Multiplexing Analysis of Millimeter-Wave Massive MIMO Systems

Dian-Wu Yue, Ha H. Nguyen and Shuai Xu

***Abstract*—This paper is concerned with spatial multiplexing analysis for millimeter-wave (mmWave) massive MIMO systems. For a single-user mmWave system employing distributed antenna subarray architecture in which the transmitter and receiver consist of K_t and K_r subarrays, respectively, an asymptotic multiplexing gain formula is firstly derived when the numbers of antennas at subarrays go to infinity. Specifically, assuming that all subchannels have the same number of propagation paths L, the formula states that by employing such a distributed antenna-subarray architecture, an exact multiplexing gain of N_s can be achieved, where $N_s \leq K_r K_t L$ is the number of data streams. This result means that compared to the co-located antenna architecture, using the distributed antenna-subarray architecture can scale up the maximum multiplexing gain proportionally to $K_r K_t$. In order to further reveal the relation between diversity gain and multiplexing gain, a simple characterization of the diversity-multiplexing tradeoff is also given. The multiplexing gain analysis is then extended to the multiuser scenario. Moreover, simulation results obtained with the hybrid analog/digital processing corroborate the analysis results.**

***Index Terms*—Millimeter-wave communications, massive MIMO, multiplexing gain, diversity gain, diversity-multiplexing tradeoff, distributed antenna-subarrays, hybrid precoding.**

I. Introduction

Recently, millimeter-wave (mmWave) communication has gained considerable attention as a candidate technology for 5G mobile communication systems and beyond [1]–[3]. The main reason for this is the availability of vast spectrum in the mmWave band (typically 30-300 GHz) that is very attractive for high data rate communications. However, compared to communication systems operating at lower microwave frequencies (such as those currently used for 4G mobile communications), propagation loss in mmWave frequencies is much higher, in the orders-of-magnitude. Fortunately, given the much smaller carrier wavelengths, mmWave communication systems can make use of compact massive antenna arrays to compensate for the increased propagation loss.

Nevertheless, the large-scale antenna arrays together with high cost and large power consumption of the mixed analog/digital signal components makes it difficult to equip a separate radio-frequency (RF) chain for each antenna and perform all the signal processing in the baseband. Therefore, research on *hybrid* analog-digital processing of precoder and combiner for mmWave communication systems has attracted very strong interests from both academia and industry [4] – [16]. In particular, a lot of work has been performed to address challenges in using a limited number of RF chains. For example, the authors in [4] considered single-user precoding in mmWave massive MIMO systems and established the optimality of beam steering for both single-stream and multi-stream transmission scenarios. In [10], the authors showed that hybrid processing can realize any fully digital processing if the number of RF chains is twice the number of data streams. However, due to the fact that mmWave signal propagation has an important feature of multipath sparsity in both the temporal and spatial domains [17]–[20], it is expected that the potentially available benefits of diversity and multiplexing are indeed not large if the deployment of the antenna arrays is co-located. In order to enlarge diversity/multiplexing gains in mmWave massive MIMO communication systems, this paper consider the use of a more general array architecture, called *distributed antenna subarray architecture*, which includes lo-located array architecture as a special case. It is pointed out that distributed antenna systems have received strong interest as a promising technique to satisfy such growing demands for future wireless communication networks due to the increased spectral efficiency and expanded coverage [21] − [25].

It is well known that diversity-multiplexing tradeoff (DMT) is a compact and convenient framework to compare different MIMO systems in terms of the two main and related system indicators: data rate and error performance [26]–[31]. This tradeoff was originally characterized by Zheng and Tse [26] for MIMO communication systems operating over i.i.d. Rayleigh fading channels. The framework has then ignited a lot of interests in analyzing various communication systems and under different channel models. For a mmWave massive MIMO system, how to quantify the diversity and multiplexing performance and further characterize its DMT is a fundamental and open research problem. In particular, to the best of our knowledge, until now there is no unified multiplexing gain analysis for mmWave massive MIMO systems that is applicable to both co-located and distributed antenna array architectures.

To fill this gap, this paper investigates the multiplexing performance of mmWave massive MIMO systems with the proposed distributed subarray architecture. The focus is on the asymptotical multiplexing gain analysis in order to find out the potential multiplexing advantage provided by multiple

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distributed antenna arrays. The obtained analysis can be used conveniently to compare various mmWave massive MIMO systems with different distributed antenna array structures.

The main contributions of this paper are summarized as follows:

- For a single-user system with the proposed distributed subarray architecture, a multiplexing gain expression is obtained when the number of antennas at each subarray increases without bound. This expression clearly indicates that one can obtain a large multiplexing gain by employing the distributed subarray architecture.
- A simple DMT characterization is further given. It can reveal the relation between diversity gain and multiplexing gain and let us obtain insights to understand the overall resources provided by the distributed antenna architecture.
- The multiplexing gain analysis is then extended to the multiuser scenario with downlink and uplink transmission.
- Simulation results are provided to corroborate the analysis results and show that the distributed subarray architecture yields significantly better multiplexing performance than the co-located single-array architecture.

The remainder of this paper is organized as follows. Section II describes the massive MIMO system model and hybrid processing with the distributed subarray architecture in mmWave fading channels. Section III and Section IV provides the asymptotical achievable rate analysis and the multiplexing gain analysis for the single-user mmWave system, respectively. In Section V, the multiplexing gain analysis is extended to the multiuser scenario. Section VI concludes the paper.

Throughout this paper, the following notations are used. Boldface upper and lower case letters denote matrices and column vectors, respectively. The superscripts $(\cdot)^T$ and $(\cdot)^H$ stand for transpose and conjugate-transpose, respectively. $\mathrm{diag}\{a_1, a_2, \ldots, a_N\}$ stands for a diagonal matrix with diagonal elements $\{a_1, a_2, \ldots, a_N\}$. The expectation operator is denoted by $\mathbb{E}(\cdot)$. $[\mathbf{A}]_{ij}$ gives the (i, j)th entry of matrix $\mathbf{A}$. $\mathbf{A} \bigotimes \mathbf{B}$ is the Kronecker product of $\mathbf{A}$ and $\mathbf{B}$. We write a function $a(x)$ of x as $o(x)$ if $\lim_{x\to 0} a(x)/x = 0$. We use $(x)^+$ to denote $\max\{0, x\}$. Finally, $\mathcal{CN}(0, 1)$ denotes a circularly symmetric complex Gaussian random variable with zero mean and unit variance.

II. System Model

Consider a single-user mmWave massive MIMO system as shown in Fig. 1. The transmitter is equipped with a distributed antenna array to send N_s data streams to a receiver, which is also equipped with a distributed antenna array. Here, a distributed antenna array means an array consisting of several remote antenna units (RAUs) (i.e., antenna subarrays) that are distributively located, as depicted in Fig. 2. Specifically, the antenna array at the transmitter consists of K_t RAUs, each of which has N_t antennas and is connected to a baseband processing unit (BPU) by fiber. Likewise, the distributed antenna array at the receiver consists of K_r RAUs, each having N_r antennas and also being connected to a BPU by fibers. Such a MIMO system shall be referred to as a (K_t, N_t, K_r, N_r) distributed MIMO (D-MIMO) system. When $K_t = K_r = 1$, the system reduces to a conventional co-located MIMO (C-MIMO) system.

