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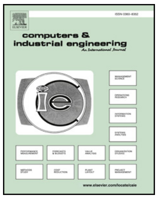

Value of on-site rework in a coordinated two-stage supply chain with supply imperfection

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ABSTRACT

This paper considers a decentralized supply chain with one supplier with supply imperfection and one manufacturer. The supplier performs outbound inspection to ensure that its components comply with the quality specification (QS), and makes efforts to improve inspection reliability. Once receiving the supplier's components, the manufacturer begins the assembly production process, requiring that every component meet the quality requirement (QR). The components that do not meet the QS or QR are reworked by the supplier. The location at which the latter group of the components is reworked becomes a strategic choice, provided that the option of on-site rework, i.e. rework performed at the manufacturer's production site, is available. We show that on-site rework may be beneficial to the supply chain even if it is more costly than in-house rework, i.e. rework performed in the supplier's in-house repair center. In addition, coordinating the decentralized supply chain with the option of on-site rework yields maximum supply chain performance over a certain cost range of on-site rework. Finally, we show the similarities and differences of the effects of QS and QR on the coordinated results.

1. Introduction

A number of production (and/or assembly) processes across a supply chain are involved in producing products for customers. In the presence of production imperfection, the quality of the products depends not only on a manufacturer's production quality but also on the quality of the components provided by its suppliers. The supply chain members therefore need to take actions to improve production processes, or deploy quality control mechanisms such as inspection and rework, or both, to counteract such production imperfection. Recent studies have examined how the supply chain members can coordinate their quality and inspection decisions in a decentralized supply chain with supply imperfection. However, these studies usually assume that rework is performed in the same location, e.g. the supplier's factory, and no decision-making for rework location is allowed for. The maintenance literature and observations from various industries, nevertheless, demonstrate that rework may take place in different locations, such as a local site, a service center, and a factory. In the liquid crystal display (LCD) manufacturing industries, for example, some suppliers of backlight units perform rework in their in-house repair center (in-house rework) when nonconforming units are identified in their production plants, and perform rework on their customers' production sites (on-site rework) where nonconforming units are identified. This study is motivated to incorporate the decision of rework location into supply chain coordination and explore the strategic value of rework location to the individual supply chain members, as well as the supply chain as a whole.

We base our analysis on a decentralized supply chain model with one manufacturer and one supplier having supply imperfection. The supplier establishes a quality specification (QS) to ensure the conformance of the produced components. The QS is enforced in a contract between the supplier and the manufacturer such that the supplier must deliver it. The supplier performs outbound inspection to identify the components that fail to meet the QS, and rework them in its in-house repair center. Due to imperfect outbound inspection, some nonconforming components will enter the manufacturer's production and the components that fail to meet the manufacturer's quality requirement (QR) will cause non-conformance of the products, and will be reworked by the supplier. The location at which these components are reworked then becomes a strategic choice, if the option of on-site rework is available. In the supply chain considered in this study, we differentiate the supplier's QS from the manufacturer's QR, because they may be established with distinct sets of objectives and purposes.

Earlier studies on supply chain coordination did not consider rework location. Whether on-site rework benefits a coordinated supply chain is yet to be explored. Furthermore, they did not differentiate the

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measures of quality conformance adopted by different supply chain members. The effects of varying these measures therefore remain unanswered. This study intends to answer these questions by investigating the interactive dynamics between the supplier and the manufacturer in a decentralized setting, and coordinating the supply chain members to achieve maximum chain performance, in light of the availability of on-site rework. Firstly, we find that on-site rework may be beneficial to the supply chain even if it is more costly than in-house rework. Secondly, coordinating the decentralized supply chain with the option of on-site rework leads to maximum supply chain performance over a certain cost range of on-site rework. Coordinating the supply chain outside this range can still yield maximum supply chain performance, if it is possible to negotiate lower anticipated profits for the supply chain members. Thirdly, QR and QS have different effects on supply chain profit and inspection reliability. We find that a looser QR or QS is always beneficial to the supply chain, and a looser QR leads to lower inspection reliability whereas a looser QS leads to higher inspection reliability. Furthermore, in cases where the supply chain profit in the presence of on-site rework is greater than the supply chain profit in the absence of on-site rework, QR has the opposite effects on the difference between these two supply chain profits. However, a looser QS always leads to a smaller difference of these supply chain profits.

