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Dynamic corporate investment and liquidity management under model uncertainty☆

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HIGHLIGHTS

- We incorporate model uncertainty into a dynamic model of investment and liquidity management.
- Model uncertainty induces under(over)-investment when the firm's liquidity is high(low).
- An increasing in model uncertainty accelerates firm's payout.
- Model uncertainty significantly lowers a firm's average q and marginal q.

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ABSTRACT

We extend the model of dynamic investment and liquidity management for financially constrained firms (Bolton et al., 2011) by incorporating model uncertainty. Our theoretical model predicts that different from traditional business risk model uncertainty and concerning about model misspecification have ambiguous effects on the investment behavior and liquidity management, which depends on the firm's liquidity measured by firm's cash–capital ratio $w = W/K$. It shows that model uncertainty induces the firm to under-invest when the firm has sufficient cash. However, the firm prefers over-investing as its liquidity is essentially low. Moreover, an increasing in model uncertainty accelerates firm's payout while an increasing in business risk will delay the firm to pay out cash. Finally, it shows that model uncertainty significantly lowers a firm's average q and marginal q as well as marginal value of liquidity.

1. Introduction

In most existing models about corporate investment, the distribution of firm's productivity shock is known to the decision maker. However, there are two good reasons for us to think about departures from this assumption. First, the Ellsberg (1961) paradox and related experimental evidence demonstrate that people deal with risk and ambiguity in different ways. Risk refers to the case where the probability distribution over the state of the world is known, while ambiguity refers to the situation where the distribution itself may be unknown to the economic agents. Second, as Hansen and Sargent (2001) pointed out, economic agents believe that the observed economic data come from a set of unspecified models. Concerns about model misspecification make a decision maker to desire robust decision rules.

The goal of this paper is to investigate how model uncertainty distorts firm's dynamic corporate investment and liquidity management. Our model has two essential building blocks, which are: (i) the workhorse neoclassical q model of investment with liquidity constraints; (ii) belief distortions induced by model uncertainty. For the first block, our model is based on the liquidation case in Bolton et al. (2011) (Henceforth BCW). In their model, firm faces liquidity constraints and cash holding costs at the same time. These two opposite effects jointly determine the endogenous payout boundary. For the second building block, we use entropy

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to measure model discrepancies, which is widely used in statistics and econometrics for model detection. More importantly, this approach is analytically tractable and especially suitable for our continuous time framework.

We assume there exist concerns about model misspecification for firm's productivity shock, thus alternative models are considered. Firm chooses the investment and liquidity management policies to maximize firm value in the worst-case scenario. We model the firm's objective as the multiplier preferences proposed by Anderson et al. (2003). Essentially, the firm solves a maxmin problem.

We find the following main novel results. First, our model predicts that concerns about model misspecification have ambiguous effects on the investment behavior, which depends on the firm's cash inventory. When the cash–capital ratio $w = W/K$ approaches the payout boundary, model uncertainty leads to underinvestment. In the high cash inventory region, the shareholders with ambiguity aversion are more eager to cash payment comparing to investing in firm's capital. This effect acts as a disincentive to firm's investment. However, the firm prefers underdisinvestment as the cash–capital ratio is close to zero. In this region, model uncertainty lowers firm's continuation value and weakens the shareholders' incentives to sustain the project. Hence model uncertainty results in underdisinvestment in this case.

Second, the effect of model uncertainty on firm's liquidity management is shown to be drastically different from that of traditional uncertainty in the form of risk. Specifically, an increase in uncertainty accelerates firm's payout while an increase in risk delays it. As the saying goes, "a bird in the hand is worth two in the bush". Since cash payout can be treated as a channel to extricate from model uncertainty, the shareholders prefer cash in hand to cash in firm's account. However, due to the precautionary motive an increase in risk always delays firm's payout. Finally, it shows that model uncertainty significantly lowers a firm's average q and marginal q.

