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A study of reactive power marginal price in electricity market

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Abstract

Developing an accurate and feasible method for reactive power pricing is significant in the electricity market. The reactive power price cannot be obtained accurately by conventional optimal power flow models which usually ignore the production cost of reactive power. In this paper, the authors include the production cost of reactive power into the objective function of the optimal power flow problem, and use sequential quadratic programming method to solve the optimization problem and obtain reactive power marginal price accordingly. A five-bus test system is used for computer study. The results from eight study cases show clearly the effects of various factors on reactive power marginal price.

Keywords: Electricity market; Reactive power price; Optimal power flow

1. Introduction

Reactive power support plays an important role in implementation of power transactions. In electricity markets, reactive power supply is classified as a part of ancillary service of electricity. It is realized that establishing accurate prices of reactive power can not only recover the costs of reactive power production, but also provide useful information related to the urgency of reactive power supply and system voltage support. Therefore, the spot pricing of reactive power becomes a significant research topic in power system restructuring.

The spot pricing method of active power was established by Scheweppe et al. [1]. In their work, the concept of marginal price of microeconomics has been extended to power systems and taken as the spot price of electricity. The spot price can help to improve production efficiency and yield maximum social benefit. In [2,3] real-time pricing methods of reactive power are studied which are similar to the active power pricing method suggested in [1] with reactive power production cost neglected. A comprehensive study of spot pricing and its implementation are reported in [4,5]. In [6] it is pointed out that reactive power price should recover not only the operational cost, but also capital investments of capacitors. However, the reactive power production cost of generators is neglected. In [7], a detailed discussion on reactive power services is made and it is shown that the capital costs should be included in reactive power price. Several methods are proposed to evaluate the reactive power price. In [8] investigation is conducted on reactive power pricing by using the objective function of maximizing social benefit instead of minimizing the production cost. A latest paper [9] introduces opportunity cost as a reactive power production cost of generator but the computation of the cost is difficult.

In this paper, we are going to study the effects of various factors on reactive power spot price with reactive production cost considered. As a first step we neglect the electricity consumer competition and assume the loads are known from load forecasting. Taking the power flow equations as constraints, the reactive power-pricing problem becomes a typical optimal power flow (OPF) problem. In order to investigate the effects of various factors on reactive power price accurately, both reactive power production cost of generators and capital investment cost of capacitors are included in the objective function of total system operation cost. The advanced sequential quadratic programming (SQP) method is applied to solve the OPF problem and obtain reactive power marginal price ac-

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cordingly. A five-bus test system is used for computer study. The results from eight study cases show clearly the effects of various factors, such as objective function, system operation point, load power factor, profit rate and bus voltage control etc., on reactive power marginal price.

In Section 2, the mathematical model for reactive power pricing is presented and the SQP method is outlined. Section 3 presents the computer results from case study with conclusions made in Section 4.

2. Mathematical model

2.1. Objective function and constraints

In our study we assume the active and reactive power of loads is known from load forecasting and kept constant during optimization. The objective function for the optimization problem is to minimize the overall costs of active and reactive power production of generators with the capital investment of capacitor referred to as an equivalent production cost. The cost of network management and system maintenance will not be included in the objective function, which should be considered independently. Based on the assumption of constant loads, to minimize the total production cost is equivalent to maximize the total social benefits. The suggested hourly cost function takes the form:

$$C = \sum_{i\in\{G\}} [C_{pgi}(P_{Gi}) + C_{qgi}(Q_{Gi})] + \sum_{i\in\{C\}} C_{cj}(Q_{Cj}) \tag{1}$$

where $\{G\}$ is the generator set; $\{C\}$ is the capacitor set; $C_{pgi}(P_{Gi})$ is the active power production cost of generator i; $C_{qgi}(Q_{Gi})$ is the reactive power production cost of generator i; $C_{cj}(Q_{Cj})$ is the equivalent production cost of capacitor j to be explained below; P_{Gi}, Q_{Gi} are the active and reactive power output of the generator on bus i; and Q_{Cj} is the reactive power output of capacitor j.

