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A practical method of transient stability analysis of stochastic power systems based on EEAC

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ABSTRACT

With the diversification of power systems and the application of power electronic technology, the uncertainty of power systems is becoming more and more serious, and the traditional deterministic transient stability analysis methods encounter severe challenges. In this paper, a practical method is proposed for transient stability analysis of stochastic multi-machine systems based on EEAC. Firstly, the stochastic differential equation model of multi-machine systems is established. Secondly, the stochastic multi-machine system is equivalent to a stochastic single-machine infinite bus system according to coherency. Finally, the acceleration and deceleration area is constructed, and Heun's method is used to seek the critical clearing time of the system. The method is applied to analyse transient stability of four machine system. Compared with the numerical simulation based on Monte Carlo method, the simplicity and effectiveness of the method are verified. The impact of stochastic disturbance intensity on stability is also presented.

1. Introduction

Power systems stability is influenced by the characteristics of each main component in the system [1–4]. Traditional power systems exist stochastic disturbance such as load fluctuation, mechanical power stochastic torsional oscillation and measurement noise. With the continuous expansion of the scale of the power grid, the access to renewable energy, the application of power electronic devices and the popularity of electric vehicles, the stochasticity of power systems cannot be ignored [5–7]. Therefore, the traditional deterministic transient stability analysis method is facing severe challenges, and the power system stochastic model and the stability analysis method considering stochastic factors have attracted wide attention [7,8].

The stochasticity in the dynamic equations of power system is mainly expressed in 3 aspects [9,10]:

(a) The stochasticity of the initial value, caused by the uncertainty of equilibrium points of power systems.
(b) The stochasticity of the parameters and coefficients of the state equation is caused by the variation of the operating state and the internal structure.
(c) Stochastic external excitation, such as conventional load fluctuation, renewable energy generation fluctuation, and charging load fluctuation of electric vehicles.

The parameter and the initial value is constant for a dynamic process, and the solution of these two types is the Probabilistic differential equation model, assuming the initial value and the parameters or coefficients obey a certain probability distribution, the probability of system stability or the envelope of the corresponding trajectory is obtained. And stochastic external excitation is characterized by rapid change, and in the dynamic process it is considered to be a time variable. It needs to introduce stochastic differential equation theory for modelling and analysis [11]. In this paper, the impact of stochastic external excitation on power systems is studied.

Time-domain simulation and direct methods are used to analyse power system transient stability [12,13]. The direct methods mainly includes the EEAC methods and the energy function methods. EEAC as a practical transient stability criterion was first proposed in [14]. These deterministic methods need to be modified in order to apply to the analysis of power systems with stochastic disturbances. And EEAC is constantly being improved and studied [15–17]. In the past decade, numerical integration methods have significant development to simulate stochastic differential equations [18]. And a method using transient energy function for stochastic power systems is proposed in [19]. The first idea to use EEAC to analyse the probabilistic stability is proposed in [20], and a probability approach to discuss the probabilistic aspects of transient stability evaluation of power systems combined EEAC and probability density function (PDF) is proposed in [21], where the stochastic disturbance is a constant value instead of a time variable in the transient process, and the stochastic differential equation theory is not introduced into the power system research to frame stochastic form of EEAC. An analysis of the evolution of the PDF of dynamic trajectories of a single infinite bus power system is proposed in [22], where the time

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varying stability probability is decided by the PDF of state variables. In the transient analysis of actual systems. It is necessary to calculate the stability probability and the critical clearing time more quickly and accurately. In this paper, the theory of EEAC and the stochastic differential equation theory will be merged and modified to present a practical approach to study the transient stability.

2. Stochastic power system model and numerical method

2.1. Itô stochastic differential equation model of power systems

In the dynamic process of power systems, the stochastic external excitation is mainly divided into the following categories:

(a) Mechanical power fluctuations;
(b) Renewable energy output fluctuations;
(c) Load fluctuations.

These fluctuations can cause fluctuations in the voltage, current and power angle, and interfere with the stability of the power system. Suppose we write the disturbance vector as $\boldsymbol{W}(t)$. The state variable of the power system is $\boldsymbol{X}$, the state equation matrix with random excitation can be written as:

$$\dot{\boldsymbol{X}} = \boldsymbol{A}(t, \boldsymbol{X}) + \boldsymbol{\zeta}(t, \boldsymbol{X})\boldsymbol{W}(t) \tag{1}$$

where $\boldsymbol{A}(t, \boldsymbol{X})$ is the deterministic state derivative function, and $\boldsymbol{\zeta}(t, \boldsymbol{X})$ is the disturbance function matrix, and $W(t) \in \boldsymbol{W}(t)$ is generally assumed that it has the following properties:

(a) $W(t)$ is independent of each other at any two different time points;
(b) $\{W(t)\}$ is stable, and its joint distribution is independent of t;
(c) $E[W(t)] = 0$.

In order to meet the above requirements, $W(t)$ can be regarded as a generalized stochastic process, that is, the white noise process. In this section, suppose the stochastic disturbance source is the load fluctuation of the power system, and the load fluctuation is generally considered to be a normal distribution [21]. Suppose $W(t)\tilde{\mathrm{N}}(0, \sigma^2 t)$, when $\sigma = 1$, that is $W(t) \sim \sqrt{\mathrm{t}}\,\mathrm{N}(0, 1)$. So $W(t)$ is a standard Gauss white noise process. Discrete form of one dimensional diffusion of (1):

$$X_{k+1} - X_k = A(t_k, X_k)\Delta t_k + \zeta(t_k, X_k)W_k\Delta t_k \tag{2}$$

where $X_j = X(t_j)$, $W_k = W(t_k)$, $\Delta t_k = t_{k+1} - t_k$. Replace $W_k\Delta t_k$ with $\Delta B_j = B_{j+1} - B_j$, and it is easy to get that $B(t)$ has a stable independent increment of mean zero. It is only Brownian motion that both meets the above properties and has a continuous path. $B(t)$ follows the Brownian movement, the project is also known as the Wiener process.

According to (2), the expression of state vector is:

$$X_k = X_0 + \sum_{j=0}^{k-1} A(t_j, X_j)\Delta t_j + \sum_{j=0}^{k-1} \zeta(t_j, X_j)\Delta B_j \tag{3}$$

when $\Delta t_j \to 0$, the continuous form of (3) is:

$$X_t = X_0 + \int_0^t A(s, X_s)ds + \int_0^t \zeta(s, X_s)dB_s \tag{4}$$

Which is also the integral form of the stochastic differential equation of power systems. State vector can be regarded as a diffusion process in (4), where A is the drift coefficient, ζ is the diffusion coefficient. When step size $\Delta t = 0.1ms$, The path of one dimensional standard Gauss process and one dimensional standard Brownian motion is shown in Fig. 1.

