Energy 119 (2017) 384–391

Contents lists available at ScienceDirect

Energy

journal homepage: www.elsevier.com/locate/energy

Benefit allocation for distributed energy network participants applying game theory based solutions

Qiong Wu [a, *], Hongbo Ren [a, **], Weijun Gao [b], Jianxing Ren [a]

[a] College of Energy and Mechanical Engineering, Shanghai University of Electric Power, 200090 Shanghai, China
[b] Faculty of Environmental Engineering, The University of Kitakyushu, 808-0135 Kitakyushu, Japan

ARTICLE INFO

Article history:
Received 10 August 2016
Received in revised form
19 December 2016
Accepted 20 December 2016

Keywords:
MILP model
Optimization
Distributed energy network
Cooperative game theory
Benefit allocation

ABSTRACT

This study develops a mixed-integer linear programming (MILP) model integrating energy system optimization and benefit allocation scheme of the building distributed heating network. Based on the proposed model, the minimized annual total cost, energy generators configuration, optimal operation strategy and heating pipeline lay-out of the distributed energy network can be determined. Moreover, four benefit allocation schemes (Shapely, the Nucleolus, DP equivalent method, Nash-Harsanyi) based on cooperative game theory are employed to deal with the benefit (reduced annual cost) assignment among the building clusters, while considering the stability and fairness of each scheme. As a case study, a local area including three buildings located in Shanghai, China is selected for analysis. The simulation results indicate that the ground coalition in which all buildings cooperate with each other by sharing and interchanging the thermal energy yields the best economic performance for the distributed energy network as a whole. In addition, different allocation schemes may result in diversified outcomes in terms of the fairness and stability, which are measured by the Shapley-Shubik Power Index and the Propensity to Disrupt value, respectively. For the current case study, the Shapely value method is recognized to be the most acceptable allocation scheme from both viewpoints.

1. Introduction

Growth in population, enhancement of building services and comfort levels, together with the rise in time spent inside buildings, have raised building energy consumption to the levels of transport and industry [1]. According to EIA (energy information administration), the building sector accounts for around 20.1% of total primary energy consumption worldwide in 2016. An effective alternative method to deal with the increasing energy using in buildings is the adoption of building combined heat and power (BCHP) system [2–4]. As a typical distributed generation system, the BCHP system produces electric and thermal energy simultaneously on-site or near site, and can convert as much as 75–80% of the fuel source into useful energy [4]. The BCHP systems have been introduced into commercial buildings such as hospitals, hotels, offices and so on.

Based on the distributed BCHP system, the generated surplus energy can be shared among the buildings to realize the energy interchange network. In the energy network, to maximize own payoff, a consumer may seek to displace expensive generator by importing energy from neighboring consumers with lower purchasing cost compared with the utility grid. Likewise, a player with excess generator capacity can choose to export energy and receive an immediate return on its initial investment. Therefore, through the distributed energy network, not only the energy performance of the whole system can be improved, but also the unbalance problem between supply and demand sides within each building can be resolved. However, to realize the best performance of a distributed energy network, optimal generator location, management of system operating and energy interacting strategy is critical. Numerous of research has studied the operation of the distributed energy network, for the purpose of energy saving, cost reduction as well as reliability improving. Yang et al. [5] constructed a superstructure based MILP model to achieve simultaneous optimization of capacity, number, and location of energy generator as well as energy distribution network structure of the entire system; two kinds of prime movers (gas engine and gas turbine) were considered as the alternative technology for the BCHP system. Bracco et al.

* Corresponding author.
** Corresponding author.
E-mail addresses: wuqiongrff@163.com (Q. Wu), tjrhb@163.com (H. Ren).

http://dx.doi.org/10.1016/j.energy.2016.12.088
0360-5442/

[6] adopted a model to optimally design and operate a hybrid heating network in a building cluster equipped with small-size CHP plants, while considering the economic saving and emissions reduction simultaneously. Casisi et al. [7] proposed an optimization model to deal with the optimal design and operation of a distributed cogeneration system with a district heating network while considering the energy saving and emissions reduction at the same time; not only a set of micro-gas turbines located inside public buildings, but also a centralized CHP system based on Internal Combustion Engine is taken into account. Besides the building scale energy network, some studies focused on the district level energy supply system. Obara et al. [8] considered the construction of a Syowa Base energy network, aiming at reducing the fuel consumption and increasing green energy utilization compared with the conventional energy supply system. Weber et al. [9] presented the DESDOP tool to determine the optimal mix of energy technologies for a small city considering distributed energy network, aiming at decreasing the emissions while at the same time guaranteeing the resilience of energy supply.

Summarizing these studies, even though most of them have optimized the energy consumption and/or cost of the whole distributed energy network, how to award the payoff from the cooperation to each player, which is the key question in a cooperation, is paid little attention. If one building can obtain more profits through collaborating with others in some coalition, it will prefer to collaborate to form this coalition rather than act individually, and vice versa. Once the buildings begin to cooperate with others in providing energy demands, the coalition is formed, and all of the consumers can be considered as the multiple stakeholders. Thus, the coordination of their interests is necessary. The purpose of this study is to strive to begin addressing this gap by considering a fair economic settlement scheme for participants in a distributed energy network based on cooperative game theory, which has been widely used to deal with the allocation of cost/gain to incentivize the stakeholders who are cooperating [10–15]. In this study, a MILP model integrating the energy system optimization and benefit (reduced annual cost) allocation scheme of the distributed energy network is proposed and verified through a case study.