The transmitter accepts as its input N_s data streams and is equipped with $N_t^{(\mathrm{rf})}$ RF chains, where $N_s \le N_t^{(\mathrm{rf})} \le N_t K_t$. Given $N_t^{(\mathrm{rf})}$ transmit RF chains, the transmitter can apply a low-dimension $N_t^{(\mathrm{rf})} \times N_s$ baseband precoder, $\mathbf{W}_t$, followed by a high-dimension $K_t N_t \times N_t^{(\mathrm{rf})}$ RF precoder, $\mathbf{F}_t$. Note that amplitude and phase modifications are feasible for the baseband precoder $\mathbf{W}_t$, while only phase changes can be made by the RF precoder $\mathbf{F}_t$ through the use of variable phase shifters and combiners. The transmitted signal vector can be written as

$$\mathbf{x} = \mathbf{F}_t \mathbf{W}_t \mathbf{P}_t^{1/2} \mathbf{s}, \tag{1}$$

where $\mathbf{P}_t = [p_{ij}]$ is a diagonal power allocation matrix with $\sum_{l=1}^{N_s} p_{ll} = 1$ and $\mathbf{s}$ is the $N_s \times 1$ symbol vector such that $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = P\mathbf{I}_{N_s}$. Thus P represents the average total input power. Considering a narrowband block fading channel, the $K_r N_r \times 1$ received signal vector is

$$\mathbf{y} = \mathbf{H}\mathbf{F}_t \mathbf{W}_t \mathbf{P}_t^{1/2} \mathbf{s} + \mathbf{n} \tag{2}$$

where $\mathbf{H}$ is $K_r N_r \times K_t N_t$ channel matrix and $\mathbf{n}$ is a $K_r N_r \times 1$ vector consisting of i.i.d. $\mathcal{CN}(0, 1)$ noise samples. Throughout this paper, $\mathbf{H}$ is assumed known to both the transmitter and receiver. Given that $N_r^{(\mathrm{rf})}$ RF chains (where $N_s \le N_r^{(\mathrm{rf})} \le N_r K_r$) are used at the receiver to detect the N_s data streams, the processed signal is given by

$$\mathbf{z} = \mathbf{W}_r^H \mathbf{F}_r^H \mathbf{H}\mathbf{F}_t \mathbf{W}_t \mathbf{P}_t^{1/2} \mathbf{s} + \mathbf{W}_r^H \mathbf{F}_r^H \mathbf{n} \tag{3}$$

where $\mathbf{F}_r$ is the $K_r N_r \times N_r^{(\mathrm{rf})}$ RF combining matrix, and $\mathbf{W}_r$ is the $N_r^{(\mathrm{rf})} \times N_s$ baseband combining matrix. When Gaussian symbols are transmitted over the mmWave channel, the the system achievable rate is expressed as

$$R = \log_2 |\mathbf{I}_{N_s} + P\mathbf{R}_n^{-1} \mathbf{W}_r^H \mathbf{F}_r^H \mathbf{H}\mathbf{F}_t \mathbf{W}_t \mathbf{P}_t \mathbf{W}_t^H \mathbf{F}_t^H \mathbf{H}^H \mathbf{F}_r \mathbf{W}_r| \tag{4}$$

where $\mathbf{R}_n = \mathbf{W}_r^H \mathbf{F}_r^H \mathbf{F}_r \mathbf{W}_r$.

Furthermore, according to the architecture of RAUs at the transmitting and receiving ends, $\mathbf{H}$ can be written as

$$\mathbf{H} = \begin{bmatrix} \sqrt{g_{11}}\mathbf{H}_{11} & \cdots & \sqrt{g_{1K_t}}\mathbf{H}_{1K_t} \\ \vdots & \ddots & \vdots \\ \sqrt{g_{K_r1}}\mathbf{H}_{K_r1} & \cdots & \sqrt{g_{K_rK_t}}\mathbf{H}_{K_rK_t} \end{bmatrix}. \tag{5}$$

In the above expression, g_{ij} represents the large scale fading effect between the ith RAU at the receiver and the jth RAU at the transmitter, which is assumed to be constant over many coherence-time intervals. The normalized subchannel matrix $\mathbf{H}_{ij}$ represents the MIMO channel between the jth RAU at the transmitter and the ith RAU at the receiver. We assume that all of $\{\mathbf{H}_{ij}\}$ are independent mutually each other.

A clustered channel model based on the extended Saleh-Valenzuela model is often used in mmWave channel modeling and standardization [4] and it is also adopted in this paper. For simplicity of exposition, each scattering cluster is assumed to

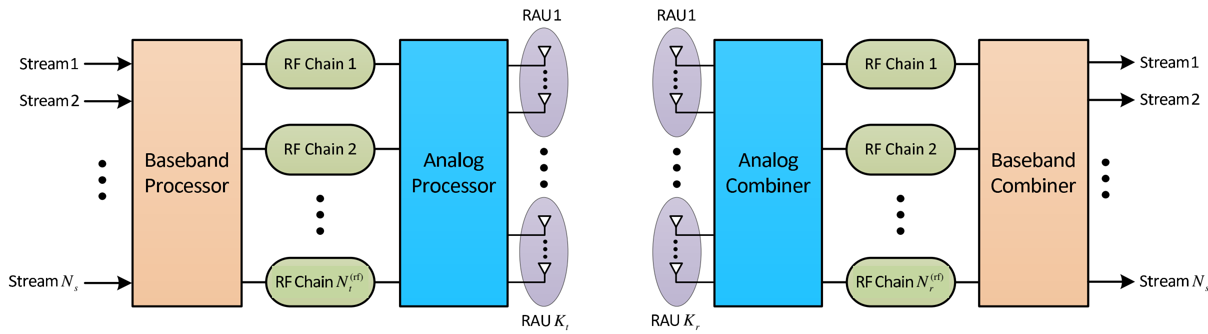

Fig. 1. Block diagram of a mmWave massive MIMO system with distributed antenna arrays.