Ito and Riemann have different rules of differentiation. Suppose X is an Ito process as follows:

$$dX = udt + vdB(t) \tag{5}$$

If $g(t, x) \in C^2([0, \infty) \times \mathbf{R})$, $Y = g(t, X)$ is a ITô process, which is satisfied to:

$$dY = \frac{\partial g}{\partial t}(t, X)dt + \frac{\partial g}{\partial x}(t, X)dX + \frac{1}{2}\frac{\partial^2 g}{\partial x^2}(t, X)\cdot(dX)^2 \tag{6}$$

where $(dX)^2 = (dX)\cdot(dX)$ is computed according to the rules:

$$dt\cdot dt = dt\cdot dB(t) = dB(t)\cdot dt = 0, \quad dB(t)\cdot dB(t) = dt \tag{7}$$

2.2. Numerical methods of ITô stochastic differential equation

Itô stochastic differential equations are the same as ordinary differential equations, only partial equations can be solved analytically, and the other part can only be solved by numerical methods. And the specific results of the analytical solution of the stochastic differential equation still need to be solved by numerical methods [23]. The main difference of numerical methods of the stochastic differential equation compared with the numerical methods of ordinary differential equations is that the special properties of the Wiener process and the stochastic integral must be considered.

At present, the numerical integration method is commonly used: Euler-Maruyama (EM) method, Milstein method, Huen's method, and Runge-Kutta method [18]. EM method is the simplest numerical method, but it has lower convergence order. Huen's method is based on the trapezoidal formula, which improves the convergence order by first predicting the post correction. For the diffusion process of (1), the numerical iterative scheme of the Huen's method is:

(a) First prediction:

$$\hat{\boldsymbol{X}}_{i+1} = \boldsymbol{X}_i + \boldsymbol{A}(\boldsymbol{X}_i)\Delta t + \boldsymbol{\zeta}(\boldsymbol{X}_i)\Delta\boldsymbol{B}_i \tag{8}$$

(b) Correction to get the iteration value:

$$\boldsymbol{X}_{i+1} = \boldsymbol{X}_i + \frac{\boldsymbol{A}(\boldsymbol{X}_i) + \boldsymbol{A}(\hat{\boldsymbol{X}}_{i+1})}{2}\Delta t + \frac{\boldsymbol{\zeta}(\boldsymbol{X}_i) + \boldsymbol{\zeta}(\hat{\boldsymbol{X}}_{i+1})}{2}\Delta\boldsymbol{B}_i \tag{9}$$

where $\boldsymbol{A}(\boldsymbol{X}_i) = \boldsymbol{A}(t_i, \boldsymbol{X})$, $\boldsymbol{\zeta}(\boldsymbol{X}_i) = \boldsymbol{\zeta}(t_i, \boldsymbol{X})$, $\boldsymbol{A}(\hat{\boldsymbol{X}}_{i+1})$ and $\boldsymbol{\zeta}(\hat{\boldsymbol{X}}_{i+1})$ is predictive value, Taking white Guassian noise as an example, $\Delta\boldsymbol{B}_i$ is the Brownian movement increment, that is $\Delta\boldsymbol{B}_i \sim \sqrt{\Delta\mathrm{t}}\,\mathbf{N}(0, 1)$. Huen's method is also applicable to the simulation under other white non-Gauss noise (such as Poisson white noise), but it needs to pay attention to the difference among different types of white noise in the Monte Carlo simulation.

3. The equivalence method of stochastic power systems

3.1. Stochastic model of multi machine system

Rotor equation is an effective tool of power system transient stability research. In this paper, the transient stability is considered only if the first swing is unstable, where the effect of the damping is neglected. Suppose that the transient potential E'_i remains constant during the transient process. In the case of stochastic disturbance, the rotor motion equation of the first i generator is:

$$M_i\frac{d^2\delta_i}{dt^2} = P_{M,i} - P_{e,i} \tag{10}$$

The electromagnetic power of the first i generator is:

$$P_{e,i} = E'^2_i G_{ii} + E'_i \sum_{i=1, j\neq i}^{n} E'_j Y_{ij}\sin(\delta_i - \delta_j - \alpha_{ij}) \tag{11}$$

The performance of external stochastic excitations is a stochastic power disturbance on the rotor motion equation. The rotor equations of the first i generator with stochastic disturbance are considered:

$$M\frac{d^2\delta_i}{dt^2} = P_{M,i} - P_{e,i} + P_{c,i} \tag{12}$$

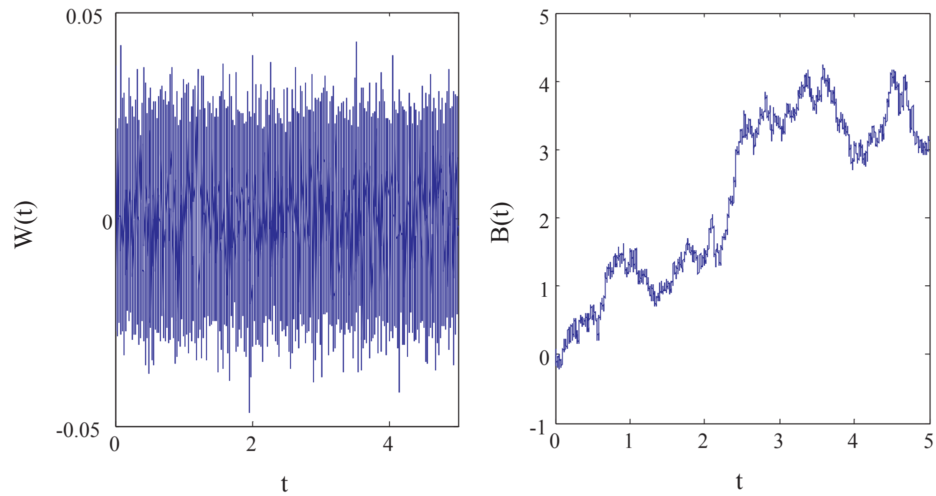

Fig. 1. Standard Gauss white noise and Corresponding path of Brownian motion.

where $P_{c,i}$ is stochastic power fluctuation of the first i generating set caused by the external stochastic excitations. Load fluctuation is chosen as the source of stochastic disturbance, and the load fluctuation is generally regarded as a Gauss process [24]. Let $P_{c,i} = \zeta_i W(t)$. According to the operation data of power system, selecting the appropriate sample and the sufficient sample quantity, the complete probability space can be established by analysing the law of stochastic power fluctuation, and the stochastic disturbance intensity ζ_i can be obtained by the similarity principle [25]. The rotor equations can be written as:

$$M\frac{d^2\delta_i}{dt^2} = P_{M,i} - P_{e,i} + \zeta_i W(t) \tag{13}$$

In this paper, we consider only the impact of white Guassian noise which is normal distribution, but other distributions can be incorporated. For instance, wind generation is often suggested to be a Weibull distribution [26], whereas Plug-in electric vehicles have been supposed to be Poisson distribution [5].