The rest of the study proceeds as follows. In Section 2, the framework of the integrated programming model is introduced. Section 3 describes the model for optimal design and operation of the distributed energy network, as well as different profit allocation schemes in detail. Sections 4 and 5 discussed the input data and results of the case study. Finally, several conclusions are deduced and summarized in Section 6.

2. Problem definition

Fig. 1 shows the overall framework of the integrated programming models including the energy supply system optimization model, as well as fair benefit allocation model based on cooperative game theory. In the first model, the input data include energy load, fuel prices, and characteristic data of various alternative technologies. The objective function is to minimize the annual total cost while considering various constraints. Through the first model, the minimized annual cost, optimal running strategy including technology selection as well as heating pipeline lay-out of all possible coalitions can be deduced. Then, based on the output of the first model, four gain/cost assignment schemes, namely the Nucleolus, the Shapley value, the Nash-Harsanyi (N-H) solution as well as Propensity to Disrupt (DP) equivalent method are considered for the allocation of the reduced cost through cooperation of the buildings. Following which, a comparison analysis is included for different allocation methods. Finally, by employing the Shapely-Shubik Power index and DP methods, the fairness and stability of each allocation scheme can be measured. It is important for the participants to decide whether to join the coalition or not. This is because, if the gain/cost allocation is not stable and fair, the cooperation will not persist, the analysis of fairness and stability can thus be helpful for the stakeholders who are making long-term decisions.

3. Mathematical formulation

Generally, in the distributed energy supply network, the electric power demand of each building is satisfied by the BCHP unit if installed, and the deficiency can be supplied by the external utility grid. As to the thermal demand, there are many types of heat resources. The recovered heat from the BCHP unit is one option, backup boiler is another option. Moreover, heat can be interchanged among the building consumers via a distributed heating pipeline and the line distances among the buildings are calculated prior to the optimization. Note that, the cooling demand is also served by the electric power using compression chillers. On the other hand, in order to promote the BCHP unit adoption, the surplus electricity generated can be sold back to the utility grid to make a profit.

3.1. Energy supply system optimization model

The aims of the energy system optimization model include: defining the type and number of BCHP unit in each building, determining the optimal operation strategy of the whole system, as well as deciding the optimal lay-out of the distributed heating network. The objective function is to minimize the total annual cost ($Cost_{tot}$) which consists of annualized initial investment cost ($Cost_{equip}$), the sustained external fuel purchasing cost ($Cost_{fuel}$), the annual maintenance cost ($Cost_{main}$), the annualized energy transfer line cost ($Cost_{dhn}$) and minus profits from the selling of excess electricity to the macro-grid ($Cost_{sal}$), all the year long.

$$Cost_{tot} = Cost_{equip} + Cost_{fuel} + Cost_{main} + Cost_{dhn} - Cost_{sal} \quad (1)$$

Commonly, the energy flows, the equipment characteristics, and the operation mode constitute the constraints in the optimization problem. Hence, the objective function must be minimized subjecting to the following constraints [5,16,17] formulated for each time period:

- The electric and thermal energy input must be equal to the output;
- The performance constraints of the equipment components, e.g. BCHP unit and boiler have to be followed;
- The trade-off constraints with the utility grid, as well as the interchange constraints among the building clusters must be satisfied.

It is worth noting that, the equipment selection and placement from the alternatives, as well as the distributed heating pipeline options are defined as binary variables in the formulas. Based on the concepts introduced above, an energy system optimization model is established, through which the minimized annual cost of each coalition formed by the buildings can be deduced. The detailed information can refer our previous studies [18,19].

3.2. Basic concepts of cooperative game theory

Generally, if there are more than one decision-makers pursuing their own profits at the same time, a decision-making process is called a game. The game theory has been proved to be an effective

