

# A Time Delay Neural Network for Dynamical System Control

X. Xu, L.M. Wan, X.L. Wang, L.K. Wang and Y.C. Liang

**Abstract**—A novel time delay neural network is proposed for dynamical system control. In this work, A continuous recurrent neural network with time delay neurons in hidden layer is constructed, and the novel training algorithm and control law independent of delay are developed based on Lyapunov's stability approach. Using the proposed method, the control error converges to a range near the zero point and remains within the domain throughout the course of the execution. The usefulness and validity of the presented algorithm are examined by numerical experiments.

## I. INTRODUCTION

An artificial neural network (ANN) follows the biological neural cells in the brain and consists of a number of neurons and weighted links [1]. ANN has strong ability to approximate the input-output mapping of a dynamical system; it hence has been applied widely to the various problems in science and engineering, such as identifications and control of nonlinear systems [2]. To embody the dynamical properties of a system, some memorial properties are needed to introduce into the Ann. These memorial schemes usually include the use of recurrent neural network with self-recurrent neurons, and the introducing of additional neuron units with past states of neurons [3, 4]. However, ANN based on these conventional schemes not only is sensitive to the external noise but also produce much more complicated network structure, which increases the computational complexity.

Stabilization is a key problem in the analysis and design of systems control. It is however very difficult to achieve a stable and on-line control using the conventional ANN methods due to external disturbances and modeling errors. Although neural control schemes based on Lyapunov stability theorem are proposed to guarantee the stability of a system, they are usually constrained to a special system and suffered from strictly restricted conditions. Hence, the construction of the stable and adaptive control scheme based on ANN is very important in theoretical studies and practical applications.

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This paper proposed a continuous recurrent neural network with delayed neurons in hidden layer for dynamical system control. We first give the mathematical description for the model and then develop the training algorithm of weights and control law on the basis of the Lyapunov's stability approach. The proposed method extend the results in Ref. [5], does not need to know too much priori knowledge of systems and can make the control error not only converge but also remain in a range near the zero point.

## II. DYNAMICAL NEURON WITH TIME DELAY

We consider a dynamical neuron [6, 7] which is of the form

$$U(t + \tau) = H \left[ M(t) - \int_{-\infty}^t k(t-s)U(s) ds - c \right] \quad (1)$$

where  $U(t)$  denotes the neuron response assumed to be zero or one,  $M(t)$  denotes the external stimulus to the neuron,  $c$  denotes the neuronal threshold,  $k(t)$  denotes the refractoriness of the neuron after it has fired or responded, and  $\tau$  denotes the delay of the system, and

$$H(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Adopting Dirac function as kernel, Gopalsamy and Leung [8] generalized system (1) to the continuous case, and obtained the scalar delay differential equations of the form

$$\frac{dz(t)}{dt} = -z(t) + a \tanh[z(t) - bz(t-\tau) - c] \quad (2)$$

where the constant  $a$  denotes the range of the continuous variable  $z(\cdot)$ ; while  $h$  denotes a measure of the inhibitory influence of the past history.

Let  $x(t) \equiv z(t) - hz(t-\tau)$  and from Eq. (2) we obtain that

$$\frac{dx(t)}{dt} = -x(t) + a \tanh x(t) + h \tanh x(t-\tau) \quad (3)$$

where  $h = ab$ .

Introducing time delay neuron detailed in Eq. (3) into hidden layer, we construct a three-layer neural network with  $m$  input neurons,  $n$  hidden neurons and one output neuron as shown in Figure 1. In Fig. 1,  $u_i$  is the control input, and the neuron in the output layer is a linear neuron. For simplification, only the diagonal interactions are investigated in the hidden layer and thus there are no interactions among different hidden neurons.

According to Eq. (3), we can obtain the mathematical description for the proposed model as follows

$$\begin{cases} \dot{x}_j = -x_j + a_j s[x_j(t)] + h_j s[x_j(t-\tau)] + \sum_i b_{ji} u_i, \\ y(k) = \sum_j p_j(k) x_j(k) \end{cases} \quad (4)$$

where  $x_j(t)$  denotes the output of the  $j$ th hidden neuron at time  $t$ ,  $u_i(t)$  the  $i$ th control input,  $a_j(t)$ ,  $h_j(t)$ ,  $b_{ji}(t)$  and  $p_j(t)$  denote linking weights of the network;  $s(x) = \tanh(x)$  is the activation function for the hidden neurons.

