Mesh node placement in wireless mesh network based on multiobjective evolutionary metaheuristic

Oumarou Mamadou Bello* and
Kolyang Dina Taiwe

Department of Computer Science,
Higher Teachers' Training College,
University of Maroua,
Maroua, Cameroon
Email: oumaroumb@gmail.com
Email: dtaiwe@yahoo.fr

Abstract: To achieve important properties of wireless mesh networks (WMNs) such as coverage and reliability, the placement of mesh nodes plays an important role. The impact of the mesh node placement on the performance of WMNs has been carried out in the past years. To improve such properties, we propose a novel scheme for mesh node placement in WMNs. We have developed a multi-objective optimisation model for node placement where the coverage, reliability and the total installation cost in terms of nodes to be deployed are the three objectives to optimise simultaneously. We first applied the two well-known evolutionary algorithms, namely the non-dominated sorting genetic algorithm-II (NSGA-II) and multi-objective genetic algorithm (MOGA) to generate the number and positions of the communication nodes. Subsequently, we developed algorithms that determine the cluster formation, gateway selection and relay nodes selection. The results showed satisfactory performance.

Keywords: wireless mesh networks; WMNs; node placement problem; coverage; reliability; multiobjective optimisation; non-dominated sorting genetic algorithm-II; NSGA-II; multi-objective genetic algorithm; MOGA.

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Biographical notes: Oumarou Mamadou Bello received his BE degree in 2007 and MS degree in 2009 in Information Science and Computer Engineering both from the Southern Federal University, City of Rostov-on-Don, Russian Federation. Currently, he is working towards his PhD degree at the Department of Computer Science of Higher Teachers' Training College at the University of Maroua, Cameroon. His research topic mainly focuses on metaheuristics techniques for wireless mesh networks design.

Kolyang Dina Taiwe is a Professor at the University of Maroua, Cameroon. He received his PhD from the University of Bremen, Germany in 1997. After a Post-doctorate at the University of Bremen, Germany, he received the habilitation to supervise research and took his position of an Associate Professor at the University of Bremen in Germany in 2008. Since 2015 he has been nominated as a Research Ambassador for the University of Bremen, Germany. His research interests include formal methods for software verification, software engineering, e-learning, information and communication technology for development, technology transfer, solidarity economy and development.

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1 Introduction

The use of wireless networks continues to grow, driven by the growth in the number of wireless connection devices. Estimates by the Wifi alliance (Biswas et al., 2015) have shown that more than 10 billion Wifi devices were sold in total in 2015 and that more than 4.5 billion of these devices are in use today. Wireless technology enables these electronic devices to be able to connect to the network, including the concept of all connected objects or the internet of things. Expand the use of wireless radio communication means whatever the environment is essential nowadays. Several types of wireless networks have been developed, including wireless mesh networks (WMNs).

WMNs (Akyildiz and Wang, 2009) are known as multi hop communication networks and hierarchically organised. It consist of mesh clients and mesh routers, where the mesh routers form a wireless infrastructure/backbone and interwork with the wired networks to provide multihop wireless connectivity to the mesh clients (Hossain et al., 2008). Mesh clients can be typically cell phone, PDA users, laptops, tablets and others wireless devices and mesh routers basically are the combination of access points, mesh relays and gateways. These three main components constitute the core of the networks. The path redundancy is a robust feature of mesh topology and makes WMN very reliable networks to potential node failures. Potential application scenarios for WMNs include backhaul support for cellular networks, home networks, enterprise networks, community networks, internet access, public safety, building automation, electric utility automation, information sharing and intelligent transport system networks (Akyildiz and Wang, 2009). Figure 1 (in Appendix A) illustrates WMN architecture.

The development of wireless networks has created new challenges that require improving connectivity and network scale strategies. Different optimisation problems have been studied in WMNs, in order to optimise its performance. These include coverage, reliability, quality of service in terms of throughput and delay, interference, etc. The first service a network must provide is access and generally a global coverage of the area of interest is required to provide that access. Coverage is therefore an essential objective and it is either to be maximised or expressed as a constraint to be met in order to ensure quality of service. We also note that the guarantee of coverage and the reliability of the network are linked. In fact, the access nodes are mostly fixed and make them possible to convey the data of the users from one point to another. A path must therefore be set up in the wireless distribution system between each end user and at least one gateway in order to guarantee connectivity constraint in the event of failure of a node or path. To efficiently provide network connectivity, optimised positions and configurations of mesh routers are required. The different solutions proposed depend heavily on the choice of the positions or locations of the infrastructures of the network and their number.

Nodes are strategically placed to achieve the desired hops and to avoid forming long hops in the backbone. Nodes placement problem in the literature (Xhafa et al., 2015) is closely related to facility location problem, which in a general setting seeks an optimal placement of a number of facilities that give service to a certain number of clients. The similarity with mesh routers node placement in WMNs is straightforward by considering mesh routers as facilities, which give service, i.e. Internet connectivity, to mesh client's nodes. Basic facility location models related to discrete network location models is presented in (Current et al., 2001) and even the most basic location models are classified as NP-hard (Garey and Johnson, 1979). Realistic planning is usually to optimise several often conflicting objectives; such that the simultaneous optimisation of radio coverage and minimising interference or trade-off between the minimisation of the cost of deployment and the maximisation of network performance. Therefore, a multicriteria optimisation approach is the best one that reflects this problem. Such a problem is called multiobjective optimisation problem (Coello et al., 2007). The multiobjective optimisation approach produces several non-dominated solutions. It is proven in (Mountassir et al., 2013) that WMN planning optimisation problem is NP-Hard. In this work we use non-dominated sorting genetic algorithm-II (NSGA-II) (Deb et al., 2002) and multi-objective genetic algorithm (MOGA) (Fonseca and Fleming, 1993) methods for solving nodes placement problem, consisting in the maximisation of the network reliability (connectivity), coverage maximisation and cost minimisation. These multiobjective genetic algorithms (GA) have the main task of generating the initial positions of the nodes and their number. Thereafter, we propose a three step approach to guarantee reliability of the network through clustering the nodes based on these two multiobjective metaheuristics.