The production cost of active power generation, i.e. the first item in Eq. (1), is modeled by a quadratic function where a, b and c are predetermined coefficients:

$$C_{pgi}(P_{Gi}) = a + bP_{Gi} + cP_{Gi}^2 \tag{2}$$

The reactive power cost of generator, i.e. the second item in Eq. (1), is the so-called opportunity cost [9]. The reactive power output of a generator will reduce its active power generation capability which can serve at least as spinning reserve, and the corresponding implicit financial loss to generator is modeled as an opportunity cost. Actually it is difficult to determine the real value of opportunity cost. For simplicity, we consider it approximately as

$$C_{qgi}(Q_{Gi}) = [C_{pgi}(S_{Gi,\max}) - C_{pgi}(\sqrt{S_{Gi,\max}^2 - Q_{Gi}^2})]k \tag{3}$$

where $S_{Gi,\max}$ is the nominal apparent power of the generator i; k is the profit rate of active power generation, usually between 5 and 10%. Here we assume $P_{Gi,\max} = S_{Gi,\max}$.

The third item of Eq. (1) represents the equivalent production cost for capital investment return of capacitors, which is expressed as their depreciate rate (the life-span of capacitors is 15 years):

$$\begin{aligned} C_{cj}(Q_{Cj}) &= Q_{Cj} \cdot \$11\ 600/\text{MVar} \\ &\quad \div (15 \times 365 \times 24 \times h)\ \text{h} \\ &= Q_{Cj} \cdot \$13.24/(100\ \text{MVar}h) \end{aligned} \tag{4}$$

where h represents the average usage rate of capacitors taken as 2/3. Q_{Cj} is in per unit on 100 MVA base. Eq. (4) is a linear cost function with the slope of $\mathrm{d}C_{cj}(Q_{Cj})/\mathrm{d}Q_{Cj} = \$13.24/(100\ \text{MVar}h)$ representing approximately the capacitor investment impacts on reactive pricing.

The equality constraints are load flow equations:

$$g_{pi} = P_{Gi} - P_{Di} - \sum_{j\in N} V_i V_j Y_{ij} \cos(\delta_{ij} - \theta_{ij}) = 0$$

$$g_{qi} = Q_{Gi} - Q_{Di} - \sum_{j\in N} V_i V_j Y_{ij} \sin(\delta_{ij} - \theta_{ij}) = 0$$

where N is the total number of buses in the system; P_{Gi}, Q_{Gi}, P_{Di}, Q_{Di} are the active and reactive power generation and demand on bus i; $Y_{ij}\angle\theta_{ij}$ is the element in the bus admittance matrix; $\dot{V}_i = V_i \angle \delta_i$ is the bus voltage at bus i and $\delta_{ij} = \delta_i - \delta_j$.

The inequality constraints used are:

- Generation limits: $P_{Gi,\min} \le P_{Gi} \le P_{Gi,\max}$ and $P_{Gi}^2 + Q_{Gi}^2 \le S_{Gi,\max}^2$, where $j \in \{G\}$
- Reactive power output limit of capacitor: $0 \le Q_{Cj} \le Q_{Cj,\max}$, where $i \in \{C\}$
- Transmission power limits: $S_{ij} \le S_{ij,\max}$, where $S_{ij} = |\dot{V}_i[(\dot{V}_i - \dot{V}_j)y_{ij}]^*| + \dot{V}_i(\dot{V}_i(y'_{ij}/2)^*)$, $y_{ij} = (1/z_{ij})$, y'_{ij} are line series and charging admittance, respectively.
- Voltage limits: $V_{i,\min} \le V_i \le V_{i,\max}$

Based on the above mathematical model the corresponding Lagrangian function of this optimization problem takes the form:

$$\begin{aligned} L = &\sum_{i\in\{G\}} [C_{pgi}(P_{Gi}) + C_{qgi}(Q_{Gi})] + \sum_{j\in\{C\}} C_{cj}(Q_{Cj}) \\ &- \sum_{i\in N} \lambda_{pi}\left[P_{Gi} - P_{Di} - \sum_{j\in N} V_i V_j Y_{ij} \cos(\delta_{ij} - \theta_{ij})\right] \\ &- \sum_{i\in N} \lambda_{qi}\left[Q_{Gi} - Q_{Di} - \sum_{j\in N} V_i V_j Y_{ij} \sin(\delta_{ij} - \theta_{ij})\right] \\ &+ \sum_{i\in\{G\}} \mu_{pi,\min}(P_{Gi,\min} - P_{Gi}) \\ &+ \sum_{i\in\{G\}} \mu_{pi,\max}(P_{Gi} - P_{Gi,\max}) \\ &+ \sum_{i\in\{G\}} \mu_{si}(P_{Gi}^2 + Q_{Gi}^2 - S_{Gi,\max}^2) \\ &+ \sum_{j\in\{C\}} \mu_{cj,\min}(Q_{Cj,\min} - Q_{Cj}) \\ &+ \sum_{j\in\{C\}} \mu_{cj,\max}(Q_{Cj} - Q_{Cj,\max}) \end{aligned}$$