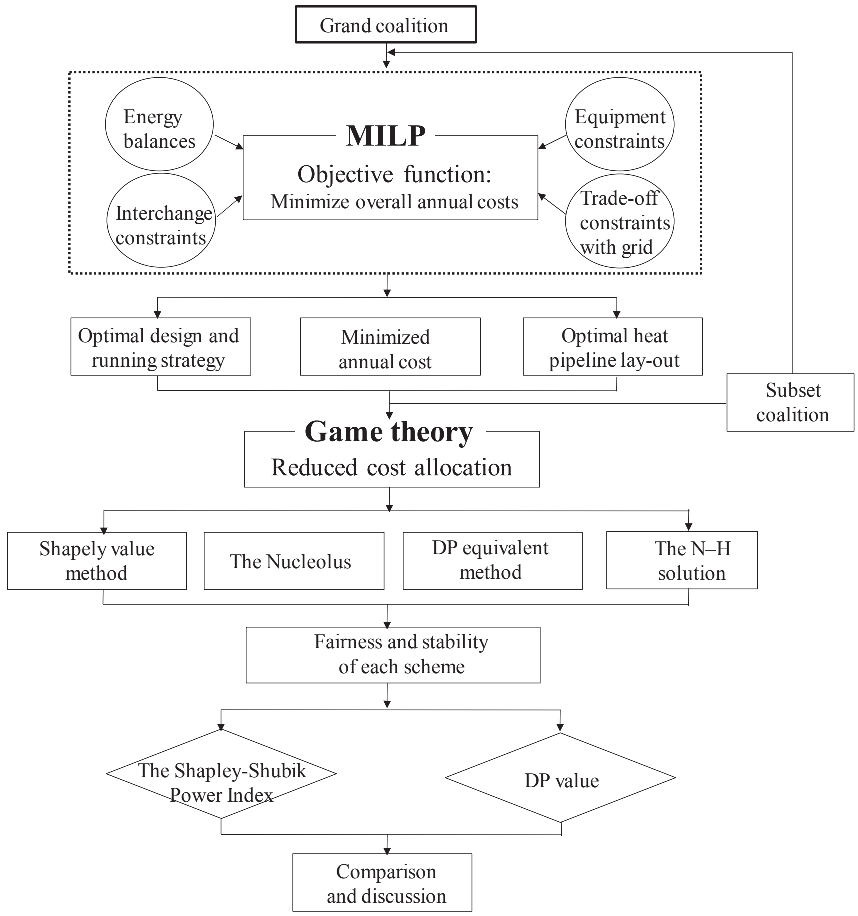

Fig. 1. The overall framework of the integrated programming models.

method to analyze the conflict and cooperation problems between rational decision-makers [10,20–22]. Usually, a game can be divided into two forms: a cooperative game and a non-cooperative game [23] depending on the existing of a binding agreement among the players or not. In addition, different from non-cooperative games, cooperative games emphasize the cooperation and bargaining between players, and care about the problem how to assign the payoffs through cooperation. Furthermore, cooperative games can be further divided into transferable and non-transferable payoff. In this study, we assume the payoff is transferable among the buildings involved through the energy interchanging cost within the distributed energy network.

Analysis based on the cooperative game theory is centered on two major issues: coalition formation and gain/cost assignment through cooperation. Undoubtedly, each participant wants to obtain its maximum profit in the coalition, thus, a satisfactory and reasonable gain/cost allocation scheme is of vital importance. In the following, the concepts and algorithms of various cooperative game theory based benefit assignment methods, as well as the fairness and stability evaluation methods are introduced one by one.

3.3. The core

In a cooperative game, the Core [24] is a set of stable imputations, placing a reasonable upper limit on the final payment to each participant, under which no player has an incentive to disrupt the coalition and no coalition can improve upon or block.

Assuming $1, 2, ...n$ is all players in a game, and let N denote the set of the players, $N = \{1, 2, ...n\}$. A coalition S, is defined to be a subset of N, $S \subset N$, which means part of the players act jointly in the game and cooperate with each other to make blinding agreements. By convention, the empty set is called empty coalition, and the set N is also a coalition, called the grand coalition. In all, for n players, there are 2^n coalitions in a game with the empty coalition and grand coalition involved. In addition, let X be a payoff vector: $X = \{x_1, x_2, ...x_n\}$, in which x_i means that through the cooperative coalition, the allocated profit received by player i is x_i. It is worth noting that, a payoff vector cannot be considered as a reasonable candidate for a solution unless it satisfies the individual rationality and group rationality [15], which are shown in Eq. (2) and Eq. (3), respectively. The individual rationality indicates that no player could agree to receive the profit less than that player could obtain when acts alone. Again, the group rationality means that, the possible maximum overall profit of the game must be equal to the sum of the obtained profit of each player. V is a real-valued function, called the characteristic function of the game, which indicates the total gain/reduced cost through cooperation.

$$x_i \geq V(\{i\}) \quad \forall i \in N \tag{2}$$

$$\sum_{i=1}^{n} x_i = V(N) \quad \forall i \in N \tag{3}$$

Furthermore, besides the individual and group rationality, any imputation in a Core should satisfy the subgroup coalition rationality. It means that for the members of coalition S, the sum of the profit assigned to each member in any subset of N should be no less that the overall profit received by any disjoint set of coalitions they could form. This condition is strict, so the existence of a Core cannot be guaranteed for a cooperative game, and at the same time, there may be more than one imputation in a Core.

$$\sum_{i=1}^{s} x_i \geq V(S) \quad \forall S \subset N \tag{4}$$

3.4. Fairness and stability of the solution

As mentioned above, each player has an acceptable cost/gain boundary that forms the Core. However, the fulfillment of the Core conditions is only a necessary condition in a gain/cost assignment scheme for its acceptability by the players. In other words, although the cost/gain allocations are included in the Core, some players may still view these solutions as unfair and may not accept the assignments. Therefore, it is necessary to analyze the fairness and stability of different solutions for the long-term sustainability of player cooperation. To measure the fairness, the Shapley-Shubik Power Index formulated by Lloyd Shapley and Martin Shubik in 1954 to measure the powers of players in a voting game is adopted, and to evaluate the fairness of a given allocation scheme among all players [25,26]. The index compares the gains to a player with the gains to the coalition, which can be expressed as:

$$\alpha_i = \frac{x_i - v(\{i\})}{\sum_{i}^{n}(x_i - v(\{i\}))} \quad i \in N; \sum_{i}^{n} \alpha_i = 1 \tag{5}$$

α_i represents the Power Index. Based on the deduced Power Index, the Fairness Index can be calculated through the following formula:

$$FI_\alpha = \sigma_\alpha / \overline{\alpha} \quad 0 \leq FI_\alpha \leq 1 \tag{6}$$

where, FI_α indicates the Fairness Index. σ_α is the standard variance and $\overline{\alpha}$ represents the average value. The greater the value of FI_α, the lower the fairness of the allocation scheme, and vice versa.