For the stochastic disturbance of white Poisson noise type, the expression is:

$$C(t) = \sum_{i=1}^{N(t)} R_i \delta(t - T_i) \tag{14}$$

where $N(t)$ is the Poisson counting process, the number of pulses arriving in [0, t], and the average arrival rate $\lambda > 0$, $\{R_i, i \geqslant 1\}$ are expressed as the pulse amplitude that obeys a certain probability distribution. The numerical simulation method is shown in the literature [24], and R_i is taken as the normal distribution. The rotor motion equation is changed into:

$$M\frac{d^2\delta_i}{dt^2} = P_{M,i} - P_{e,i} + \zeta_i C(t) \tag{15}$$

3.2. Equivalent method of stochastic multi machine systems

Because of the direct application of Monte Carlo method have some limitations on dimension and computation time, it is very important to study the new method to analyse transient stability. Inspired by the practical stability criterion EEAC of the deterministic system, the stochastic multi-machine system is simplified and the equivalence is carried out. When a fault occurs or a fault is clear, according to Coherence and the results of large step integration the generators can be divided into group S and group a. Assume that damping is ignored, the equivalent value can be obtained by the following method:

Assuming that the stochastic disturbances are independent of each other, the equations of motion of each generator set in group S can be added together:

$$\frac{d^2}{dt^2}\sum_{i\in S} M_i\delta_i = \sum_{i\in S}(P_{M,i} - P_{e,i}) + \sum_{i\in S}\zeta_i W(t) \tag{16}$$

Because the linear combination of multiple independent Gauss processes is still a Gauss process, so let $\zeta_S W(t) = \sum_{i\in S}\zeta_i W(t)$, (16) could be transformed to:

$$M_S\frac{d^2\delta_S}{dt^2} = P_{M,S} - P_{e,S} + \zeta_S W(t) \tag{17}$$

where

$$M_S = \sum_{i\in s} M_i,\ \delta_S = \frac{\sum_{i\in s} M_i\delta_i}{\sum_{i\in s} M_i},\ P_{M,S} = \sum_{i\in S} P_{M,i},\ P_{e,S} = \sum_{i\in S} P_{e,i}$$

In the same way, the equivalent rotor motion equation of group a:

$$M_a\frac{d^2\delta_a}{dt^2} = P_{M,a} - P_{e,a} + \zeta_a W(t) \tag{18}$$

(17) and (18) can be regarded as a two machine system, $\zeta_S W(t)$ and $\zeta_a W(t)$ are equivalent stochastic disturbance of equivalent units of corresponding groups. Let $\delta = \delta_s - \delta_a$, that is $\frac{d^2\delta}{dt^2} = \frac{d^2\delta_S}{dt^2} - \frac{d^2\delta_a}{dt^2}$. (17) minus (18), we have

$$M\frac{d^2\delta}{dt^2} = P_M - P'_e + \zeta W(t) \tag{19}$$

where

$$M = \frac{M_S M_a}{M_S + M_a}$$

$$P_M = \frac{M_a}{M_S + M_a}P_{M,S} - \frac{M_S}{M_S + M_a}P_{M,a}$$

$$P'_e = \frac{M_a}{M_S + M_a}P_{e,S} - \frac{M_S}{M_S + M_a}P_{e,a}$$

And the equivalence of random disturbance intensity:

$$\zeta W(t) = \frac{M_a}{M_S + M_a}\sum_{i\in S}\zeta_i W(t) - \frac{M_S}{M_S + M_a}\sum_{j\in a}\zeta_j W(t) \tag{20}$$

At this point, based on the EEAC equivalence principle, the equivalent method of stochastic disturbance is derived. $\zeta W(t)$ is equivalent stochastic disturbance of the equivalent single-machine infinite system (SMIB) system, ζ is equivalent stochastic disturbance intensity, it follows the algorithm of stochastic variables. Because of $P'_e = P_{me} + P_{\max}\sin(\delta - \upsilon)$, which contains the invariant part, it is separated into the equivalent mechanical power, we can get the equation:

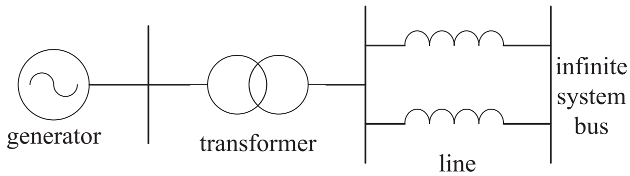

Fig. 2. The diagram of a SMIB system.

$$M\frac{d^2\delta}{dt^2} = P_m - P_e + \zeta W(t) \tag{21}$$

The specific process of equivalence is shown in EEAC part of the appendix. The equivalent method is still applicable to the other additive white noise, such as Poisson white noise, and it is worth noting that different types of white noise are different when calculating (20).

4. Modified EEAC

Stochastic complex systems can be equivalent to a stochastic SMIB system by the method of the previous section, and the transient stability of the equivalent SMIB system can be studied by Equal-Area Criteria [27]. Due to the time-varying stochastic disturbance, the UEP of the system cannot be directly determined, that is, the maximum deceleration area can not be calculated by the deterministic method. The research object in this section is a SMIB system. The structure is shown in Fig. 2. The detailed parameters are shown in the article [28]. Load fluctuation is also a stochastic disturbance source, damping neglected, considering whether the first swing is unstable.