The rest of the paper is organised as follows. In Section 2 we review some optimisation problems related to node placement problems in WMNs. We describe and formulate the optimisation problems in Section 3. In Section 4, we present NSGA-II and MOGA methods that we have used for solving node placement problems. We also present a three step approach to guarantee reliability of the network through clustering the nodes. Section 5 presents the numerical results. In Section 6, the conclusions and perspectives are outlined.

2 Related works

We review in this section some techniques provided in the literature about mesh node placement. Clustering technique is widely used for mesh backbone formation in designing a WMN to achieve user coverage and ensure network's reliability.

Bejerano (2004) has started to develop the way of WMN design using clustering technique. He breaks the problem into two sub-problems:

1. finding the minimal number of disjoint connected clusters that contain all the nodes and satisfying the delay constraint
2. dividing the clusters that violate the cluster size constraint.

But by splitting a cluster without considering re-assigning those wireless mesh routers to existing clusters may create some unnecessary clusters and therefore increase significantly the number of clusters. Aoun et al. (2006) tried to minimise the disadvantages of Bejerano's technique by combining the two sub-problems, where the spanning tree and cluster coverage evolve in parallel subject to satisfy the quality of service constraints. It recursively computes minimum dominating sets of the graph resulting from the previous iteration in each recursive iteration. The algorithm first produces an adjacency matrix. Then, greedy selection is used to select a node that covers as many un-selected nodes as possible and builds the spanning tree. They also proposed to increase throughput by introducing multiple gateways. But placement of multiple gateways throughout the mesh does not always result in more throughput as proved by Pandey et al. (2012). A novel method for clustering the nodes and load sharing amongst the clusters based on graph partitioning approach has been presented in Pandey et al. (2012). Seyedzadegan et al. (2013) elaborated the importance of nodes degree and clustering for the efficient operation of backbone WMNs. A novel zero-degree algorithm is proposed for clustering the backbone WMN based on degree/number of wireless routers' connections, while ensuring delay, relay load and cluster size constraints.

Nodes placement problem has been also formulated as an optimisation problem and several heuristics and metaheuristics approaches have been proposed. In fact, the different versions of the problem can be obtained depending on the types of nodes to be deployed as well as the objectives to be optimised.

Amaldi et al. (2008) have proposed an optimisation models for planning WMNs, where the objective is to minimise the network installation cost while providing full coverage to wireless mesh clients. They have proposed and evaluated a relaxation-based heuristic to solve a mixed integer linear programming (ILP) models that allow to select the number and positions of mesh routers and access points, while accurately taking into account traffic routing, interference, rate adaptation and channel assignment. But they have considered only cost as objective to optimise rather than multiobjective optimisation model. De Marco (2009) presented an evolutionary algorithm, for node placement problem in WMN, which optimises the graph topology by minimising the node degree and maximising the user coverage percentage, while allowing cycles in the graphs, i.e., by allowing non-minimum spanning tree (MST) graphs. A generic multiobjective optimisation framework of a WMNs planning problem was devised by Benyamina et al. (2012). The goal is to minimise the cost and maximise the overall network performance prior to its deployment. They proposed three multiobjective models for WMN problem, namely load-balanced model, Interference model and flow-capacity model. They used ILP optimisation approach and devised an evolutionary swarm based algorithm that is a hybrid combination of multi-objective particle swarm optimisation (MOPSO) and GA to solve the three models.

Mountassir et al. (2013) proposed a mutliobjective model for WMNs planning by optimising four objective functions simultaneously including: cost minimisation, coverage maximisation, links congestion minimisation and gateways congestion minimisation subject to a set of constraints to take into account namely interference, robustness and load balancing. They used the MOPSO method to provide interesting results and let the network planner decide which solution responds to his requirements. In Chung-Chen (2013) particle swarm optimisation (PSO) based model has been proposed to solve mesh routers placement in dynamic network. Model considers the mobility of both mesh routers and clients so that mesh clients can change the network access to on or off. The model aimed to maximise the network connectivity and users' coverage based on mathematical formulation. However, the performance of the PSO based algorithm has been evaluated by discussing the influences of the different parameters on the network design to present the convergence in the PSO toward the solution.

The work done by Abdelkhalek et al. (2015) presented a novel multiobjective node placement problem that optimises concurrently four objectives: maximising communication coverage, minimising the active structures' costs, maximising the total capacity bandwidth and minimising the noise level in the network. They have applied a multiobjective variable-length genetic algorithm (VLGA) that simultaneously searches for the optimal number, positions and nature of heterogeneous nodes and communication devices. Fendji et al. (2015) have presented an approach based on metropolis algorithm to solve the problem of mesh nodes placement in rural WMN. The goal is to determine an optimal number and positions of mesh router nodes while maximising the coverage of areas of interest, minimising the coverage of optional areas and ensuring connectivity of all mesh router nodes. In the study of Xhafa et al. (2015), they presented the implementation and evaluation of Tabu search (TS) for the problem of mesh router node placement in WMNs. Given a number of router nodes to deploy, a deployment area and positions of client nodes in the area, an optimisation problem was formulated aiming to find the placement of router nodes so as to maximise network connectivity and user coverage. The experimental evaluation showed the efficiency of TS in solving a benchmark of instances. Wang et al. (2015) have formulated the relay placement problem for content-centric WMNs as an integer linear program in order to maximise the network throughput. A near-optimal approximation algorithm based on linear programming relaxation has been developed to optimally solve the problem.

The problem addressed in this paper has characteristics that are similar to those of the problems reviewed above. However, we consider that mesh clients do not act as routers or as a gateway, i.e., mesh clients must go through mesh routers to communicate with other nodes.

3 Problem description and formulation

3.1 Problem description

Let N be the number of nodes to be deployed on a geographical surface S where each point is identified by its coordinates. Initially, we model the WMN by a non-oriented graph $G = (V, E)$, where the set of vertices V represents the wireless nodes and the set of edges E represents the communications links between the nodes of the network. The set of vertices of the graph is decomposed into three subsets $V = V_{ap} \cup V_{rr} \cup V_{gw}$, where V_{ap} represents the set of access points (used to collect traffic from the demand points to backhaul), V_{rr} is the set of relays (used to extend communication coverage and relay traffic from access points to / from gateways) and V_{gw} represents the set of gateways that are connected to the internet or to a wired network. Each node $n_j \in V$ has several radio interfaces R corresponding to the number of links that this node can establish with its neighbours. The set of neighbours of n_j denoted by $K_{n_j} \in V,$ is the set of nodes that are within the transmission range of n_j. The arc $(n_j, n_k) \in E$ represents the wireless link between the node n_j and the node n_k. A link can be established between two nodes n_j and

n_k only when the distance $d(n_j, n_k)$ which separates them is less than the transmission range of each node and each node is assumed to have same radio coverage. Hops are built over wireless links between mesh client and a gateway. The restriction is that the number of wireless hops must be limited to an upper bound as proved in Andrade et al. (2015). We consider that, only relay node can be served by the gateway because of their proximity.