On the other hand, for the stability of a coalition, the concept of DP value introduced by Gately in 1974 [27,28] is employed to measure whether the coalition will be disrupt or not based on the allocation scheme. The DP value of player i is defined as the loss of the members except player i in coalition N, compared to the loss of player i if i refuses to cooperate and disrupts the grand coalition. This ratio can be expressed as:

$$G_i = \frac{\sum_{j \neq i} x_j - v(N-i)}{x_i - v(i)} = \frac{v(N) - v(N-i)}{x_i} - 1 \quad \forall i, j \in N \tag{7}$$

where, G_i is the DP value for player i. $\sum_{j \neq i} x_j$ is the sum of the allocated cost/gain of each player except i through the grand coalition. $v(N-i)$ indicates the total benefit deduced from the cooperation formed by all players other than i. Clearly, the higher the DP value for a given player, the greater player i will disrupt the grand coalition unless his allocation is improved. In detail, if the DP value is more than 1, it means that the loss of the members except player i is larger than the loss of player i if i refuses to cooperate in coalition N. In other words, due to the important role of player i, the cooperation is easily to be disrupted unless player i could receive more benefits. On the contrary, if the value is less than 1, even becomes negative, it reflects the enthusiasm for the allocation [25], and the coalition is relatively stable.

3.5. Gain/cost allocation solutions

According to the above analysis, allocating cooperative gain/cost to induce cooperation of the stakeholders is necessary. In this study, four gain/cost allocation schemes will be considered: the Nucleolus, the Shapley value, DP equivalent method as well as the N–H solution. These schemes are selected for this study because they have been widely used [25,29–32] and are relatively easy to be implemented. In the following, a short review of the four main existing approaches based on cooperative game theory is provided.

3.5.1. The shapely value

In game theory, the Shapley value was proposed by Shapely in 1953 [13,30]. Generally, allocations are proportional to the marginal contribution of a player to the total cost/gain. The Shapley value uses the notion that the marginal contribution depends on the order in which a player joins the coalition, to the greatest extent by evaluating the marginal cost/gain of each player for every possible order of recruitment. The Shapley value allocation for player i in an n-person game is defined as follows:

$$\varphi_i(v) = \sum_{\substack{S \in N \\ i \in S}} W(|S|)[v(S) - v(S - \{i\})] \forall i = 1, ..., n \tag{8}$$

$$W(|S|) = \frac{(|S|-1)!(n-|S|)!}{n!} \tag{9}$$

where, $\varphi_i(v)$ represents the Shapely value to player i from playing in the game, $|S|$ is the number of players within each coalition S, $v(S - \{i\})$ is the gain/cost value related to the coalition which is formed by all members of S but i. $W(|S|)$ is the weight factor which represents the percentage of marginal benefit that should be allocated to each player.

3.5.2. The Nucleolus

Another interesting value function for a cooperative game may be found in the Nucleolus, which is a single point inside the Core (if the Core is non-empty) [31,33]. In order to understand the rationale of the Nucleolus, let $e(S, x)$ denote the excess, which measures the inequity of an imputation $\{x_i\}$ for a coalition S: $V(S) - \sum_{i=1}^{s} x_i$. In addition, to obtain the Nucleolus, the ε-core is defined to be the set of allocations that would be in the Core if each coalition is given a subsidy at the level of ε. The Nucleolus tries to find an imputation that minimizes the ε of the various coalitions which also called the least core, and can be determined according to Eq. (10) and Eq. (11).

It is worth noting that, since the Nucleolus is always in the Core if it exists, the calculation of the Nucleolus must be also subject to the Core conditions as shown in Eq. (2)~(4).

$$\min \varepsilon \tag{10}$$

$$e(S, x) = V(S) - \sum_{i=1}^{s} x_i \leq \varepsilon \quad \forall S \subset N \tag{11}$$

3.5.3. DP equivalent method

As mentioned above, the concept of DP value has been utilized to judge the stability of a coalition. In this method, the allocation scheme can be deduced assuming that the DP value of each building involved is the same, thus, the degree of preference for the participants in the profit allocation strategy is the same.

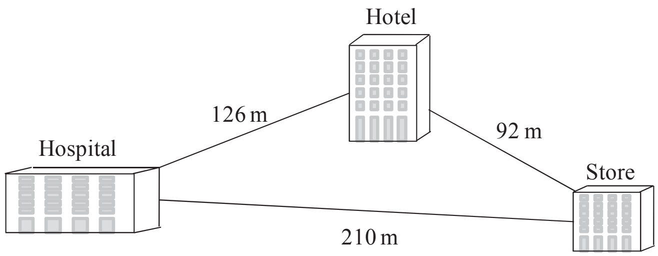

Fig. 2. The geographic lay-out of the buildings.

$$DP_1 = DP_2 = \ldots\ldots = DP_n \tag{12}$$

3.5.4. The N–H solution

The N–H allocation [29] maximizes the difference between the allocated profit gained from cooperation in grand coalition and the no-cooperation case, by equating the gains of all players. Meanwhile, the solution should be subjected to the Core conditions as mentioned in Eq. (2)~(4). The formulation of this solution concept is as follows:

$$\max \prod_{i \in N} (x(i) - v(i)) \tag{13}$$

4. Case study

This study applies the proposed optimization framework for a building cluster with three categories of buildings located in Shanghai, China. There are store, hotel and hospital with a floor area of 20,000 m^2 for each of them. The geographic lay-out and distance of the buildings are shown in Fig. 2.