Considering one dimensional stochastic disturbance, the two order model of the generator in the transient process:

$$M\frac{d^2\delta}{dt^2} = P_m - P_{ei} + \zeta W(t) \tag{22}$$

where $i = 1, 2, 3$ respectively indicate pre-fault, during the fault and after fault clearance. Assuming that the system is in steady state at pre-fault, the accelerating area under stochastic disturbance can be calculated by stochastic differential formula. According to (5)–(7), the accelerating area can be written:

$$S_a = M\left(\int_{\omega_0}^{\omega_c} \omega d\omega + \frac{1}{2}\int_{\omega_0}^{\omega_c} (d\omega)^2\right) \tag{23}$$

The acceleration area can also be written as follows:

$$S_a = \int_{t_0}^{t_c} (P_m - P_{e2}\sin\delta)\omega dt + \zeta\int_{t_0}^{t_c} \omega dB(t) + \frac{1}{2M}\zeta^2\int_{t_0}^{t_c} dt \tag{24}$$

Similarly, deceleration area can be written as the following form:

$$S_d = \int_{t_c}^{t_u} (P_{e3}\sin\delta - P_m)\omega dt - \zeta\int_{t_c}^{t_u} \omega dB(t) - \frac{1}{2M}\zeta^2\int_{t_c}^{t_u} dt \tag{25}$$

As mentioned above, a formula for calculating the acceleration and deceleration area with $It\hat{o}$ integral is proposed. When t_u corresponds to the return point, S_d is the deceleration area. When t_u corresponds to the nearest unstable equilibrium point, S_d is the maximum deceleration area. When the system is stable or critical stable, the acceleration area of the system is equal to the deceleration area and smaller than the maximum deceleration area. The latter two terms of (24) and (25) reflect the difference between Ito integral and Riemann integral. Stochastic disturbance caused unpredictable area fluctuation, so the analytical method cannot solve the acceleration and deceleration area. The above deduction is aimed at the stochastic disturbance of Gauss white noise type. If the stochastic disturbance is Poisson white noise or other types of white noise, we need to derive the corresponding stochastic differential rule. Huen's method has good numerical stability and strong convergence for solving stochastic differential equations, so it can be solved by the Huen's method to solve the acceleration and deceleration area.

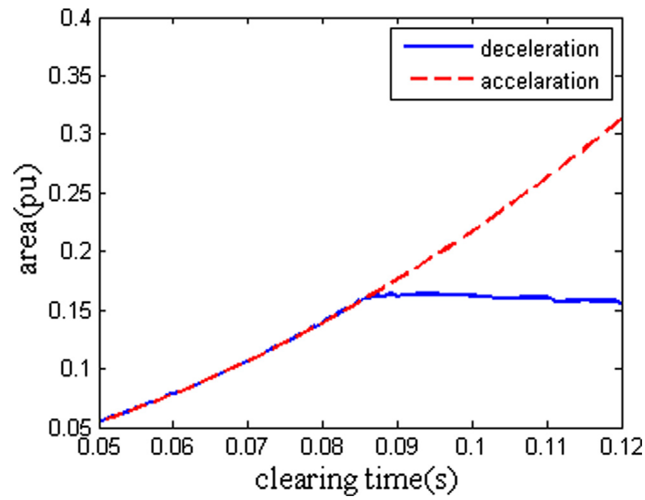

Fig. 3. The curves of Acceleration and deceleration area when ζ is 0.001 (1 run).

This paper presents a modified EEAC method for the transient stability analysis of stochastic multi machine systems:

1) According to the complex system operation parameters and fault conditions, all generators are divided into two coherent groups in different periods, and take simplification and equivalence on systems respectively, Then parameters of the equivalent SMIB system are obtained, so acceleration area S_a and deceleration area S_d are able to be calculated;
2) If the range of CCTs is known, as shown in Fig. 3, $t_{cr} \in [0.06, 0.12]$, and the step size is dt. From the time of resection, use (18)–(19) and Huen's method to obtain the corresponding trajectory of the acceleration and deceleration area;
3) If areas are equal, set the clearing time for $t_{crb} + dt$. Loop until the first time when $S_a - S_d > 0$ (i.e., the corresponding separation point of the acceleration and deceleration area in Fig. 3). And the corresponding clearing time is the CCT on this stochastic process.

The accuracy of CCTs is decided by the step size of the clearing time. When the stochastic disturbance intensity is 0.001, the acceleration area and deceleration area changes as shown in Fig. 3. The corresponding clearing time is the CCT on this stochastic process.

As shown in Fig. 4, due to the existence of stochastic disturbance, the acceleration area and deceleration area have a fluctuation, the separation points under 10 runs have also been a big wave. This shows that the CCTs of power systems is not a fixed value in the case of stochastic disturbance.

5. Case study

The system of four machines two areas is used for case study, as shown in Fig. 5, and specific parameters are shown in appendix [28]. Fault is set in one line of double line on bus 6 to bus 7 which is near to bus 7, the fault type is three-phase short circuit.

Under this fault, group S contains no. 1, 2 machines, other machines are in group a. Fault clearing time is set to be 0.195 s, and the relative angle curve of G1 and G3 without stochastic disturbance is calculated, as thick lines shown in Fig. 6, indicating that the system is stable. In the case of stochastic disturbance, the angle curves after the 100 runs are shown in Fig. 6. It can be seen that under some stochastic processes, the system is stable, but in other cases it is unstable. This phenomenon indicates that the stability of the system is no longer a single stable and unstable state in the case of stochastic disturbance, and it needs to be described by the stability probability. Under different stochastic paths,

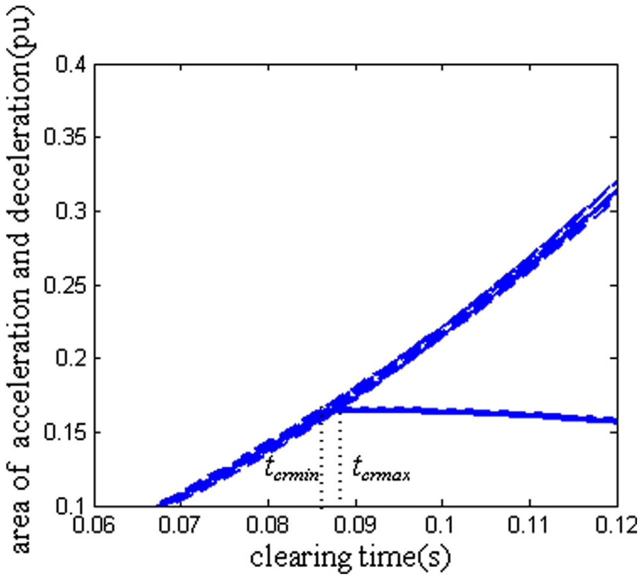

Fig. 4. The results of t_{cr} (10 runs).

