A Price-Based Demand Response Scheduling Model in Day-Ahead Electricity Market

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***Abstract*—The variation of power consumption on the demand side is a challenging issue for the real time balancing of power systems. To tackle this problem, various demand response programs have been introduced to help the Independent System Operator (ISO) in mitigating the demand fluctuation. Typically, they include demand curtailment programs and price responsive demand programs. This paper presents a scheduling model at the day-ahead stage incorporating the price-based demand bidding that enables the price-elastic feature of demand. The bidding mechanism is visualized and the mathematical representation of the scheduling model are presented. By simulating the model on the IEEE 30-bus system, it is shown that integrating price-elastic demand bids into day-ahead scheduling can effectively reduce the demand to average demand ratio. In addition, the proposed model not only brings surplus to the participating load serving entities (LSEs), but also increase the social welfare of the power system.**

***Index Terms*—Day-ahead scheduling, demand response, demand variation, price-elastic demand bid, social welfare.**

I. Introduction

In the operation of power system, the key issue is the real time balancing of active and reactive power between generation and demand. The variation of power consumption on the demand side is a main challenge for the ISO. Traditionally, most of the power systems in the world are operated in the way that the variation of demand is followed and balanced by the generation, which is called “load following”. However, the ability of generators to follow demand fluctuation is constrained by their inherent physical limits, such as ramping limits, minimum up and down time, etc. Furthermore, with more and more renewable energy resources being integrated into the grid, the intermittency and uncertainty in power systems is expected to reach a very high level. This has put more challenge to ISOs in balancing the power system.

In recent years, the introduction of demand response (DR) programs has brought new solutions for ISOs to tackle this issue at the day-ahead scheduling stage. DR is defined as the tariffs or programs that motivate changes in electricity consumption by customers within the electricity market scheme [1]. For ISOs, the responsive demand can be utilized as an important resource for system operation [2]. Currently, ISOs in the U.S. offer 2 major types of DR programs: the demand curtailment programs and the price responsive demand programs. In demand curtailment programs offered by PJM and NYISO, LSEs are compensated for bidding demand reduction into day-ahead market [3], [4]. In price responsive demand programs offered by PJM and CAISO, the aggregated demand of LSEs can be modified when price changes, without being centrally dispatched by ISOs [3], [5].

Day-ahead scheduling of price-based demand has recently been studied in [6]-[9]. In [6] and [9], demand shifting in response to the high price is studied. In [7], curtailable demands and shifting demands are treated separately in day-ahead unit commitment model. In [8], shifting demand is evaluated against security constraints in power systems.

In this paper, the bidding mechanism of the price-based demand is presented to study how the demand profile will be influenced. In addition, the gross surplus gained by LSEs from the price-elastic demand bidding is evaluated. The contributions in this paper are summarized as follows:

- The cleared price-elastic demands in the day-ahead market is effective in mitigating the demand fluctuation, which is incurred by the fixed demand.
- The bidding framework not only brings surplus to the participating LSEs that represents the consumers who are willing to adjust their demands, but also increase the social welfare in the power system.
- Influence to the market price is studied to show that the bill-saving intention of price-elastic demand bidding may lead to increase in price at some point.

The rest of the paper is organized as follows, Section II is a detailed description of how the price-based demand bidding is formulated. Section III describes the mathematical representation of day-ahead market clearing model. In Section IV, case studies are performed to evaluate the influence of price-based demand bidding. In the end, the paper is concluded in Section V.

II. Formulation of Price-Based Demand Bidding

During the day-ahead market bid period, LSEs can submit hourly demand bids to ISO. Traditionally, without DR programs offered by the ISO, the demand bids submitted from LSE are the fixed demand bids. With the availability of DR

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programs, the demand bids submitted from each LSE may include a new portion of demand that is responsive to the market price. This new portion of demand is called the price-elastic demand.

In the fixed demand bids, the bidding demand is inelastic to the market price. In other words, the customers are ready to accept any price for this portion of demand that is to be consumed in real time.