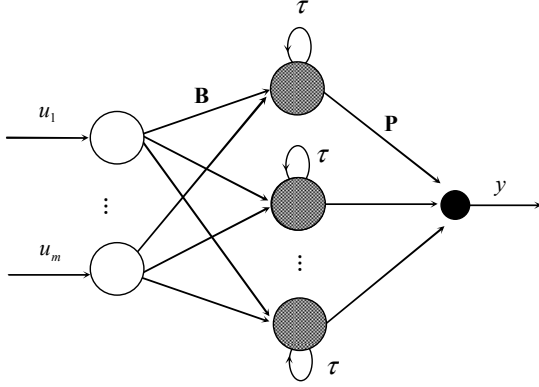


Fig. 1. Architecture of neural network with delayed neuron in hidden layer.

Eq. (4) can be written in a matrix form as

$$\begin{cases} \dot{\mathbf{x}} = -\mathbf{x} + \mathbf{A}\mathbf{S}(\mathbf{x}) + \mathbf{h}\mathbf{S}(\mathbf{x}_\tau) + \mathbf{B}\mathbf{u}, \\ y = \mathbf{P}\mathbf{x} \end{cases} \quad (5)$$

where  $\mathbf{x}_\tau = \mathbf{x}(t-\tau)$ ,  $\mathbf{S}(\mathbf{x}) = \text{diag}\{s(x_1), \dots, s(x_n)\}$ . It is easy to see that  $\|\mathbf{S}(\mathbf{x})\| \leq s_0$  where  $s_0 > 0$ .

If  $h = 0$ , Eq. (5) is exact the system studied in Ref. [5]. In this work, we extend the studies in Ref. [5] to  $h \neq 0$ . In the practical system, the values of time delays are very difficult to obtain and measure. Hence, only the delay-independent control laws are considered in this paper.

### III. CONTROL LAW

Due to the approximation capabilities of the neural network [1], we assume that there exist ideal weight values  $\mathbf{A}^* \in R^{n \times n}$ ,  $\mathbf{h}^* \in R^{n \times n}$ ,  $\mathbf{B}^* \in R^{n \times m}$  and  $\mathbf{P}^* \in R^{1 \times n}$  satisfying  $\|\mathbf{A}^*\| \leq A_0^*$ ,  $\|\mathbf{h}^*\| \leq h_0^*$ ,  $\|\mathbf{B}^*\| \leq B_0^*$ ,  $\|\mathbf{P}^*\| \leq P_0^*$  such that the following system

$$\begin{cases} \dot{\mathbf{x}} = -\mathbf{x} + \mathbf{A}^*\mathbf{S}(\mathbf{x}) + \mathbf{h}^*\mathbf{S}(\mathbf{x}_\tau) + \mathbf{B}^*\mathbf{u} + \varepsilon(x, u) \\ Y = \mathbf{P}^*\mathbf{x} \end{cases} \quad (6)$$

approximates the input-output mapping of the system, where  $\varepsilon(x, u)$  is the modeling error or external disturbance and  $\|\varepsilon(x, u)\| \leq \varepsilon_0$  ( $\varepsilon_0 > 0$ ).

For simplification, we assume that the desired value  $y_d$  and its derivative with respect to time  $\dot{y}_d$  are bounded, that is  $|y_d| \leq y_0$  and  $|\dot{y}_d| \leq \bar{y}_0$  ( $y_0 > 0$ ,  $\bar{y}_0 > 0$ ).