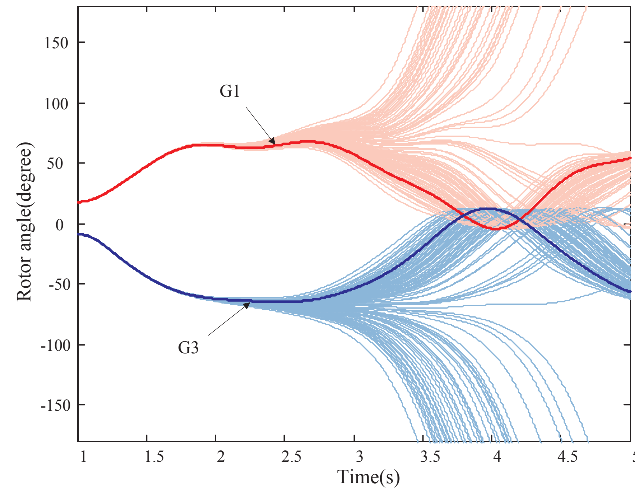

Fig. 6. The relative rotor angle curves of G1 and G3 with and without stochastic disturbance (100 runs).

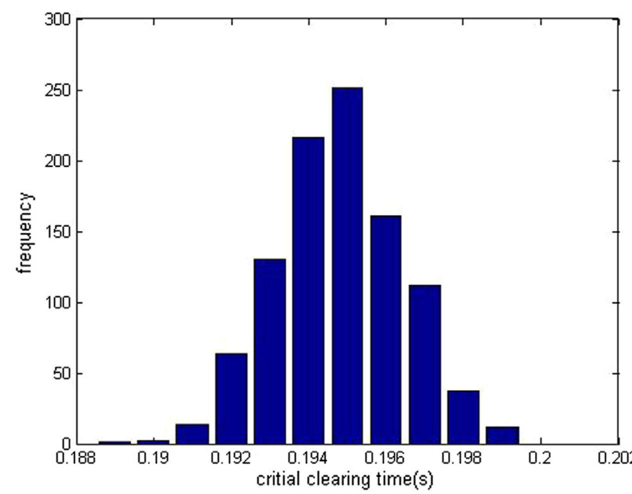

Fig. 7. Frequency distribution histogram of t_{cr} based on results of modified EEAC.

the critical clearing time is also different. After 1000 runs, the CCTs are counted, and the frequency distribution diagram is shown in Fig. 7. Therefore, the stochastic disturbance has an impact on transient stability. The impact need to be quantified.

In order to verify the effectiveness of the method proposed in this paper, the numerical simulation based on Monte Carlo method is compared with modified EEAC. The results obtained by using the MC method are shown in Fig. 8. The results of the two methods are shown in Table 1. It can be seen that the mean value, standard deviation, maximum value and minimum value of the statistics are consistent within the error range, which verifies the correctness and effectiveness of the proposed method.

Two methods are conducted in .m file of Matlab R2012a, where the CPU is Intel Core i5, the memory is 8 G, and the operating system is Windows 10. The contrast of cputime shows the advantage of the proposed method in terms of algorithm efficiency. And this advantage will become more evident when the system continues to expand.

The stochastic disturbance intensity are set to different times of the initial value to study its impact on stability using modified EEAC. By 1000 runs, the maximum and minimum variation is shown in Fig. 9 under different stochastic disturbance intensity. It can be seen that, with the increase of the intensity, the uncertainty of CCTs is also increased. The statistics of CCTs are shown in Table 2. As shown in Fig. 10, the standard deviation becomes larger with the increase of the stochastic disturbance intensity. And according to the phenomenon of this approximate linear growth trend, the standard deviation CCTs could be estimated under a certain stochastic disturbance intensity.

According to the law of large number of Bernoulli, the probability density distribution can be approximated by the frequency distribution.

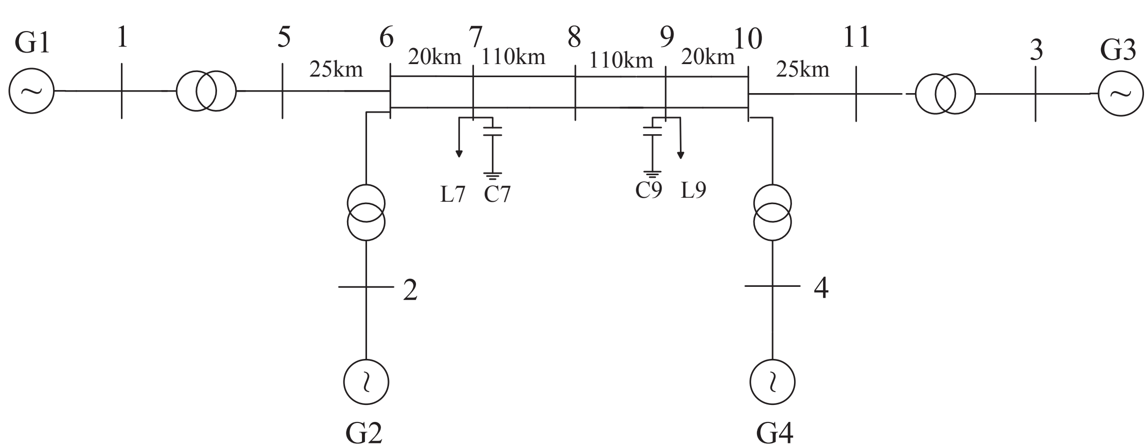

Fig. 5. The diagram of the case system.

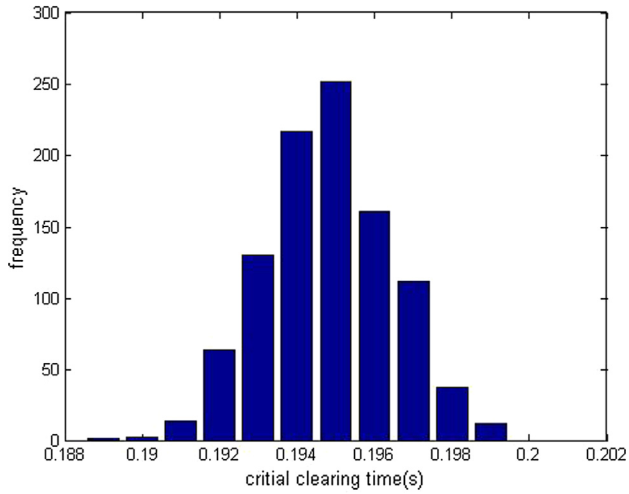

Fig. 8. Frequency distribution histogram of t_{cr} based on results of Monte Carlo method.