Our paper is related to a fast growing literature on dynamic corporate finance in continuous time. Our paper is most closely related to BCW. They propose a model of dynamic investment, financing, and risk management for financially constrained firms. Our main contribution is to introduce robustness into their models and study corporate investment and liquidity management implications. Although model uncertainty has been extensive discussed in asset pricing,[1] the impact on corporate finance decisions has not received enough attention. Nishimura and Ozaki (2007) introduce Knight uncertainty into the standard real option framework. Miao and Rivera (2016) study how to design robust contracts with hidden action in a dynamic environment. In contrast, our paper incorporates model uncertainty into a dynamic model of corporate investment and liquidity management.

The remainder of the paper is organized as follows: Section 2 describes the model setup, which includes the firm's production technology, liquidity management, model uncertainty and first-best solutions with model uncertainty. The solution to the model is derived in Section 3. In Section 4, we provide the quantitative results and discussions. Finally, Section 5 concludes the paper.

2. Model setup

We incorporate belief distortions due to concerns about model misspecification into BCW. First, we describe firm's production technology and liquidity management. Then we introduce belief distortions and the concerns for model uncertainty.

[1] For instance, Maenhout (2004) considers the dynamic portfolio and consumption problem of an investor who worries about model uncertainty. Ju and Miao (2012) propose a novel generalized recursive smooth ambiguity model and apply this model to a consumption-based asset-pricing model.

2.1. Production technology and liquidity management

The firm employs physical capital for production. We denote the capital stock and investment level as K and I, respectively. Then the firm's capital stock evolves as

$$dK_t = (I_t - \delta K_t)\, dt, \tag{1}$$

where $\delta \geq 0$ is the depreciation rate.

As in the neoclassical investment model, investment entails adjustment cost. We take the conventional assumption that the adjustment cost $G(I, K)$ is convex in investment I and homogeneous of degree one in investment level I and capital stock K. Hence we can write $I + G(I, K) = c(i) K$, where $c(i)$ denotes the total investment cost (including the adjustment cost) per unit of capital and $i = I/K$ is investment–capital ratio. Specifically, we assume $c(i)$ takes the standard quadratic form as

$$c(i) = i + \frac{1}{2}\phi i^2, \tag{2}$$

where ϕ measures the adjustment cost for investment.

We assume the firm's cumulative productivity evolves according to

$$dA_t = \mu dt + \sigma dB_t, \tag{3}$$

where B is a standard Brownian motion under the probability measure $\mathbb{P}$. Over time increment dt, the firm's operating revenue is given by $K_t dA_t$, which is often referred to as the "AK" technology in the macroeconomics literature.

Then the firm's cumulative operating profit evolves as

$$dY_t = K_t\,(dA_t - c(i_t)\, dt), \tag{4}$$

where the first term is the incremental gross output and the second term is total cost of investment. Finally, we assume that the firm can liquidate its assets at any time. To preserve the linear homogeneity of our model, we assume the liquidation value L_t is proportional to the firm's capital with $L_t = lK_t$.

Now we turn to discuss the firm's cash inventory W_t. Following BCW, we model the cash holding cost generated by the agency problem in reduced-form. We assume the rate of return that the firm earns on W_t is the risk-free rate r minus a carry cost $\lambda > 0$. Hence the parameter λ measures the cost of cash holding. Besides cash accumulation, the firm can distribute cash to the shareholders as well. We denote U_t as the firm's cumulative payout to shareholders. Therefore, the firm's cash inventory evolves according to

$$dW_t = dY_t + (r - \lambda)\, W_t dt - dU_t, \tag{5}$$

which is a general accounting identity. In this paper, we only consider the liquidation case in BCW and leave the external financing case for the future research.