4.1. Energy demands

Detailed and accurate information about the energy demands is one of the most important inputs to the optimization model. Fig. 3 shows the hourly electric and thermal energy demands of each building involved on a typical day in winter. It can be concluded that, both of hotel and hospital have electricity and heating demands throughout the whole day for their specific utilization forms, whereas store has thermal demand only during the daytime. In addition, all the three buildings illustrate similar electricity load profiles, in which high demand occurs during the daytime, and low demand occurs at night. Furthermore, hotel enjoys the highest thermal demand, followed by hospital and store.

Table 1
Technical and economical characteristics of the equipment candidates.

Technology	Capacity (kW)	Rated efficiency/COP	Unit capital cost ($/kW)	Lifetime (years)
Natural gas boiler	≦1000	83.0%	100.00	30
Gas engine	100	27.0%	1900.00	20
	230	33.0%	1905.63	20
	470	34.2%	1699.73	20
	633	34.5%	1790.00	20
	1121	36.8%	1475.00	20
	2000	43.7%	1348.19	20
Compression chiller	–	4.73	102.00	25
Heating pipeline	–	–	787.00	20

4.2. Energy tariffs

Market data, such as electricity tariff and fuel price is another important data for the economic performance of the energy system. Here, the energy tariffs in Shanghai are investigated and employed for this case study. More specifically, according to the investigation, as the fuel for BCHP unit and gas boiler, natural gas price is constant, which is 0.057 $/kWh for commercial users. On the other hand, a time of use tariff structure for electric power has been employed in Shanghai, which is 0.156 $/kWh during the peak time period (AM 6:00~PM 10:00) and 0.072 $/kWh during the off-peak period (PM 11:00~PM 12:00, AM 1:00~AM 5:00). Furthermore, in order to promote the adoption of the BCHP system, assuming that the surplus electricity generated on site can be sold back to the utility grid to make a profit. The buy-back price is assumed as 0.06 $/kWh.

4.3. Alternative equipment options

Table 1 lists the technical and economical characteristics of the equipment candidates covered in this case study, including capacity, generation efficiency, unit cost and lifetime [5,8,17]. Generally, the type and capacity alternatives of the BCHP units have been given in advance, whereas other technologies are only given the boundaries. Moreover, it is worth noting that, in order to keep the linearity of the proposed model, the energy generation efficiency of the BCHP unit is considered to be constant [17]. Although actually, it may vary as the change of the unit capacity, the partial load factor as well as outside temperature.

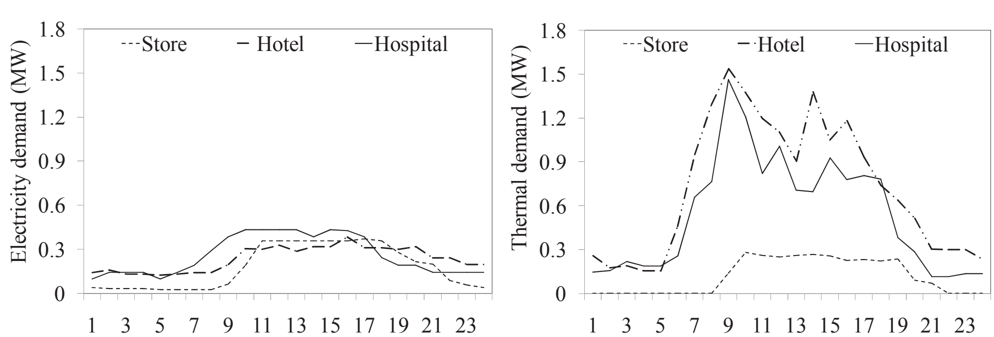

Fig. 3. The load profile of each building on a typical day in winter.

Table 2
Generator combination and corresponding capacity in each coalition.

S	CHP unit (kW)			Boiler (kW)			Compression chiller (kW)		
	1	2	3	1	2	3	1	2	3
1	100	–	–	119	–	–	154	–	–
2	–	1121	–	–	1000	–	–	137	–
3	–	–	633	–	–	782	–	–	450
12	230	230	–	615	790	–	154	137	–
13	230	–	230	90	–	1000	154	–	450
23	–	1121	230	–	1000	813	–	137	450
123	230	230	230	742	1000	935	154	137	

5. Results and discussion

To solve the benefit allocation problem based on cooperative game theory, each building can be regarded as one player in a game. The set of players in the game is represented as $N = \{1, 2, 3\}$, in which 1, 2 and 3 implies store, hotel and hospital respectively. As mentioned in Section 3, in theory, these buildings can partially cooperate in seven nonempty combinations as follows: $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{1, 2, 3\}$. Each subset forms a coalition in which different combinations of consumers take part in the distributed energy system with thermal interchange. The empty subset is the situation in which there is no building participant in the system and all buildings act alone. Furthermore, it is worth noting that, this study utilized Linear Interactive and General Optimizer (Lingo) to solve the MILP models both for the energy network optimization and fair profit allocation among the players.