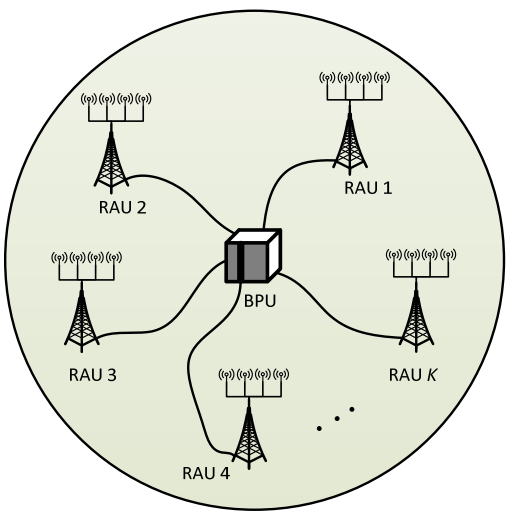

Fig. 2. Illustration of distributed antenna array deployment.

contribute a single propagation path.[1] Using this model, the subchannel matrix $\mathbf{H}_{ij}$ is given by

$$\mathbf{H}_{ij} = \sqrt{\frac{N_t N_r}{L_{ij}}} \sum_{l=1}^{L_{ij}} \alpha_{ij}^l \mathbf{a}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl}) \mathbf{a}_t^H(\phi_{ij}^{tl}, \theta_{ij}^{tl}), \tag{6}$$

where L_{ij} is the number of propagation paths, α_{ij}^l is the complex gain of the lth ray, and ϕ_{ij}^{rl} (θ_{ij}^{rl}) and ϕ_{ij}^{tl} (θ_{ij}^{tl}) are its random azimuth (elevation) angles of arrival and departure, respectively. Without loss of generality, the complex gains α_{ij}^l are assumed to be $\mathcal{CN}(0,1)$. [2] The vectors $\mathbf{a}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl})$ and $\mathbf{a}_t(\phi_{ij}^{tl}, \theta_{ij}^{tl})$ are the normalized receive/transmit array response vectors at the corresponding angles of arrival/departure. For an N-element uniform linear array (ULA) , the array response vector is

$$\mathbf{a}^{\mathrm{ULA}}(\phi) = \frac{1}{\sqrt{N}} \left[1, \mathrm{e}^{j2\pi \frac{d_u}{\lambda} \sin(\phi)}, \ldots, \mathrm{e}^{j2\pi (N-1) \frac{d_u}{\lambda} \sin(\phi)}\right]^T \tag{7}$$

where λ is the wavelength of the carrier and d_u is the inter-element spacing. It is pointed out that the angle θ is not included in the argument of $\mathbf{a}^{\mathrm{ULA}}$ since the response for an ULA is independent of the elevation angle. In contrast, for a uniform planar array (UPA), which is composed of N_h and N_v antenna elements in the horizontal and vertical directions, respectively, the array response vector is represented by

$$\mathbf{a}^{\mathrm{UPA}}(\phi, \theta) = \mathbf{a}_h^{\mathrm{ULA}}(\phi) \otimes \mathbf{a}_v^{\mathrm{ULA}}(\theta), \tag{8}$$

where

$$\mathbf{a}_h^{\mathrm{ULA}}(\phi) = \frac{1}{\sqrt{N_h}} \left[1, \mathrm{e}^{j2\pi \frac{d_h}{\lambda} \sin(\phi)}, \ldots, \mathrm{e}^{j2\pi (N_h-1) \frac{d_h}{\lambda} \sin(\phi)}\right]^T \tag{9}$$

and

$$\mathbf{a}_v^{\mathrm{ULA}}(\theta) = \frac{1}{\sqrt{N_v}} \left[1, \mathrm{e}^{j2\pi \frac{d_v}{\lambda} \sin(\theta)}, \ldots, \mathrm{e}^{j2\pi (N^v-1) \frac{d_v}{\lambda} \sin(\theta)}\right]^T. \tag{10}$$

III. Asymptotic Achievable Rate Analysis

From the structure and definition of the channel matrix $\mathbf{H}$ in Section II, there is a total of $L_s = \sum_{i=1}^{K_r} \sum_{j=1}^{K_t} L_{ij}$ propagation paths. Naturally, $\mathbf{H}$ can be decomposed into a sum of L_s rank-one matrices, each corresponding to one propagation path. Specifically, $\mathbf{H}$ can be rewritten as

$$\mathbf{H} = \sum_{i=1}^{K_r} \sum_{j=1}^{K_t} \sum_{l=1}^{L_{ij}} \tilde{\alpha}_{ij}^l \tilde{\mathbf{a}}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl}) \tilde{\mathbf{a}}_t^H(\phi_{ij}^{tl}, \theta_{ij}^{tl}), \tag{11}$$

where

$$\tilde{\alpha}_{ij}^l = \sqrt{g_{ij} \frac{N_t N_r}{L_{ij}}} \alpha_{ij}^l, \tag{12}$$

$\tilde{\mathbf{a}}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl})$ is a $K_r N_r \times 1$ vector whose bth entry is defined as

$$[\tilde{\mathbf{a}}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl})]_b = \begin{cases} [\mathbf{a}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl})]_{b-(i-1)N_r}, & b \in Q_i^r \\ 0, & b \notin Q_i^r \end{cases} \tag{13}$$

where $Q_i^r = ((i-1)N_r, iN_r]$. And $\tilde{\mathbf{a}}_t(\phi_{ij}^{tl}, \theta_{ij}^{tl})$ is a $K_t N_t \times 1$ vector whose bth entry is defined as

$$[\tilde{\mathbf{a}}_t(\phi_{ij}^{tl}, \theta_{ij}^{tl})]_b = \begin{cases} [\mathbf{a}_t(\phi_{ij}^{tl}, \theta_{ij}^{tl})]_{b-(j-1)N_t}, & b \in Q_j^t \\ 0, & b \notin Q_j^t. \end{cases} \tag{14}$$

where $Q_j^t = ((j-1)N_t, jN_t]$. Regarding $\{\tilde{\mathbf{a}}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl})\}$ and $\{\tilde{\mathbf{a}}_t(\phi_{ij}^{tl}, \theta_{ij}^{tl})\}$, we have the following lemma from [32].

Lemma 1: Suppose that the antenna configurations at all RAUs are either ULA or UPA. Then all L_s vectors

[1]This assumption can be relaxed to account for clusters with finite angular spreads and the results obtained in this paper can be readily extended for such a case.

[2]The different variances of α_{ij}^l can easily accounted for by absorbing into the large scale fading coefficients g_{ij}.

$\{\tilde{\mathbf{a}}_r(\phi_{ij}^{rl},\theta_{ij}^{rl})\}$ are orthogonal to each other when $N_r \to \infty$. Likewise, all L_s vectors $\{\tilde{\mathbf{a}}_t(\phi_{ij}^{tl},\theta_{ij}^{tl})\}$ are orthogonal to each other when $N_t \to \infty$.

Mathematically, the distributed massive MIMO system can be considered as a co-located massive MIMO system with L_s paths that have complex gains $\{\tilde{\alpha}_{ij}^l\}$, receive array response vectors $\{\tilde{\mathbf{a}}_r(\phi_{ij}^{rl},\theta_{ij}^{rl})\}$ and transmit response vectors $\{\tilde{\mathbf{a}}_t(\phi_{ij}^{tl},\theta_{ij}^{tl})\}$. Furthermore, order all paths in a decreasing order of the absolute values of the complex gains $\{\tilde{\alpha}_{ij}^l\}$. Then the channel matrix can be written as

$$\mathbf{H} = \sum_{l=1}^{L_s} \tilde{\alpha}^l \tilde{\mathbf{a}}_r(\phi^{rl},\theta^{rl})\tilde{\mathbf{a}}_t(\phi^{tl},\theta^{tl})^H, \tag{15}$$

where $\tilde{\alpha}^1 \geq \tilde{\alpha}^2 \geq \cdots \geq \tilde{\alpha}^{L_s}$.