2.2. Belief distortions and model uncertainty

We now introduce belief distortions and the concerns for model uncertainty. The shareholders treat the probability measure $\mathbb{P}$ as the reference model. Suppose that the shareholders do not trust this model and consider alternative models to protect themselves from model misspecifications. Let $\mathbb{P}^h$ denotes the probability measure of the alternative model, where ξ_t is its Radon–Nikodym derivative with respect to $\mathbb{P}$,

$$\frac{d\xi_t}{\xi_t} = h_t dB_t, \tag{6}$$

where h_t is a real-valued process satisfying $\int_0^t h_s^2 ds < \infty$ for all $t > 0$, and where $\xi_0 = 1$. Then the process B_t^h defined by $dB_t^h =$

$dB_t - h_t dt$, is a standard Brownian motion under the measure $\mathbb{P}^h$. Under the new measure $\mathbb{P}^h$, the firm's cumulative productivity evolves as

$$dA_t = (\mu + \sigma h_t)\, dt + \sigma dB_t^h. \tag{7}$$

Following Anderson et al. (2003), Hansen et al. (2006), and Hansen and Sargent (2012), we employ the discounted relative entropy to measure the discrepancy between $\mathbb{P}^h$ and $\mathbb{P}$,

$$r\mathbb{E}^{\mathbb{P}}\left[\int_0^\infty e^{-rt}\xi_t \ln \xi_t dt\right] = \frac{1}{2}\mathbb{E}^{\mathbb{P}}\left[\int_0^\infty e^{-rt}\xi_t h_t^2 dt\right] = \frac{1}{2}\mathbb{E}^{\mathbb{P}^h}\left[\int_0^\infty e^{-rt} h_t^2 dt\right]. \tag{8}$$

To incorporate a concern for robustness of belief distortions, we present the firm's objective to maximize

$$\inf_h \mathbb{E}^{\mathbb{P}^h}\left[\int_0^\tau e^{-rt} dU_t + e^{-r\tau}\left(lK_\tau + W_\tau\right)\right] + \frac{1}{2\theta}\mathbb{E}^{\mathbb{P}^h}\left[\int_0^\tau e^{-rt} h_t^2 K_t dt\right], \tag{9}$$

where θ can be interpreted as an ambiguity aversion parameter. A small θ implies a small degree of concern for robustness. For instance, as θ converges to zero, the firm's objective reduces to the benchmark case without ambiguity as

$$\mathbb{E}^{\mathbb{P}}\left[\int_0^\tau e^{-rt} dU_t + e^{-r\tau}\left(lK_\tau + W_\tau\right)\right]. \tag{10}$$

It is worth mentioning that we introduce the capital stock into the second term of (9) to preserve the linear homogeneity of our model.

3. Model solution

Since firm value depends on two state variables, its cash inventory W and capital stock K, we denote $P(K, W)$ as firm value. Intuitively, due to the existence of carry cost, the firm should distribute cash to the shareholders for a sufficient high cash inventory. Denote $\overline{W}$ as the endogenous payout boundary. Then for $W < \overline{W}$, firm value $P(K, W)$ satisfies the following Hamilton–Jacobi–Bellman– Isaacs (HJBI) equation[2]

$$rP(K,W) = \max_I \inf_h (I - \delta K) P_K + [(r-\lambda) W + (\mu + \sigma h - c(i_t)) K] P_W + \frac{\sigma^2 K^2}{2} P_{WW} + \frac{h^2}{2\theta} K. \tag{11}$$

