pi : 0.1 : 2\pi]$	252 : x *in* $[-2\pi : 0.05 : 2\pi]$

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