Reliability of the network can be measured by the value of the clustering coefficient (CC) in order to improve robustness to failure (Watts and Strogatz, 1998). The CC measures the degree of how strongly nodes are clustered in a network. As defined in Brust et al. (2012) the local CC (CC_{nj} of a node n_j with K_{n_j} neighbors is

$$CC_{n_j} = \frac{\left|E\left(L_{n_j}\right)\right|}{K_{n_j}\left(K_{n_j} - 1\right)}$$

where $|E(L_{n_j})|$ is the number of links in the neighbourhood of n_j and $K_{n_j}(K_{n_j} - 1)$ is the total number of possible links in the neighbourhood of n_j. The local CC determines the degree of the connectedness of the node's neighbours. And the global CC of a graph $G = (V, E)$ is the average of all local CCs in the network denoted as $CC = \frac{1}{N}\sum_{n_j} CC_{n_j}$ where N is the number of nodes in G denoted as $N = |V|$ (Brust et al., 2012). A high CC value means that the network consists of a high number of locally clustered nodes, i.e., CC reflects the probability that a randomly chosen pair of nodes n_1, $n_2 \in V$ that are connected,
$(n_1, n_2) \in E$ have a mutual neighbour $n_3 \in V$ with $(n_1, n_2) \in E$ and $(n_2, n_3) \in E$.

The objective is to create disjoint clusters such that the trade-off between number of deployed nodes, the user coverage and reliability of the network be the best possible. Our graph is based on geographic information of the area of deployment and takes into consideration maximum coverage range of nodes. In many studies user coverage mainly depends on how the users were distributed on the specific area. Since the distribution of real mesh clients can not be predicted, we assume that the locations of mesh clients are fixed in the deployment area by uniform distribution.

3.2 Formulation

As cited earlier let a graph $G = (V, E)$, where $|V| = N$ is the set of wireless nodes and E describes the set of links between pair of routers nodes. The sets of access points, relays and gateways are referred as V_{ap}, V_{rr}, V_{gw} respectively with $V = V_{ap} \cup V_{rr} \cup V_{gw}$. Each node has R radio interfaces. Let a set $P = \{p_1, p_2, p_3, \ldots, p_n\}$ represents the n candidates locations or position where nodes can be installed and $U = \{u_1, u_2, u_3, \ldots, u_m\}$ the set of m positions where users or clients are distributed. We stated that $j = 1, 2, 3, \ldots, n$ and $i = 1, 2, 3, \ldots, m$. We define the following notation:

N number of wireless nodes

n number of candidates location

m number of clients positions

R number of radio interface per node

$C_{v_{ap}}$ access point installation cost

$C_{v_{rr}}$ relay router installation cost

$C_{v_{gw}}$ gateway installation cost

h_j maximum number of wireless hops to gateway j

$Z^{i}_{j,k}$ hop associated of arc (j, k) in level l to h_j

C_a node's radio interface capacity for access communication

C_b node's radio interface capacity for backbone forwarding

$C_{i,j}$ cost associated if the assignment of the user location i to node j is done

A_j installation of an access point at location j

M_j installation of mesh relay at location j

G_j installation of a gateway at j

k_g set of wireless links of a gateway

$k_{h,j}$ k- hop neighbours of node j

Δ_j number of nodes supported by a gateway j

r_j reliability of node j

$x_{i,j}$ assignment of the user location i to node j

$Cov_{i,j}$ coverage of location i by node j

T_i: traffic generated by users in i position

T_j flow outgoing through gateway i

$T_{j,k}$ flow between node j and node k through arc (j, k)

Consider the following decision variables:

$$A_j = \begin{cases} 1 & \text{if a router relay is installed at j} \\ 0 & \text{otherwise} \end{cases}.$$

$$M_j = \begin{cases} 1 & \text{if a router relay is installed at j} \\ 0 & \text{otherwise} \end{cases}.$$

$$G_j = \begin{cases} 1 & \text{if a gateway relay is installed at j} \\ 0 & \text{otherwise} \end{cases}.$$

$$Cov_{i,j} = \begin{cases} 1 & i\text{f location i is covered by node j} \\ 0 & \text{otherwise} \end{cases}.$$

$$x_{i,j} = \begin{cases} 1 & if \text{ location i is assigned to node j} \\ 0 & \text{otherwise} \end{cases}.$$

$$Z_{j,k}^{l} = \begin{cases} 1 & \text{if arc (j, k) is in level l to gateway} \\ 0 & \text{otherwise} \end{cases}.$$

Setting as an objective the minimisation of the cost, the maximisation of user coverage and the maximisation of the reliability, the mathematical model can be formulated as follows:

$$\min\left(\sum_{j=1}^{n}\left(C_{V_{ap}}A_j + C_{V_{rr}}M_j + C_{V_{gw}}G_j\right) + \sum_{j=1}^{m} C_{i,j}X_{i,j}\right) \tag{1}$$

$$\max \sum_{j=1}^{n}\sum_{i=1}^{m} Cov_{i,j}x_{i,j} \tag{2}$$

$$\max \sum_{j\in J}\left(A_j + M_j + G_j\right)r_j \tag{3}$$

Subject to

$$\sum_{i=1}^{m} x_{i,j}A_j \geq 1, \;\; j = 1, \ldots, n \tag{4}$$

$$\sum_{j\in P} A_j + M_j + G_j \leq N \tag{5}$$

$$\sum_{l=0}^{h_j} Z_{j,k}^{l} \leq 1, \;\; \forall (j,l)\in E, \, j\in V_{ap}, k\in V_{rr}\cup V_{gw} \tag{6}$$

$$\sum k_{h_j} \leq \Delta_j G_j, \;\; k_{h_j} \in V_{ap}\cup V_{rr} \tag{7}$$

$$A_j + G_j \leq 1 \tag{8}$$

$$x_{i,j} \leq A_j \tag{9}$$

$$\sum k_g \leq RG_{j'} \tag{10}$$

$$T_i \leq C_a A_j \tag{11}$$

$$T_{j,k} \leq \sum_{l=1}^{h_j} Z_{j,k}^{l} C_b \tag{12}$$

$$\sum_{i\in U} T_i x_{i,j} + \sum_{j,k\in P}\left(T_{j,k} - T_{k,j}\right) - T_j = 0 \tag{13}$$

$$A_j, M_j, G_j, Cov_{i,j}, Z_{j,k}^{l} \in \{0,1\} \;\; \forall i\in U, \, j,k\in P, l\in h_j \tag{14}$$