In the price-elastic demand bids, the demand is regarded as elastic to market price. The customers may select a portion of demand that they consider less critical to be price-elastic for the sake of bill saving. Currently, large customers (e.g. industrial factories, commercial buildings, etc.) are the main participants of demand bidding programs. Because their demands are more predictable to the ISO, they are more likely to perform DR as agreed if the bid is secured in the market, and the bill saving effect for them is enormous. But as electricity markets become transparent in different countries, small customers (e.g. residential customers) are also encouraged to participate, preferably through an aggregator such as LSE. For small customers, they may consider to bid a non-critical part of their demand as price elastic (e.g. electric heaters, air-conditioning, refrigeration, etc.) Bidding information of the price-elastic demand consists of the maximum price, the minimum price and the corresponding maximum price elastic demand. The ISO will schedule the volume of price-elastic demand for each LSE in each time slot using the ISO's market clear engine. The cleared price-elastic demand will be dispatched in real time.

The demand bidding curve of each LSE is shown in Figure 1. and 2. The bidding curve of demand can be separated into 2 parts. In Part 1, demand is a fixed value irrespective of price change. This part represents the fixed portion of total demand that customers must consume as base load. To satisfy this portion of demand, LSEs are willing to bid at any price to purchase it in the wholesale market. In Part 2, the price elastic demand decreases as the price arises. When the price is below the maximum price λ_e^{max} that is acceptable by the customers, customers may choose to increase their demand, on top of the fixed demand. Thus the price-elastic demand appears in the first quadrant as a positive value. The bidding curve for the price elastic demand can be mathematically represented as:

$$\lambda_{ej}^t = \lambda_{ej}^{max} - k_{ej} \times P_{ej}^t$$
$$= \lambda_{ej}^{max} - k_{ej} \times \left(P_{dj}^t - P_{fj}^t\right) \quad (1)$$

where λ_{ej}^t is the bidding price of elastic portion of demand for the LSE at bus j in hour t. λ_{ej}^{max} is maximum bidding price of elastic demand for the the LSE at bus j in hour t. $-k_{ej}$ is the slope of the price-elastic portion of the demand bidding curve for the LSE at bus j. P_{dj}^t is the total bidding demand for the LSE at bus j in hour t. P_{fj}^t is the fixed demand bid for the LSE at bus j in hour t.

The shaded area shown in Figure 2. represents the gross surplus for LSEs which is brought by the cleared price-elastic demand bids in the day-ahead market. Commonly, LSE's benefit (or profit) is defined as its revenue gained by selling electricity at the retail price, less the purchasing cost at the

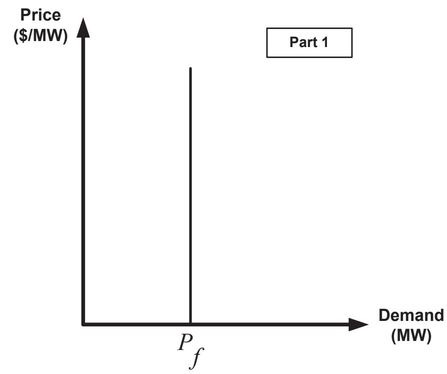

Figure 1. LSE's bidding curve of fixed demand.

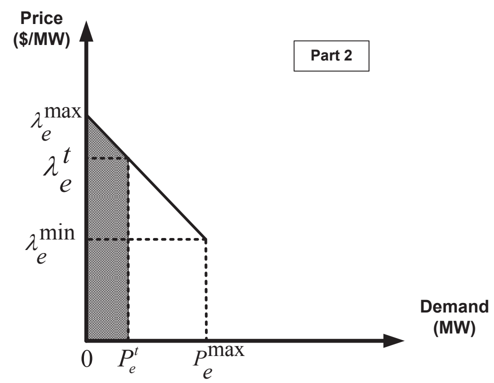

Figure 2. LSE's bidding curve of price-elastic demand.

wholesale market price [10]. For the end customers, they pay for their electricity consumption at the retail price. However, the scope of this paper is not the interaction between the LSE and its customers (e.g. time dependent tariffs, aggregation of demand response resources, etc.), but rather the ISO's scheduling of generators and LSEs at the bus level. Hence the retail price of electricity to customers is not further illustrated in this paper.