1.1. Literature review

A number of recent studies have examined production policies involving inspection and rework (Chen, 2013; Hu & Zong, 2009; Konstantaras, Goyal, & Papachristos, 2007; Wang, 2005; Yeh & Chen, 2006; Yang & Cho, 2014; Yoo, Kim, & Park, 2012). Yeh and Chen (2006), for instance, determined the optimal lot size and production inspection policy for a deteriorating production system with products sold with a free minimal repair warranty. Wang (2005) investigated the production run length and product inspection policy for a deteriorating production system, where inspections take place at the end of the production run. Hu and Zong (2009) extended the work of Wang (2005) by considering an inspection policy with which inspection is performed in the middle of the production run, and defective items identified during inspection and all items produced after inspection are reworked. Konstantaras et al. (2007) examined a joint lot sizing and inspection inventory model in which each lot received by the buyer has a random proportion of defective (imperfect) units. These imperfect units can be either reworked at some cost and restored to good quality units, or sold to a secondary market as a single batch at a lower price. They then determined the buyer's decision of ordering and batching for both options. More recently, Yoo et al. (2012) studied an unreliable production and inspection system with customer returns and the disposal of defective items, and developed an optimal lot sizing model that considers different types of quality costs. Chen (2013) investigated the optimal inspection interval, inspection frequency, and production quantity in an imperfect production process with both rework and errors in preventive maintenance. Yang and Cho (2014) formulated an optimization problem for minimizing the aggregate cost of inspection and rework in an interconnected inspection-rework system, and then used an enumeration method to determine the cost-minimizing frequency of inspection cycles. However, the above studies focused on a single supply chain member, i.e. the manufacturer (or the buyer), and did not examine the interactive dynamics between supply chain members.

Inspections have strategic value in supply chains where the members compete with each other (Hsieh & Liu, 2010; Tapiero, 2001) or downstream chain members attempt to influence their suppliers' quality decisions (Baiman, Fischer, & Rajan, 2000; Balachandran & Radhakrishnan, 2005; Hwang, Radhakrishnan, & Su, 2006; Wan & Xu, 2008). Hwang et al. (2006), for instance, studied a supply chain in which the supplier's quality effort is neither observable nor verifiable, and the buyer uses an appraisal arrangement, for which the buyer inspects the supplier's units, or a certification arrangement, for which the supplier obtains vendor certification and the buyer does not perform inspection, to ensure the supplier's quality effort. Wan and Xu (2008) investigated a two-echelon supply chain in which the manufacturer purchases components from the supplier and determines an inbound inspection policy and a damage cost-sharing contract to affect the supplier's decision of component quality improvement. There is also a stream of literature concerning inspection games (Deutsch, Golany, & Rothblum, 2011; Rothenstein & Zamir, 2002). For instance, Deutsch et al. (2011) considered a game setting in which the inspection agency with limited resources inspects multiple parties to verify whether they comply with its regulations. They established all possible Nash equilibria, and identified the situations in which there exists a unique Nash equilibrium. Although these studies have explored the strategic value of inspections in an interactive context, they did not probe into the strategic value of rework location in decision-making.

By contrast, maintenance-related studies have paid considerable attention to the decision as to whether defective components shall be repaired or discarded, and the location at which repair and discard shall be performed (Alfredsson, 1997; Basten, Schutten, & van der Heijden, 2009; Basten, van der Heijden, & Schutten, 2012; Brick & Uchoa, 2009), an issue which is known as the level of repair analysis (LORA) problem. For instance, Basten et al. (2012) proposed an algorithm to jointly solve the LORA problem and the spare parts stocking problem, and demonstrated that this approach leads to a lower cost than solving these two problems sequentially. This line of research, nevertheless, centered on a single firm, and did not examine the interactive dynamics between supply chain members.

In summary, earlier studies on supply chain coordination did not explore the value of rework location, nor did they probe into the effects of varying the measures of quality conformance adopted by different supply chain members. The remainder of the paper is organized as follows. Section 2 introduces the decentralized supply chain with in-house rework, establishes the chain members' profit functions and their decisions in this decentralized supply chain, and examines the conditions under which coordination is beneficial to each individual supply chain member. Section 3 extends Section 2 by allowing for on-site rework and explores whether coordination with the option of on-site rework leads to maximum supply chain performance. Section 4 analyzes the effects of quality specification, quality requirement, and some other model parameters on inspection reliability and supply chain profit in the absence and presence of on-site rework, and their effects on coordination performance when coordination does not deliver maximum supply chain performance. Finally, Section 5 concludes this paper with a brief summary.