The function objective (1) minimises the total installation cost and exploitation of the network. Term (2) is the maximisation of user coverage and Term (3) is the maximisation of the reliability. Constraint (4) assigns a position i to at least one access point A_j. Constraint (5) limits the number of deployed nodes. Constraint (6) limits an arc to be in more than one level. Constraint (7) requires that if a gateway is installed, it should have a number of supported nodes Δ_j. Constraint (8) an access point and a gateway should not be deployed close to each other. Constraint (9) ensure that a position *i* can only be assigned to a node if the node is installed at location *j*. Constraint (10) states that the number of links emanating from a gateway is limited by the number of its radio interfaces. Constraints (11) and (12) ensure that access points and node-to-node capacities are respected. Constraint (13) defines the flow balance.

4 Resolution approach

Node placement problems are known to be computationally hard to solve to optimality in their general formulations and thus evolutionary algorithms have proven to be useful approach to cope in practice for their resolution (Barolli et al. 2011; Benyamina et al., 2012; Abdelkhalek et al., 2015). We make a brief introduction to evolutionary algorithms with special emphasis on methods used in our study, including multiobjective optimisation methods NSGA-II and MOGA.

NSGA-II proposed by Deb et al. (2002) is a popular and efficient multiobjective genetic algorithm for solving real world engineering problems. A brief description of NSGA-II is given bellow and we refer to Deb et al. (2002) for more details. NSGA-II starts with a set of acceptable solutions (population of parents P_t of size N) and by applying the typical genetic operators namely selection, crossover and mutation a new population (population of children Q_t of size N) is obtained. Population of parent P_t and population of children Q_t are assembled to form population R_t of size $2N$. The population of R_t are sorted according to their rank, i.e., solutions (individuals) are classified into Pareto fronts and the best solutions are chosen to create a new relative population (P_{t+1}). This new relative population is formed by adding classified fronts as long as they do not exceed N. In the case of having to select some solutions with the same rank, a density estimation based on measuring the crowding distance to the surrounding solutions belonging to the same rank is used to get the most promising solutions. Once the solutions in the population P_{t+1} are identified, a new child population (Q_{t+1}) is created by selection, crossover and mutation. The process is repeated until a stopping criterion is met.

MOGA was proposed by Fonseca and Fleming (1993). MOGA is a method in which each individual (solution) is checked for its domination in the population of size N. Then, the algorithm uses a performance calculation function for taking into account the rank of the individual and non-dominated solutions are assigned a rank equal to 1 and the maximum rank cannot be larger than the size of the population N. Since no solution would dominate a non-dominated solution in a population, all individuals of the same rank have the same performance. In order to maintain the diversity among non-dominated solutions, niching among solutions of each rank are introduced. This method provides a disadvantage risk of premature convergence because of the great pressure exerted by the selection for represented solutions in any rank. To avoid this problem, the authors introduced a performance share function to better distribute the solutions along the Pareto frontier. This procedure is continued until all ranks are processed. Selection, crossover and mutation operators are applied to create a new population.

We describe in this section the algorithms we have developed.

Some work presented in Benyamina (2010) proposed to fix the number of nodes to be installed in search algorithms during deployment. But when the number of nodes is overestimated, the configuration of the nodes obtained leads to an operation sometimes degraded by interference between channels. Hence the need to find the optimal number of nodes to be deployed. Initially, the NSGA-II and MOGA algorithms generate on the grid a graph formed only of isolated vertices (N nodes) on the different coordinates of the cells of the grid. We then design a clustering algorithm, according to Algorithm 1, that is to say a graph composed of several connected components that are not linked together. This algorithm iteratively identifies a group by dividing the graph into clusters. We then select, with Algorithm 2, a gateway in each cluster. Afterwards the relay nodes are chosen, according to Algorithm 3 based on the algorithm of Prim (1975) to leverage the reliability issues. To account for quality of service, we limit the number of hops between each user node and the nearest gateway. We consider following notation:

$G = (V, E)$ Graph G

N *t*he set of nodes with avec $N = \{n_1, n_2, \ldots, n_p\}$

C_i cluster i

n_j node j inside cluster C_i

$n(C_i)$ number of nodes in a cluster ($n(C_i)\Sigma n_j$)

d_{max} maximum distance between two nodes

$d_{j,k}$ distance matrix d(n_j, n_k) of $G\ \forall j = 1, \ldots, p$ and $k = 1, \ldots, p$

clustNb set of r disjoint clusters in the network ($|clsutNb| = C_i$ where i = 1, …, r)

$S_{j,k}$ cost of connexion between node j and node k

$TC(n_j)$ total cost when all nodes are linked through n_j

Gw_j cluster gateway

$m_{j,k}$ adjacency matrix ($m_{j,k}$ if (n_j, n_k) $\in E$ and $m_{j,k}$ otherwise)

h_k number of arc at the node n_k

Algorithm 1 Formation of clusters

Input: N, $n(C_i)$, d_{max}, $d_{j,k}$
Output: *clsutNb*
clustNb $\leftarrow \varnothing$
$C_i \leftarrow \varnothing$
    if $N > 0$ **then**
        for $i = 1$ to r
            Create a new cluster C_i
        while $C_i \leq n(C_i)$
    find any two nodes (n_j, n_k)such as $d_{j,k} \leq d_{max}$ // $\forall j = 1, \ldots, p$ and $k = 1, \ldots, p$
        $C_i \leftarrow C_i + \{n_j, n_k\}$
    if ($C_i \geq n(C_i)$ OR $d_{j,k} > d_{max}$) **then**
    $C_i \leftarrow n(C_i)$