$$+\sum_{i\in N}\sum_{\substack{i\in N\\ j\neq i}}\eta_{ij}(S_{ij}-S_{ij,\max})+\sum_{i\in N}v_{i,\min}(V_{i,\min}-V_i)$$

$$+\sum_{i\in N}v_{i,\max}(V_i-V_{i,\max})$$

According to the theory of microeconomics, the marginal prices for active power and reactive power on bus i are λ_{pi} and λ_{qi}, respectively, in the above Lagrangian function and will be taken as the corresponding spot prices in electricity markets [2,3,8].

2.2. Sequential quadratic programming method for OPF solution

The sequential quadratic programming (SQP) is applied to solve the OPF problem because it is one of the best methods in nonlinear programming developed recently [10]. In the SQP method, the Lagrangian function is built up first, then the original problem is converted to an approximate quadratic programming (QP) sub-problem. Through iterations, the QP problem solutions converge to a final optimum which is proved to be the optimum of the original problem.

The SQP method can be outlined as follows. For an optimization problem:

$$\min_{\substack{X\\ s.t.}} f(X) \tag{5}$$

$$g_i(X)=0 \quad i=1,2,\ldots,p$$

$$h_j(X)\leq 0 \quad j=1,2,\ldots,m$$

The corresponding Lagrangian function is defined as

$$L(X,\lambda)=f(X)+\sum_{i=1}^{p}\lambda_i g_i(X)+\sum_{j=1}^{m}\lambda_{p+j}h_j(X) \tag{6}$$

where λ_i $(i=1,2\ldots,p+m)$ is the Lagrangian multiplier for the ith constraint with $\lambda_{p+j}=0$ when $h_j(X)$ does not hit the limit. The optimum of Eq. (6) can be reached through sequentially solving the approximate quadratic programming sub-problem:

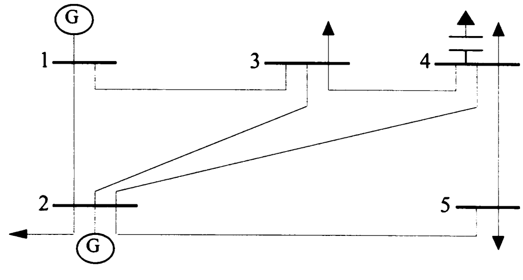


Fig. 1. The five-bus study system diagram.

$$\min_{\substack{X\\ s.t.}} Q=\nabla f^{T}\Delta X+\frac{1}{2}\Delta X^{T}[\nabla^2 L]\Delta X \tag{7}$$

$$g_i+\nabla g_i^{T}\Delta X=0 \quad i=1,2,\ldots,p$$

$$h_j+\nabla h_j^{T}\Delta X\leq 0 \quad j=1,2,\ldots,m$$

where $[\nabla^2 L]$ denotes the Hessian matrix of the Lagrangian function. The details of the SQP method can be found in [10]. For our problem $f(X)$ is presented in Eq. (1). $g_i(X)=0$ is the load flow equation at a certain bus and the corresponding λ_i is the corresponding marginal cost of active or reactive power at that bus. $h_j(X)$ is an inequality constraint such as generation limit, capacity limit of a capacitor, transmission power limit or voltage limit mentioned above.

3. Computer test results

3.1. Sample system and test cases

A five-bus power system [11] is used for computer study (Fig. 1). There are two generators on buses 1 and 2, respectively. The nominal apparent power output of each generator is 125 MVA. The lower and upper limits of power generation is 20 and 125 MW. The active power production cost of each generator is:

$$C_{gpi}(P_{Gi})=75+750P_{Gi}+420P_{Gi}^2 \ (\$/\mathrm{h})$$

If not particularly specified, all the parameters stated here are in per unit on a 100 MVA base.