The notations used throughout the paper are as follows.

s, m	subscripts denoting the supplier and the manufacturer, respectively
q	probability that the supplier produces a Type A component
p	probability of a Type B component meeting the manufacturer's QR
r	inspection reliability (i.e. the probability of a Type B component being correctly identified)
r^*	optimal inspection reliability in the absence of on-site rework
r^{o*}	optimal inspection reliability in the presence of on-site rework
w	manufacturer's unit purchase price
V	manufacturer's value of each component that meets its quality requirement
M	product quantity
c_I	supplier's unit cost of improving inspection reliability; $c_I = \alpha r^2/2$, $\alpha > 0$
c_S	supplier's unit production cost
c_L	supplier's unit external failure cost
c_R	supplier's unit in-house rework cost
c_R^o	supplier's unit on-site rework cost

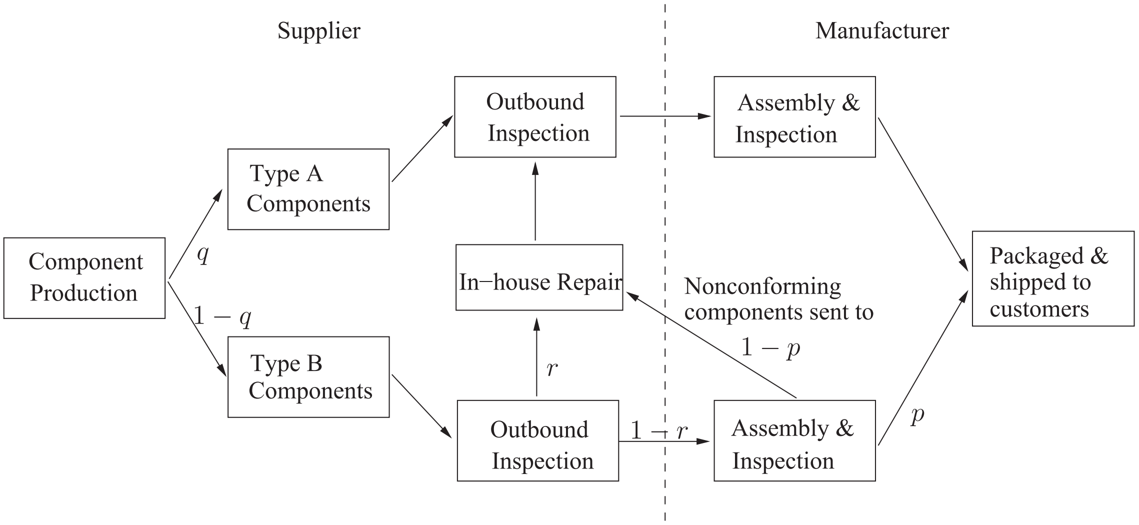

Fig. 1. Component flow with in-house repair.

c_H	manufacturer's unit handling cost in the absence of on-site rework
c_H^o	manufacturer's unit handling cost in the presence of on-site rework
ρ	the ratio $c_R^o/(c_R + c_L)$
π_i	the profit of chain member $i \in \{s,m\}$ in the absence of on-site rework
π_i^o	the profit of chain member $i \in \{s,m\}$ in the presence of on-site rework
π_i^d	the decentralized and uncoordinated profit of chain member $i \in \{s,m\}$ in the absence of on-site rework
π_i^{od}	the decentralized and uncoordinated profit of chain member $i \in \{s,m\}$ in the presence of on-site rework
π_J	the supply chain profit in the absence of on-site rework
π_J^o	the supply chain profit in the presence of on-site rework
π_J^*	the optimal supply chain profit in the absence of on-site rework
π_J^{o*}	the optimal supply chain profit in the presence of on-site rework
π_J^c	the coordinated supply chain profit in the absence or presence of on-site rework
χ	supplier's share of the coordinated supply chain profit

2. The modeling framework

We consider a parsimonious supply chain setting with one manufacturer and one supplier. The manufacturer purchases components over a planned time horizon from the supplier at a unit price w to produce M units of a single product, each requiring one unit of the supplier's components. Without loss of generality, we let $M = 1$. The supplier has production imperfection, although it also has an in-control process, and establishes a quality specification (QS) for the produced components to ensure their conformance. Quality specifications are "the desired measurements for the quality characteristics of the components and subassemblies that make up the product" (Montgomery, 2009, p. 8), and are enforced in a contractual agreement between the supplier and the manufacturer such that the supplier has to deliver them. One example of QS for manufactured components is that the diameter of a metal shaft in a disk-drive unit shall be within 0.2500 ± 0.0015 in (Montgomery, 2009, p. 84). Another example is that the surface luminance of AUO's 42-inch Full-HD color TFT-LCD module is at least $400\,\text{cd/m}^2$ (http://www.beyondinfinite.com/library.html).

The supplier verifies the conformance of the produced components at the outbound inspection site, as illustrated in the component flow of Fig. 1. The components that meet the QS are conforming and denoted by Type A components, and those that do not meet the QS are nonconforming and denoted by Type B components. We let q and $1{-}q$ denote the probability of a produced component being conforming (Type A) and nonconforming (Type B), respectively. If a Type B component is identified by outbound inspection, it will be sent to the in-house repair center for rework. We refer to this as in-house rework. We later refer to rework of a Type B component which is performed at the manufacturer's production site as on-site rework in Section 3. We assume that both in-house and on-site rework corrects Type B components so that they meet the QS (Konstantaras et al., 2007; Montgomery, 2009; Sonntag & Kiesmüller, 2018).