13 end **if**
14 end **while**
15 $N \leftarrow N - n(C_i)$
16 $clsutNb \leftarrow clsutNb + i$
17 end **for**
18 $clustNb \leftarrow$ the set of r clusters
19 end **if**

As input of the algorithm, we have the set of nodes N and the distance matrix $d_{j,k}$ between nodes. The first step of algorithm is to verify there are not empty nodes, after that clusters are iteratively identified. A new cluster is created on the basis of an analysis of the distance matrix from the coordinates of potential locations of nodes. The position of the nodes is determined by its coordinates on the grid. The selection of a subset of nodes (which the distance between them should be less than d_{max}) among a discrete set N is performed and any cluster must satisfy the constraints imposed by the cluster' size ($n(C_i)$). At the end of the algorithm, we obtain a set of clusters (*clsutNb*). The aim of the cluster' formation is to minimise hop boundary from each node and to create an effective topology.

Algorithm 2 Selecting gateway in a cluster

1 **Input**: r, $n(C_i)$, $d_{j,k}$, $S_{j,k}$, n_j, $TC(n_j)$
2 **Output**: Gw_j
3 **for** $c = 1$ to r **do** // for each cluster C_i
4 **for** $n_j = 1$ to $n(C_i)$ **do** // for each node n_j of cluster C_i
5 **Connect** all others nodes of the cluster C_i to the node n_j
6 **Evaluate** the cost of connecting all the other nodes through the node n_j

$$TC(n_j) \leftarrow \sum_{k=2}^{n(C_i)} S_{j,k} \cdot d_{j,k}$$

7 end **for**
8 **Return** *min* $TC(n_j)$
9 $Gw_j \leftarrow n_j$
10 **if** not all clusters were visited **then**
Increase c and go to step 3
11 end **if**
12 end **for**
13 **Return** the Gw_j of each cluster C_i

Assuming that the location of the gateway is potentially equivalent to the location of any node inside the cluster, the selection of a node n_j as a gateway (Gw_j) of a cluster C_i is based on the cost ($TC(n_j)$) of all possible links or connection centred in that node n_j. The Algorithm 2 is based on a sequential search of all clusters of the Algorithm 1 with a gateway in a given node (n_j). The initial data is a list of: r clusters, the number of nodes ($n(C_i)$) in each cluster, the distance matrix $d_{j,k}$ and the cost of single connection between two nodes $S_{j,k}$. Sequentially search through all possible variants of minimum value of

($TC(n_j)$) is performed and the variant with the least cost value ($TC(n_j)$) is chosen from all options. The aim is to minimise the number of gateways as well as reduce the number of supported node. Thereafter, we assume that a connected graph is built, in which every vertex ($n_j \in C_i$ is equipped with at least two radio interfaces. By using two interfaces, a node can simultaneously send and receive frames in both directions, i.e., uplink and downlink. WMNs performance can be optimised by limiting the number of connected nodes associated with the same gateway. The maximum possible mesh routers must be connected to the gateways to create an effective topology. To do this, we have implemented the Algorithm 3 based on Prim's algorithm for the selection of relay nodes to associate with a gateway.

Algorithm 3 Tree covering in a cluster

Input: $n(C_i)$, C_i, r, h_k, $m_{j,k}$, Gw_j, n_j, n_k
Output: $T \leftarrow$ Tree covering in each cluster C_i
  for $c = 1$ to r **do** // for each cluster C_i
  , $m_{j,k} \leftarrow 0$, $Gw_j \leftarrow n_j$, $T \leftarrow Gw_j$
  While ($n_k \leq n(C_i)$) **do** // $\forall j = [1], k = [2..n(C_i)]$
if ($d_{j,k} = =$ min $d_{j,k}$) **then**
$T \leftarrow T \cup \{n_k\}$
$m_{j,k} \leftarrow 1$
  $n(C_i) \leftarrow n(C_i) - \{n_k\}$
end **if**
if ($n(C_i) \neq \varnothing$) **then**
  $k \leftarrow k + 1$
end **if**
  end **while**
end **for**
Come out tree T
  if $h_k \geq 3$ **then** // $h_k \in T$
Choose the node n_k as a relay
  else
Do not consider n_k as a relay
  end **if**

The number of nodes in each cluster ($n(C_i)$, distance matrix ($d_{j,k}$), adjacency matrix ($m_{j,k}$), gateway (Gw_j) and r clusters are collected as the input information of the algorithm. We referred to Prim algorithm (Prim, 1975) for finding the minimum weight tree. This algorithm starts with a completely disconnected graph, i.e., with an empty network, without communication links. We highlight the tree's concentration point and we choose a node n_k which have up to h_k incidents arc as the relay's nodes.

5 Used parameters and results

We describe the methodology we followed for our experiments.

We consider the area of deployment as a grid that should be covered by a set of nodes N (where $N = \{n_1, n_2, \ldots, n_p\}$). Each cell of this grid represents a candidate placement of a node n_j. We assume that a point in the cell can be covered by a node $n_j \in N$, if the distance between the node n_j and this point is less than transmission range of n_j (minimum distance of 100 meters has been fixed in our simulation).

The first step of an evolutionary algorithm is the initialisation of the population, allowing creating a starting set to optimisation. We represent the grid by a chromosome vector modelled as a binary string (with a size equal to $W \times H$ where W represents the width and H the length of the deployment area) corresponding to the number of positions in the grid. Each element of the chromosome is a Boolean representing the presence of a node n_j deployed in the corresponding position i. That is to say at each position i of the grid is allocated a binary decision variable, corresponding to the presence or not of a node deployed on position i and a cell can contain only one node at the same time. We thus generate a set of chromosomes which will constitute the initial population for the two algorithms NSGA-II and MOGA. Both algorithms are initialised with a population size of 100. Each chromosome is evaluated according to the dominance rule by the performance functions. The second step consists in applying the selection, crossing and mutation operators in order to carry out a simulation of the natural evolution. We have used the binary tournament for the selection of individuals, with the single-point recombination and bit-flip mutation operators. Then, the obtained sets of solutions are used to assess the performance of our proposed algorithms. Table 1 (in Appendix B) resumes the parameters of configuration for NSGA-II and MOGA algorithms.