There are capacitors installed on bus 4 with the total capacity of 50 MVA. We assume the reactive power output of capacitors can be adjusted continuously. The system loads on buses 2–5 are listed in Table A1 of the Appendix A with a common power factor of 0.9. The transmission line impedance and charging admittance are given in Table A2 of the Appendix A. The other system operation limits used are transmission power limit: $S_{ij}\leq 1.8$, voltage limit: $0.95\leq V_i\leq 1.05$, and swing bus settings: $V_1 1.05$ and $\delta_1=0°$.

In order to study the impacts of various factors on the marginal price of reactive power, the following eight cases are studied. (1) The objective function (see Eq. (1)) has only the first item. This case is taken as the base case for comparison. (2) The objective function has only the first two items with capacitor cost neglected. (3) The objective function has only the first and the third items with reactive power production cost of generators neglected. (4) The objective function has all the three items as described in Eq. (1). Based on case 4, cases (5)–(8) are designed to study the impacts of various factors on reactive power marginal price, including load power factor, daily load fluctuation (i.e. system operation point change), voltage control and the profit rate k used in opportunity cost (see Eq. (2)).

Table 1
Test results of cases 1–4

	Case 1 ($Q_{C4} \neq 0$)	Case 1 ($Q_{C4} = 0$)	Case 2	Case 3	Case 4
Objective function	$\Sigma_{i\in G} C_{pgi}(P_{Gi})$	$\Sigma_{i\in G} C_{pgi}(P_{Gi})$	$\Sigma_{i\in G} C_{pgi}(P_{Gi}) + C_{qgi}(Q_{Gi})$	$\Sigma_{i\in G} C_{pgi}(P_{Gi}) + \Sigma_{i\in C} C_{ci}(Q_{Ci})$	$\Sigma_{i\in G} [C_{pgi}(P_{Gi}) + C_{qgi}(Q_{Gi})] + \Sigma_{j\in C} C_{cj}(Q_{Cj})$
$S_{Gi} = P_{Gi}+jQ_{Gi}$ $(i = 1, 2)$	$0.8170-0.1090j$	$0.8282-0.0510j$	$0.8270+0.0020j$	$0.8269-0.1034j$	$0.8267-0.0584j$
	$0.8693+0.2624j$	$0.8617+0.6658j$	$0.8599+0.1024j$	$0.8604+0.5072j$	$0.8602+0.1823j$
Reactive power output of capacitor on bus 4	0.4447	0	0.4975	0.2000	0.3636
Total active power cost of generators	US$2012.4/h	US$2017.4/h	US$2013.0/h	US$2013.6/h	US$2013/h
Total reactive power cost of generators			US$0.3462/h		US$6.0258/h
Total capacitor cost				US$2.6478/h	US$4.8141/h
Marginal price λ_p of active power at buses 1–5 (US$/MW h)	14.439	14.458	14.446	14.450	14.444
	14.727	14.738	14.724	14.726	14.726
	14.882	15.227	15.203	15.213	15.218
	14.913	15.275	15.248	15.261	15.266
	14.992	15.408	15.393	15.419	15.411
Marginal price λ_q of reactive power at buses 1–5 (US$/MVar h)	0	0.000007	0.001345	0	0.03859
	0	0.000010	0.067537	0.0001	0.1206
	0.047493	0.247995	0.053786	0.1523	0.1683
	0.000581	0.260485	0.000002	0.1324	0.1324
	0.241188	0.326345	0.283788	0.2851	0.3697
$\rho_{q-\mathrm{avg}}$			US$0.0043/MVar h	US$0.03311/MVar h	US$0.0754/MVar h

3.2. Computer test results for cases 1–4

The computer test results from cases 1–4 (see Table 1) are used to study the impacts of objective functions on reactive power marginal price (RPMP) under normal operation conditions, where $\rho_{q-\mathrm{avg}}$ is the average cost of reactive power of the whole network, which is obtained through dividing the total system reactive power cost by the total reactive power demand. A similar set of tests are conducted and denoted as cases 1′–4′ (see Table 2). In cases 1′–4′ the loads on buses 1–5 increase to [0, 0.6, 0.9, 0.8, 1.0] per unit with same power factor of 0.9 and the power generation limits of the two generators are set to be 200MVA. The system is quite stressed.

