



Dynamic output feedback control for a class of switched delay systems under asynchronous switching

Rui Wang^{a,*}, Zhi-Gang Wu^a, Peng Shi^{b,c}

^a School of Aeronautics and Astronautics, State Key Laboratory of Structural Analysis of Industrial Equipment, Dalian University of Technology, Dalian 116024, China

^b School of Electrical and Electronic Engineering, The University of Adelaide, Adelaide, SA 5005, Australia

^c School of Engineering and Science, Victoria University, Melbourne, 8001 Vic, Australia

ARTICLE INFO

Article history:

Received 24 May 2012

Received in revised form 20 September 2012

Accepted 17 October 2012

Available online 10 November 2012

Keywords:

Switched delay systems

Dynamic output feedback control

Average dwell time

Asynchronous switching

ABSTRACT

In this paper, the problem of output feedback stabilization is considered for a class of switched delay systems under asynchronous switching. When the switching signal of the switched controller involves delay, by constructing a novel Lyapunov functional which is allowed to increase during the running time of active subsystems with the mismatched controller, sufficient conditions for exponential stability are developed for a class of switching signals based on the average dwell time method. Moreover the stabilizing output feedback controllers are designed. Finally, an example is given to demonstrate the feasibility and effectiveness of the proposed design techniques.

© 2012 Elsevier Inc. All rights reserved.

1. Introduction

Switching systems, as a class of hybrid dynamical systems, consist of a set of time-varying subsystems and a switching signal that orchestrates the switching between them. Such control systems appear in many applications, such as communication networks, flight and air traffic control and robot manipulators [6,15,3]. Therefore switched systems have drawn considerable attention in recent years [1,7,8,19,21,27,31,32]. As is well known, time-delay phenomenon is very common in practical engineering control and is frequently a source of instability and performance deterioration [4,5,16]. At present, there has been increasing interest in switched delay systems [2,12,13,18,20].

On the other hand, in the ideal case, the switching of the controllers coincides exactly with that of corresponding subsystems, that is to say, the controllers are switched synchronously with the subsystems. In actual operation, however, since it takes time to identify the active subsystem and apply the matched controller, the switching time of controllers may lag behind that of practical subsystems, which results in asynchronous switching between the controllers and system modes. Therefore, it is significant to study the problem of asynchronous switching and some valuable results have been obtained [17,23,24,26]. In [28], the asynchronously switched control problem for a class of switched linear systems with average dwell time was investigated. [30] studied the problems of stability, L_2 -gain and asynchronous H_∞ control for a class of discrete-time switched systems. The robust control problem for uncertain switched delay systems under asynchronous

* Corresponding author.

E-mail address: ruiwang@dlut.edu.cn (R. Wang).

switching was considered in [24]. However, the references mentioned above did not consider the dynamic output feedback control. In practical applications, the system states may be not measured due to some reasons, therefore they cannot be used for feedback control. Therefore it is very significant to design the dynamic output feedback control for this case. To the best of our knowledge, no attention has been paid to the asynchronously switched control problem of switched delay systems via dynamic output feedback controllers.

In this paper, we study the dynamic output feedback stabilization problem for a class of switched delay systems under asynchronous switching. Through constructing a piecewise Lyapunov functional which can be allowed to increase during the running time of the active subsystem with the mismatched controller, based on the average dwell time method, a solution for dynamic output feedback controllers are derived in terms of LMIs such that the resulting closed-loop system is exponentially stable. The main contributions of the paper are as follows: First, the dynamic output feedback controllers are designed while on existing work, the state feedback controller design problem was considered. Second, both the delayed state and the delayed switching signal are considered. Since this two kind of delays lie in two different types of sets, how to deal with the case, where the state delays and switching delays coexist is a challenging issue.

The paper is organized as follows. In section 2, preliminaries and problem formulation are introduced. Section 3 gives the sufficient conditions of exponential stability and the controller design algorithm of the system. It is the main result of this paper. In Section 4, an example is given to illustrate the effectiveness of the proposed approach. The conclusions are summarized in Section 5.

Notations: Throughout this paper, \mathbb{R}^n denotes the n -dimensional Euclidean space, $P > 0$ means that P is a positive definite, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of P , I is the identity matrix with appropriate dimensions, $\|\cdot\|$ denotes Euclidean vector norm, $*$ denotes the symmetric block in one symmetric matrix, $diag\{\dots\}$ stands for a block-diagonal matrix.

2. Preliminaries and problem formulation

Consider a class of switched delay systems of the form

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + E_{\sigma(t)}x(t-h), \\ x(\theta) &= \psi(\theta), \theta \in [-h, 0], \\ y(t) &= C_{\sigma(t)}x(t), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^q$ is the control input, $y \in \mathbb{R}^p$ is the measurement output, $\sigma(t): [0, +\infty) \rightarrow M = \{1, 2, \dots, m\}$ is the switching signal. Specifically, denote $\sigma(t): \{(t_0, \sigma(t_0)), \dots, (t_k, \sigma(t_k)), \dots, |k = 0, 1, 2, \dots\}$, where t_0 is the initial switching instant, and t_k is the k th switching instant. A_i, B_i, C_i, E_i are constant matrices with appropriate dimensions, $\psi(\theta)$ is a differentiable vector-valued initial function on $[-h, 0]$, $h > 0$ denotes the state delay. If delay h is neglected, system (1) will reduce the model presented in [10].