We describe the setup of the computational experiments performed to analyse the presented algorithms. We vary the size of the grid by 7, 14 and 30 km^2 with respectively 150, 300 and 500 distribution of test points and we set the range of each node to 100 m. Our proposed algorithms try to maximise the reliability objective in addition with these solutions found by NSGA-II and MOGA to satisfy cost and coverage objectives. Table 2 (in Appendix C), Table 3 (in Appendix D) and Table 4 (in Appendix E) show the obtained potential solutions for respectively instance 7 km^2, 14 km^2 and 30 km^2 to which are associated the cost (in term of number of nodes), coverage and reliability. The results are obtained after averaging the performances obtained by each of the algorithms following 20 independent simulations.

We plotted in Figures 2–4 (in Appendix F–G) the set of Pareto front approximations found by NSGA-II and MOGA, together with the results provided by our proposed algorithms for different area size. We observe that the solutions reported are characterised by the number of nodes to deploy in order to cover all the area of interest and also constrained by the reliability value of the network. Both NSGA-II and MOGA algorithms try to minimise the number of deployed nodes to satisfy coverage constraint and cost. We can argue that the number of nodes and the size of area have an impact effect on the quality of the solution. The more area size we have to cover, the more expense is the cost of deployment (in term of number of nodes). However more nodes we have, the more covered is the area of deployment. It can be seen that these solutions give a coverage rate between (92% and 96%) and a reliability rate between (0.7744 and 0.8839).

Figure 2 (in Appendix F) shows the Pareto 3D fronts found by the NSGA-II and MOGA algorithms for the area size of 7 km^2. These figures show the different solutions obtained i.e. the compromise between the coverage rate and the reliability of the network with the number of nodes. The results show a better coverage rate obtained by the MOGA algorithm compared to the NSGA-II algorithm, with a margin of more than 4% average (88% to 98% for MOGA compared with 65% to 96% for NSGA-II). But NSGA-II generates a better rate characterising the network reliability of the order of 0.5 to 0.89 compared with the order of 0.2 to 0.86 for MOGA.

Figure 3 (in Appendix F) shows the 3D Pareto fronts obtained for the area size of 14 km^2. This instance proposes a network coverage rate between 84% and 98% for both NSGA-II and MOGA algorithms and on the other hand, the reliability rate of NSGA-II is better than 3% for MOGA. It should also be noted that the two algorithms obtained an equal number of non-dominated solutions for the generation of the Pareto front.

The Pareto 3D fronts obtained for the area size of 30 km^2 instance are shown in figure 4 (in Appendix G). The results show a high coverage rate (> 90% average) obtained by the NSGA-II algorithm compared to the MOGA algorithm (between 85% and 98%). It is also observed that the reliability rate obtained with NSGA-II is significantly higher than that obtained with MOGA. It can be seen logically in Figure 5 (in Appendix G) that as the nodes increase the cost increases.

5.1 Performance indicators for multi-objective algorithms

The major difficulty of multiobjective optimisation assessment is that the output of the optimisation process is not a single solution but a set of solutions representing an approximation of the Pareto front. To evaluate the performances of different multiobjective metaheuristics, one needs to compare sets of solutions forming non-dominated sets. Several performance indicators are used in the literature. To evaluate the performance of the different metaheuristics, we have adopted, in our experiments, the generational distance for the convergence measure, the Spread indicator as the diversity indicator and the hypervolume as the hybrid indicator.

5.1.1 Generational distance

The generational distance I_{GD} computes the average distance between the approximated set A and a reference set R (Veldhuizen et al., 2000). The reference set is generally represented by the exact Pareto front $\mathcal{PF}^*$. At a given iteration t, the distance between the two sets is averaged over the pair wise minimum distances:

$$I_{GD}(A, R) = \frac{\left(\sum_{u \in A} \min_{v \in R} \|F(u) - F(v)\|^2\right)^{1/2}}{|R|}.$$

The generational distance measures how far the compromise surface is a set of solutions. Where the distance represents the Euclidean distance in the objective space. If the approximated front A is included in the reference set R, the generational distance will be equal to 0.

5.1.2 Spread

The spread indicator I_s combines the distribution and cardinality to measure the dispersion of the approximated set A (Zitzler et al., 2000):

$$I_s = \frac{\sum_{u \in A} \left| \left\{ u' \in A : \left\| F(u) - F(u') \right\| > \sigma \right\} \right|}{|A| - 1}$$

where $\sigma > 0$ is a neighbourhood parameter. The closer is the measure to 1 the better is the spread of the approximated set A.

5.1.3 Hypervolume

The indicator of the hypervolume (I_H) measuring the volume of the portion weakly dominated by a set of point A, in the objective space (Zitzler and Thiele, 1999). The calculation of this volume requires the designation of a reference point, which is preferably dominated by all the points of the set A. The indicator of the hypervolume is often calculated relative to a reference set R, this indicator is noted $I_{\bar{H}}$ and defined as follows (Zitzler and Thiele, 1999):

$$I_{\bar{H}}(A) = I_H(R) - I_H(A)$$

where higher values of $I_{\bar{H}}$ is smaller, the quality is better. The indicator of the hypervolume can take into account both the convergence of the algorithm and the variety of solutions found. However, the computational cost is high; the complexity is exponentially proportional to the number of objectives.

The results of these indicators are summarised in Table 5 (in Appendix H) and we have chosen on all three instances the coverage rate between 85% and 98% and an average reliability rate of 0.7. We can see in Table 5 (in Appendix H) that MOGA surpasses NSGA-II in all instances according to the generational distance metric. According to the spreading metric NSGA-II and MOGA are close on instances 14 km^2 and 30 km^2 with the exception of the instance 7 km^2 where NSGA-II surpasses MOGA. We notice that NSGA-II in all three cases, obtained the best results for hypervolume indicators. However the three instances give MOGA results as close as those of NSGA-II for hypervolume indicators.