Suppose that  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{h}$  and  $\mathbf{P}$  are the approximated values of  $\mathbf{A}^*$ ,  $\mathbf{h}^*$ ,  $\mathbf{B}^*$  and  $\mathbf{P}^*$ , respectively. Define  $\tilde{\mathbf{A}} = \mathbf{A} - \mathbf{A}^*$ ,  $\tilde{\mathbf{h}} = \mathbf{h} - \mathbf{h}^*$ ,  $\tilde{\mathbf{B}} = \mathbf{B} - \mathbf{B}^*$  and  $\tilde{\mathbf{P}} = \mathbf{P} - \mathbf{P}^*$  then Eq. (6) can be written as

$$\begin{cases} \dot{\mathbf{x}} = -\mathbf{x} + (\mathbf{A} - \tilde{\mathbf{A}})\mathbf{S}(\mathbf{x}) + (\mathbf{h} - \tilde{\mathbf{h}})\mathbf{S}(\mathbf{x}_\tau) + (\mathbf{B} - \tilde{\mathbf{B}})\mathbf{u} + \varepsilon(x, u) \\ Y = (\mathbf{P} - \tilde{\mathbf{P}})\mathbf{x} \end{cases}$$

In order to guarantee the stability of the control system in the sense of Lyapunov stability [9], we construct the update law for the weights in the neural network and the control inputs are as follows

$$\dot{\mathbf{A}} = -k_a \mathbf{A} + k \mathbf{P}^T e \mathbf{S}^T(\mathbf{v}) \quad (7)$$

$$\dot{\mathbf{h}} = -k_h \mathbf{h} + k \mathbf{P}^T e \mathbf{S}^T(\mathbf{x}_\tau) \quad (8)$$

$$\dot{\mathbf{B}} = -k_b \mathbf{B} + k \mathbf{P}^T e \mathbf{u}^T \quad (9)$$

$$\dot{\mathbf{P}} = -\frac{\mathbf{x}^T}{1 + \|\mathbf{x}\|} \quad (10)$$

$$\mathbf{u} = \frac{(\mathbf{P}\mathbf{B})^T [\frac{1}{\lambda_1} \mathbf{A}\mathbf{S}(\mathbf{x}) + \frac{1}{\lambda_2} \mathbf{h}\mathbf{S}(\mathbf{x}_\tau)]}{(1 + \|\mathbf{P}\mathbf{B}\|^2)(1 + \|\mathbf{A}\| + \|\mathbf{h}\|)} \quad (11)$$

where,  $e = z - y_d \approx \mathbf{P}^* \mathbf{x} - y_d$  is the control error, and  $k$ ,  $k_a$ ,  $k_h$ ,  $k_b$ ,  $\lambda_1$ ,  $\lambda_2$  are all constant to be determined later.

From Eqs. (10) and (11), it is clear that both weight  $\mathbf{P}$  and control law  $\mathbf{u}$  are bounded. We then suppose that  $|\mathbf{P}| \leq p_0$ ,  $|\tilde{\mathbf{P}}| \leq \tilde{p}_0$ ,  $|\mathbf{P}^*| \leq p_0^*$  and  $|\mathbf{u}| \leq u_0$ .

**Theorem:** Assume the input-output of a dynamical system can be approximated by neural network (6). Adopt equations (7-11) as the training algorithm of the weights and control law.

If the constant  $k$ ,  $k_a$ ,  $k_h$ ,  $k_b$ ,  $\lambda_1$ ,  $\lambda_2$  satisfy

(1)  $k = k_0 + k_1 + k_2$ , where  $k_0$ ,  $k_1$ ,  $k_2$  are all positive;

(2)  $\lambda_1 \geq \frac{kp_0 s_0}{\sqrt{2k_1 k_a - kp_0 s_0}}$ ,  $\lambda_2 \geq \frac{kp_0 s_0}{\sqrt{2k_2 k_h - kp_0 s_0}}$ ,

then the control error of the system converges to a range near the zero point and remains within the domain throughout the execution. Furthermore, all signals in the closed loop are bounded.

**Proof:**

Take the Lyapunov function as

$$L = \frac{k}{2} [e^T e + \text{tr}(\tilde{\mathbf{A}}^T \tilde{\mathbf{A}}) + \text{tr}(\tilde{\mathbf{h}}^T \tilde{\mathbf{h}}) + \text{tr}(\tilde{\mathbf{B}}^T \tilde{\mathbf{B}})] \quad (12)$$

where  $\text{tr}(\mathbf{A}^T \mathbf{A}) = \sum_{i,j} (a_{ij})^2 = \|\mathbf{A}\|^2$ .