When the controllers are switched synchronously with the subsystems, the dynamic output feedback controllers are formed as

$$\begin{aligned} \dot{\xi}(t) &= G_{\sigma(t)}\xi(t) + L_{\sigma(t)}y(t), \\ u(t) &= K_{\sigma(t)}\xi(t). \end{aligned} \tag{2}$$

where ξ is the state of the controllers, G_i, L_i, K_i are constant matrices.

However, in practical engineering, since it takes time to identify the active subsystem and apply the matched controller, the switching time of controllers may lag behind that of practical subsystems, which results in asynchronous switching between the controllers and system modes. Thus, we need to take the switching delay into account.

Remark 1. Because we may not know the initial mode and the subsequent modes of the system in advance, the switchings of the controllers may not coincide exactly with those of system modes. If a wrong controller is used over a specified amount of time, the solution to the system might escape to infinity before a correct controller is switched into action [25].

We now consider the dynamic output feedback controllers of the following form:

$$\begin{aligned} \dot{\xi}(t) &= G_{\sigma(t)}\xi(t) + L_{\sigma(t)}y(t), \\ u(t) &= K_{\sigma(t-\tau_d)}\xi(t), \end{aligned} \tag{3}$$

where τ_d is the delay of switched controllers to system modes.

The following definitions will be used in the sequel.

Definition 1 [13]. The equilibrium $x^* = 0$ of system (1) is said to be exponentially stable under $\sigma(t)$ if the solution $x(t)$ of system (1) satisfies

$$\|x(t)\| \leq k\|x(t_0)\|e^{-\lambda(t-t_0)}, \quad \forall t \geq t_0,$$

for constants $k \geq 1$ and $\lambda > 0$.

Definition 2 ([1,7]). For switching signal σ and any $t \geq t_0 \geq 0$, let $N_\sigma(t_0, t)$ denote the number of switching of σ over the time interval (t_0, t) . If

$$N_\sigma(t_0, t) \leq N_0 + \frac{t - t_0}{\tau_a}. \quad (4)$$

holds for $N_0 \geq 0$, $\tau_a > 0$, then τ_a is called the ‘‘average dwell-time’’ and N_0 is the chatter bound. As commonly used in the literature, for convenience, we choose $N_0 = 0$ in this paper.

3. Main results

In this section, we will give stability analysis, synthesis conditions and a design algorithm.

3.1. Stability analysis

Applying the dynamic output feedback controllers (3) to system (1), we have the closed-loop system

$$\dot{\bar{x}}(t) = \bar{A}_{\sigma(t)}\bar{x}(t) + \bar{E}_{\sigma(t)}\bar{x}(t - h) \quad (5)$$

where

$$\bar{x} = \begin{bmatrix} x \\ \zeta \end{bmatrix}, \quad \bar{A}_{\sigma(t)} = \begin{bmatrix} A_\sigma & B_\sigma K_{\sigma(t-\tau_d)} \\ L_\sigma C_\sigma & G_\sigma \end{bmatrix}, \quad \bar{E}_{\sigma(t)} = \begin{bmatrix} E_{\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix},$$

The following result presents a sufficient condition of exponentially stability for system (5).

Theorem 1. For given positive constants α and β , if there exist matrices $P_i > 0$, $Q_i > 0$, $\forall i \in M$ such that

$$\bar{\Sigma}_i = \begin{bmatrix} \bar{A}_i^T P_i + P_i \bar{A}_i + Q_i + \alpha P_i & P_i \bar{E}_i \\ * & -e^{-\alpha h} Q_i \end{bmatrix} < 0, \quad (6)$$

$$\bar{\Pi}_i = \begin{bmatrix} \bar{A}_{ij}^T P_i + P_i \bar{A}_{ij} + Q_i - \beta P_i & P_i \bar{E}_i \\ * & -e^{\beta h} Q_i \end{bmatrix} < 0, \quad (7)$$

then dynamic output feedback controllers (3) make system (5) exponentially stable under asynchronous switching for any switching signal satisfying average dwell time

$$\tau_a \geq \tau_a^* = \frac{\ln \mu + (\alpha + \beta)\tau_d}{\alpha}, \quad (8)$$

where $\mu \geq 1$ satisfies

$$P_i \leq \mu P_j, \quad Q_i \leq \mu Q_j, \quad \forall i, j \in M. \quad (9)$$

Proof. Due to the switching delay, the j th subsystem has been switched to the i th subsystem, and the controller K_j is still active for τ_d . Thus, we have

$$\dot{\bar{x}}(t) = \begin{cases} \bar{A}_{ij}\bar{x}(t) + \bar{E}_i\bar{x}(t - h), & \forall t \in [t_i, t_i + \tau_d); \\ \bar{A}_i\bar{x}(t) + \bar{E}_i\bar{x}(t - h), & \forall t \in [t_i + \tau_d, t_{i+1}). \end{cases} \quad (10)$$

where

$$\bar{A}_{ij} = \begin{bmatrix} A_i & B_i K_j \\ L_i C_i & G_i \end{bmatrix}, \quad \bar{A}_i = \begin{bmatrix} A_i & B_i K_i \\ L_i C_i & G_i \end{bmatrix}.$$

When $\forall t \in [t_k + \tau_d, t_{k+1})$, the Lyapunov functional candidate

$$V_{1\sigma}(t) = \bar{x}^T(t) P_\sigma \bar{x}(t) + \int_{t-h}^t \bar{x}^T(s) e^{\alpha(s-t)} Q_\sigma \bar{x}(s) ds, \quad (11)$$

where P_i, Q_i are positive definite matrices satisfying (6), (7) and (9).