6 Conclusions and perspectives

We have developed in this paper, a multi-objective optimisation model for node placement in WMNs, where the coverage, reliability and the total installation cost in terms of nodes to be deployed are the three objectives to optimise simultaneously. We applied two well-known evolutionary algorithms, namely the NSGA-II and MOGA to generate the number and positions of the communication nodes. Subsequently, we developed algorithms that determine the cluster formation, gateway selection and relay nodes selection. The results showed satisfactory performance. Knowing that multi-criteria optimisation depends mostly on the decision of the decision maker, some criteria may be included to improve the solutions presented here. Results show that the mesh node placements are able to efficiently improve some performance metrics as coverage, reliability when designing or planning WMNs.

In our future work, we intend to use the simulation based system to study the different network configurations for WMNs performance and also to compare our proposed algorithms with some existing one in the literature as an alternative to the mathematical modelling.

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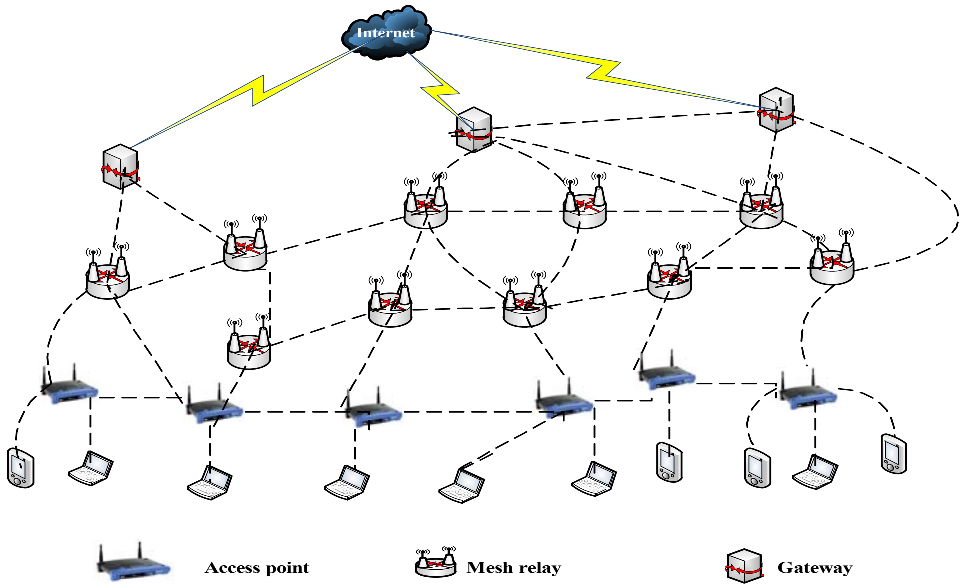
Appendix A

Figure 1 WMN's architecture (see online version for colours)



Appendix B

Table 1 NSGA-II and MOGA used parameters

	NSGA-II	*MOGA*
Number of iterations	4,000	500
Population size	100	100
Crossover probability (single point)	0.9	0.9
Mutation probability (bit flip)	0.06	0.1
Selection	Binary tournament	Binary tournament
Number of independent runs	20	20

Appendix C

Instance details

Table 2 Experimental results for area 7 km^2

Area size (km^2)	*Number of run*	*Distribution of tests points*	*Number of nodes*	*Number of clusters*	*Average relay by cluster*	*Coverage*	*Reliability*
7	1	150	43	4	2	90%	0.9375
	2		45	3	2	91%	0.6337
	3		43	3	2	96%	0.9733
	4		41	3	2	93%	0.7744
	5		48	3	2	91%	0.7518
	6		42	4	2	95%	0.6349
	7		44	5	2	98%	0.8628
	8		42	4	2	97%	0.9733
	9		41	5	2	95%	0.7744
	10		46	3	2	96%	0.7518
	11		47	2	2	96%	0.6349
	12		45	3	2	98%	0.8628
	13		44	4	2	97%	0.8628
	14		40	3	2	98%	0.8628
	15		42	3	2	98%	0.9733
	16		45	3	2	99%	0.9733
	17		41	3	2	98%	0.9733
	18		48	4	2	98%	0.7744
	19		47	4	2	97%	0.8628
	20		43	4	2	99%	0.8628

Notes: Average node $N = 43$, average coverage = 96%, average reliability = 0.7744.

Appendix D

Instance details

Table 3 Experimental results for area 14 km^2

Area size (km^2)	*Number of run*	*Distribution of tests points*	*Number of nodes*	*Number of clusters*	*Average relay by cluster*	*Coverage*	*Reliability*
14	1	300	73	5	3	82%	0.8971
	2		72	5	3	96%	0.8132
	3		81	5	2	89%	0.8403
	4		81	4	3	96%	0.9244
	5		75	5	2	98%	0.9239
	6		83	4	3	96%	0.9646
	7		78	4	3	89%	0.8132
	8		88	4	2	89%	0.8403
	9		79	4	2	87%	0.8971
	10		88	4	2	96%	0.8971
	11		78	4	3	98%	0.8971
	12		92	4	3	97%	0.8971
	13		90	3	3	99%	0.8971
	14		96	3	3	86%	0.8971
	15		89	3	2	89%	0.8971
	16		89	4	2	89%	0.8971
	17		79	5	2	98%	0.8403
	18		78	3	1	98%	0.8403
	19		79	4	3	98%	0.8403
	20		78	4	3	97%	0.9646

Notes: Average node $N = 83$, average coverage = 93%, average reliability = 0.8839.