From Tables 1 and 2 we can observe the following:

- When the system is operating at normal condition, the total active power production cost and the active power marginal prices at various buses have only small changes (no more than 3% for the latter) along with the objective function changes. Therefore, the active power pricing subproblem can be studied approximately with reactive power production cost neglected. In cases 1′–4′, the active and reactive power marginal prices increase dramatically, however the above observation is still valid.
- For each test case, active power marginal prices at various buses are in the same order while the RPMP fluctuates significantly from bus to bus. Generally in non-stressed power systems, the active power marginal price is much higher than the RPMT. In our case it is ~ 100 times as much as RPMP. However, in a stressed system even with high power factors, the reactive power marginal prices may rise significantly. Some test shows that reactive power marginal prices may be higher than active power marginal prices if some buses in a stressed system have poor power factors and the system hits some operation limits. The corresponding results will be presented in case 5.
- The total reactive power production cost changes apparently along with the objective function changes. Although the cost is small under normal operation condition, it can cumulate into a large amount.
- When the capacitor cost and/or the reactive power generation cost is neglected, the corresponding reactive power source bus(es) will have zero or very little RPMP(s) for the free reactive power available locally. The nearby buses also get benefits and have small RPMPs. For example bus 3 of case 2, which is close to bus 4 where the capacitor is installed, has much smaller RPMP as compared with bus 5 which is far from reactive sources. When all the reactive power production costs are taken into consideration, the corresponding RPMP increases noticeably (see case 4) which gives the load an incentive to reduce its reactive power demand. Besides, the revenue to the

reactive power producer will encourage them to invest and provide enough reactive power. Similar observations can be made from cases 1′ to 4′.

- The revenue from reactive power marginal price will be much higher than that from the system average price of reactive power. Some adjustment should be made accordingly if RPMP is to be used.

3.3. Computer test results for cases 5–8

Cases 5–8 study the impacts of various factors on RPMP. Computer test results from cases 5–8 are shown in Figs. 2–5, respectively.

In case 5 the impact of load power factor is studied for both normal load (Fig. 2(a–c)) and heavy load (Fig. 2(d–e)) conditions.

From Fig. 2(a–c) of normal load condition we can see:

- When the load power factor reduces from 1.0 to 0.7, the RPMP increases greatly while the average price increases very slowly (Fig. 2(a)). Therefore, the RPMP can provide clear economic information to loads to improve their power factors.
- When bus 5 reaches its minimum voltage of 0.95 per unit at a lower power factor (see Fig. 2(c)), the corresponding RPMP of bus 5 increases dramatically (see Fig. 2(a)), which can act as an index of the urgency of the reactive power supply and voltage support on bus 5.
- In Fig. 2(b) when the power factor is close to 1, the total system reactive power demand including reactive power losses of transmission lines can be supported by line charging capacitors. Therefore, the reactive power output of the two generators becomes negative which means that the system has surplus reactive power and the generators are asked to absorb reactive power. The corresponding RPMPs in Fig. 2(a) are very small.
- When the cheaper and local reactive source of capacitor is used up (see Fig. 2(b)), the load bus voltages (see buses 3 and 4 in Fig. 2(c)) will reduce quickly along with the power factor reduction and the corresponding RPMPs will increase rapidly simultaneously (Fig. 2(a)).
- The revenue of reactive power supply based on the marginal price will be much more than those based on the average price especially at lower power factors (see Fig. 2(a)). Therefore, some adjustment should be made if RPMP is going to be used.