Table 1
The statistics comparison of t_{cr}.

Method	MC simulation	Modified EEAC	Error (%)
Mean value (s)	0.194	0.195	0.57
Standard deviation (ms)	1.711	1.649	3.06
Maximum (s)	0.199	0.199	/
Minimum (s)	0.188	0.188	/
Cputime (s)	616.7656	18.6125	/

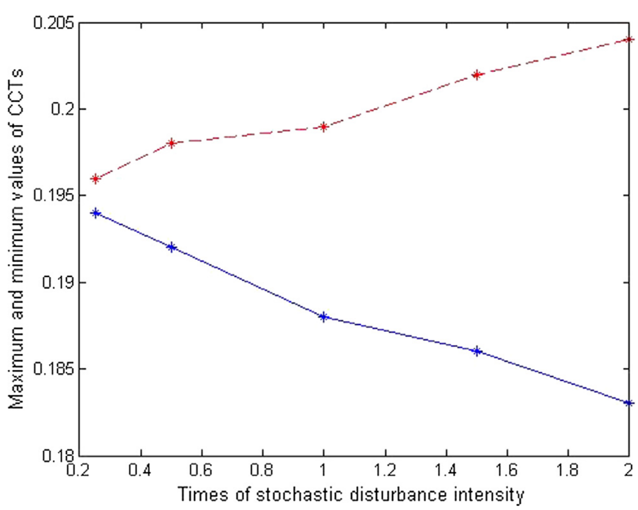

Fig. 9. Maximum and minimum variation of t_{cr} under different stochastic disturbance intensity.

Table 2
The statistics comparison of t_{cr} under different random excitation intensities.

Times of initial intensity	Mean value (s)	Standard deviation (ms)	maximum (s)	minimum (s)
0.25	0.195	0.480	0.196	0.194
0.5	0.195	0.876	0.198	0.192
1	0.195	1.649	0.199	0.188
1.5	0.195	2.513	0.202	0.186
2	0.194	3.436	0.204	0.183

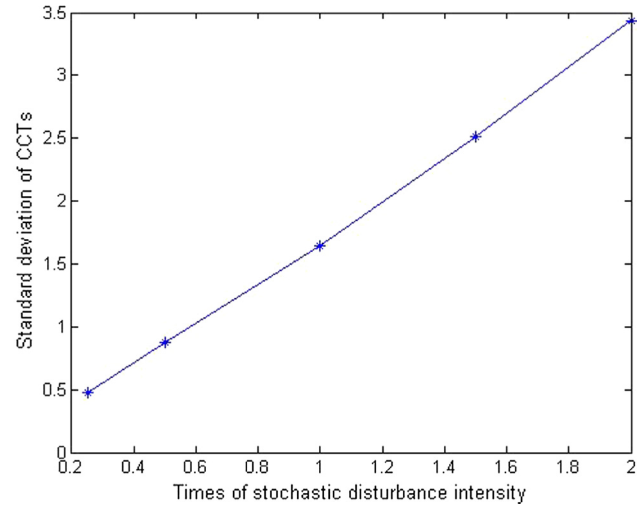

Fig. 10. Standard deviation change of t_{cr} under different stochastic disturbance intensities.

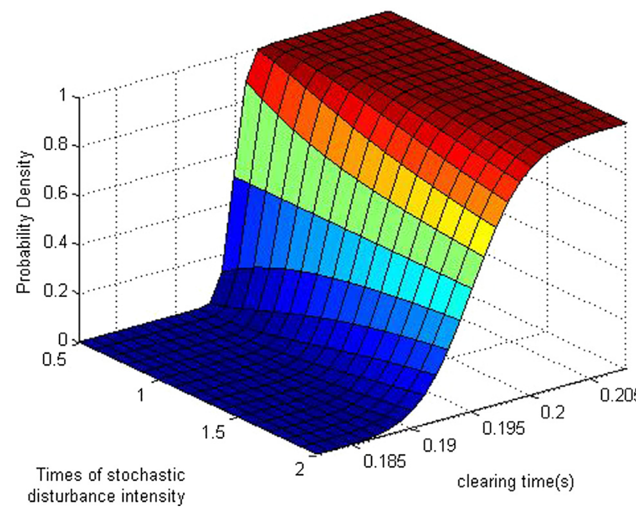

Fig. 11. Probability of instability under different t_{cr} and different stochastic disturbance intensities.

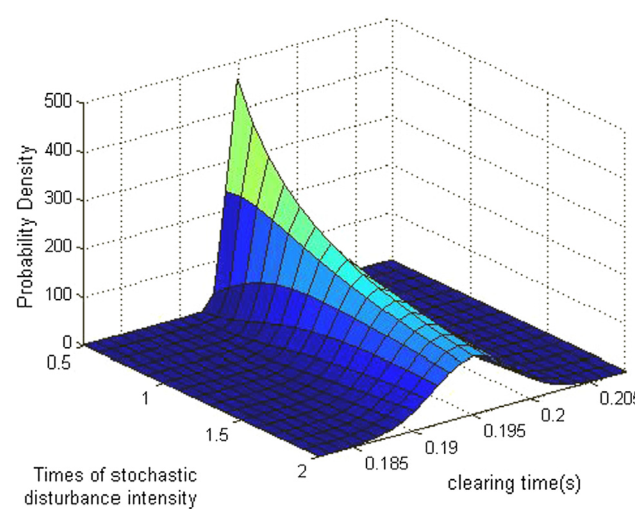

Fig. 12. Probability density under different t_{cr} and different stochastic disturbance intensities.