Differentiating (12) with respect to time gives

$$\dot{L} = ke^T \dot{e} + k \text{tr}(\tilde{\mathbf{A}}^T \dot{\tilde{\mathbf{A}}}) + k \text{tr}(\tilde{\mathbf{h}}^T \dot{\tilde{\mathbf{h}}}) + k \text{tr}(\tilde{\mathbf{B}}^T \dot{\tilde{\mathbf{B}}}) \quad (13)$$

Using Eq. (6) we have

$$\dot{e} = \dot{Y} - \dot{y}_d = \mathbf{P}^* (-\mathbf{x} + \mathbf{A}^* \mathbf{S}(\mathbf{x}) + \mathbf{h}^* \mathbf{S}(\mathbf{x}_\tau) + \mathbf{B}^* \mathbf{u} + \varepsilon) - \dot{y}_d \quad (14)$$

Substituting (14) into (13) gives

$$\begin{aligned}
\dot{L} &= ke^T \mathbf{P}^* [-\mathbf{x} + \mathbf{A}^* \mathbf{S}(\mathbf{x}) + \mathbf{h}^* \mathbf{S}(\mathbf{x}_\tau) + \mathbf{B}^* \mathbf{u} + \varepsilon] - e^T \dot{y}_d \\
&\quad + k\text{tr}(\tilde{\mathbf{A}}^T \tilde{\mathbf{A}}) + k\text{tr}(\tilde{\mathbf{h}}^T \tilde{\mathbf{h}}) + k\text{tr}(\tilde{\mathbf{B}}^T \tilde{\mathbf{B}}) \\
&= -ke^T \mathbf{P}^* \mathbf{x} + ke^T \mathbf{P}^* [\mathbf{A}^* \mathbf{S}(\mathbf{x}) + \mathbf{h}^* \mathbf{S}(\mathbf{x}_\tau) + \mathbf{B}^* \mathbf{u} + \varepsilon] \\
&\quad - ke^T \dot{y}_d + k\text{tr}(\tilde{\mathbf{A}}^T \tilde{\mathbf{A}}) + k\text{tr}(\tilde{\mathbf{h}}^T \tilde{\mathbf{h}}) + k\text{tr}(\tilde{\mathbf{B}}^T \tilde{\mathbf{B}}) \\
&= -ke^T e + ke^T (\mathbf{P} - \tilde{\mathbf{P}}) [\mathbf{A}^* \mathbf{S}(\mathbf{x}) + \mathbf{h}^* \mathbf{S}(\mathbf{x}_\tau) + \mathbf{B}^* \mathbf{u} + \varepsilon] \\
&\quad - ke^T (y_d + \dot{y}_d) + k\text{tr}(\tilde{\mathbf{A}}^T \tilde{\mathbf{A}}) + k\text{tr}(\tilde{\mathbf{h}}^T \tilde{\mathbf{h}}) + k\text{tr}(\tilde{\mathbf{B}}^T \tilde{\mathbf{B}}) \\
&= -ke^T e + ke^T \mathbf{P} [\mathbf{A}^* \mathbf{S}(\mathbf{x}) + \mathbf{h}^* \mathbf{S}(\mathbf{x}_\tau) + \mathbf{B}^* \mathbf{u} + \varepsilon] + w \\
&\quad - ke^T (y_d + \dot{y}_d) + k\text{tr}(\tilde{\mathbf{A}}^T \tilde{\mathbf{A}}) + k\text{tr}(\tilde{\mathbf{h}}^T \tilde{\mathbf{h}}) + k\text{tr}(\tilde{\mathbf{B}}^T \tilde{\mathbf{B}})
\end{aligned} \tag{15}$$

where  $w = -ke^T \tilde{\mathbf{P}} [\mathbf{A}^* \mathbf{S}(\mathbf{x}) + \mathbf{h}^* \mathbf{S}(\mathbf{x}_\tau) + \mathbf{B}^* \mathbf{u} + \varepsilon]$  and

$$\begin{aligned}
\|w\| &\leq k|e| \cdot \|\tilde{\mathbf{P}}\| \cdot (\|\mathbf{A}^*\| \cdot \|\mathbf{S}\| + \|\mathbf{h}^*\| \cdot \|\mathbf{S}\| + \|\mathbf{B}^*\| \cdot \|\mathbf{u}\| + \varepsilon_0) \\
&\leq k|e| \tilde{p}_0 (A_0^* s_0 + h_0^* s_0 + B_0^* u_0 + \varepsilon_0) = k|e| w_0
\end{aligned} \tag{16}$$

where  $w_0 = \tilde{p}_0 (A_0^* s_0 + h_0^* s_0 + B_0^* u_0 + \varepsilon_0)$ .