We further consider that the supplier's outbound inspection is not perfectly reliable: a Type A component is correctly identified as being Type A with probability one, whereas a Type B component may be incorrectly identified as being Type A with probability $(1{-}r), 0 < r < 1$. Here, the probability r represents inspection reliability, as depicted in Fig. 1. The supplier is able to make efforts to improve the inspection reliability r. In line with the operations management literature on quality and reliability improvement (Baiman et al., 2000; Balachandran & Radhakrishnan, 2005; Tapiero, 2001), we assume that the unit cost, c_I, of improving inspection reliability is an increasing convex function of inspection reliability r, i.e. the marginal cost of improving inspection reliability is increasing with inspection reliability. The choice of an increasing convex cost function in inspection reliability is not un-intuitive. For instance, in the example of the metal shaft supplier, higher inspection reliability can be achieved by allowing more measurements to be made. However, because this activity will slow down the throughout, the supplier has to invest increasingly in order to achieve the targeted inspection reliability, while maintaining the same throughput. To achieve mathematical tractability, we adopt a quadratic form of c_I, $c_I = \alpha r^2/2$, $\alpha > 0$, which is the simplest polynomial function with the desired property. Nevertheless, the qualitative results obtained through parametric analysis in this study remain valid for c_I taking on different increasing convex functions.

The supplier's components, once passing the outbound inspection, are sent to the manufacturer's production site for the product assembly operations. After product assembly, online inspection of the products is performed, and then the conforming products are packaged and shipped to customers. The components that do not meet the manufacturer's quality requirement (QR) will cause non-conformance of the products produced with them, and will be identified by online inspection.[1] These components are removed from production, gathered, and

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[1] The manufacturer is able to verify the cause of nonconforming products, and the supplier has no responsibility for nonconforming products that are attributed to the

then sent back to the supplier for in-house rework.

With regard to asymmetry of component quality requirements in the supply chain, we assume that the supplier's QS and the manufacturer's QR are set independently and that the supplier's QS is tighter than the manufacturer's QR.[2] The reason behind the assumptions is that the supplier's QS is established in light of various factors such as its own commitment to deliver a certain level of quality, common industry practice, or the manufacturer's requests, and hence likely differs from the manufacturer's QR. Especially when the supplier has technological advantages, it will adopt quality differentiation strategies (Beal & Lockamy, 1999; Kouvelis & Mukhopadhyay, 1995; Shetty, 1987) by setting the QS to be tighter than the manufacturer's QR. To illustrate that the supplier's QS is tighter than the manufacturer's QR, consider the earlier examples that a supplier supplies metal shafts to a disk-drive manufacturer and that a panel manufacturer supplies 42-in. TFT-LCD modules to a TV manufacturer. In the first example, the supplier sets the QS for the diameter of a metal shaft to be 0.2500 ± 0.0015 in., whereas the disk-drive manufacturer's QR is 0.2500 ± 0.0020 in.. Because the tolerances $\pm$ 0.0015 in the QS are narrower than the tolerances $\pm$ 0.0020 in the QR, the supplier's QS is considered to be tighter than the disk-drive manufacturer's QR. In the second example, the panel manufacturer sets the luminance of a 42-in. TFT-LCD module to have the minimum value $400\,\mathrm{cd/m^2}$, which is larger than the minimum value $370\ \mathrm{cd/m^2}$ required by the TV manufacturer's QR. In this case, the panel manufacturer's QS is considered to be tighter than the TV manufacturer's QR.

With the supplier's QS being tighter than the manufacturer's QR, Type A components definitely meet the manufacturer's QR, whereas Type B components may or may not satisfy the manufacturer's QR. We let p and $1-p$ denote the probability of a Type B component meeting and not meeting the manufacturer's QR, respectively. The use of p and q allows us to characterize the relative discrepancy of the QS and QR. For a given QR, a tighter QS is equivalent to a smaller probability q of a Type A component, while keeping fixed the probability $(1-p)(1-q)$ that a component fails to meet the QR. This means that both p and $p(1-q)$ increase as the QS becomes tighter. On the other hand, for a given QS, a tighter QR is equivalent to a smaller probability p, while keeping q fixed, and hence a larger probability $(1-p)(1-q)$ and a smaller probability $p(1-q)$. Next, we establish the supplier's profit and the manufacturer's profit with in-house rework.