One can rewrite $\mathbf{H}$ in a matrix form as

$$\mathbf{H} = \mathbf{A}_r \mathbf{D} \mathbf{A}_t^H \tag{16}$$

where $\mathbf{D}$ is a $L_s \times L_s$ diagonal matrix with $[\mathbf{D}]_{ll} = \tilde{\alpha}^l$, and $\mathbf{A}_r$ and $\mathbf{A}_t$ are defined as follows:

$$\mathbf{A}_r = [\tilde{\mathbf{a}}_r(\phi^{r1},\theta^{r1}),\ldots,\tilde{\mathbf{a}}_r(\phi^{rL_s},\theta^{rL_s})] \tag{17}$$

and

$$\mathbf{A}_t = [\tilde{\mathbf{a}}_t(\phi^{t1},\theta^{t1}),\ldots,\tilde{\mathbf{a}}_t(\phi^{tL_s},\theta^{tL_s})]. \tag{18}$$

Since both $\{\tilde{\mathbf{a}}_r(\phi^{rl},\theta^{rl})\}$ and $\{\tilde{\mathbf{a}}_t(\phi^{tl},\theta^{tl})\}$ are orthogonal vector sets when $N_r \to \infty$ and $N_t \to \infty$, $\mathbf{A}_r$ and $\mathbf{A}_t$ are asymptotically unitary matrices. Then one can form a singular value decomposition (SVD) of matrix $\mathbf{H}$ as

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H = [\mathbf{A}_r|\mathbf{A}_r^{\perp}]\boldsymbol{\Sigma}[\tilde{\mathbf{A}}_t|\tilde{\mathbf{A}}_t^{\perp}]^H \tag{19}$$

where $\boldsymbol{\Sigma}$ is a diagonal matrix containing all singular values on its diagonal, i.e.,

$$[\boldsymbol{\Sigma}]_{ll} = \begin{cases} |\tilde{\alpha}^l|, & \text{for } 1 \leq l \leq L_s \\ 0, & \text{for } l > L_s \end{cases} \tag{20}$$

and the submatrix $\tilde{\mathbf{A}}_t$ is defined as

$$\tilde{\mathbf{A}}_t = [\mathrm{e}^{-j\psi_1}\tilde{\mathbf{a}}_t(\phi^{t1},\theta^{t1}),\ldots,\mathrm{e}^{-j\psi_{L_s}}\tilde{\mathbf{a}}_t(\phi^{tL_s},\theta^{tL_s})] \tag{21}$$

where ψ_l is the phase of complex gain $\tilde{\alpha}^l$ corresponding to the lth path. Based on (19), the optimal precoder and combiner are chosen, respectively, as

$$[\mathbf{F}_t\mathbf{W}_t]_{\text{opt}} = [\mathrm{e}^{-j\psi_1}\tilde{\mathbf{a}}_t(\phi^{t1},\theta^{t1}),\ldots,\mathrm{e}^{-j\psi_{L_s}}\tilde{\mathbf{a}}_t(\phi^{tN_s},\theta^{tN_s})] \tag{22}$$

and

$$[\mathbf{F}_r\mathbf{W}_r]_{\text{opt}} = [\tilde{\mathbf{a}}_r(\phi^{r1},\theta^{r1}),\ldots,\tilde{\mathbf{a}}_r(\phi^{rN_s},\theta^{rN_s})]. \tag{23}$$

To summarize, when N_t and N_r are large enough, the massive MIMO system can employ the optimal precoder and combiner given in (22) and (23), respectively.

Now suppose that $\tilde{\alpha}^l = \tilde{\alpha}_{ij}^{l'} = \sqrt{g_{ij}\frac{N_tN_r}{L_{ij}}}\alpha_{ij}^{l'}$ for a given l. We introduce two notations:

$$\tilde{\gamma}_l = Pp_{ll}g_{ij}\frac{N_tN_r}{L_{ij}} \tag{24}$$

and

$$\tilde{\beta}_l = \alpha_{ij}^{l'}. \tag{25}$$

Then it follows from the above SVD analysis that the instantaneous SNR of the lth data stream is given by

$$\mathrm{SNR}_l = Pp_{ll}|\tilde{\alpha}^l|^2 = \tilde{\gamma}_l|\tilde{\beta}_l|^2, \quad l = 1,2,\ldots,N_s. \tag{26}$$

So we obtain another lemma.

Lemma 2: Suppose that both sets $\{\tilde{\mathbf{a}}_r(\phi_{ij}^{rl},\theta_{ij}^{rl})\}$ and $\{\tilde{\mathbf{a}}_t(\phi_{ij}^{tl},\theta_{ij}^{tl})\}$ are orthogonal vector sets when $N_r \to \infty$ and $N_t \to \infty$. Let $N_s \leq L_s$. In the limit of large N_t and N_r, then the system achievable rate is given by

$$R = \sum_{l=1}^{Ns}\log_2(1+\tilde{\gamma}_l|\tilde{\beta}_l|^2). \tag{27}$$

Remark 1: (22) and (23) indicate that when N_t and N_r is large enough, the optimal precoder and combiner can be implemented fully in RF using phase shifters [4]. Furthermore, (13) and (14) imply that for each data stream only a couple of RAUs needs the operation of phase shifters at each channel realization.

Remark 2: By using the optimal power allocation (i.e., the well-known waterfilling power allocation [33]), the system can achieve a maximum achievable rate, which is denoted as R_o. We use $R_e(P/N_s)$ to denote the achievable rate obtained by using the equal power allocation, namely, $p_{ll} = \frac{P}{N_s}$, $l = 1,2,\ldots,N_s$. Then

$$R_e(P/N_s) \leq R_o \leq R_e((PN_s)/N_s) = R_e(P). \tag{28}$$

By doing expectation operation on (28), (28) becomes,

$$\bar{R}_e(P/N_s) \leq \bar{R}_o \leq \bar{R}_e(P). \tag{29}$$

In what follows, we derive an asymptotic expression of the ergodic achievable rate with the equal power allocation, $\bar{R}_e(P/N_s)$ (or $\bar{R}_e$ for simplicity). For this reason, we need to define an integral function

$$\begin{aligned}\Delta(x) &= \int_0^{+\infty}\log_2(1+t)e^{-t/x}\frac{1}{x}dt \\ &= \log_2(e)e^{1/x}E_1(1/x)\end{aligned} \tag{30}$$

where $E_1(\cdot)$ is the exponential integral of a first-order function defined as [34], [35]

$$\begin{aligned}E_1(y) &= \int_1^{+\infty}\frac{e^{-yt}}{t}dt \\ &= -E+\ln(y)-\sum_{k=1}^{\infty}\frac{(-y)^k}{k\cdot k!}\end{aligned} \tag{31}$$

with E being the Euler constant.