From the above illustrations, the gross surplus for the LSEs that is bidding price-elastic demand is equivalently the shaded area in Figure 2. Thus it can be mathematically represented as follows:

$$S\left(P_{dj}^t\right) = \int_{P_{fj}^t}^{P_{dj}^t} \lambda_{ej}^t \, dP_{dj}^t$$

$$= \left(\lambda_{ej}^{max} \times P_{dj}^t - 0.5k_{ej} \times {P_{dj}^t}^2 + k_{ej} \times P_{fj}^t \times P_{dj}^t\right) \Big|_{P_{fj}^t}^{P_{dj}^t}$$

$$= -0.5k_{ej} \times {P_{dj}^t}^2 + \left(k_{ej} \times P_{fj}^t + \lambda_{ej}^{max}\right) \times P_{dj}^t$$
$$-0.5k_{ej} \times {P_{fj}^t}^2 - \lambda_{ej}^{max} \times P_{fj}^t \quad (2)$$

This gross surplus function is a quadratic function with the variable of P_{dj}^t. In this function, P_{fj}^t is regarded as given. This function will be formulated in the objective function of the day-ahead market clearing model, which is described in detail in Section III.

III. Day-Ahead Market-Clearing Model

The accepted generation offers and demand bids will be cleared in the day-ahead market by the ISO. Traditionally, the objective for the ISO in the market clearing model is to achieve the least-cost means of generator commitment for the 24 hours on operating day. This is due to the fact that from the ISO's point of view, the cost to satisfy the forecasted demand comes entirely from generation, which is the only dispatchable resource in the scheduling. However, with the implementation of DR programs, the price-elastic demand has become another 'controllable' factor in power system scheduling. In this case, the new day-ahead market clearing model will aim for the maximization of social welfare. In this paper, market participants are considered to be GENCOs and LSEs. The social welfare is defined the difference between the value attached to the electrical energy purchased and the cost of producing this electricity [11]. The constraints of this optimization model consists of the active power flow equation, generator lower and upper limits, generator ramp up and down limits, the bus voltage limits and the demand limits.

The market clearing results will provide the ISO with the schedule of generation and price-elastic demand in each hour interval for the 24 hours in the operating day. In addition, the clearing price at which the market equilibrium is achieved will be calculated. In terms of the ISO, the advantages for the penetration of price-elastic demand are twofolds:

- The clearing price may fall on the price-elastic segment of the demand bidding curve, resulting in lower market price;
- The price-elastic demand may mitigate the variation of fixed demand, resulting in a more 'flat' demand profile that is desirable for real time balancing.

From the perspective of LSEs, involving in the price-elastic demand bidding will provide the consumers with the opportunity to adjust their electricity consumption in advance to real time operation. Currently, most of the real time DR programs assume that the participating customers are willing to adjust their activities in a very short duration (e.g. 5 minutes) after they receive the price signal before real time dispatch. But in fact, this assumption may not be practical as for most of the customers, deliberately turning ON/OFF the appliances or levelling up/down the power demand in such a short time may bring discomfort.

However, if the information of cleared price-elastic demand and the corresponding clearing price is issued to the participating customers at the day ahead stage, it is reasonable to assume that customers will follow the schedule and adjust their consumption in real time accordingly. As for the customers, if they are willing to bid a portion of their demand into the market to vary according to the market price, then this action will reduce their electricity bills, compared to bidding this portion of demand to be fixed.