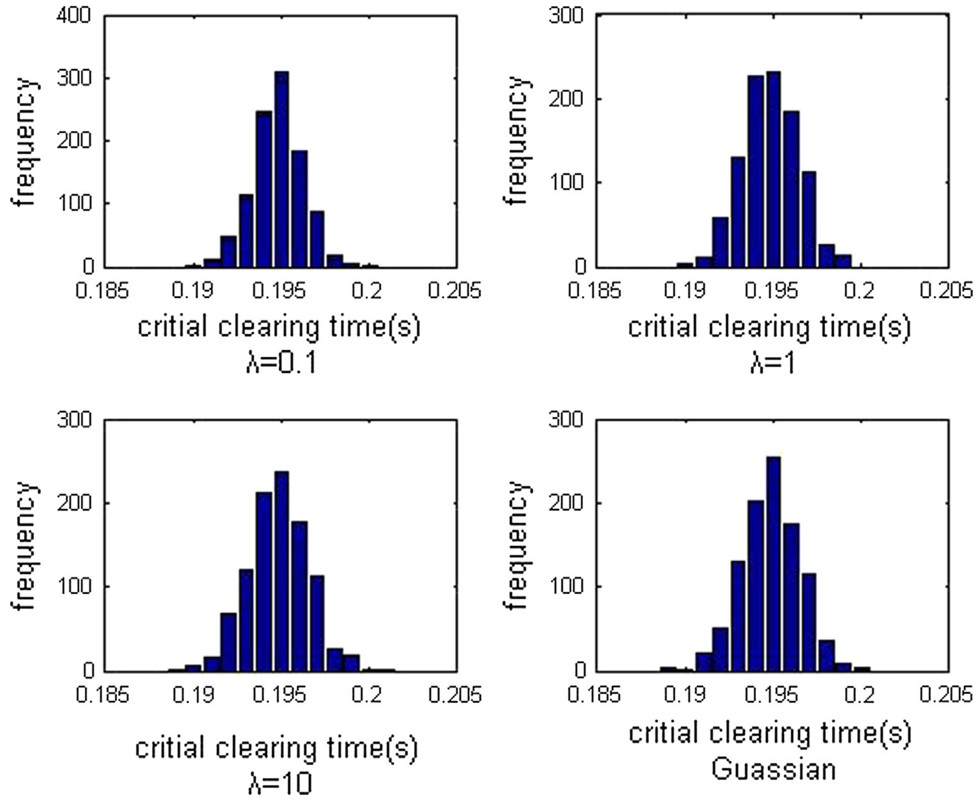

Fig. 13. Frequency distribution histogram of t_{cr} under different types of white noise.

Applied probability fitting, the relationship between different stochastic disturbance and stability probability is obtained. As shown in Figs. 11,12, the earlier the fault is removed, the greater the probability of stability. According to the results of the deterministic method, $t_{cr} = 0.195s$, and taking into account the error of action, generally set action time of circuit breakers to be 0.190 s in the project. Under the 0.5, 1 times of the initial intensity, the probability of transient instability is about 0%, but the probability is 1.23% under 1.5 times of the initial intensity, and the probability of instability is 4.97% under 2 times of the initial intensity. The greater the stochastic disturbance intensity, the larger the standard deviation. In order to meet the requirements of stability, the action time should be in advance.

Let $\lambda = 0.1,1,10$, Using the proposed method, CCTs of the system with white Poisson noise are calculated and compared with the white Guassian noise. As shown in Fig. 13 and Table 3, with the increase of Poisson counting strength λ, the result is closer to the intensity of white Guassian noise. This is consistent with the result that when the white Poisson noise counts tend to infinity, its limit is the result of white Guassian noise. With the emerging electric vehicle load, intermittent renewable energy and various power electronic devices connected to the power system, Stochasticity of power system is increasing, study the impact of stochastic disturbance on the transient stability of power system especially the relay protection action time setting is necessary.

Table 3
The statistics comparison of t_{cr} under different types of white noise.

Types	Mean value (s)	Standard deviation (ms)	maximum (s)	minimum (s)
$\lambda = 0.1$	0.195	1.405	0.200	0.190
$\lambda = 1$	0.195	1.611	0.198	0.190
$\lambda = 10$	0.195	1.725	0.199	0.189
Guassian	0.195	1.681	0.204	0.189

6. Conclusion

Stochasticity has become an important factor in the stability analysis of power system. In this paper, based on the EEAC theory and the stochastic differential equation model of the power system, a modified EEAC is established to study the transient stability of the stochastic power system. The modified EEAC was applied to calculate the CCTs of the 4 machine system. Compared with the numerical simulation based on Monte Carlo method, the validity of the method is verified. Finally, the impact of different stochastic disturbance intensity on stability is discussed. The proposed method is based on EEAC, stochastic differential equation theory and Monte Carlo method, which can simplifies the system model and reduce computation without losing accuracy. Although this method have advantages of rapidity and simplicity, if the rotors swing curves of the multi-machine system show a poor two-group characteristics, the accuracy of this method will deteriorate, which is same as traditional EEAC. The focus of further research is:

1) Establish the transient stability margin assessment system of the stochastic complex multi machine system, and to study and analyse the measures to improve the transient stability of stochastic power systems based on the stochastic control theory.
2) Stochastic disturbance intensity has a great influence on the accuracy of power system modelling, but there is few research published on how to measure the intensity accurately.
3) The proposed method is still a Monte Carlo based method. The analytical method of stochastic transient stability which is independent of Monte Carlo method remains to be studied.

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Appendix

EEAC part:

$$P_{ei}(t) = E_i(t)\sum_{k\in S,a} E_k(t)(G_{ik}\cos\delta_{ik}(t) + B_{ik}\sin\delta_{ik}(t))$$

The inner node potential of generator keeps unchanged during transient process, so:

$$E_i(t) = E_i E_k(t) = E_k$$

let

$$\xi_i = \delta_i - \delta_s \qquad \forall\, i \in S$$

$$\xi_j = \delta_j - \delta_a \qquad \forall\, j \in a$$

$$\xi_{ij} = \xi_i - \xi_j$$

We have

$$\begin{gathered} P_{ei}(t) = E_i(t)\textstyle\sum_{k\in S} E_k(t)(G_{ik}\cos\xi_{ik}(t) + B_{ik}\sin\xi_{ik}(t)) \\ + E_i(t)\textstyle\sum_{j\in a} E_j(t)((G_{ij}\cos\xi_{ij}(t) + B_{ij}\sin\xi_{ij}(t))\cos\delta(t) + (B_{ij}\cos\xi_{ij}(t) - G_{ij}\sin\xi_{ij}(t))\sin\delta(t)) \\ \forall\, i\in S \end{gathered}$$

$$\begin{gathered} P_{ej}(t) = E_j(t)\textstyle\sum_{l\in a} E_l(t)(G_{jl}\cos\xi_{jl}(t) + B_{jl}\sin\xi_{jl}(t)) \\ + E_j(t)\textstyle\sum_{i\in S} E_i(t)((G_{ji}\cos\xi_{ji}(t) + B_{ji}\sin\xi_{ji}(t))\cos\delta(t) - (B_{ji}\cos\xi_{ji}(t) - G_{ji}\sin\xi_{ji}(t))\sin\delta(t)) \\ \forall\, j\in a \end{gathered}$$