Theorem 1: Suppose that both sets $\{\tilde{\mathbf{a}}_r(\phi_{ij}^{rl},\theta_{ij}^{rl})\}$ and $\{\tilde{\mathbf{a}}_t(\phi_{ij}^{tl},\theta_{ij}^{tl})\}$ are orthogonnal vector sets when $N_r \to \infty$ and $N_t \to \infty$. Let $N_s \leq L_s$ and $\tilde{\gamma}_1 = \tilde{\gamma}_2 = \ldots = \tilde{\gamma}_{L_s} = \tilde{\gamma}$. Then in the limit of large N_t and N_r, the ergodic achievable rate with homogeneous coefficient set $\{\tilde{\gamma}_l\}$, denoted $\bar{R}_{eh}$, is given by

$$\bar{R}_{eh} = \sum_{l=1}^{N_s}\sum_{k=0}^{L_s-l}\frac{(-1)^{L_s-l-k}L_s!}{(L_s-l)!(l-1)!}\binom{L_s-l}{k}\frac{\Delta(\frac{\tilde{\gamma}}{L_s-k})}{L_s-k}. \tag{32}$$

When $N_s = L_s$, $\bar{R}_{eh}$ can be simplified to

$$\bar{R}_{eh} = L_s\Delta(\tilde{\gamma}). \tag{33}$$

Proof: Due to the assumptions that each complex gain α_{ij}^l is $\mathcal{CN}(0,1)$ and the coefficient set $\{\tilde{\gamma}_l\}$ is homogeneous, thus the instantaneous SNRs in the L_s available data streams are i.i.d.. Let $F(\gamma)$ and $f(\gamma)$ denote the cumulative distribution function (CDF) and the probability density function (PDF) of the unordered instantaneous SNRs, respectively. Then $\tilde{\gamma}$ is just the average receive SNR of each data stream. Furthermore, $F(\gamma)$ and $f(\gamma)$ can be written as

$$F(\gamma) = 1 - \mathrm{e}^{-\frac{\gamma}{\tilde{\gamma}}}, \quad f(\gamma) = \frac{1}{\tilde{\gamma}}\mathrm{e}^{-\frac{\gamma}{\tilde{\gamma}}}. \tag{34}$$

For the lth best data stream, based on the theory of order statistics [36], the PDF of the instantaneous receive SNR at the receiver, denoted γ_l, is given by

$$f_{l:L_s}(\gamma_l) = \frac{L_s!}{(L_s-l)!(l-1)!}[F(\gamma_l)]^{L_s-l}[1-F(\gamma_l)]^{l-1}f(\gamma_l). \tag{35}$$

Inserting (34) into (35), we have that

$$f_{l:L_s}(\gamma_l) = \sum_{k=0}^{L_s-l} \frac{L_s!(-1)^{L_s-l-k}}{(L_s-l)!(l-1)!}\binom{L_s-l}{k}\frac{e^{-\gamma_l(L_s-k)/\tilde{\gamma}}}{\tilde{\gamma}}. \tag{36}$$

By the definition of the function $\Delta(\cdot)$, the ergodic available rate for the lth data stream can therefore be written as

$$\begin{aligned} R_{eh}^{(l)} &= \int_0^{+\infty} \log_2(1+\gamma_l) f_{l:L_s}(\gamma_l) d\gamma_l \\ &= \sum_{k=0}^{L_s-l}\binom{L_s-l}{k}\frac{L_s!(-1)^{L_s-l-k}}{(L_s-l)!(l-1)!}g_k(\tilde{\gamma}) \\ &= \sum_{k=0}^{L_s-l}\binom{L_s-l}{k}\frac{L_s!(-1)^{L_s-l-k}}{(L_s-l)!(l-1)!}\frac{\Delta(\frac{\tilde{\gamma}}{L_s-k})}{L_s-k} \end{aligned} \tag{37}$$

where

$$g_k(\tilde{\gamma}) = \int_0^{+\infty} \log_2(1+\gamma_l)\frac{e^{-\gamma_l(L_s-k)/\tilde{\gamma}}}{\tilde{\gamma}} d\gamma_l. \tag{38}$$

So we can obtain the desired result (32).

Finally, when $N_s = L_s$, we can readily prove (33) with the help of the knowledge of unordered statistics. □

Remark 3: Now let $N_s = L_s$ and assume that $L_{ij} = L$ for any i and j (i.e., all subchannels $\mathbf{H}_{ij}$ have the same number of propagation paths). When $N_r \to \infty$ and $N_t \to \infty$, the ergodic achievable rate of the distributed MIMO system, $\bar{R}_{eh}$, can be rewritten as

$$\bar{R}_{eh}(K_t, K_r) = K_t K_r L \Delta(\tilde{\gamma}(K_t, K_r)). \tag{39}$$

Furthermore, consider a co-located MIMO system in which the numbers of transmit and receiver antennas are equal to $K_t N_t$ and $K_r N_r$, respectively. Then its asymptotic ergodic achievable rate can be expressed as

$$\bar{R}_{eh}(1,1) = L\Delta(K_t K_r \tilde{\gamma}(K_t, K_r)). \tag{40}$$

Remark 4: Generally, the coefficient set $\{\tilde{\gamma}_l\}$ is inhomogeneous. Let $\tilde{\gamma}_{max} = \max\{\tilde{\gamma}_l\}$ and $\tilde{\gamma}_{min} = \min\{\tilde{\gamma}_l\}$. Then the ergodic achievable rate with inhomogeneous coefficient set $\{\tilde{\gamma}_l\}$, $\bar{R}_e$, has the following upper and lower bounds:

$$\bar{R}_{eh}(\tilde{\gamma}_{min}) \le \bar{R}_e(\{\tilde{\gamma}_l\}) \le \bar{R}_{eh}(\tilde{\gamma}_{max}). \tag{41}$$

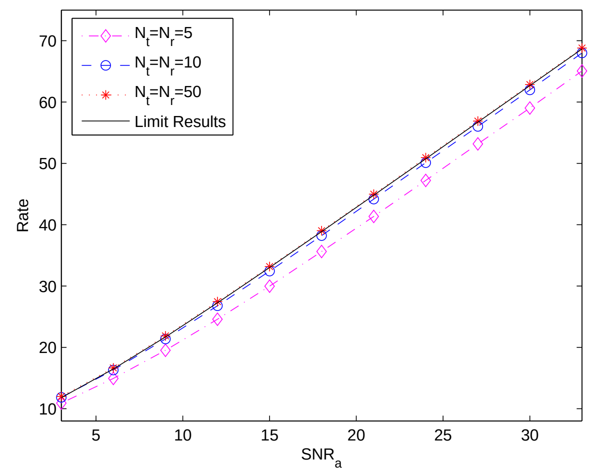

Fig. 3. Rate versus SNR_a for different numbers of antennas.

Assuming that $N_t^{(\mathrm{rf})} = N_r^{(\mathrm{rf})} = 2N_s$ [10] and $N_r = N_t = N$, Fig.3 plots the ergodic achievable rate curves versus $\mathrm{SNR}_a = \tilde{\gamma}$ for different numbers of antennas, $N = 5, 10, 50$. In Fig.3, we set $N_s = 6$, $K_r = K_t = 2$, and $L = 3$. As expected, it can be seen that the rate performance is improved as N increases. Obviously, the rate curve with $N = 10$ is very close to the curve with limit results obtained based on (30) while the rate curve with $N = 50$ is almost the same as the curve with limit results. This verifies Theorem 1.