2.1. The chain members' profits with in-house rework

The supplier has the unit production cost c_S. For every Type B component identified by the supplier's outbound inspection, the supplier incurs the unit in-house rework cost c_R. For every component that fails to meet the manufacturer's QR, the supplier incurs the unit external failure cost c_L, in addition to the in-house rework cost c_R, and the manufacturer incurs the unit handling cost c_H. Here, the unit external failure cost c_L accounts for factors such as the administration, handling, and logistics of a failed component. We further let V denote the manufacturer's value of each component that meets the QR. In assessing the supplier's profit, we find that the supplier obtains $w-c_S-c_I$ from each Type A component (with probability q), $w-c_S-c_I-c_R$ from each Type B component identified by outbound inspection (with probability $(1-q)r$), $w-c_S-c_I$ from each Type B component that is not identified by outbound inspection but meets the manufacturer's QR (with probability $p(1-q)(1-r)$), and $w-c_S-c_I-c_R-c_L$ from each Type B component that is not identified by outbound inspection and does not meet the manufacturer's QR (with probability $(1-p)(1-q)(1-r)$). Summing these profit terms weighted by the corresponding probabilities yields the supplier's profit π_s:

$$\pi_s = w-c_S-c_I-c_R(1-q)(1-p(1-r))-c_L(1-p)(1-q)(1-r). \tag{1}$$

Similarly, the manufacturer obtains $V-w$ from each component that meets the QR (with probability $1-(1-p)(1-q)(1-r)$), and $V-w-c_H$ from each component that fails to meet the QR (with probability $(1-p)(1-q)(1-r)$). Summing these profit terms weighted by the corresponding probabilities yields the manufacturer's profit π_m:

$$\pi_m = V-w-c_H(1-p)(1-q)(1-r). \tag{2}$$

The supply chain profit π_J is then defined as the sum of the supplier's profit π_s and the manufacturer's profit π_m:

$$\pi_J = V-c_S-c_I-c_R(1-q)(1-p(1-r))-(c_L+c_H)(1-p)(1-q)(1-r). \tag{3}$$

2.2. Decentralized and centralized decisions with in-house rework

We focus on the following interactive dynamics between the supplier and the manufacturer in a decentralized and uncoordinated setting: The manufacturer first chooses the purchase price w, and the supplier then decides the inspection reliability r, with both chain members aiming to maximize their individual profits independently.[3] The manufacturer's choice of w shall be such that the supplier's profit is no less than its reservation profit, for otherwise the supplier will not supply components to the manufacturer. Furthermore, because improving inspection reliability r is increasingly costly and we are interested in interior solutions of the inspection reliability r (i.e. $0<r<1$), we make the following assumption for α and c_L throughout the paper.

Assumption 1. $\alpha > (c_H+c_L-p(c_H+c_L+c_R))(1-q)$, and $c_L > (p/(1-p))c_R$.

To obtain the chain members' decentralized (and uncoordinated) decisions, we first solve the first-order condition of π_s in (1) for r to obtain the supplier's best response r^d[4]:

$$r^d = \frac{(c_L-p(c_L+c_R))(1-q)}{\alpha}. \tag{4}$$

We observe from (4) that the supplier's best response r^d is independent of the manufacturer's purchase price w. When maximizing the profit π_m in (2) with $r=r^d$ substituted, the manufacturer will lower the purchase price as much as possible, such that the supplier's profit is equal to its reservation profit. We let $\hat{\pi}_s$ denote the supplier's reservation profit, and w^d denote the value of the purchase price at which $\pi_s|_{r=r^d} = \hat{\pi}_s$. The decentralized and uncoordinated profits of the supplier and the manufacturer are then given, respectively, by

$$\pi_s^d = \hat{\pi}_s = w^d-c_S-(c_L+c_R)(1-p)(1-q) + \frac{(c_L-p(c_L+c_R))^2(1-q)^2}{2\alpha}, \tag{5}$$

$$\pi_m^d = V-w^d-c_H(1-p)(1-q)\left[1-\frac{(c_L-p(c_L+c_R))(1-q)}{\alpha}\right]. \tag{6}$$

Note that these decentralized profits π_s^d and π_m^d also apply to situations in which the purchase price w is pre-determined, e.g. by the market, and is treated as an exogenous variable, as long as $\pi_s|_{r=r^d} \geqslant \hat{\pi}_s$ at this w.

As a benchmark, we first establish the centralized (system optimal) decision for the supply chain. The centralized decision results when the supplier and the manufacturer jointly make the decision of the inspection reliability in maximizing the supply chain profit π_J in (3). The following lemma establishes the optimal inspection reliability and the optimal chain profit in a supply chain with in-house rework.

(footnote continued)
imperfection of the manufacturer's assembly production process.

[2] If the QS were looser than the QR, insufficient supplier component quality could have contributed to various failure modes in the manufacturer's production process, inevitably harming the manufacturer's product quality and damaging the manufacturer's profit. We thus focus in this paper on the setting with the QS being tighter than the QR.

[3] It applies to settings in which the manufacturer has a superior bargaining position.