The right second item of Eq. (15) can be written as

$$\begin{aligned}
&ke^T \mathbf{P} [-\mathbf{x} + \mathbf{A}^* \mathbf{S}(\mathbf{x}) + \mathbf{h}^* \mathbf{S}(\mathbf{x}_\tau) + \mathbf{B}^* \mathbf{u}] \\
&= ke^T \mathbf{P} [-\mathbf{x} + (\mathbf{A} - \tilde{\mathbf{A}}) \mathbf{S}(\mathbf{x}) + (\mathbf{h} - \tilde{\mathbf{h}}) \mathbf{S}(\mathbf{x}_\tau) + (\mathbf{B} - \tilde{\mathbf{B}}) \mathbf{u}]
\end{aligned} \tag{17}$$

Substituting Eqs. (7)-(11) and (17) into (15), we have

$$\begin{aligned}
\dot{L} &= -ke^T e + ke^T \mathbf{P} [\mathbf{A} \mathbf{S}(\mathbf{x}) + \mathbf{h} \mathbf{S}(\mathbf{x}_\tau) + \mathbf{B} \mathbf{u}] - ke^T (y_d + \dot{y}_d) \\
&\quad + w - k_a \text{tr}(\tilde{\mathbf{A}}^T \tilde{\mathbf{A}}) - k_h \text{tr}(\tilde{\mathbf{h}}^T \tilde{\mathbf{h}}) - k_b \text{tr}(\tilde{\mathbf{B}}^T \tilde{\mathbf{B}}) \\
&= -ke^T e + ke^T \mathbf{P} [\mathbf{A} \mathbf{S}(\mathbf{x}) + \mathbf{h} \mathbf{S}(\mathbf{x}_\tau)] - ke^T (y_d + \dot{y}_d) \\
&\quad + w + ke^T \mathbf{P} \mathbf{B} \frac{(\mathbf{P} \mathbf{B})^T [\frac{1}{\lambda_1} \mathbf{A} \mathbf{S}(\mathbf{x}) + \frac{1}{\lambda_2} \mathbf{h} \mathbf{S}(\mathbf{x}_\tau)]}{(1 + \|\mathbf{P} \mathbf{B}\|^2)(1 + \|\mathbf{A}\| + \|\mathbf{h}\|)} \\
&\quad - k_a \text{tr}(\tilde{\mathbf{A}}^T \tilde{\mathbf{A}}) - k_h \text{tr}(\tilde{\mathbf{h}}^T \tilde{\mathbf{h}}) - k_b \text{tr}(\tilde{\mathbf{B}}^T \tilde{\mathbf{B}})
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
\dot{L} &\leq -k|e|^2 + k|e| p_0 [(1 + \frac{1}{\lambda_1}) s_0 \|\mathbf{A}\| + (1 + \frac{1}{\lambda_2}) s_0 \|\mathbf{h}\|] + k|e| w_0 \\
&\quad + k|e| (y_0 + \bar{y}_0) - k_a \text{tr}(\tilde{\mathbf{A}}^T \tilde{\mathbf{A}}) - k_h \text{tr}(\tilde{\mathbf{h}}^T \tilde{\mathbf{h}}) - k_b \text{tr}(\tilde{\mathbf{B}}^T \tilde{\mathbf{B}})
\end{aligned} \tag{19}$$

Moreover, from Ref. [10] we have

$$\begin{cases} \text{tr}(\mathbf{A}^T \tilde{\mathbf{A}}) = \frac{1}{2} (\|\mathbf{A}\|^2 + \|\tilde{\mathbf{A}}\|^2 - \|\mathbf{A}^*\|^2) \\ \text{tr}(\mathbf{h}^T \tilde{\mathbf{h}}) = \frac{1}{2} (\|\mathbf{h}\|^2 + \|\tilde{\mathbf{h}}\|^2 - \|\mathbf{h}^*\|^2) \\ \text{tr}(\mathbf{B}^T \tilde{\mathbf{B}}) = \frac{1}{2} (\|\mathbf{B}\|^2 + \|\tilde{\mathbf{B}}\|^2 - \|\mathbf{B}^*\|^2) \end{cases} \tag{20}$$