Along the trajectory of (10) we have

$$\dot{V}_{1i} + \alpha V_{1i} = \bar{x}^T(t) [P_i \bar{A}_i + \bar{A}_i^T P_i + Q_i + \alpha P_i] \bar{x}(t) + 2\bar{x}^T(t) P_i \bar{E}_i \bar{x}(t - h) - \bar{x}^T(t - h) e^{-\alpha h} Q_i \bar{x}(t - h) = \zeta^T(t) \bar{\Sigma}_i \zeta(t),$$

where $\zeta(t) = [\bar{x}^T(t) \quad \bar{x}^T(t - h)]^T$.

From (6), we can get

$$\dot{V}_{1i} + \alpha V_{1i} \leq 0. \tag{12}$$

When $\forall t \in [t_k, t_k + \tau_d)$, the Lyapunov functional candidate

$$V_{2\sigma}(t) = \bar{x}^T(t)P_\sigma\bar{x}(t) + \int_{t-h}^t \bar{x}^T(s)e^{\beta(t-s)}Q_\sigma\bar{x}(s)ds, \tag{13}$$

where P_i, Q_i are positive definite matrices satisfying (6), (7) and (9).

Along the trajectory of (10), we have

$$\dot{V}_{2i} - \beta V_{2i} = \bar{x}^T(t)[P_i\bar{A}_{ij} + \bar{A}_{ij}^T P_i + Q_i - \beta P_i]\bar{x}(t) + 2\bar{x}^T(t)P_i\bar{E}_i\bar{x}(t-h) - \bar{x}^T(t-h)e^{\beta h}Q_i\bar{x}(t-h) \leq \zeta^T(t)\bar{\Pi}_i\zeta(t).$$

From (7), we can get

$$\dot{V}_{2i} - \beta V_{2i} \leq 0. \tag{14}$$

Obviously

$$\int_{t-h}^t \bar{x}^T(s)e^{\alpha(s-t)}Q_i\bar{x}(s)ds \leq \int_{t-h}^t \bar{x}^T(s)Q_i\bar{x}(s)ds \leq \int_{t-h}^t \bar{x}^T(s)e^{\beta(t-s)}Q_i\bar{x}(s)ds, \tag{15}$$

Thus, combining (11), (13) and (15), it holds that

$$V_{1i}(t) \leq V_{2i}(t). \tag{16}$$

Considering the whole interval $[t_0, t)$, the Lyapunov functional candidate is the combination of (11) and (13)

$$V(t) = \begin{cases} V_{1\sigma}(t), & t \in [t_k + \tau_d, t_{k+1}), \quad k = 0, 1, 2, \dots; \\ V_{2\sigma}(t), & t \in [t_k, t_k + \tau_d), \quad k = 0, 1, 2, \dots. \end{cases} \tag{17}$$

For $t \in [t_k + \tau_d, t_{k+1})$, integrating both sides of (12) from $t_k + \tau_d$ to t , and combining (4), (9) and (16), we have

$$\begin{aligned} V(t) &\leq e^{-\alpha(t-(t_k+\tau_d))}V_{1i}((t_k + \tau_d)^+) \\ &\leq e^{-\alpha(t-(t_k+\tau_d))}V_{2i}((t_k + \tau_d)^-) \\ &\leq e^{-\alpha(t-(t_k+\tau_d))}e^{\beta\tau_d}V_{2i}(t_k^+) \\ &\leq \mu e^{-\alpha(t-(t_k+\tau_d))}e^{\beta\tau_d}V_{2i}(t_k^+) \\ &\leq \dots \\ &\leq \mu^k e^{(k+1)\beta\tau_d} e^{-\alpha[t-t_0-(k+1)\tau_d]} V(t_0) \\ &\leq e^{(\alpha+\beta)\tau_d} e^{\lceil \frac{\ln \mu + (\alpha+\beta)\tau_d}{\tau_d} - \alpha \rceil (t-t_0)} V(t_0). \end{aligned} \tag{18}$$

Similarly, for $t \in [t_k, t_k + \tau_d)$, we obtain

$$\begin{aligned} V(t) &\leq e^{\beta(t-t_k)}V_{2i}(t_k^+) \\ &\leq \mu e^{\beta(t-t_k)}V_{1i}(t_k^-) \\ &\leq \mu e^{\beta\tau_d} e^{-\alpha(t_k-t_{k-1}-\tau_d)}V_{1i}((t_{k-1} + \tau_d)^-) \\ &\leq \dots \\ &\leq \mu^k e^{(k+1)\beta\tau_d} e^{-\alpha[t-t_0-(k+1)\tau_d]} V(t_0) \\ &\leq e^{(\alpha+\beta)\tau_d} e^{\lceil \frac{\ln \mu + (\alpha+\beta)\tau_d}{\tau_d} - \alpha \rceil (t-t_0)} V(t_0). \end{aligned} \tag{19}$$