Due to the homology of each generator in the group, the power angle difference of each generator in each group is kept constant during the whole dynamic process, so the offset angle of the part of the inertial center power angle of the disturbed trajectory is constant. The upper form can also be written as:

$$\begin{aligned} P_{ei}(t) &= H_i + C_i\cos\delta(t) + D_i\sin\delta(t) \\ H_i &= E_i(t)\textstyle\sum_{k\in S} E_k(t)(G_{ik}\cos\xi_{ik}(t) + B_{ik}\sin\xi_{ik}(t)) \\ C_i &= E_i(t)\textstyle\sum_{j\in a} E_j(t)(G_{ij}\cos\xi_{ij}(t) + B_{ij}\sin\xi_{ij}(t)) \\ D_i &= E_i(t)\textstyle\sum_{j\in a} E_j(t)(B_{ij}\cos\xi_{ij}(t) - G_{ij}\sin\xi_{ij}(t)) \\ &\forall\, i\in S \end{aligned}$$

$$\begin{aligned} P_{ej}(t) &= H_j + C_j\cos\delta(t) + D_j\sin\delta(t) \\ H_j &= E_j(t)\textstyle\sum_{l\in a} E_l(t)(G_{jl}\cos\xi_{jl}(t) + B_{jl}\sin\xi_{jl}(t)) \\ C_j &= E_j(t)\textstyle\sum_{i\in S} E_i(t)(G_{ji}\cos\xi_{ji}(t) + B_{ji}\sin\xi_{ji}(t)) \\ D_j &= E_j(t)\textstyle\sum_{i\in S} E_i(t)(G_{ji}\sin\xi_{ji}(t) - B_{ji}\cos\xi_{ji}(t)) \\ &\forall\, j\in a \end{aligned}$$

The electromagnetic power of the two machine system:

$$P_{e,S} = \sum_{i\in S} P_{ei} = H_S + C_S\cos\delta + D_S\sin\delta$$

$$P_{e,a} = \sum_{j\in a} P_{ej} = H_a + C_a\cos\delta + D_a\sin\delta$$

where

$$H_S = \sum_{i\in S} H_i$$

$$C_S = \sum_{i\in S} C_i$$

$$D_S = \sum_{i\in S} D_i$$

$$H_a = \sum_{j\in a} H_j$$

$$C_a = \sum_{j\in a} C_j$$

$$D_a = \sum_{j \in a} D_j$$

Then the two machine system is transformed into SMIB system by using the formula. The electromagnetic power of the equivalent SMIB system can be expressed as

$$P'_e = P_c + P_{\max} \sin(\delta - \upsilon)$$

where

$$P_c = \frac{M_a H_S - M_S H_a}{M_S + M_a}$$

$$P_{\max} = \sqrt{C^2 + D^2}$$

$$\upsilon = \tan^{-1}(-\frac{C}{D})$$

$$C = \frac{M_a C_S - M_S C_a}{M_S + M_a}$$

$$D = \frac{M_a D_S - M_S D_a}{M_S + M_a}$$

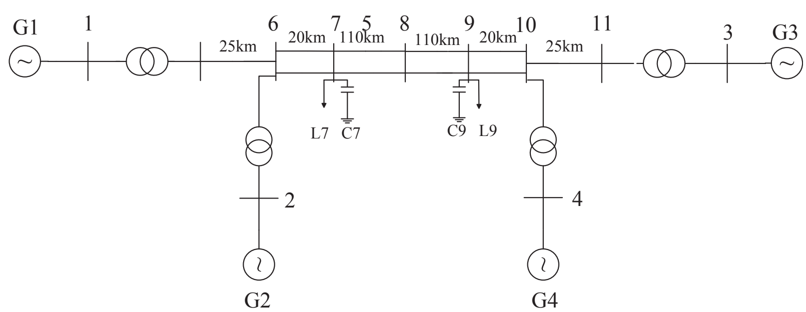
The mechanical power of the equivalent SMIB system can be expressed as

$$P_M = \frac{M_a P_{M,S} - M_S P_{M,a}}{M_S + M_a}$$

Under the assumption of classical model and two-group model, P_M, P_c, $P_{\max}$, υ is constant, so we have

$$M\frac{d^2\delta}{dt^2} = P_M - P_c - P_{\max}\sin(\delta - \upsilon) + \zeta W(t)$$

Data part:



For the 4 machine system, the rating capacity of each generator is 900MVA, and the rated voltage is 20 kV. The generator parameters under rating capacity (MVA) and rating voltage (kV) are:

$X_d = 1.8$

$X_q = 1.7$

$X_l = 0.2$

$X'_d = 0.3$

$X'_q = 0.55$

$X''_d = 0.25$

$X''_q = 0.\ 25$

$R_a = 0.0025$

$H = 6.5$ (G1 and G2)
$H = 6.175$ (G3 and G4)

$K_D = 0$

Stochastic disturbance intensity $\zeta = 0.001$
Impedance of each step-up transformer under the reference of 900MVA and 20/230 kV:

$Z_T = 0 + j0.15$;

And the ratio is 1.0;

The length of the line has been marked, and the p.u. value of the line parameters under the reference of 900MVA and 230 kV is:

$r = 0.0001 \quad \text{pu/km}$

$x_L = 0.001 \quad \text{pu/km}$

$b_c/2 = 0.00175\ \text{pu/km}$

Electric potential of the inner node of a generator and active power under rating capacity (MVA) and rating voltage (kV):

G1: $E'_q = 0.9466 + j0.5891 P = 0.7873$

G2: $E'_q = 1.0293 + j0.4251 P = 0.7847$

G3: $E'_q = 1.1087 + j0.1016\ P = 0.8034$

G4: $E'_q = 1.0988 - j0.0950\ P = 0.7718$

Parameter of constant impedance model equalled to the load and reactive power of shunt capacitors of node 7 and 9:

$y_7 = 1.0744 + j0.1111$

$y_9 = 1.9633 + j0.2778$

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