Substituting Eqs. (20) into (19) reads

$$\begin{aligned}
\dot{L} &\leq -k|e|^2 + k|e| p_0 [(1 + \frac{1}{\lambda_1}) s_0 \|\mathbf{A}\| + (1 + \frac{1}{\lambda_2}) s_0 \|\mathbf{h}\|] + k|e| c_0 \\
&\quad - \frac{k_a}{2} \|\mathbf{A}\|^2 - \frac{k_h}{2} \|\mathbf{h}\|^2 + \frac{k_a}{2} \|\mathbf{A}^*\|^2 + \frac{k_h}{2} \|\mathbf{h}^*\|^2 + \frac{k_b}{2} \|\mathbf{B}^*\|^2,
\end{aligned}$$

where  $c_0 = y_0 + \bar{y}_0 + w_0$ .

If we choose  $k = k_0 + k_1 + k_2$ ,  $\lambda_1 \geq \frac{k p_0 s_0}{\sqrt{2k_1 k_a - k p_0 s_0}}$ , and

$\lambda_2 \geq \frac{k p_0 s_0}{\sqrt{2k_2 k_h - k p_0 s_0}}$ , then we have

$$\dot{L} \leq -k_0 |e|^2 + k|e| c_0 + J \tag{21}$$

where  $J = \frac{1}{2} \|\mathbf{A}_0^*\|^2 + \frac{1}{2} \|\mathbf{h}_0^*\|^2 + \frac{1}{2} \|\mathbf{B}_0^*\|^2$ .

Define a range:

$$\Omega_e := \{e : |e|^2 - \bar{c}|e| - J \leq 0\} \tag{22}$$

and a scalar functions

$$H(\phi) = k_0 \phi^2 - k c_0 \phi - J.$$

Then, Eq. (22) can be written as

$$\dot{L} \leq -H(|e|) \tag{23}$$

In the following, it will be shown that control error will converge to the range  $\Omega_e$  and will remain within this range. Let  $e(t)$  be a solution corresponding to the initial conditions  $e(t_0)$ . For the discussion later we distinguish the following three possible cases:

**Case a:** From Eq. (23), it is easy to see that  $H(|e|) > 0$  when error  $e(t)$  is out of the range  $\Omega_e$ . Hence according to Eq. (23) we have  $\dot{L} < 0$ . if  $e(t) \notin \Omega_e$  holds for  $\forall t \in [t_0, +\infty)$ , then the strictly negative of  $\dot{L}$  indicates that there exists a time  $T_0 > 0$  such that  $\dot{L} \leq -\beta < 0$  when  $t > T_0$ .

Integrating  $\dot{L}$  from  $T_0$  to  $t$ , we have

$\int_{T_0}^t \dot{L} dt \leq \int_{T_0}^t -\beta dt$  and  $L(t) \leq L(T_0) - \beta(t - T_0)$ . It can be seen that  $L(t) < 0$  when  $t \rightarrow +\infty$ . This contradicts the definition of  $L$  from Eq. (12). So error  $e(t)$  will converge to  $\Omega_e$  in finite time.

**Case b:** If  $e(t_0) \in \Omega_e$ , the above mentioned discussions show that  $e(t)$  will be always kept in the range  $\Omega_e$ , which indicate that the control error is bound.

Note that control error  $e$ , control law  $\mathbf{u}$  and weights  $\mathbf{P}$  are all bounded, from Eqs. (7-9), we have weights  $\mathbf{A}, \mathbf{B}, \mathbf{h}$  and thus state variable  $\mathbf{x}$  are also bounded, which completes the proof. #

According to the theorem, though the ideal values  $\mathbf{A}^*$ ,  $\mathbf{h}^*$ ,  $\mathbf{B}^*$  and  $\mathbf{P}^*$  cannot be approached exactly, the control error can converge into the range  $\Omega_e$  near zero and stay in that domain throughout the execution. In addition, because the initial weights in the neural network may be far from the ideal weights, the control system may be unstable in the transient state of identification process. Therefore, the weights can be updated off-line in the neural network according to the static input-output information to guarantee the stability of the control system in the transient state.