Notice (11) and (13), it obviously holds that

$$\begin{aligned} a\|\bar{x}(t)\|^2 &\leq V_{1i}(t) \leq b\|\bar{x}(t)\|^2, \quad t \in [t_k + \tau_d, t_{k+1}); \\ a\|\bar{x}(t)\|^2 &\leq V_{2i}(t) \leq b\|\bar{x}(t)\|^2, \quad t \in [t_k, t_k + \tau_d), \end{aligned} \tag{20}$$

where

$$a = \min_{i \in M} \{\lambda_{\min}(P_i)\}, \quad b = \max\{b_1, b_2\},$$

where

$$\begin{aligned} b_1 &= \max_{i \in M} \{\lambda_{\max}(P_i)\} + h \max_{i \in M} \{\lambda_{\max}(Q_i)\}, \\ b_2 &= \max_{i \in M} \{\lambda_{\max}(P_i)\} + h e^{\beta h} \max_{i \in M} \{\lambda_{\max}(Q_i)\}. \end{aligned}$$

Then, applying (18)–(20) yields

$$\|x(t)\| \leq \sqrt{\frac{b}{a}} e^{\frac{(\alpha+\beta)\tau_d}{2}} e^{\frac{\ln \mu + (\alpha+\beta)\tau_d}{2\tau_a} - \frac{\gamma}{2}(t-t_0)} \|x(t_0)\|, \quad \forall t \in [t_k, t_{k+1}). \quad (21)$$

From (8), system (5) is exponentially stable. \square

Remark 2. Although the Lyapunov functional constructed in Theorem 1 is allowed to increase both at the switching instants t_k and during the running time of active subsystems with the mismatched controllers $[t_k, t_k + \tau_d)$, by restricting the lower bound of the average dwell time, the Lyapunov functional is decreasing as a whole and hereby the system stability is guaranteed.

Remark 3. Theorem 1 provides a sufficient condition for exponential stability of system (1) (or for system (5) under control law (3)). However, inequalities (6) and (7) are not in the form of LMIs if the controller gains are to be determined. We will give LMIs conditions for determining the controller gains in the next subsection.

3.2. Synthesis conditions

This section will give some LMIs conditions for the controller design.

Theorem 2. Given positive numbers α , β and γ , if there exist symmetric matrices X_i , Y_i , T_i , Z_i and matrices \widehat{A}_i , \widehat{B}_i , \widehat{C}_i , \widehat{A}_{ij} ($\forall i, j \in M$) such that the following matrix inequalities

$$\begin{bmatrix} X_i & I \\ * & Y_i \end{bmatrix} > 0, \quad (22)$$

$$\begin{bmatrix} \Xi_i & A_i + \widehat{A}_i^T + \gamma X_i + \alpha I & E_i & 0 & X_i \\ * & \Omega_i + \alpha Y_i & Y_i E_i & 0 & 0 \\ * & * & -e^{-\alpha h} \gamma I & 0 & 0 \\ * & * & * & -e^{-\alpha h} T_i & 0 \\ * & * & * & * & -\gamma^{-1} I \end{bmatrix} < 0, \quad (23)$$

$$\begin{bmatrix} \Xi_{ij} & A_i + \widehat{A}_{ij}^T + \gamma X_i - \beta I & E_i & 0 & X_i \\ * & \Omega_i - \beta Y_i & Y_i E_i & 0 & 0 \\ * & * & -e^{\beta h} \gamma I & 0 & 0 \\ * & * & * & -e^{\beta h} T_j & 0 \\ * & * & * & * & -\gamma^{-1} I \end{bmatrix} < 0, \quad (24)$$

hold, where

$$\Xi_i = A_i X_i + X_i A_i^T + B_i \widehat{C}_i + \widehat{C}_i^T B_i^T + \alpha X_i + Z_i,$$

$$\Omega_i = Y_i A_i + \widehat{B}_i C_i + A_i^T Y_i + C_i^T \widehat{B}_i^T + \gamma I,$$

$$\Xi_{ij} = A_i X_i + X_i A_i^T + B_i \widehat{C}_j + \widehat{C}_j^T B_i^T - \beta X_i + Z_j,$$

then dynamic output feedback controllers (3) make the resulting switched system exponentially stable under asynchronous switching corresponding to any switching signal with average dwell time τ_a satisfying (8) and the controller parameters are given by

$$\begin{aligned} K_i &= \widehat{C}_i (M_i^T)^{-1}, \\ L_i &= N_i^{-1} \widehat{B}_i, \\ G_i &= N_i^{-1} (\widehat{A}_i - Y_i A_i X_i - N_i L_i C_i X_i - Y_i B_i K_i M_i^T) (M_i^T)^{-1}, \end{aligned} \quad (25)$$

where M_i and N_i satisfy the constraint

$$M_i N_i^T = I - X_i Y_i, \quad (26)$$

the constant $\mu \geq 1$ satisfies

$$R_i^{-1} S_i \leq \mu R_j^{-1} S_j, \quad T_i \leq \mu T_j \quad (27)$$

with

$$R_i = \begin{bmatrix} X_i & I \\ M_i^T & 0 \end{bmatrix}, \quad S_i = \begin{bmatrix} I & Y_i \\ 0 & N_i^T \end{bmatrix}, \quad \forall i, j \in M.$$