IV. Multiplexing Gain Analysis and Diversity-Multiplexing Tradeoff

A. Multiplexing Gain Analysis

Definition 1: Let $\bar{\gamma} = \frac{1}{L_s}\sum_{l=1}^{Ls}\tilde{\gamma}_l$. The distributed MIMO system is said to achieve spatial multiplexing gain G_m if its ergodic date rate with optimal power allocation satisfies

$$G_m(\bar{R}_o) = \lim_{\bar{\gamma}\to\infty}\frac{\bar{R}_o(\bar{\gamma})}{\log_2\bar{\gamma}}. \tag{42}$$

Theorem 2: Assume that both sets $\{\tilde{\mathbf{a}}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl})\}$ and $\{\tilde{\mathbf{a}}_t(\phi_{ij}^{tl}, \theta_{ij}^{tl})\}$ are orthogonal vector sets when N_r and N_t are very large. Assume that N_r and N_t are always very large but fixed and finite when $\bar{\gamma}\to\infty$. Let $N_s \le L_s$. Then the spatial multiplexing gain is given by

$$G_m(\bar{R}_o) = N_s. \tag{43}$$

Proof: We first consider the simple homogeneous case with $\tilde{\gamma}_1 = \tilde{\gamma}_2 = \ldots = \tilde{\gamma}_{L_s} = \tilde{\gamma}$ and derive the spatial multiplexing gain with respect to $\bar{R}_{eh}$. In this case, $\bar{\gamma} = \tilde{\gamma}$. Obviously,

$$G_m(\bar{R}_{eh}) = \lim_{\bar{\gamma}\to\infty}\frac{\bar{R}_{eh}(\bar{\gamma})}{\log_2\bar{\gamma}} = \sum_{l=1}^{N_s}\lim_{\bar{\gamma}\to\infty}\frac{\bar{R}_{eh}^l(\bar{\gamma})}{\log_2\bar{\gamma}}. \tag{44}$$

For the lth data stream, under the condition of very large N_t and N_r, the individual ergodic rate can be written as

$$R_{eh}^{(l)}(\bar{\gamma}) = \mathbb{E}\log_2(1+\bar{\gamma}|\tilde{\beta}_l|^2). \tag{45}$$

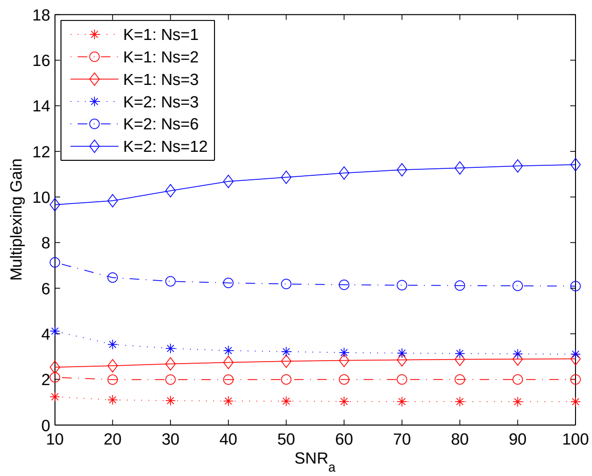

Fig. 4. Multiplexing gain versus SNR$_a$ for different numbers of data streams.

Noting that

$$\mathbb{E}\log_2(|\tilde{\beta}_l|^2) \leq \mathbb{E}\log_2(1+|\tilde{\beta}_l|^2) = R_{eh}^{(l)}(1) \quad (46)$$

and $R_{eh}^{(l)}(1)$ is a finite value, we can have that

$$\begin{aligned} G_m^{(l)}(\bar{R}_{eh}) &= \lim_{\bar{\gamma}\to\infty} \frac{R_{eh}^{(l)}(\bar{\gamma})}{\log_2\bar{\gamma}} \\ &= \lim_{\bar{\gamma}\to\infty} \frac{\log_2\bar{\gamma} + \mathbb{E}\log_2(|\tilde{\beta}_l|^2)}{\log_2\bar{\gamma}} \\ &= 1. \end{aligned} \quad (47)$$

Therefore, $G_m(\bar{R}_{eh}) = \sum_{l=1}^{N_s} G_m^{(l)}(\bar{R}_{eh}) = N_s$.

Now we consider the general inhomogeneous case with equal power allocation. Because $c_{min} = \tilde{\gamma}_{min}/\bar{\gamma}$ and $c_{max} = \tilde{\gamma}_{max}/\bar{\gamma}$ be finite when $\bar{\gamma} \to \infty$. Consequently, it readily follows that both of the two systems with the achievable rates $\bar{R}_{eh}(\tilde{\gamma}_{min})$ and $\bar{R}_{eh}(\tilde{\gamma}_{max})$ can achieve a multiplexing gain of $G_m(\bar{R}_{eh}) = N_s$. So we conclude from (41) that the distributed MIMO system with the achievable rate $\bar{R}_e(\bar{\gamma})$ can achieve a multiplexing gain of $G_m(\bar{R}_e) = N_s$.

Finally, it can be readily shown that the system with the optimal achievable rate $\bar{R}_o(\bar{\gamma})$ can only achieve multiplexing gain $G_m(\bar{R}_o) = N_s$ since both of the equal power allocation systems with the achievable rates $\bar{R}_e(P/N_s)$ and $\bar{R}_e(P)$ have the same spatial multiplexing gain N_s. □

Remark 5: Assume that $L_{ij} = L$ for any i and j. Theorem 2 implies that the distributed massive MIMO system can obtain a maximum spatial multiplexing gain of $K_r K_t L$ while the co-located massive MIMO system can only obtain a maximum spatial multiplexing gain of L.

Now we let $N_t^{(\mathrm{rf})} = N_r^{(\mathrm{rf})} = 2N_s$ [10] and $K_r = K_t = K$, and set $L = 3$ and $N_r = N_t = 50$. We consider the homogeneous case and define $\Psi(\bar{\gamma}) = \frac{\bar{R}_{eh}(\bar{\gamma})}{\log_2\bar{\gamma}}$. In order to verify Theorem 2, Fig.4 plots the curves of $\Psi(\bar{\gamma})$ versus SNR$_a$ $= \bar{\gamma}$ for different numbers of data streams, namely, $N_s = 1, 2, 3$ when $K = 2$ and $N_s = 3, 6, 12$ when $K = 2$. It can be seen that for any given N_s, the function $\Psi(\bar{\gamma})$ converges to the limit value N_s as $\bar{\gamma}$ grows large. This observation is expected and agrees with Theorem 2.

B. Diversity-Multiplexing Tradeoff

The previous subsection shows how much the maximal spatial multiplexing gain we can extract for a distributed mmWave-massive MIMO system while our previous work in [32] indicates how much the maximal spatial diversity gain we can extract. However, maximizing one type of gain will possibly result in minimizing the other. We need to bridge between these two extremes in order to simultaneously obtain both types of gains. We firstly give the precise definition of diversity gain before we carry on the analysis.