#### IV. NUMERICAL SIMULATION AND DISCUSSIONS

An ultrasonic motor (USM) is a newly developed motor, which has some excellent performances and useful features such as high torque at low speeds, compactness in size, no electromagnetic interference, short start-stop times, and many others. The USM has strongly nonlinear speed characteristics that vary with the driving conditions [2, 11]. It is therefore difficult to construct a precise model of the USM. In this section, numerical simulations are performed using the proposed method for the speed control of a longitudinal

vibration ultrasonic motor [11] shown in Figure 2. Some parameters of this USM model are taken as: driving frequency 27.8 kHz, amplitude of driving voltage 300V, allowed output moment 2.5 kg.cm, and rotation speed 3.8 m/s.

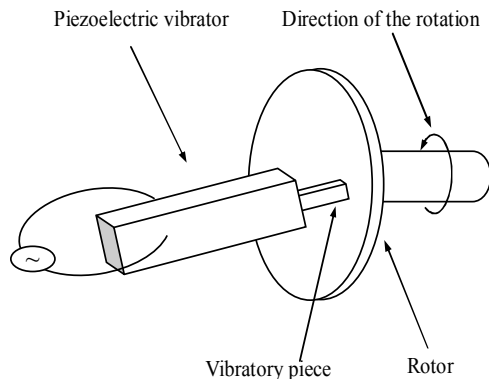


Fig. 2. Schematic diagram of the ultrasonic motor.

Figure 3 shows the speed control curves using the scheme base on DRNN [4] and the proposed scheme in this paper. From the figure it can be seen that the time of convergence using the proposed method is much less than that using the method based on DRNN. In addition when a parameter of the USM varies suddenly, the large fluctuation can be eliminated after a short time control using this scheme.

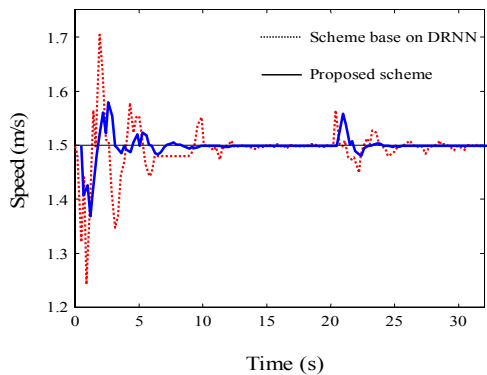


Fig. 3. Comparison of speed control curve for two schemes.

Figures 4 show the case of the fluctuation using the method based on DRNN [4] and the proposed scheme in this paper, where the fluctuation is defined as

$$\zeta = (V_{\max} - V_{\min}) / V_{\text{aver}} \times 100\%,$$

where  $V_{\max}$ ,  $V_{\min}$  and  $V_{\text{aver}}$  represent the maximum, minimum and average values of the speeds, respectively. It can be seen that the control precision can be increased around 3 times when the proposed method is employed.

Figure 5 shows the comparison of the average errors using different neural network models in the control system. From the comparison it can be seen that the time and

precision of convergence using the proposed method is much superior to that obtained using the methods based on DRNN [4] and BP [2] network.

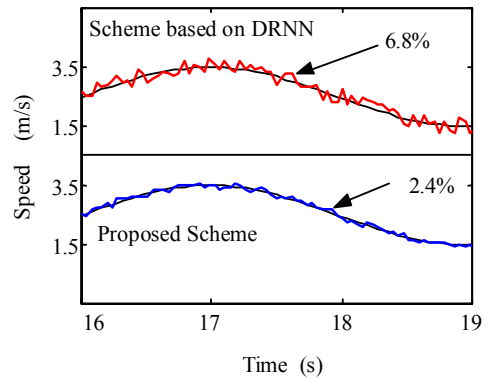


Fig. 4. Comparison of fluctuation for different schemes.