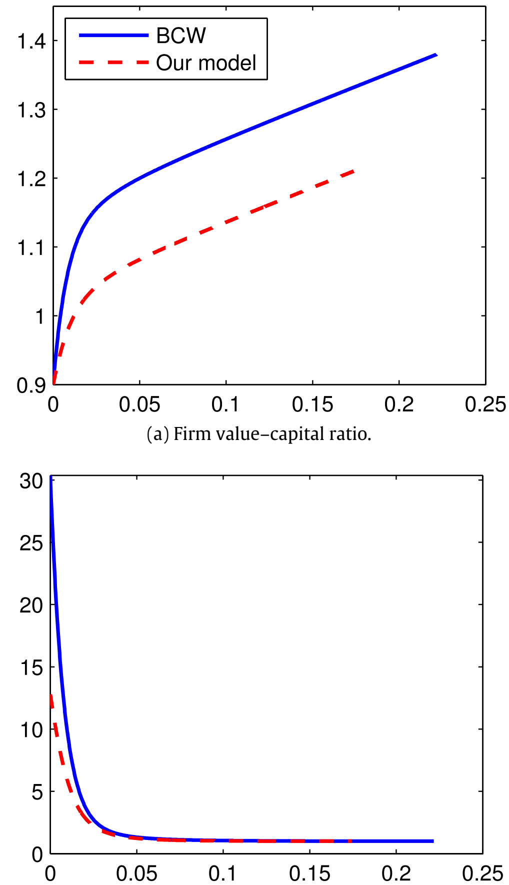
We find the right side of (11) is convex in h, thus there exists a unique solution for minimization $h = -\theta\sigma P_W$, which is time-varying and depends on the marginal value of cash. Substituting it back into (11) yields

$$rP(K,W) = \max_I (I - \delta K) P_K + [(r-\lambda) W + (\mu - c(i_t)) K] P_W + \frac{\sigma^2 K^2}{2} P_{WW} - \frac{\theta\sigma^2}{2} K P_W^2. \tag{12}$$

The convexity of the physical adjustment cost implies the optimal investment is an interior solution. Then the investment–capital ratio i satisfies the following first-order condition

$$c'(i) = \frac{P_K(K,W)}{P_W(K,W)}. \tag{13}$$

[2] As documented in Miao and Rivera (2016), it is technically challenging to provide a rigorous derivation of the HJBI equation. Such an analysis is beyond the scope of our paper.



(a) Firm value–capital ratio.

(b) Marginal value of cash.

Fig. 1. Firm value–capital ratio $p(w)$ and marginal value of cash $p'(w)$.

By virtue of homogeneity, we simplify our two-state optimization to a one-state problem. Thus we rewrite the firm value as

$$P(K,W) = K \cdot p(w), \tag{14}$$

where $w = W/K$ is the firm's cash–capital ratio. Note that the marginal value of capital is $P_K(K,W) = p(w) - wp'(w)$, the marginal value of cash is $P_W(K,W) = p'(w)$ and $P_{WW} = p''(w)/K$. Substituting these terms into (12), we obtain the following ODE

$$(r + \delta - i(w))\, p(w) = [(r + \delta - \lambda - i(w))\, w + \mu - c(i(w))]\, p'(w) + \frac{\sigma^2}{2} p''(w) - \frac{\theta\sigma^2}{2} p'(w)^2, \tag{15}$$