Definition 2: Let $\bar{\gamma} = \frac{1}{L_s}\sum_{l=1}^{Ls}\tilde{\gamma}_l$. The distributed MIMO system is said to achieve spatial diversity gain G_d if its average error probability satisfies

$$G_d(\bar{P}_e) = -\lim_{\bar{\gamma}\to\infty} \frac{\bar{P}_e(\bar{\gamma})}{\log_2\bar{\gamma}}. \quad (48)$$

or its outage probability satisfies

$$G_d(\bar{P}_{out}) = -\lim_{\bar{\gamma}\to\infty} \frac{\bar{P}_{out}(\bar{\gamma})}{\log_2\bar{\gamma}}. \quad (49)$$

With the help of a result of diversity analysis from [32], we can derive the following DMT result.

Theorem 3: Assume that both sets $\{\tilde{\mathbf{a}}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl})\}$ and $\{\tilde{\mathbf{a}}_t(\phi_{ij}^{tl}, \theta_{ij}^{tl})\}$ are orthogonal vector sets when N_r and N_t are very large. Assume that N_r and N_t are always very large but fixed and finite when $\bar{\gamma} \to \infty$. Let $N_s \leq L_s$. For a given $d \in [0, L_s]$, by using optimal power allocation, the distributed MIMO system with N_s data streams can reach the following maximum spatial multiplexing gain at diversity gain $G_d = d$

$$G_m = \sum_{l=1}^{L_s}(1 - \frac{d}{L_s - l + 1})^+. \quad (50)$$

Proof: We first consider the simple case where the distributed system is the one with equal power allocation and the channel is the one with homogeneous large scale fading coefficients. The distributed system has L_s available link paths in all. For the lth best path, its individual maximum diversity gain is equal to $G_d^{(l)} = L_s - l + 1$ [32]. Due to the fact that each path can not obtain a multiplexing gain of $G_m^{(l)} > 1$ [31], we therefore design its target data rate $R^{(l)} = r_l \log_2\bar{\gamma}$ with $0 \leq r_l \leq 1$. Then the individual outage probability is expressed as

$$\begin{aligned} P_{out}^{(l)} &= \mathbb{P}(\log_2(1+\bar{\gamma}|\tilde{\beta}_l|^2) < r_l\log_2\bar{\gamma}) \\ &= \mathbb{P}(|\tilde{\beta}_l|^2 < \frac{\bar{\gamma}^{r_l}-1}{\bar{\gamma}}). \end{aligned} \quad (51)$$

From [37], [38], the PDF of the parameter $\mu = |\tilde{\beta}_l|^2$ can be written as

$$f_\mu = a\mu^{L_s - l} + o(\mu^{L_s - l}) \quad (52)$$

where a is a positive constant. So $P_{out}^{(l)}$ can be rewritten as

$$P_{out}^{(l)} = (c\bar{\gamma})^{-(L_s-l+1)(1-r_l)} + o((\bar{\gamma})^{-(L_s-l+1)(1-r_l)}) \quad (53)$$

where c is a positive constant. This means that this path now can obtain diversity gain

$$G_d^{(l)} = (L_s - l + 1)(1 - r_l). \quad (54)$$

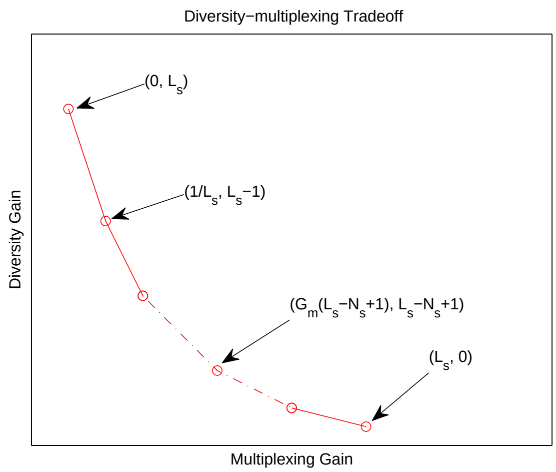

Fig. 5. Diversity-multiplexing tradeoff $G_m(d)$ for a general integer d.

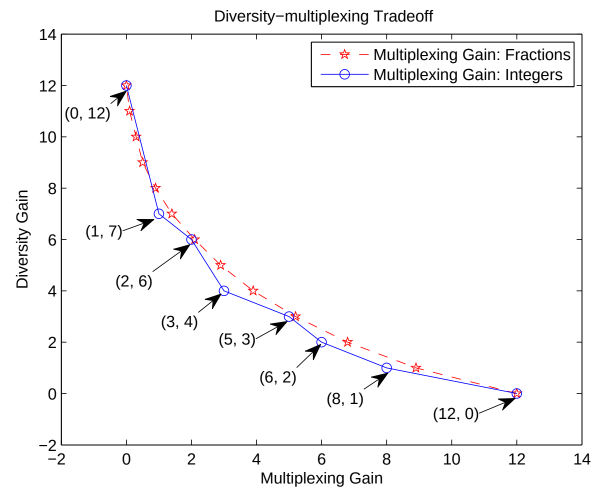

Fig. 6. Diversity-multiplexing tradeoff when $L = 3$ and $K_t = K_r = 2$.

Since the distributed system requires the system diversity gain $G_d \geq d$, this implies that

$$G_d^{(l)} = (L_s - l + 1)(1 - r_l) \geq d \tag{55}$$

or say

$$r_l \leq (1 - \frac{1}{L_s - l + 1})^+. \tag{56}$$

To this end, under the condition that the diversity gain satisfies $G_d = d$, the maximum spatial multiplexing gain of the distributed system must be equal to

$$G_m(\bar{R}_{eh}) = \sum_{l=1}^{L_s} r_l = \sum_{l=1}^{L_s} (1 - \frac{d}{L_s - l + 1})^+. \tag{57}$$

This proves that (50) holds under the special case. We readily show that for a general case, the lth best path can also reach a maximum diversity gain of $L_s - l + 1$. So applying (41) and (29) leads to the desired result. □

Remark 6: When d is an integer, $G_m(d)$ can be expressed simply. In particular, $G_m(0) = L_s$ if $d = 0$; $G_m(1) = \sum_{l=1}^{L_s-1} \frac{L_s - l}{L_s - l + 1}$ if $d = 1$; $G_m(L_s - 1) = \frac{1}{L_s}$ if $d = L_s - 1$; $G_m(L_s) = 0$ if $d = L_s$. In general, if $d = L_s - N_s + 1$ for a given integer $N_s \leq L_s$, then

$$G_m(L_s - N_s + 1) = \sum_{l=1}^{N_s - 1} \frac{N_s - l}{L_s - l + 1}. \tag{58}$$

The function $G_m(d)$ is plotted in Fig.5. Note that $G_m(L_s - N_s) - G_m(L_s - N_s + 1) = \sum_{l=1}^{N_s} \frac{1}{L_s - l + 1}$. Generally, when $d \in [L_s - N_s, L_s - N_s + 1)$, the multiplexing gain is given by

$$G_m(d) = N_s - \sum_{l=1}^{N_s} \frac{d}{L_s - l + 1}. \tag{59}$$

Example 1: We set that $L = 3$ and $K_t = K_r = 2$. So $L_s = 12$. The DMT curve with fractional multiplexing gains is shown in Fig.6. If the multiplexing gains be limited to integers, the corresponding DMT curve is also plotted in Fig.6 for comparison.