Proof. Motivated by the method in [22,9,11], we define matrices

$$P_i = \begin{bmatrix} Y_i & N_i \\ N_i^T & W_i \end{bmatrix}, \quad (i = 1, 2),$$

where $W_i > 0$. Then, $P_i^{-1} = \begin{bmatrix} X_i & M_i \\ M_i^T & Z_i \end{bmatrix}$ with $Z_i > 0$. We can easily obtain $P_i \begin{bmatrix} X_i & I \\ M_i^T & 0 \end{bmatrix} = \begin{bmatrix} I & Y_i \\ 0 & N_i^T \end{bmatrix}$, that is $P_i R_i = S_i$ and thus $P_i = R_i^{-1} S_i$. Here define $Q_i = \text{diag}\{\gamma I, T_i\}$, where $T_i > 0$ and γ is positive scalar to be chosen.

We first prove that matrix inequality (6) is equivalent to LMI (23).

Pre- and post-multiplying both sides of inequality (6) by $\text{diag}\{R_i^T, I\}$ and $\text{diag}\{R_i, I\}$ yield the following matrix inequality

$$\begin{bmatrix} R_i^T \bar{A}_i^T P_i R_i + R_i^T P_i \bar{A}_i R_i + R_i^T Q_i R_i + \alpha R_i^T P_i R_i & R_i^T P_i \bar{E}_i \\ * & -e^{-\alpha h} Q_i \end{bmatrix} < 0. \tag{28}$$

A straight forward computation gives the following equalities.

$$R_i^T P_i \bar{A}_i R_i = \begin{bmatrix} A_i X_i + B_i K_i M_i^T & A_i \\ Y_i A_i X_i + N_i L_i C_i X_i + Y_i B_i K_i M_i^T + N_i G_i M_i^T & Y_i A + N_i L_i C_i \end{bmatrix},$$

$$R_i^T P_i \bar{E}_i = \begin{bmatrix} E_i & 0 \\ Y_i E_i & 0 \end{bmatrix}, \quad R_i^T P_i R_i = S_i^T R_i = \begin{bmatrix} X_i & I \\ Y_i X_i + N_i M_i^T & Y_i \end{bmatrix} = \begin{bmatrix} X_i & I \\ I & Y_i \end{bmatrix},$$

$$R_i^T Q_i R_i = \begin{bmatrix} \gamma X_i X_i + M_i T_i M_i^T & \gamma X_i \\ \gamma X_i & \gamma I \end{bmatrix}.$$

Define the following transformation of variables:

$$\begin{aligned} \hat{A}_i &= Y_i A_i X_i + N_i L_i C_i X_i + Y_i B_i K_i M_i^T + N_i G_i M_i^T, \\ \hat{B}_i &= N_i L_i, \quad \hat{C}_i = K_i M_i^T, \quad Z_i = M_i T_i M_i^T. \end{aligned} \tag{29}$$

So, from (28) and (29), we have

$$\begin{bmatrix} \Xi_i + \gamma X_i X_i & \hat{A}_i^T + A_i + \alpha I + \gamma X_i & E_i & 0 \\ * & \Omega_i & Y_i E_i & 0 \\ * & * & -e^{-\alpha h} \gamma I & 0 \\ * & * & * & -e^{-\alpha h} T_i \end{bmatrix} < 0. \tag{30}$$

According to Schur complement Lemma, matrix inequality (30) is equivalent to LMI (23). Therefore, (6) is equivalent to (23).

In the following, we will deduce (7) from matrix inequalities (24). Pre- and post-multiplying both sides of inequality (7) by $\text{diag}\{R_i^T, I\}$ and $\text{diag}\{R_i, I\}$ yield the following matrix inequality

$$\begin{bmatrix} R_i^T \bar{A}_{ij}^T P_i R_i + R_i^T P_i \bar{A}_{ij} R_i + R_i^T Q_i R_i - \beta R_i^T P_i R_i & R_i^T P_i \bar{E}_i \\ * & -e^{\beta h} Q_i \end{bmatrix} < 0. \tag{31}$$

From (31), we have

$$\begin{bmatrix} \Xi_{ij} + \gamma X_i X_i & \hat{A}_{ij}^T + A_i - \beta I + \gamma X_i & E_i & 0 \\ * & \Omega_i - \beta Y_i & Y_i E_i & 0 \\ * & * & -e^{\beta h} \gamma I & 0 \\ * & * & * & -e^{\beta h} T_j \end{bmatrix} < 0, \tag{32}$$

where

$$\begin{aligned} \hat{A}_{ij} &= Y_i A_i X_i + N_i L_i C_i X_i + Y_i B_i K_j M_j^T + N_i G_i M_j^T, \\ \hat{C}_j &= K_j M_j^T, \quad Z_j = M_j T_j M_j^T. \end{aligned} \tag{33}$$

According to Schur complement Lemma, matrix inequality (32) is equivalent to LMI (24). Therefore, (7) is equivalent to (24).