Table 2 shows the optimized design of the distributed energy network including the generator combination and the corresponding capacity in each possible coalition. The short dash means that the technology is not adopted. According to the table, it can be found that, the selected capacities of BCHP units for store, hotel and hospital when they act individually are 100 kW, 1121 kW, 633 kW respectively. Meanwhile, the boiler installation capacities are 119 kW, 1000 kW and 782 kW. Again, as to the cooperation of two/three players, a BCHP unit with 230 kW is preferred and selected in all situations except for hotel when cooperated with hospital. Furthermore, note that the introduced capacity of compression chiller for each player is the same in all coalitions. This is because the capacity of the compression chiller is only determined by the peak value of the hourly cooling demand in each building.

5.1. Costs for various degrees of cooperation

The annual total costs and reduced costs of all possible coalitions: non-cooperation, partial cooperation, and full cooperation for the three buildings involved, are displayed in Table 3. The result reveals that the costs of all partly and fully cooperative coalitions are lower than those of non-cooperation, for all of the reduced cost are no less than 0. The minimal reduced cost is 24236 $ appearing with cooperation between 1 (store) and 3 (hospital), whereas the maximal reduced cost is 112207 $ under the situation with full cooperation. Furthermore, compared with the situation that each building acts alone, the overall cooperation results in an annual energy cost reduction ratio of about 7.6%. It should be indicated that, this value may enlarge even more while compared with conventional energy system without distributed energy adoption.

Table 3
Annual total cost and reduced cost for various degrees of cooperation.

Scenario	Coalition	Annual total cost ($)	Annual reduced cost ($)	Reduction ratio of energy consumption (%)
Act alone	1	326115	0	0
	2	630958		
	3	518262		
Subset coalition	12	869248	87825	4
	13	820141	24236	1
	23	1120648	28572	7
Grand coalition	123	1363128	112207	8

Table 4
The Core of the grand cooperation.

Player	The core	
	Lower limit ($)	Upper limit ($)
1	0	83635
2	0	87972
3	0	24382

Moreover, the reduction ratios of annual energy consumption for different scenarios are also listed in Table 3. It can be concluded that, compared with the non-cooperation situation, both partly and fully cooperative coalitions result in reduced annual energy consumption. In detail, the reduction ratios are 4%, 1%, 7% and 8% for 12 cooperation (store and hotel), 13 cooperation (store and hospital), 23 cooperation (hotel and hospital) and 123 grand cooperation (store, hotel and hospital), respectively.

5.2. The core of the cooperation game

The allocation of the reduced cost of all three buildings cannot migrate outside of the Core. In the following, the Core of the game will be presented firstly, since as mentioned in Section 3, all the allocation schemes are evaluated with regard to the Core allocations. Table 4 lists the Core of the grand cooperation in the distributed energy network, and shows the lower and upper limit of reduced cost allocation within each building involved.

5.3. Reduced cost allocation schemes based on four solutions

By applying the introduced four methods of cooperative game theory to the case study, the reduced cost assignment strategy of each method can be deduced and summarized in Table 5. Apparently, the sum of the allocated value of each building is equal to the total reduced cost (112,207 $) through the grand coalition. Meanwhile, all of the allocation schemes satisfy the Core requirements as shown in Table 4. Comparing the values of x_1, x_2, x_3 which represent the assigned reduced costs for store, hotel and hospital, it can be found that, generally, store and hotel enjoy higher benefit than hospital in all schemes. In addition, the amounts of reduced cost allocated to players in Shapely value method and DP equivalent method are approximately similar. However, there also exist some differences in the results of the four allocation schemes. For example, store enjoys the highest allocation produced by the Nucleolus method, whereas replaced by hotel produced by Shapely value method and DP equivalent method. Again, hospital prefers the assignment strategy of the N–H method due to the highest allocated benefit compared with other three methods, and may be against the Nucleolus in which the allocated benefit is 0.

5.4. Fairness and stability for different allocation schemes

In the following, in order to investigate the fairness and stability for different allocation schemes, as mentioned in Section 3, Shapley-Shubik Power Index and DP value are calculated and compared with each other.

Table 5
Reduced cost assignment using different schemes.

Solution scheme	Reduced cost allocation ($)			In core
	x_1	x_2	x_3	
Shapely value method	46555	48723	16929	Yes
The nucleolus	59253	52954	0	Yes
DP equivalent method	47882	50365	13960	Yes
The N–H solution	43913	43913	24382	Yes

Table 6
The fairness evaluation of each scheme.

Solution scheme	The Shapley-Shubik power index			Fairness index
	α_1	α_2	α_3	
Shapely value method	0.41	0.43	0.16	0.37
The Nucleolus	0.53	0.47	0.00	0.71
DP equivalent method	0.43	0.45	0.12	0.44
The N–H solution	0.39	0.39	0.22	0.25

Table 6 shows the Power Index within each scheme by utilizing Eq. (5), and based on which the Fairness Index can be calculated through Eq. (6). It can be found that, generally, the sum of Shapley-Shubik Power Index for each player is equal to 1. In addition, the Power Index for hospital in each allocation arrangement is the smallest in comparison with other two buildings. Especially in the Nucleolus method, the value is equal to 0. This is because, the allocated benefit for hospital is 0 when adopting the Nucleolus method (see Table 5). Furthermore, Shapely value method and DP equivalent method have similar Power Indexes for all buildings.