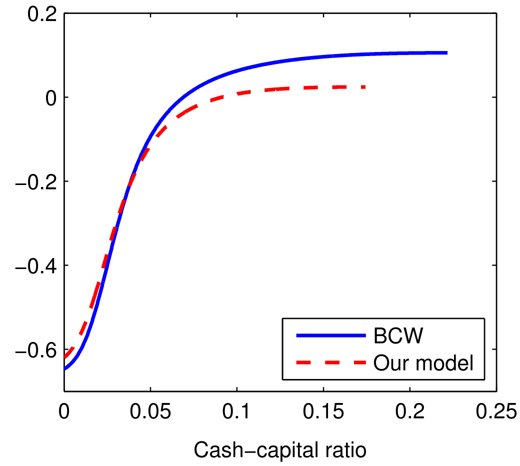
where

$$c'(i(w)) = \frac{p(w)}{p'(w)} - w. \tag{16}$$

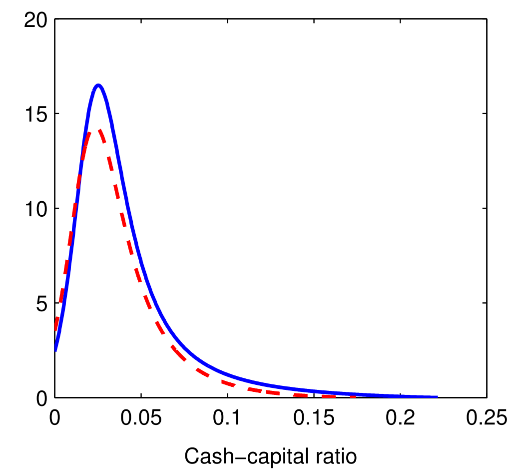
To completely characterize the solution for $p(w)$, we should determine the boundary condition. For $w > \overline{w}$, it is optimal for the firm to distribute the excess cash as a lump sum and bring the cash–capital ratio back to $\overline{w}$. Thus we have the following equation

$$p(w) = p(\overline{w}) + (w - \overline{w}). \tag{17}$$

Since (17) holds for all $w > \overline{w}$, we take the limit and derive the equation for the endogenous upper boundary $p'(\overline{w}) = 1$. Since the payout boundary is optimally chosen, we also have the

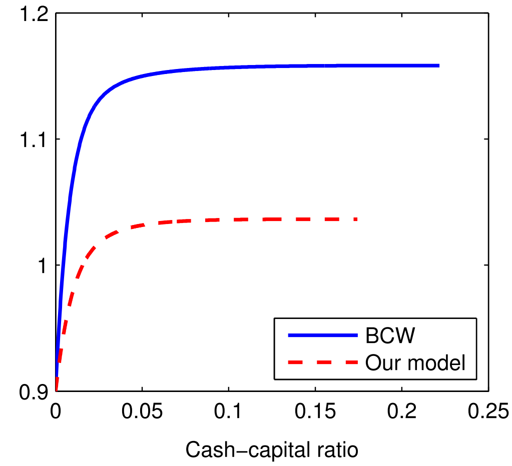


(a) Investment–capital ratio.

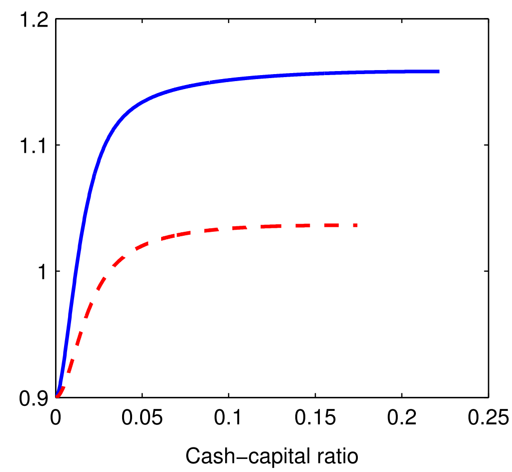


(b) Investment–cash sensitivity.

Fig. 2. Investment–capital ratio $i(w)$ and investment–capital sensitivity $i'(w)$.



(a) Average *q*.



(b) Marginal *q*.

Fig. 3. Average *q* and marginal *q*.

"super contact" condition (Dumas, 1991) $p''(\overline{w}) = 0$. For the lower boundary, BCW shows that it is optimal for the firm to wait until it runs out of cash. This argument still holds in our model. At the lower barrier $\underline{w} = 0$, the firm has to liquidate its assets, thus we have $p(0) = l$.

4. Quantitative results

In this section, we turn to analysis the quantitative results. Most of the parameter values are borrowed from BCW: the mean and volatility of the risk-adjusted productivity shock are $\mu = 18\%$ and $\sigma = 9\%$; the risk free rate is $r = 6\%$; the rate of depreciation is $\delta = 10.07\%$; the adjustment cost parameter is $\phi = 1.5$; the cash-carrying cost parameter is $\lambda = 1\%$ and the liquidation value is $l = 0.9$. In addition, we take the ambiguity aversion parameter as $\theta = 3$.