If LMIs (22)–(24) have feasible solutions $\widehat{A}_i, \widehat{B}_i, \widehat{C}_i, X_i, Y_i, Z_i$, then we can get matrices M_i, N_i from (26) and (29). Therefore, controller matrices (25) can be obtained.

From LMIs (22)–(24) and Theorem 1, we know that system (1) with dynamic output feedback controllers (3) is exponentially stable under asynchronous switching for any switching signal satisfying (8) and (9). This completes the proof.

If switching delay $\tau_d = 0$, that is to say, the controllers are switched synchronously with the subsystems, we can derive the following result. \square .

Corollary 1. Consider the switched delay system (1). Given positive numbers α and γ , if there exist symmetric matrices X_i, Y_i, T_i, Z_i and matrices $\widehat{A}_i, \widehat{B}_i, \widehat{C}_i$ ($\forall i \in M$) such that (22), (23) hold, then dynamic output feedback controllers (2) make the resulting switched system exponentially stable corresponding to any switching signal with average dwell time τ_a satisfying $\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha}$, where the controller parameters are given by (25) and constant $\mu \geq 1$ satisfies (27).

If $h = 0$, switched delay system (1) degenerates into non-delay switched system, we have the following corollary.

Corollary 2. Consider the switched system (1) with $h = 0$. Given positive numbers α, β , if there exist symmetric matrices X_i, Y_i and matrices $\widehat{A}_i, \widehat{B}_i, \widehat{C}_i, \widehat{A}_{ij}$ ($\forall i, j \in M$) such that LMIs (22),

$$\begin{bmatrix} \Xi_i & A_i + \widehat{A}_i^T + \alpha I \\ * & \Omega_i + \alpha Y_i \end{bmatrix} < 0,$$

$$\begin{bmatrix} \Xi_{ij} & A_i + \widehat{A}_{ij}^T - \beta I \\ * & \Omega_i - \beta Y_i \end{bmatrix} < 0,$$

hold, where

$$\Xi_i = A_i X_i + X_i A_i^T + B_i \widehat{C}_i + \widehat{C}_i^T B_i^T + \alpha X_i,$$

$$\Omega_i = Y_i A_i + \widehat{B}_i C_i + A_i^T Y_i + C_i^T \widehat{B}_i^T,$$

$$\Xi_{ij} = A_i X_i + X_i A_i^T + B_i \widehat{C}_j + \widehat{C}_j^T B_i^T - \beta X_i,$$

then dynamic output feedback controllers (2) make the resulting switched system exponentially stable under asynchronous switching corresponding to any switching signal with average dwell time τ_a satisfying (8), where the controller parameters are given by (25) and constant $\mu \geq 1$ satisfies $R_i^{-1} S_i \leq \mu R_j^{-1} S_j$.

3.3. Algorithm

Based on Theorem 2, we present an algorithm for the design of dynamic output controllers.

Step I. Given α, β and γ , solve LMIs (22)–(24) to obtain $X_i, Y_i, T_i, Z_i, \widehat{A}_i, \widehat{B}_i, \widehat{C}_i$.

Step II. Then obtain matrices M_i and N_i by (26) and (29).

Step III. calculate matrices K_i, L_i and G_i according to (25).

Step IV. From $P_i = R_i^{-1} S_i, Q_i = \text{diag}\{\gamma I, T_i\}$, calculate μ by the following optimization approach

$$\begin{aligned} & \text{minimize } \mu \\ & \text{s.t. } P_i \leq \mu P_j, Q_i \leq \mu Q_j, \forall i, j \in M. \end{aligned} \quad (34)$$

Step V. Calculate the average dwell time bound based on (8).

Then for any switching signal with average dwell time satisfying (8), the dynamic output controllers given by (3) make system (1) exponentially stable under asynchronous switching.

4. An example

In this section, an example is presented to demonstrate the effectiveness of proposed design method.

Consider the switched system (1) consisting of two subsystems described by

$$A_1 = \begin{bmatrix} -9 & 0.2 \\ 0.3 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.5 & 2 \\ 0.1 & 0.9 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.3 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 4 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}, \quad h = 0.4.$$

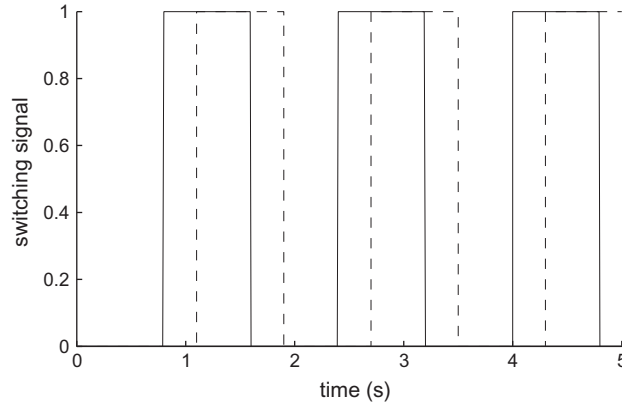


Fig. 1. switching signals (solid line—switching signal of subsystems, dashed line—switching signal of controllers).