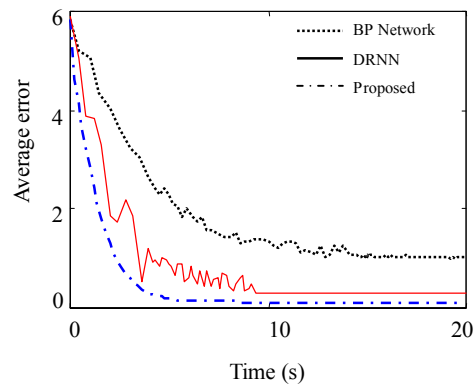


Fig. 5. Comparison of average errors for different methods.

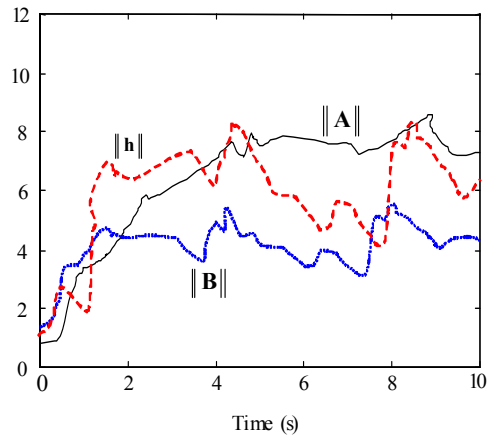


Fig. 6. Variation of  $\|A\|$ ,  $\|h\|$  and  $\|B\|$  of the proposed scheme.

Figure 6 shows the variation of the norm of the weights. It can be seen that all the weights of proposed scheme are bounded.

Figure 7 shows the speed control results when parameters are changed strongly for the load at 50%, elastic stiffness at 2 times and the bend modulus of elasticity at 2 times when  $t > 10.6s$ . It can be seen that the proposed method can compensate for the effect of parameter variation with time. Although the strong disturbance makes the error out of zero point, the error can return near zero for a short time control. This shows the proposed method has good adaptive performance with respect to parameter variation.

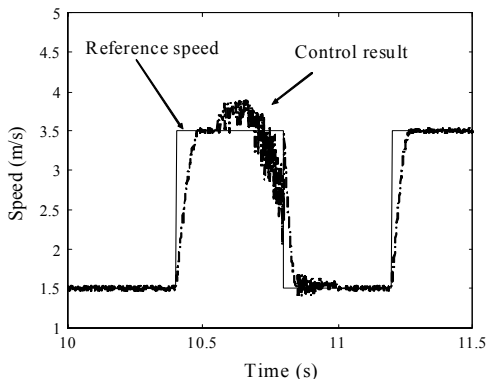


Fig. 7. The adaptive performance of the proposed scheme.

Figure 8 illustrates the control results using the scheme given in [5] for the same parameters and drive conditions with that from Figure 7. The comparison of two figures indicates that the proposed scheme based on the time delay neuron model has much more adaptive performance than that of the scheme given in Ref. [5].

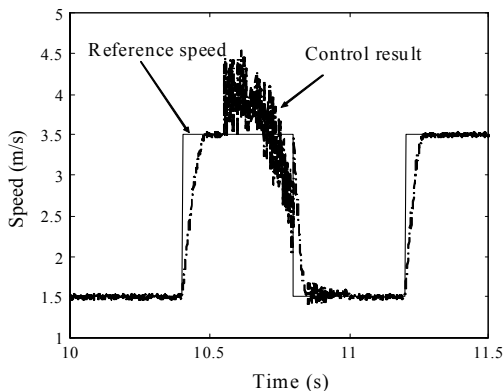


Fig. 8. The adaptive performance using the scheme in Ref. [5].

## V. CONCLUSIONS

A stable adaptive speed control scheme of nonlinear dynamical systems based on a time delay neuron network is

proposed. A newly developed updating algorithm for weights and control inputs independent of delay are provided to guarantee the stability of control system. We give theoretical proof that the control error could approach a range around zero point and keeps within the domain throughout the execution. The usefulness and validity of the proposed method is examined for its on-line adapting ability, recovering ability from disturbances, and adaptive performance to parameters variation. The numerical simulations verify the theoretical results and shows that the proposed method based on the time delay neural network is an effective scheme for dynamical system control.

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