4.1. Scaled firm value

Fig. 1 plots the scaled firm value as a function of the firm's liquidity w. As is depicted in Panel A, the endogenous payout boundary in BCW is 0.22. For our model, this payout boundary drops to 0.17. As the saying goes "a bird in the hand is worth two in the bush". Since cash payout can be treated as a channel of avoiding model uncertainty, the shareholders prefer cash in hand to cash in firm's account. This effect accelerates firm's cash payout. It implies that uncertainty and business risk are treated in different ways since an increasing in business risk always induce the firm to delay its payout due to the precautionary motive. In addition, the firm value in our model is substantial lower than that in BCW due to the additional discount for model uncertainty. Panel B plots the marginal value of cash as a function of cash–capital ratio. And it shows that the marginal value of cash is significantly lower in our model with model uncertainty than that in BCW model when the firm's cash is low. For example, $p'(w)$ drops from 30 in BCW to 13 in our model when it exhausts the liquidity, i.e. $w = 0$. It is intuitive because model uncertainty lowers firm value and weakens the incentive to avoid liquidation. Thus the marginal value of cash is much lower in our model. As expected, as the firm has abundant cash, the marginal value approaches 1 both in BCW and in our model.

4.2. Optimal investment

We plot the optimal investment in Fig. 2. Panel A shows the investment–capital ratio $i(w)$. We find there exists a cutoff point w^* that $i(w) > i^{BCW}(w)$ for $w < w^*$ and $i(w) < i^{BCW}(w)$ for $w > w^*$. In other words, model uncertainty leads to both under-investment and under-disinvestment.

When the cash inventory is high, the shareholders with model uncertainty are more eager to cash payment. Thus the investment–capital ratio becomes lower due to model uncertainty. On the other way, as the firm becomes close to liquidation, it will do disinvestment to acquire cash. For the same reason, model uncertainty lowers the firm's continuation value, which weakens the shareholders' incentives to sustain the project. Hence model uncertainty leads to over-investment in this case.

Now we turn to analysis the investment–cash sensitivity. From (16), we derive

$$i'(w) = -\frac{p(w)\,p''(w)}{\phi p'(w)^2} > 0. \tag{18}$$

The concavity of p ensures that $i'(w)$ is positive, as shown in Panel B of Fig. 2. We find model uncertainty does not alter the pattern. Furthermore, the sensitivity $i'(w)$ is not monotonic in w as we cannot decide the sign of $i''(w)$, which is related to $p'''(w)$.

4.3. *Average q and marginal q*

In this subsection, we turn to discuss the model implication for average q and marginal q. Average q is the firm value net of cash divided by its capital stock $q_a(w) = \frac{P(K,W)-W}{K} = p(w) - w$. Similarly, marginal q is defined as $q_m(w) = \frac{d(P(K,W)-W)}{dK} = p(w) - wp'(w)$. Since the marginal value of cash is always larger than one, we have $q_m(w) < q_a(w)$. We plot the average q and marginal q in Fig. 3. With model uncertainty, we find both average q and marginal q are lower than their counterparts in BCW.

5. Conclusions

We incorporates model uncertainty into a dynamic model of corporate investment and liquidity management based on BCW. The firm's aversion to model uncertainty generates an endogenous belief distortion, which depends on its cash inventory. As the firm runs out of cash, it will become more pessimistic about firm's productivity. Our model predicts that concerns about model misspecification have ambiguous effects on the investment behavior, which depends on the firm's cash inventory. When the cash–capital ratio $w = W/K$ approaches the payout boundary, model uncertainty leads to underinvestment. In this region, the shareholders with ambiguity aversion are more eager to cash payment comparing to invest in firm's capital. However, the firm prefers underdisinvestment as the cash–capital ratio is close to zero. Since model uncertainty lowers firm's continuation value and weakens the shareholders' incentives to sustain the project. The effect of uncertainty on firm's liquidity management is shown to be drastically different from that of traditional uncertainty in the form of risk. Specifically, an increase in uncertainty accelerates firm's payout while an increase in risk delays it. Finally, it shows that the additional discount for model uncertainty significantly lowers a firm's average q and marginal q.

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