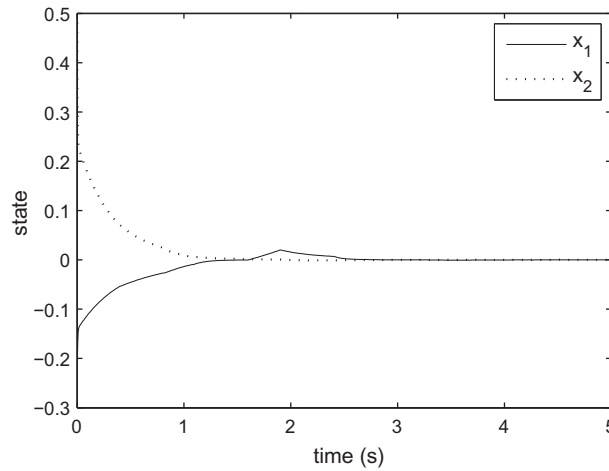


Fig. 2. state trajectories of system (1) satisfying the switching condition.

We assume the delay of asynchronous switching $\tau_d = 0.3$.

Now, we design the output feedback controllers using the algorithm. Choosing $\alpha = 4, \beta = 2, \gamma = 1.5$, we can obtain positive-definite matrices $X_i, Y_i, T_i, Z_i, \hat{A}_i, \hat{B}_i, \hat{C}_i$ ($i = 1, 2$) by solving LMIs (22)–(24). Following Step II, we get M_i and N_i from (26) and (29). According to Step III, we can obtain controller gains

$$\begin{aligned}
 K_1 &= \begin{bmatrix} -591.1 & 1308.3 \\ 67.6 & -328.1 \end{bmatrix}, & K_2 &= \begin{bmatrix} -41.2284 & 20.5746 \\ -0.7735 & -821.4389 \end{bmatrix}, \\
 L_1 &= \begin{bmatrix} -0.2648 & 0.2835 \\ 0.0948 & 2.9816 \end{bmatrix}, & L_2 &= \begin{bmatrix} 2.8610 & 0 \\ 0 & 0.8379 \end{bmatrix}, \\
 G_1 &= \begin{bmatrix} -309.6939 & -4.2060 \\ 2.9895 & -315.8485 \end{bmatrix}, & G_2 &= \begin{bmatrix} -310.3264 & 0.2892 \\ -0.7896 & -316.2327 \end{bmatrix}.
 \end{aligned} \tag{35}$$

Applying (34) produces $\mu = 2.9$, and according to Step V, we have average dwell time $\tau_a \geq \tau_a^* = \frac{\ln \mu + (\alpha + \beta)\tau_d}{\alpha} = 0.7162$. Let $\tau_a = 0.8$. Fig. 1 describes the switching signals, where solid line and dashed line represent switching signals of subsystems and controllers, respectively. Under this switching signals and dynamic output feedback controllers with parameters (35), the steady-state responses of the closed-loop system with $x_0 = [-0.3 \ 0.5]^T$ are depicted in Fig. 2.

Moreover, according to (21), we get

$$\|x(t)\| \leq 51.8189e^{-0.2096(t-t_0)} \|x(t_0)\|. \tag{36}$$

Therefore, it can be seen from Fig. 2 and (36) that the proposed dynamic output feedback controllers can guarantee that the closed-loop system is exponentially stable although there exists asynchronous switching.

5. Conclusion

We have investigated the dynamic output feedback stabilization problem for a class of switched delay systems under asynchronous switching. Time delays appear not only in the state, but also in the switching signal of the controller. Based on a novel Lyapunov functional method combined with the average dwell time scheme, we have established sufficient conditions for exponential stability in terms of LMIs. We have also designed output feedback controllers and identified a class of switching signals satisfying a specific lower bound of the average dwell time. In the future studies on this topic, an extension of these results to the case of nonlinear plant systems, networked control systems [29], or neural networks [14] would make a major step forward.

Acknowledgments

This work was partially supported by the National Science Foundation of China (61004020 and 11072044), the Program for New Century Excellent Talents in University, China (NCET-11-0054) and the Engineering and Physical Sciences Research Council, UK (EP/F029195).