[4] r^d is unique, because the second derivative of π_s with respect to r is negative; and $r^d \in (0,1)$, because of Assumption 1.

Lemma 1. *The optimal inspection reliability* r^* *and the optimal chain profit* π_J^* *in a supply chain with in-house rework are*

$$r^* = \frac{(c_H + c_L - p(c_H + c_L + c_R))(1-q)}{\alpha}, \tag{7}$$

$$\pi_J^* = V - c_S - (c_H + c_L + c_R)(1-p)(1-q) + \frac{(c_H + c_L - p(c_H + c_L + c_R))^2(1-q)^2}{2\alpha}. \tag{8}$$

The proofs of Lemma 1 and subsequent lemmas and propositions are included in Appendix A. We can show that the difference of the optimal chain profit π_J^* and the decentralized supply chain $\pi_s^d + \pi_m^d$ is

$$\pi_J^* - (\pi_s^d + \pi_m^d) = \frac{(c_H(1-p)(1-q))^2}{2\alpha} > 0 \tag{9}$$

and conclude that the decentralized supply chain does not yield maximum chain performance. Furthermore, this positive profit difference widens when improvement of inspection reliability is less costly, the manufacturer's unit handling cost increases, or the QR becomes tighter. It is, however, independent of the QS. The next section investigates whether the supplier and the manufacturer can engage in coordination to achieve better chain performance while improving their individual profits.

2.3. Coordination with in-house rework

In order for the supply chain members to voluntarily participate in coordination, their coordinated profits shall be no less than their decentralized profits. For notational convenience, we let π_J^c denote the coordinated supply chain profit, and χ and $1-\chi$ denote the supplier's share and the manufacturer's share of π_J^c, respectively. The Pareto-optimal values of the sharing parameter χ are the values of χ that satisfy $\chi\pi_J^c \geqslant \pi_s^d$ and $(1-\chi)\pi_J^c \geqslant \pi_m^d$, thereby benefiting both the supplier and the manufacturer. As such, before finalizing a coordinating contract, the supplier and the manufacturer shall negotiate the sharing parameter χ within the set of the Pareto-optimal values, based on their relative bargaining power, and reach an agreed value. Our aim in this study is to better understand whether there exists a set of the Pareto-optimal values of the sharing parameter χ under which coordination leads to system optimum, rather than to design a new coordinating contract.

In order for coordination to achieve system optimum, the supplier's decentralized decision should be aligned with the system optimal decision, and this requires that two contractual parameters in the coordinating contract be contingent on the choice of the supplier's decision. There are a number of contract designs suitable for this task. Appendix B briefly explains one such cost sharing contract used in this study. When coordination leads to system optimum, we find that there exists a set of the Pareto-optimal values of χ such that $\chi\pi_J^c \geqslant \pi_s^d$ and $(1-\chi)\pi_J^c \geqslant \pi_m^d$, because $\pi_J^c = \pi_J^* > (\pi_s^d + \pi_m^d)$. Specifically, this set of the Pareto-optimal values of χ is $[\pi_s^d/\pi_J^*, 1-\pi_s^m/\pi_J^*]$.

The above analysis focuses on a decentralized supply chain with in-house rework. When the option of on-site rework is available, how should the supply chain members coordinate their decisions to achieve better chain performance, while also increasing their individual profits? Coordination with the option of on-site rework appears to be more complicated than coordination with in-house rework, because not only is the supplier's decentralized decision required to be aligned with the system optimal decision, but the chain members' interests with regard to the adoption of on-site rework also need to be aligned. The question thus arises as to whether there always exists a Pareto-optimal set of the sharing parameter χ with which coordination leads to maximum supply chain performance in a decentralized supply chain with the option of on-site rework. These questions are addressed in the next section.

3. Value of on-site rework

In this section, we first present the chain members' profits and decisions in the supply chain with on-site rework. We then analyze the decentralized and centralized decisions in the absence and presence of on-site rework. Finally, we explore the existence of a Pareto-optimal set of the sharing parameter χ with which coordination leads to maximum supply chain performance in a decentralized supply chain with the option of on-site rework.

On-site rework is an alternative to in-house rework of the failed components that occur at the manufacturer's production site, although it should be noted that in-house rework is still performed for Type B components identified in the supplier's outbound inspection. On-site rework requires that the supplier install a repair center at the manufacturer's production site. The supplier's on-site rework cost could be greater or less than the external failure cost and in-house rework cost combined, depending on the scale and efficiency of the on-site repair center. However, the manufacturer's cost of handling the failed components is lower when the supplier's on-site repair center is present. The components that are reworked on-site return to the manufacturer's production at no additional cost.