V. Multiplexing Gain Analysis for the Multiuser Scenario

This section considers the downlink communication in a multiuser massive MIMO system as illustrated in Fig. 7. Here the base station (BS) employs K_b RAUs with each having N_b antennas and $N_b^{(\text{rf})}$ RF chains to transmit data streams to K_u mobile stations. Each mobile station (MS) is equipped with N_u antennas and $N_u^{(\text{rf})}$ RF chains to support the reception of its own N_s data streams. This means that there is a total of $K_u N_s$ data streams transmitted by the BS. The numbers of data streams are constrained as $K_u N_s \leq N_b^{(\text{rf})} \leq K_b N_b$ for the BS, and $N_s \leq N_u^{(\text{rf})} \leq N_u$ for each MS.

At the BS, denote by $\mathbf{F}_b$ the $K_b N_b \times N_b^{(\text{rf})}$ RF precoder and by $\mathbf{W}_b$ the $N_b^{(\text{rf})} \times N_s K_u$ baseband precoder. Then under the narrowband flat fading channel model, the received signal vector at the ith MS is given by

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{F}_b \mathbf{W}_b \mathbf{s} + \mathbf{n}_i,\ i = 1, 2, \ldots, K_u \tag{60}$$

where $\mathbf{s}$ is the signal vector for all K_u mobile stations, which satisfies $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \frac{P}{K_u N_s} \mathbf{I}_{K_u N_s}$ and P is the average transmit power. The $N_u \times 1$ vector $\mathbf{n}_i$ represents additive white Gaussian noise, whereas the $N_u \times K_b N_b$ matrix $\mathbf{H}_i$ is the channel matrix corresponding to the ith MS, whose entries $\mathbf{H}_{ij}$ are described as in Section II. Furthermore, the signal vector after combining can be expressed as

$$\mathbf{z}_i = \mathbf{W}_{ui}^H \mathbf{F}_{ui}^H \mathbf{H}_i \mathbf{F}_b \mathbf{W}_b \mathbf{s} + \mathbf{W}_{ui}^H \mathbf{F}_{ui}^H \mathbf{n}_i,\ i = 1, 2, \ldots, K_u \tag{61}$$

where F_{ui} is the $N_u \times N_u^{(\text{rf})}$ RF combining matrix and $\mathbf{W}_{ui}$ is the $N_u^{(\text{rf})} \times N_s$ baseband combining matrix for the ith MS.

Theorem 4: Assume that all antenna array configurations for the downlink transmission are ULA. For the ith user, let $L_s^{(i)} = \sum_{j=1}^{K_b} L_{ij}$ and $0 \leq d^{(i)} \leq L_s^{(i)}$. In the limit of large N_b and N_u, the ith user can achieve the following maximum spatial multiplexing gain when its individual diversity gain

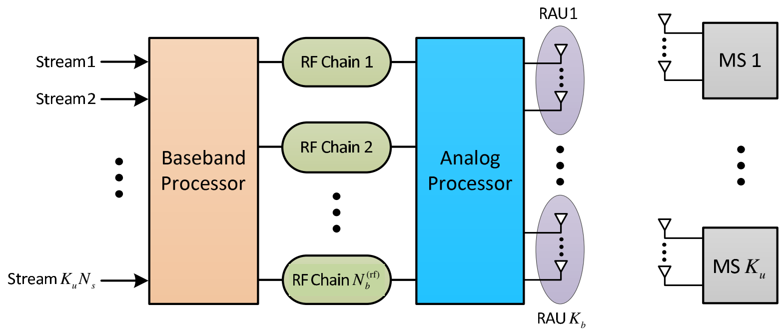

Fig. 7. Block diagram of a multiuser mmWave system with distributed antenna arrays.

satisfies $G_d^{(i)} = d^{(i)}$

$$G_m^{(i)} = \sum_{l=1}^{L_s^i}(1 - \frac{d^{(i)}}{L_s^{(i)} - l + 1})^+. \quad (62)$$

Proof: For the downlink transmission in a massive MIMO multiuser system, the overall equivalent multiuser basedband channel can be written as

$$\mathbf{H}_{\rm eq} = \begin{bmatrix} \mathbf{F}_{u1}^H & 0 & \cdots & 0 \\ 0 & \mathbf{F}_{u2}^H & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{F}_{uK_u}^H \end{bmatrix} \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_{K_u} \end{bmatrix} \mathbf{F}_b. \quad (63)$$

On the other hand, when both N_b and N_u are very large, both receive and transmit array response vector sets, $\{\tilde{\mathbf{a}}_r(\phi_{ij}^{rl}, \theta_{ij}^{rl})\}$ and $\{\tilde{\mathbf{a}}_t(\phi_{ij}^{tl}, \theta_{ij}^{tl})\}$, are orthogonal sets. Therefore the multiplexing gain for the ith user can depend only on the subchannel matrix $\mathbf{H}_i$ and the choices of $\mathbf{F}_{ui}$ and $\mathbf{F}_b$. The subchannel matrix $\mathbf{H}_i$ has a total of $L_s^{(i)}$ propagation paths. Similar to the proof of Theorem 2, by employing the optimal RF precoder and combiner for the ith user, when its diversity gain satisfies $G_d^i = d^{(i)}$, the user can achieve a maximum multiplexing gain of

$$G_m^{(i)} = \sum_{l=1}^{L_s^{(i)}}(1 - \frac{d^{(i)}}{L_s^{(i)} - l + 1})^+. \quad (64)$$

So we obtain the desired result. □

Remark 7: Consider the case that all antenna array configurations for the downlink transmission are ULA and $L_{ij} = L$ for any i and j . Let $0 \le d \le K_bL$. In the limit of large N_b and N_u, the downlink transmission in the massive MIMO multiuser system can achieve the following maximum spatial multiplexing gain at diversity gain $G_d = d$

$$G_m = \sum_{i=1}^{K_u} G_m^{(i)} = K_u \sum_{l=1}^{K_bL}(1 - \frac{d}{K_bL - l + 1})^+. \quad (65)$$

Remark 8: In a similar fashion, it is easy to prove that the uplink transmission in the massive MIMO multiuser system can also achieve simultaneously a diversity gain of $G_d = d$ ($0 \le d \le K_bL$) and a spatial multiplexing gain of

$$G_m = K_u \sum_{l=1}^{L_s}(1 - \frac{d}{K_bL - l + 1})^+. \quad (66)$$

VI. Conclusion

This paper has investigate the distributed antenna subarray architecture for mmWave massive MIMO systems and provided the asymptotical multiplexing analysis when the number of antennas at each subarray goes to infinity. In particular, this paper has derived the closed-form formulas of the asymptotical available rate and spatial maximum multiplexing gain under the assumption which the subchannel matrices between different antenna subarray pairs behave independently. The spatial multiplexing gain formula shows that mmWave systems with the distributed antenna architecture can achieve potentially rather larger multiplexing gain than the ones with the conventional co-located antenna architecture. On the other hand, using the distributed antenna architecture can also achieve potentially rather higher diversity gain. For a given mmWave massive MIMO channel, both types of gains can be simultaneously obtained. This paper has finally given a simple DMT tradeoff solution, which provides insights for designing a mmWave massive MIMO system.

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