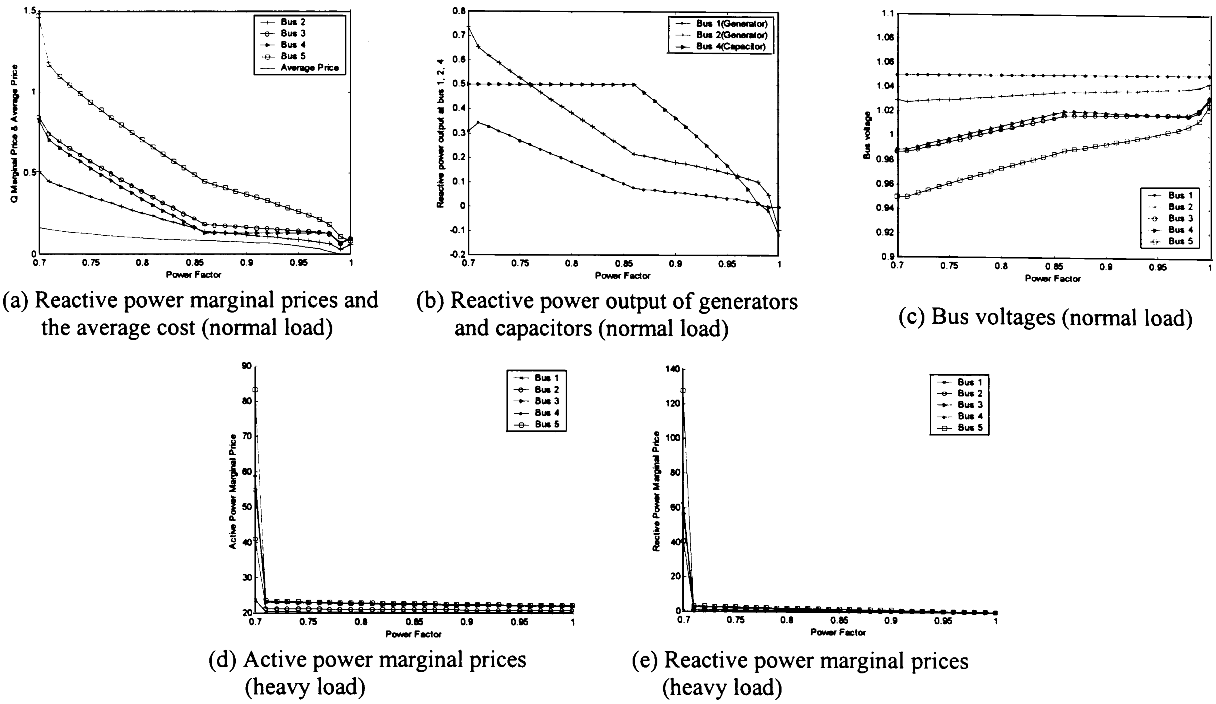
Under heavy load condition, we can see from Fig. 2(d) and (e):

- At poor power factor the reactive power marginal price might rise dramatically and even be higher than the active power marginal price for lack of reactive power and voltage support capability.

In case 6 the effects of the daily load change, or say the operation point change, is studied. Assume that the daily load percentage change is in a pattern shown in Fig. 3(a) and all the load power factor keeps as 0.9.

Table 2
Test results of cases 1′–4′

	Case 1′	Case 2′	Case 3′	Case 4′
Objective function	$\Sigma_{i\in G} C_{pgi}(P_{Gi})$	$\Sigma_{i\in G} C_{pgi}(P_{Gi}) + C_{qgi}(Q_{Gi})$	$\Sigma_{i\in G} C_{pgi}(P_{Gi}) + \Sigma_{i\in C} C_{ci}(Q_{Ci})$	$\Sigma_{i\in G} [C_{pgi}(P_{Gi}) + C_{qgi}(Q_{Gi})] + \Sigma_{j\in C} C_{cj}(Q_{Cj})$
$S_{Gi}=P_{Gi}+jQ_{Gi}$ $(i=1, 2)$	$1.6676-0.2145i$	$1.6535-0.0372i$	$1.6676-0.2286i$	$1.6535-0.0372i$
	$1.7737+1.4393i$	$1.7878+1.2659i$	$1.7737+1.4532i$	$1.7878+1.2659i$
Reactive power output of capacitor on bus 4	0.5	0.5	0.5	0.5
Total active power production cost of generators	US$5220/h	US$5222/h	US$5220/h	US$5222/h
Total reactive power production cost of generators		US$46.6/h		US$46.6/h
Total capacitor cost of capacitors			$6.62/h	$6.62/h
Marginal price λ_p of active power at buses 1–5 (US$/MW h)	21.4822	21.3897	21.5099	21.3897
	22.4237	22.5174	22.3971	22.5174
	24.0689	24.3183	23.9645	24.3183
	24.2182	24.5213	24.1068	24.5213
	24.4866	25.2110	24.3535	25.2110
Marginal price λ_q of reactive power at buses 1–5 (US$/MVar h)	0	0.0212	0	0.0212
	0	0.7519	0.0002	0.7519
	0.6085	1.4604	0.6044	1.4604
	0.5156	1.4607	0.5130	1.4607
	0.9640	2.9720	0.9650	2.9720
$\rho_{q-\mathrm{avg}}$		US$0.29/MVar h	US$0.0414/MVar h	US$0.333/MVarh