Appendix E

Instance details

Table 4 Experimental results for area 30 km^2

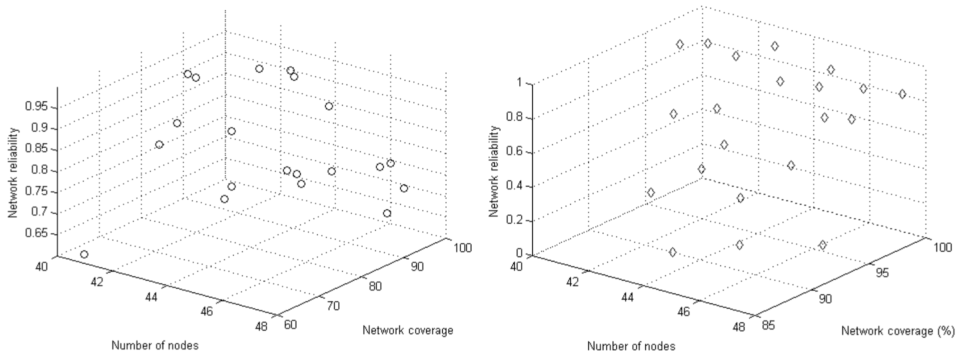
Area size (km^2)	*Number of run*	*Distribution of tests points*	*Number of nodes*	*Number of clusters*	*average relay by cluster*	*Coverage*	*Reliability*
30	1	500	114	10	3	97%	0.8412
	2		113	8	4	98%	0.8451
	3		112	8	4	94%	0.7745
	4		122	9	4	90%	0.8884
	5		122	6	4	91%	0.9181
	6		124	7	4	92%	0.9514
	7		100	8	4	95%	0.8821
	8		111	8	4	96%	0.7821
	9		120	9	4	88%	0.7945
	10		114	11	3	94%	0.8212
	11		108	9	3	89%	0.9532
	12		112	8	4	92%	0.8933
	13		115	8	2	97%	0.9456
	14		123	8	3	93%	0.7466
	15		126	9	3	95%	0.9222
	16		115	9	3	93%	0.8872
	17		116	7	3	89%	0.9113
	18		120	7	3	91%	0.9025
	19		119	6	3	93%	0.786
	20		117	8	3	97%	0.7677

Notes: Average node N = 116, average coverage = 92%, average reliability = 0.8512.

Appendix F

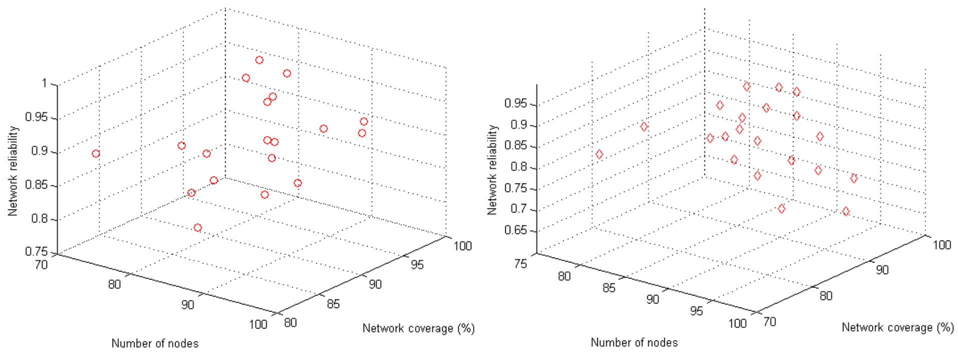
The set of Pareto front approximations found by NSGA-II and MOGA

Figure 2 Pareto fronts for area 7 km^2 found by (a) NSGA-II (b) MOGA



(a) (b)

Figure 3 Pareto fronts for area 14 km^2 found by (a) NSGA-II (b) MOGA (see online version for colours)

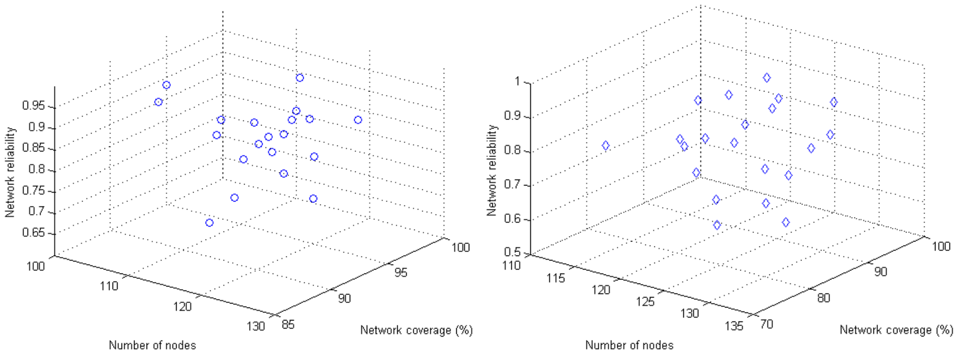


(a) (b)

Appendix G

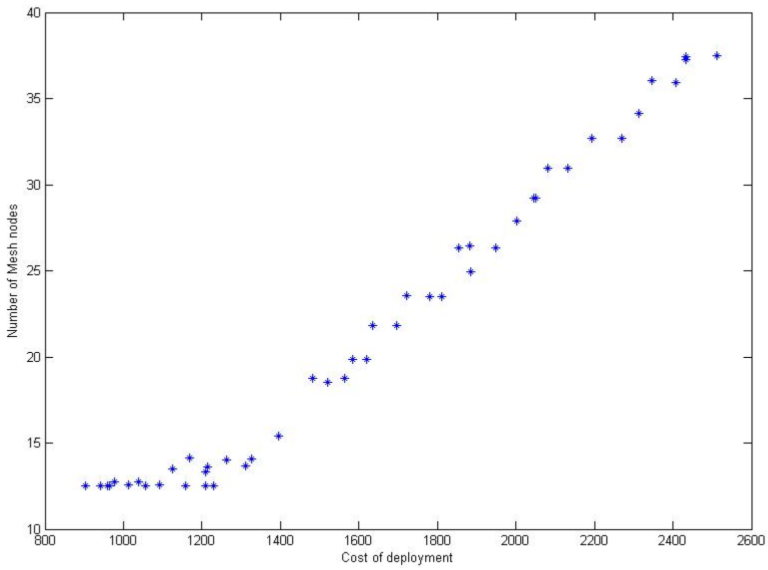
The set of Pareto front approximations found by NSGA-II and MOGA

Figure 4 Pareto fronts for area 30 km^2 found by (a) NSGA-II (b) MOGA (see online version for colours)



(a) (b)

Figure 5 Cost of deployment vs. numbers of nodes (see online version for colours)



Appendix H

Performance indicators for multi-objective algorithms

Table 5 Performance indicator for NSGA II and MOGA

Indicator metrics	*Area size*	*NSGA II*	*MOGA*
Generational distance	7 km^2	7.73e–02	8.67e–01
	14 km^2	1.68e–01	1.02e–01
	30 km^2	1.32e+01	2.68e+00
Spread	7 km^2	0.57592	0.65014
	14 km^2	0.79073	0.74075
	30 km^2	0.89007	0.64013
Hypervolume	7 km^2	0.063652	0.101285
	14 km^2	0.110741	0.128813
	30 km^2	0.050469	0.069648