The mathematical representation of the model is as follows:

Objective function that maximizes the social welfare:

$$\text{Max } SW = \sum_{t=1}^{T}\sum_{j=1}^{NB}\left[\mathrm{S}\left(P_{dj}^{t}\right) - \mathrm{C}\left(P_{gj}^{t}\right)\right] \tag{3}$$

The generator cost function is a quadratic function in the form of:

$$\mathrm{C}\left(\mathrm{P}_{gj}^{t}\right) = a_j(\mathrm{P}_{gj}^{t})^2 + b_j\mathrm{P}_{gj}^{t} + c_j \tag{4}$$

where P_{gj}^{t} is the active power generation at bus j in hour t. NB is the number of buses. T is the scheduling time horizon.

The objective function is then subject to the following constraints:

Power flow equations:

$$P_{gj}^{t} - P_{dj}^{t} = V_j^t \times \sum_{k=1}^{NB} V_k^t \times [G_{jk}\cos(\theta_{jk}) + B_{jk}\sin(\theta_{jk})] \tag{5}$$

$$Q_{gj}^{t} - Q_{dj}^{t} = V_j^t \times \sum_{k=1}^{NB} V_k^t \times [G_{jk}\sin(\theta_{jk}) - B_{jk}\cos(\theta_{jk})] \tag{6}$$

where V_j^t is the voltage magnitude at bus j in hour t, V_k^t is the voltage magnitude at bus k that is adjacent to bus j in hour t. θ_{jk} is the voltage angle difference between bus j and bus k. Q_{gj}^{t}, Q_{dj}^{t} are the reactive power generation and reactive power demand at bus j in hour t, respectively.

Power factor constraint:

$$Q_{dj}^{t} = P_{dj}^{t} \times \frac{\sqrt{1 - (cos\varphi_j^t)^2}}{cos\varphi_j^t} \tag{7}$$

where $cos\varphi_j^t$ is the power factor at bus j in hour t.

Generator limits:

$$P_{gj}^{min} \le P_{gj}^{t} \le P_{gj}^{max} \tag{8}$$

$$Q_{gj}^{min} \le Q_{gj}^{t} \le Q_{gj}^{max} \tag{9}$$

$$P_{gj}^{t+1} - P_{gj}^{t} \le \Delta P_{gj}^{up} \tag{10}$$

$$P_{gj}^{t} - P_{gj}^{t+1} \le \Delta P_{gj}^{down} \tag{11}$$

where ΔP_{gj}^{up} and ΔP_{gj}^{down} are the ramping up and down limits for the generator at bus j in hour t.

Bus voltage limits:

$$V_j^{min} \le V_j^t \le V_j^{max} \tag{12}$$

Demand limits:

$$P_{dj}^{min} \le P_{dj}^{t} \le P_{dj}^{max} \tag{13}$$

Using the above model, the ISO will be able to clear the market at the day-ahead scheduling stage. If required, further security constraints can be added by the ISO to evaluate the feasibility of the optimal solution against power system security limits. But as it is not in the scope of this paper, it will not be discussed further.

In summary, the optimal schedule achieved from model in (3) to (13) can provide least-cost generation commitment for generators, surplus gained for LSEs and also bill saving power demand consumption pattern for the participating customers. To further illustrate the influence of price-elastic demand bidding to the power system from the aspects of power system operation and power system economy, the detailed numerical results and analysis of the day-ahead market clearing model is presented in Section IV.

IV. Case Study

A. Data

In order to study the impact to various aspects of the power system with the participation of price-elastic demand bids, the market-clearing model described in Section III has been applied to the IEEE 30-bus system for simulation. The optimization model presented in (3) to (13) is programmed in GAMS [12]. In accordance with practical ISO scheduling, the time interval of the simulation is set to 1 hour and the time horizon $T = 24$ hours. In the applied 30-bus system, the 6 generators are located at buses 1, 2, 5, 8, 11 and 13. The 7 LSEs are located at buses 2, 5, 7, 8, 12, 21 and 30. Bus 1 is set to be the slack bus. The lower and upper voltage limits at all buses are set to be 0.96 p.u. and 1.06 p.u., respectively.