(a) Reactive power marginal prices and the average cost (normal load)

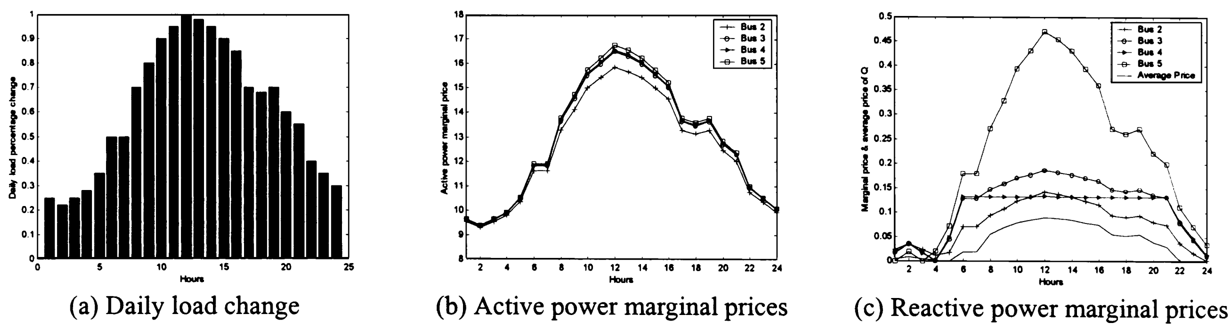
(b) Reactive power output of generators and capacitors (normal load)

(c) Bus voltages (normal load)

(d) Active power marginal prices (heavy load)

(e) Reactive power marginal prices (heavy load)

Fig. 2. Computer test results of case 5.

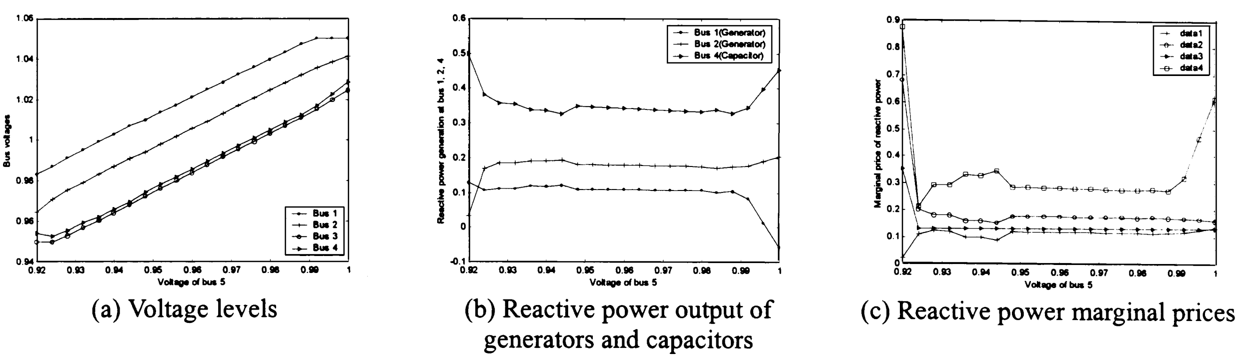


(a) Daily load change

(b) Active power marginal prices

(c) Reactive power marginal prices

Fig. 3. Computer test results of case 6.



(a) Voltage levels

(b) Reactive power output of generators and capacitors

(c) Reactive power marginal prices

Fig. 4. Computer test results of case 7.

From Fig. 3(b) we can see that the active power marginal prices are in the same order for the different buses, and their daily changes have the same shape as the daily load percentage change. However the RPMPs on various buses have quite different values and also they have quite different contours w.r.t. time as compared with that of the daily load percentage change (Fig. 3(c)). It can be noticed that when the demand is very low at night, the two generators will absorb surplus reactive power from line charging (not shown in the figure) and the RPMPs of the generator buses will rise a little at around 02:00 h at night (Fig. 3(c)) for the reactive power opportunity cost.