On the other hand, as mentioned in Section 3, the greater the value of Fairness Index, the lower the fairness of the allocation strategy. Thus, it can be concluded that, although all of the values are within a reasonable range ($0 \le FI_\alpha \le 1$) referring Eq. (6), the most fairness scheme is the N–H solution due to the lowest Fairness Index and followed by Shapely value, DP equivalent method and the Nucleolus ($0.25 < 0.37 < 0.44 < 0.71$), respectively.

Table 7 shows the DP value for each player using the four allocation methods. It can be found that, the calculated DP value for hospital employing the Nucleolus method is infinite referring Eq. (7). This is because based on this method, the allocated benefit for hospital is 0 (see Table 5). Nevertheless, as mentioned in Section 3, the smaller the DP value is, the greater willingness the player will have to join the coalition and vice versa. Again, one player will disrupt the coalition only when the DP value is less than 1. Therefore, the hospital will refuse to accept the allocation strategy based on the Nucleolus method. Likewise, in the N-H solution, the DP value for hotel is equal to 1.03, which also leads hotel to be more likely to consider defecting the cooperation. Therefore, it can be concluded that, both of the allocation schemes based on the Nucleolus method and N-H solution are unstable unless the benefit assignments among the buildings are reallocated.

On the other hand, all three buildings have interests in keeping the grand coalition when using the Shapely value and DP equivalent methods, since in both of which, all of the DP values are less than 1 and can meet the stability requirements.

Table 7
The stability evaluation of each scheme.

Solution scheme	DP value		
	DP1	*DP2*	*DP3*
Shapely value method	0.80	0.81	0.44
The Nucleolus	0.41	0.66	–
DP equivalent method	0.74	0.74	0.74
The N–H solution	0.90	1.03	0.00

According to above discussions, as to the current test case, to deal with the benefit allocation problem among the consumers within the distributed energy network, the N-H solution and the Shapely value method are more recommended when only considering the allocation fairness aspect. Again, to keep the stability of the grand coalition, both of the Shapely value method and the DP equivalent method are feasible. Therefore, in summary, the best allocation scheme can be deduced through the Shapely value method which can meet both of the fairness and stability of the allocation strategy simultaneously.

6. Conclusions

In this study, an energy supply system optimization model integrating reduced cost allocation methods through energy interchange is proposed. Four widely used game theory based cost/gain assignment schemes: Shapely value, the Nucleolus, DP equivalent method and the N–H solution are chosen to deal with the benefit allocation problem. In addition, two kinds of index: the Shapley-Shubik Power Index and DP value are selected to evaluate the performances of the four solutions. The results of the case study reveal the following conclusions:

(1) The grand cooperation in which all buildings join the energy network enjoys the best economic performance compared with other subset coalitions. Therefore, energy interchanging between different end-users may result in additional economic benefits. However, the formation of local energy network may lead to some technical and administrative complications which should be paid enough attention.
(2) Different gain/cost assignment methods show different outcomes in terms of the fairness and stability to the players, whereas the allocated reduced cost of each building is within the boundary of the Core.
(3) According to the performance evaluation results of four alternative gain/cost allocation schemes, the Shapely value method may be recognized to be the most acceptable allocation scheme for the present case study, from both the fairness and stability aspects. It should be notice that, the conclusions may be sensitive to the physical and economic features of the distributed energy network. Therefore, to verify the universality of the conclusion, additional analyses of various test cases are necessary.

In the following studies, to promote the realization and penetration of the distributed energy network, intensive study on the technical and administrative issues of the distributed energy network will be executed from both qualitative and quantitative viewpoints. From the technical viewpoint, the technology and method to integrate and manage the interacting energy sources, energy supply networks and energy demands across multiple energy vectors will be focused. From the administrative viewpoint, how to coordinate the profits (not only the economic one) of various stakeholders and break through the system and policy barriers will be paid the main attention. In addition, additional analyzes on various test cases (e.g., energy networks with difference scale or diversified combination of customers) will be executed to verify the numerical accuracy of the proposed methods.

Acknowledgements

The research work was supported by National Natural Science

Foundation of China under Grant No. 71403162, as well as The Key Fund of Science and Technology Commission of Shanghai Municipality (No. 16020500700 and No. 16020500900).