For the sake of simplicity and better comprehension of the impact brought by price-elastic demand bids, parameters related to generators should be kept the same. To be specific, generator cost functions are set to be the same between 6 generators, which is given below:

$$\mathrm{C}\left(\mathrm{P}_{gi}^{t}\right) = 0.04\left(\mathrm{P}_{gi}^{t}\right)^{2} + 7\mathrm{P}_{gi}^{t} + 10 \ \ (\$/h)$$

$$\forall i = 1,2,\dots,NG;\ t = 1,\dots,T. \quad (11)$$

In addition, the lower and upper limits of active and reactive generation of all the generators are set to be the same, which is taken from [13]. Ramping up and down limits of all the generators are set to be 50 MW/hr.

In terms of the demand side, the data settings are as follows: the lower and upper limits for the price-elastic portion of the demand of all LSEs are set to be 0 and 60 MWs, respectively. For better comparison, the fixed portion of the demand at each bus is set to be a given value that varies over time, which is produced according to the typical variation of daily demand in PJM market. The power factor is set to be 0.9 lagging, which is same at all buses in each hour. The parameter setting for the demand bidding curve of every LSE are given as: $\lambda_{el}^{max} =$ 45 \$/MWh, $k_{el} = 0.6$ \$/MW^{2}h. The maximum bidding price value is selected with reference to the maximum marginal price of the optimal power flow calculation in standard IEEE 30-bus system. Slope of the demand bidding curve is kept as same among all LSEs at each bus [8]. If required, this parameter can set to be different to show the influence of customer elasticity to the various aspects of the system.

B. Cases

Base Case:

In this case, the LSEs at all buses are only submitting fixed demand bids to the market. There is no price-elastic bid submitted into the market. The representative demand profile for the base case is the fixed demand curve, as given. Demand peak occurs around 8:00am, 12:00pm and 5:00pm of the day, which corresponds to times with potential high demand for electricity due to human activities. On the other hand, demand valley occurs around 3:00am, when most people are asleep, and 3:00pm of the day, when most people are not at home. These corresponds to the times with low demand for electricity. For clarity, Base Case demands are represented as fixed demand curves in the simulation results.

Case I:

In this case, the LSE at bus 8 is bidding both price-elastic demand and fixed demand. The LSEs at the buses 2, 5, 7, 12, 21 and 30 are bidding fixed demand only. The impact to the various aspects of power system after adding price-elastic demand at one single bus is simulated and analyzed.

Case II:

In this case, the LSEs at buses 2, 5, 7, 8, 12, 21 and 30 are all bidding both price-elastic demand and fixed demand. The impact to the various aspects of power system after adding price-elastic demand at all buses is simulated and analyzed.

C. Simulation Results

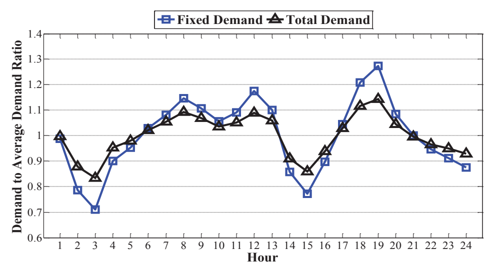

Figure 3. Demand to average demand ratio in each hour of 24 hours at bus 8.

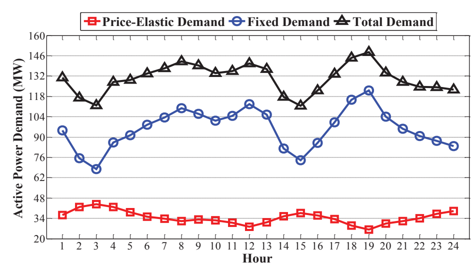

Figure 4. Scheduled power demand at each hour of 24 hours at bus 8 in Case II.

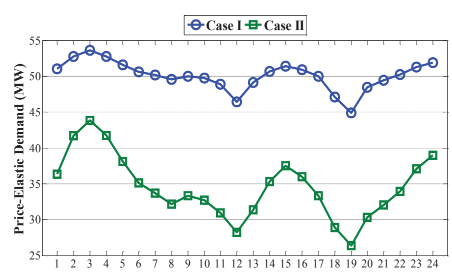

Figure 5. Cleared price-elastic demand in each hour of 24 hours at bus 8.