References

- [1] J.P. Hespanha, A.S. Morse, Stability of switched systems with average dwell-time, in: Proceedings of the 38th IEEE Conference on Decision and Control, Phoenix, Arizona, IEEE, Berlin, 1999, pp. 2655–2660.
- [2] B. Jiang, D.S. Du, V. Cocquempot, Fault detection for discrete-time switched systems with interval time-varying delays, *Int. J. Control, Automat. Syst.* 9 (2) (2011) 396–401.
- [3] D. Jeon, M. Tomizuka, Learning hybrid force and position control of robot manipulators, *IEEE Trans. Robot. Automat.* 9 (4) (1996) 423–431.
- [4] H.R. Karimi, Robust delay-dependent H_∞ control of uncertain Markovian Jump systems with mixed neutral, discrete and distributed time-delays, *IEEE Trans. Circ. Syst. I* 58 (8) (2011) 1910–1923.
- [5] H.R. Karimi, H.J. Gao, New delay-dependent exponential H_∞ synchronization for uncertain neural networks with mixed time-delays, *IEEE Trans. Syst., Man Cyber., Part B* 40 (1) (2010) 173–185.
- [6] D.K. Kim, P.G. Park, J.W. Ko, Output-feedback control of systems over communication networks using a deterministic switching system approach, *Automatica* 40 (7) (2004) 1025–1212.
- [7] D. Liberzon, *Switching in Systems and Control*, Birkhauser, Boston, 2003.
- [8] H. Lin, Antsaklis, Stability and stabilization of switched linear systems: a survey of recent results, *IEEE Trans. Automat. Control* 54 (2) (2009) 308–322.
- [9] Ju. H. Park, On design of dynamic output feedback controller for GCS of large-scale systems with delays in interconnections: LMI optimization approach, *Appl. Math. Comput.* 161 (2) (2005) 423–432.
- [10] Z.D. Sun, S.S. Ge, Dynamic output feedback stabilization of a class of switched linear systems, *IEEE Trans. Circ. Syst. I: Fundam. Theory Appl.* 50 (8) (2003) 1111–1115.
- [11] P. Shi, Z. Lin, Y. Shi, Robust output feedback control for discrete time-delay uncertain systems, *Control Intell. Syst.* 34 (1) (2006) 57–63.
- [12] X.-M. Sun, W. Wang, G.-P. Liu, J. Zhao, Stability analysis for linear switched systems with time-varying delay, *IEEE Trans. Syst. Man Cyber., Part B* 38 (2) (2008) 1528–1533.
- [13] X.-M. Sun, J. Zhao, D.J. Hill, Stability and L_2 -gain analysis for switched delay systems: a delay-dependent method, *Automatica* 42 (10) (2006) 1769–1774.
- [14] Y. Tang, S.Y.S. Leung, W.K. Wong, J.-A. Fang, Impulsive pinning synchronization of stochastic discrete-time networks, *Neurocomputing* 73 (10–12) (2010) 2132–2139.
- [15] C. Tomlin, G.J. Pappas, S. Sastry, Conflict resolution for air traffic management: a study in multi-agent hybrid systems, *IEEE Trans. Automat. Control* 43 (4) (1998) 509–521.
- [16] Y. Tang, Z.D. Wang, J.-A. Fang, Controller design for synchronization of an array of delayed neural networks using a controllable probabilistic PSO, *Inform. Sci.* 181 (20) (2011) 4715–4732.
- [17] L. Vu, M.A. Kristi, Stability of time-delay feedback switched linear systems, *IEEE Trans. Automat. Control* 55 (10) (2010) 2385–2389.
- [18] Z. Wu, P. Shi, H. Su, J. Chu, Delay-dependent stability analysis for switched neural networks with time-varying delay, *IEEE Trans. Syst., Man Cyber., Part B: Cyber.* 41 (6) (2011) 1522–1530.
- [19] X.T. Wu, L.T. Yan, W.B. Zhang, Y. Tang, Stability of stochastic nonlinear switched systems with average dwell time, *J. Phys. A: Math. Theor.* 45 (8) (2012) 085207.
- [20] L.G. Wu, W.X. Zheng, Weighted H_∞ model reduction for linear switched systems with time-varying delay, *Automatica* 45 (1) (2009) 186–193.
- [21] R. Wang, J. Zhao, Reliable guaranteed cost control for uncertain switched nonlinear systems, *Int. J. Syst. Sci.* 40 (3) (2009) 205–211.
- [22] S.Y. Xu, T.W. Chen, H_∞ output feedback control for uncertain stochastic systems with time-varying delays, *Automatica* 40 (12) (2004) 2091–2098.
- [23] G.M. Xie, L. Wang, Stabilization of switched linear systems with time-delay in detection of switching signal, *J. Math. Anal. Appl.* 305 (6) (2005) 277–290.
- [24] Z.R. Xiang, R.H. Wang, Robust control for uncertain switched non-linear systems with time delay under asynchronous switching, *IET Control Theory Appl.* 3 (8) (2008) 1041–1050.
- [25] W.X. Xie, C.Y. Wen, Z.G. Li, Input-to-state stabilization of switched nonlinear systems, *IEEE Trans. Automat. Control* 46 (7) (2001) 1111–1116.
- [26] L.X. Zhang, N.G. Cui, M. Liu, Y. Zhao, Asynchronous filtering of discrete-time switched linear systems with average dwell time, *IEEE Trans. Circ. Syst. I* 58 (5) (2011) 1109–1118.
- [27] J. Zhao, G.M. Dimirovski, Quadratic stability of a class of switched nonlinear systems, *IEEE Trans. Automat. Control* 49 (4) (2004) 574–578.
- [28] L.X. Zhang, H.J. Gao, Asynchronously switched control of switched linear systems with average dwell time, *Automatica* 46 (5) (2010) 953–958.
- [29] L.X. Zhang, H.J. Gao, O. Kaynak, Survey of studies on network-induced constraints in networked control systems, *IEEE Trans. Ind. Inform.* (in press).
- [30] L.X. Zhang, P. Shi, Stability, L_2 -Gain and asynchronous H_∞ control of discrete-time switched systems with average dwell time, *IEEE Trans. Automat. Control* 54 (9) (2009) 2193–2200.
- [31] X. Zhao, L. Zhang, P. Shi, M. Liu, Stability of switched positive linear systems with average dwell time switching, *Automatica* 48 (6) (2012) 1132–1137.
- [32] X. Zhao, L. Zhang, P. Shi, M. Liu, Stability and stabilization of switched linear systems with mode-dependent average dwell time, *IEEE Trans. Automat. Control* 57 (7) (2012) 1809–1815.