In case 7 the voltage level of bus 5 is controlled and varying from 0.92 to 1.00 (in this case bus 1 will not keep constant voltage of 1.05 pu). From the system diagram (Fig. 1) and data we know that bus 5 is far from the reactive power sources and has the most serious voltage problem. From Fig. 4(a) we can see that the system voltage level changes simultaneously with the voltage change of bus 5 and the nearby buses have similar voltage contour as bus 5. In Fig. 4(b), we can see that the reactive power output of generators and capacitors fluctuates intensively when some buses reach their voltage limits. Particularly when the voltage of bus 1 reaches its upper limit and starts to absorb reactive power in order to keep the voltage as 1.05, the reduction of its reactive power output is rapid (Fig. 4(b)). When bus 5 or its nearby buses reach their voltage limits, dramatic increase of certain RPMPs can be observed in Fig. 4(c) at the two ends of the curves.

All the previous cases assume the profit rate k in Eq. (2) is 5%. In case 8 we assume the profit rate changes from 0 to 10% in order to study how it effects on the reactive power marginal price. In Fig. 5(a) we can see the reactive power marginal price at buses 2 and 5 are affected more than that at buses 3 and 4. The possible reason is consumers at buses 2 and 5 mainly use reactive power produced by generators, and consumers at buses 3 and 4 mainly use reactive power produced by capacitors whose price are not affected by k. Fig. 5(b) shows the reactive power output of generators and capacitors while the profit rate k changes. In the figure, when k nears zero, the system tends to use reactive power produced by generators as much as it can. The generator at bus 2 is prior to produce reactive power because it is close to loads and the generator at bus 1 slightly absorbs reactive power in order to manage its voltage as a swing bus. With the profit rate increasing, which implies higher opportunity cost of reactive power produced by generators, the system tends to use the capacitors at bus 4 as much as possible, and the reactive power output of generators reaches a lower level. Although the test system is small, most of the rules observed from the system can be extended to larger power systems.

4. Conclusion

In this paper the reactive power marginal price is studied in detail. The corresponding optimal power flow problem is defined and solved by the advanced SQP method. Computer tests show the following.

(1) Under normal operation condition, the active power marginal price sub-problem can be studied with reactive power production cost neglected. The active power marginal price is usually much higher than the reactive power marginal price in non-stressed power systems.

(2) The reactive power production cost of generators and the capital investment cost of capacitors should be considered in reactive power spot pricing for their noticeable impacts on reactive power marginal price.

(3) Load power factor, daily load fluctuation, bus voltage control and limits and profit rate in reactive power opportunity cost may have significant impacts on reactive power marginal price especially when certain system operation limits are reached.

(4) RPMP can serve as a system index related to the urgency of the reactive power supply and system voltage support and an incentive to improve load power factor and reduce reactive power demand.

(5) The revenue based on RPMP is much higher than that based on reactive power average price. Therefore, some adjustment should be made in using reactive power marginal price.

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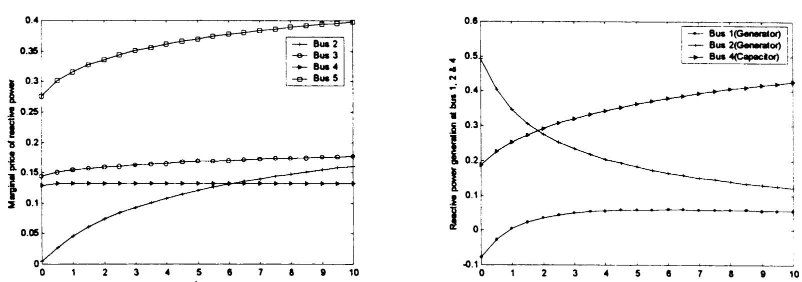

(a) Reactive power marginal prices

(b) Reactive power out put of generators and capacitors

Fig. 5. Computer test results of case 8.

Appendix A

Table A1 Test system loads

Bus	Active power	Reactive power
2	0.20	0.097
3	0.45	0.22
4	0.40	0.19
5	0.60	0.29

Table A2 Test system line data

Bus node	Line impedance z_{ij}	Line charging y'_{ij}
1–2	$0.02+j0.06$	$0.0+j0.030$
1–3	$0.08+j0.24$	$0.0+j0.025$
2–3	$0.06+j0.18$	$0.0+j0.020$
2–4	$0.06+j0.18$	$0.0+j0.020$
2–5	$0.04+j0.12$	$0.0+j0.015$
3–4	$0.01+j0.03$	$0.0+j0.010$
4–5	$0.08+j0.24$	$0.0+j0.025$

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