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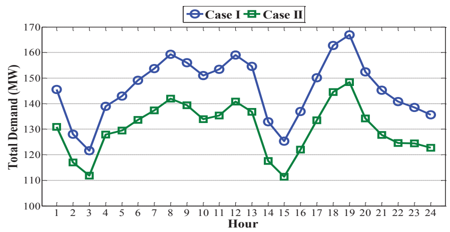

Figure 6. Total demand in each hour of 24 hours at bus 8.

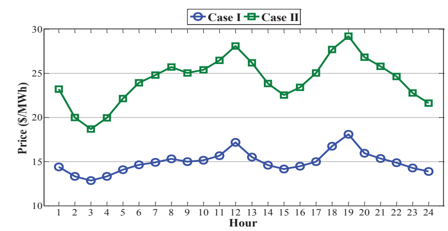

Figure 7. Clearing price of price-elastic demand bids in each hour of 24 hours at bus 8.

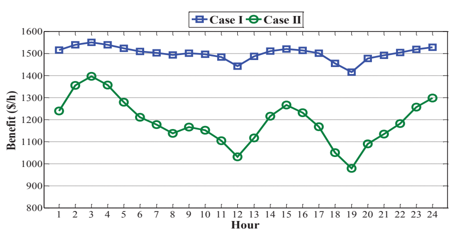

Figure 8. Gross surplus of the LSE in each hour at bus 8.

D. Discussion

In the sense of power system operation, it can be seen in Figure 3 and Figure 4 that, with the adding of price-elastic demand, the variation of the total demand is mitigated compared to that in the original fixed demand. This validates that the presented model can help the ISO in achieving a more "flat" demand curve at the day-ahead scheduling stage. Note that the demand to average demand ratio is same as in Case I and Case II, hence they are plotted in one single graph. This further indicates that the population of price-elastic demand bids in the system does not affect the degree of demand fluctuation at each bus, although the cleared demand will vary between different cases, as shown in Figure 5 and Figure 6.

From the aspect of market and economy, it can be seen from Figure 7 that, with the increase in the population of LSEs that are bidding price-elastic demand in the whole power system, the clearing price for the LSE at bus 8 is increased. This results from the reduced price-elastic demand and the corresponding total demand at bus 8, as shown in Figure 5 and Figure 6. On the other hand, the gross surplus gained by LSE at bus 8 is reduced in Case II. By comparing Figure 7 and Figure 8, it can be observed that the trends of price change and gross surplus change are contrast to each other. Figure 4 is presented to illustrate different parts of the cleared demand under Case II. It can be observed that the variation of price-elastic demand is in contrast with that in the fixed demand, mitigating the variation in the fixed demand.

From these observations, it can be concluded that from the ISO's point of view, the introduction of price-elastic demand can effectively mitigate the variation in the fixed demand. However, the increasing population of price-elastic demand bids in the system will lead to the decrease in every LSE's cleared price-elastic demand. Correspondingly, the price will increase and the gross surplus for LSEs will decrease. But from the ISO's perspective, if more LSEs are willing to bid a portion of their demand to be price-elastic, then the variation of demand at the corresponding buses will be mitigated. This may leads to lower requirement of system spinning reserve, and eventually providing more flexibility to the power system.

V. Conclusion

Under the background of increasing interests in DR programs from the major ISOs in the U.S., in this paper, the day-ahead price-based demand scheduling model is presented. It is shown in the paper that, the presented model can generate proper price-elastic demand bids that is able to effectively mitigate the variation of the original fixed demand bids. This is desirable for the ISO in the real time operation, assuming that the LSEs will follow the cleared day-ahead price-elastic demand bids in real time operation. It has also been presented in the paper that, with increasing population of price-elastic demand bids, the overall gross surplus gained from LSEs will increase, however, the clearing price will also increase, which will lead to higher electricity bills that is paid by the customers.

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