We let c_R^o and c_H^o denote the supplier's unit on-site rework cost and the manufacturer's unit handling cost of failed components in the presence of on-site rework, respectively, where the superscript o stands for on-site rework and $c_H^o < c_H$. Furthermore, we let π_s^o and π_m^o denote the supplier's profit and the manufacturer's profit, respectively, in the presence of on-site rework. By analogy to the derivation of π_s and π_m, we obtain π_s^o and π_m^o as

$$\pi_s^o = w - c_S - c_I - c_R(1-q)r - c_R^o(1-p)(1-q)(1-r), \tag{10}$$

$$\pi_m^o = V - w - c_H^o(1-p)(1-q)(1-r). \tag{11}$$

We then define the supply chain profit π_J^o in the presence of on-site rework to be the sum of $\pi_s^o + \pi_m^o$:

$$\pi_J^o = V - c_S - c_I - c_R(1-q)r - (c_R^o + c_H^o)(1-p)(1-q)(1-r). \tag{12}$$

Because we are interested in interior solutions of the inspection reliability, we make the following assumption in the presence of on-site rework.

Assumption 2. $\alpha > (c_H^o + c_R^o - c_R - p(c_H^o + c_R^o))(1-q)$, and $c_R^o > c_R/(1-p)$.

3.1. Decentralized and centralized decisions in the presence of on-site rework

To derive the chain members' decentralized decisions in the presence of on-site rework, we solve the first-order condition of π_s^o in (10) for r to obtain the supplier's best response r^{od}[5]:

$$r^{od} = \frac{(c_R^o(1-p) - c_R)(1-q)}{\alpha}, \tag{13}$$

which is independent of the manufacturer's purchase price. The decentralized and uncoordinated profits of the supplier and the manufacturer at the manufacturer's purchase price $w = w^d$ are given, respectively, by

$$\pi_s^{od} = w^d - c_S - c_R^o(1-p)(1-q) + \frac{(c_R - c_R^o(1-p))^2(1-q)^2}{2\alpha}, \tag{14}$$

$$\pi_m^{od} = V - w^d - c_H^o(1-p)(1-q)\left[1 + \frac{(c_R - c_R^o(1-p))(1-q)}{\alpha}\right]. \tag{15}$$

Here, we set the manufacturer's purchase price w to be w^d in the presence of on-site rework for two reasons. First, because on-site rework is

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[5] r^{od} is unique, because the second derivative of π_s^o with respect to r is negative; and $r^{od} \in (0,1)$, because of Assumption 2.

Table 1

Profit comparison in the absence and presence of on-site rework when $\rho_4 > \underline{\rho}$, where $\underline{\rho}$, ρ_1 and ρ_4 are given, respectively, in (19)–(21).

	(a) $\underline{\rho} < \rho < \rho_4$	(b) $\rho_4 < \rho < 1$	(c) $1 < \rho < \rho_1$	(d) $\rho_1 < \rho < \bar{\rho}$
Supply chain profit	$\pi_J^* < \pi_J^{o*}$	$\pi_J^* < \pi_J^{o*}$	$\pi_J^* < \pi_J^{o*}$	$\pi_J^* > \pi_J^{o*}$
Supplier's profit	$\pi_s^d < \pi_s^{od}$	$\pi_s^d < \pi_s^{od}$	$\pi_s^d > \pi_s^{od}$	$\pi_s^d > \pi_s^{od}$
Manufacturer's profit	$\pi_m^d > \pi_m^{od}$	$\pi_m^d < \pi_m^{od}$	$\pi_m^d < \pi_m^{od}$	$\pi_m^d < \pi_m^{od}$

optional, the sequential interaction prevents the manufacturer from using two different values of w in a decentralized and uncoordinated setting. Second, the difference between the decentralized profits in the absence and presence of on-site rework anticipated by each chain member can be attributed mainly to the adoption of on-site rework itself. The following lemma establishes the optimal (centralized) decision and the optimal chain profit for the supply chain in the presence of on-site rework.

Lemma 2. *In the presence of on-site rework, the optimal inspection reliability r^{o*} and the optimal chain profit π_J^{o*} are*

$$r^{o*} = \frac{(c_H^o + c_R^o - c_R - p(c_H^o + c_R^o))(1-q)}{\alpha}, \tag{16}$$

$$\pi_J^{o*} = V - c_S - (c_H^o + c_R^o)(1-p)(1-q) + \frac{(c_H^o + c_R^o - c_R - p(c_H^o + c_R^o))^2(1-q)^2}{2\alpha}. \tag{17}$$

We can further show that the profit difference is

$$\pi_J^{o*} - (\pi_s^{od} + \pi_m^{od}) = \frac{(c_H^o(1-p)(1-q))^2}{2\alpha} > 0$$

and that it increases when improvement of inspection reliability is less costly, the manufacturer's unit handling cost increases, or the QR becomes tighter.