References

[1] Pérez-Lombard L, Ortiz J, Pout C. A review on buildings energy consumption information. Energy Build 2008;40(3):394–8.
[2] Wang J-J, Yang K, Xu Z-L, Fu C, Li L, Zhou Z-K. Combined methodology of optimization and life cycle inventory for a biomass gasification based BCHP system. Biomass Bioenergy 2014;67:32–45.
[3] Wang J-J, Xu Z-L, Jin H-G, Shi G-h, Fu C, Yang K. Design optimization and analysis of a biomass gasification based BCHP system: a case study in Harbin. China Renew Energy 2014;71:572–83.
[4] Ruan Y, Liu Q, Li Z, Wu J. Optimization and analysis of Building Combined Cooling, Heating and Power (BCHP) plants with chilled ice thermal storage system. Appl Energy 2016;179:738–54.
[5] Yang Y, Zhang S, Xiao Y. Optimal design of distributed energy resource systems coupled with energy distribution networks. Energy 2015;85:433–48.
[6] Bracco S, Dentici G, Siri S. Economic and environmental optimization model for the design and the operation of a combined heat and power distributed generation system in an urban area. Energy 2013;55:1014–24.
[7] Casisi M, Pinamonti P, Reini M. Optimal lay-out and operation of combined heat & power (CHP) distributed generation systems. Energy 2009;34(12):2175–83.
[8] Obara Sy, Morizane Y, Morel J. A study of small-scale energy networks of the Japanese Syowa Base in Antarctica by distributed engine generators. Appl Energy 2013;111:113–28.
[9] Weber C, Shah N. Optimisation based design of a district energy system for an eco-town in the United Kingdom. Energy 2011;36(2):1292–308.
[10] Bond A, Pope J, Morrison-Saunders A, Retief F. A game theory perspective on environmental assessment: what games are played and what does this tell us about decision making rationality and legitimacy? Environ Impact Assess Rev 2016;57:187–94.
[11] Bompard E, Carpaneto E, Ciwei G, Napoli R, Benini M, Gallanti M, et al. A game theory simulator for assessing the performances of competitive electricity markets. Electr Power Syst Res 2008;78(2):217–27.
[12] Dong X, Li C, Li J, Wang J, Huang W. A game-theoretic analysis of implementation of cleaner production policies in the Chinese electroplating industry. Resour Conserv Recycl 2010;54(12):1442–8.
[13] Lo Prete C, Hobbs BF. A cooperative game theoretic analysis of incentives for microgrids in regulated electricity markets. Appl Energy 2016;169:524–41.
[14] Rosenthal EC. A game-theoretic approach to transfer pricing in a vertically integrated supply chain. Int J Prod Econ 2008;115(2):542–52.
[15] Maali Y. A multiobjective approach for solving cooperative n-person games. Int J Electr Power Energy Syst 2009;31(10):608–10.
[16] Haikarainen C, Pettersson F, Saxén H. A model for structural and operational optimization of distributed energy systems. Appl Therm Eng 2014;70(1):211–8.
[17] Omu A, Choudhary R, Boies A. Distributed energy resource system optimisation using mixed integer linear programming. Energy Policy 2013;61:249–66.
[18] Wu Q, Ren H, Gao W, Ren J. Modeling and optimization of distributed energy supply network with power and hot water interchanges. Appl Therm Eng 2016;94:635–43.
[19] Wu Q, Ren H, Gao W, Ren J. Multi-objective optimization of a distributed energy network integrated with heating interchange. Energy 2016;109:353–64.
[20] AlSkaif T, Guerrero Zapata M, Bellalta B. Game theory for energy efficiency in wireless sensor networks: latest trends. J Netw Comput Appl 2015;54:33–61.
[21] Zhang N, Yan Y, Su W. A game-theoretic economic operation of residential distribution system with high participation of distributed electricity prosumers. Appl Energy 2015;154:471–9.
[22] Zhang D, Samsatli NJ, Hawkes AD, Brett DJL, Shah N, Papageorgiou LG. Fair electricity transfer price and unit capacity selection for microgrids. Energy Econ 2013;36:581–93.
[23] Kim T, Jeon Y. Stationary perfect equilibria of an n-person noncooperative bargaining game and cooperative solution concepts. Eur J Operational Res 2009;194(3):922–32.
[24] Su W, Huang AQ. A game theoretic framework for a next-generation retail electricity market with high penetration of distributed residential electricity suppliers. Appl Energy 2014;119:341–50.
[25] Dinar A, Howitt RE. Mechanisms for allocation of environmental control cost: empirical tests of acceptability and stability. J Environ Manag 1997;49(2):183–203.
[26] Shapley LS, Shubik M. A method for evaluating the distribution of power in a committee system. Am Political Sci Rev 1954;48(3):787–92.
[27] Littlechild SC, Vaidya KG. The propensity to disrupt and the disruption nucleolus of a characteristic function game. Int J Game Theory 1976;5(2):151–61.
[28] Gately D. Sharing the gains from regional cooperation: a Game Theoretic application to planning investment in electric power. Int Econ Rev 1974;15(1):195–208.
[29] Dinar A, Nigatu GS. Distributional considerations of international water resources under externality: the case of Ethiopia, Sudan and Egypt on the Blue Nile. Water Resour Econ 2013;2–3:1–16.
[30] Yingfei Z, Shuxia Z, Xiaojing C, Fang L. Application of modified Shapley value in gains allocation of closed-loop supply chain under third-party reclaim. Energy Procedia 2011;5:980–4.
[31] Puerto J, Perea F. Finding the nucleolus of any n-person cooperative game by a single linear program. Comput Operations Res 2013;40(10):2308–13.
[32] Liao Z, Zhu X, Shi J. Case study on initial allocation of Shanghai carbon emission trading based on Shapley value. J Clean Prod 2015;103:338–44.
[33] Solymosi T, Sziklai B. Characterization sets for the nucleolus in balanced games. Operations Res Lett 2016;44(4):520–4.