3.2. Comparison of decisions in the absence and the presence of on-site rework

To measure the cost effectiveness of on-site rework, we compare the unit on-site rework cost c_R^o and the sum of the unit in-house rework cost c_R and the external failure cost c_L, and define the ratio $\rho = c_R^o/(c_R + c_L)$. A larger value of ρ implies that on-site rework is less cost effective. From Assumption 2, we find that ρ is bounded by $\underline{\rho} < \rho < \bar{\rho}$, where

$$\bar{\rho} = \frac{\alpha + (c_R - c_H^o(1-p))(1-q)}{(c_L + c_R)(1-p)(1-q)}, \tag{18}$$

$$\underline{\rho} = \frac{c_R}{(c_L + c_R)(1-p)}, \tag{19}$$

and $\bar{\rho} > 1$ and $\underline{\rho} < 1$.

With the optimal supply chain profits π_J^* in (8) and π_J^{o*} in (17), we now establish the condition under which the supply chain profit with on-site rework is greater than that without on-site rework in the following proposition.

Proposition 1. *When the ratio ρ is less (greater) than ρ_1, the supply chain profit with on-site rework π_J^{o*} is greater (less) than the supply chain profit without on-site rework π_J^*, where*

$$\rho_1 = \frac{c_H - c_H^o + c_L + c_R}{c_L + c_R}. \tag{20}$$

With Proposition 1, we can derive the following insights. Firstly, because $c_H^o < c_H$, ρ_1 in (20) is greater than 1. This means that on-site rework could be beneficial to the supply chain even if it is more costly than the sum of the external failure and in-house rework costs. Secondly, we can establish that when the ratio ρ is less (greater) than ρ_1, r^{o*} in (16) is less (greater) than r^* in (7). This means that when the supply chain with on-site rework outperforms the supply chain without on-site rework, the optimal inspection reliability for the former is less than the optimal inspection reliability for the latter.

Proposition 1 is concerned with the value of on-site rework from a chain's perspective. Proposition 2, on the other hand, adopts an individual's perspective, and establishes the conditions of ρ under which on-site rework is beneficial to each supply chain member in the decentralized and uncoordinated setting.

Proposition 2. *$\pi_s^d < \pi_s^{od}$, if $\rho < 1$; $\pi_s^d > \pi_s^{od}$ if $1 < \rho < \bar{\rho}$. $\pi_m^d > \pi_m^{od}$ if $\rho < \rho_4$; $\pi_m^d < \pi_m^{od}$, if $\rho_4 < \rho < \bar{\rho}$, where*

$$\rho_4 = \frac{-\alpha(c_H - c_H^o) + (c_H c_L + c_H^o c_R - p c_H(c_L + c_R))(1-q)}{c_H^o(c_L + c_R)(1-p)(1-q)} \tag{21}$$

and $\rho_4 < 1$.

Table 1 summarizes the results of Propositions 1 and 2 in four cases when $\rho_4 > \underline{\rho}$. Note that ρ_4 in (21) could be less than $\underline{\rho}$. When $\rho_4 < \underline{\rho}$, $\pi_m^d < \pi_m^{od}$ holds for all feasible ρ, and Table 1 shall be revised by removing the second column (case (a)) and changing $\rho_4 < \rho < 1$ to $\underline{\rho} < \rho < 1$ for case (b). It is evident from Table 1 that on-site rework is beneficial to both supply chain members in the decentralized and uncoordinated setting only in the case of $\rho_4 < \rho < 1$. This implies that when coordinating the decentralized supply chain under different cases, the Pareto-optimal values of the sharing parameter that lead to system optimum may not exist.

3.3. Coordination with the option of on-site rework

We analyze coordination for the four cases of Table 1 sequentially. We continue using π_J^c to denote the coordinated supply chain profit with the option of on-site rework, and χ and $1-\chi$ to denote the supplier's and manufacturer's shares of the coordinated supply chain profit π_J^c, respectively.

3.3.1. Case (a) of $\underline{\rho} < \rho < \rho_4$

In Case (a) of $\underline{\rho} < \rho < \rho_4$, where $\rho_4 > \underline{\rho}$, on-site rework benefits the supply chain, because $\pi_J^{o*} > \pi_J^*$. However, the supplier and manufacturer have conflicting interests in the decentralized and uncoordinated setting, because $\pi_s^d < \pi_s^{od}$ and $\pi_m^d > \pi_m^{od}$, as depicted in Table 1. In order for coordination to benefit both the supplier and manufacturer in this case, the supplier will anticipate that its coordinated profit is no less than the maximum of π_s^d and π_s^{od}, which is π_s^{od}, and the manufacturer will anticipate that its coordinated profit is no less than the maximum of π_m^d and π_m^{od}, which is π_m^d. The Pareto-optimal values of the sharing factor χ shall meet the following constraints that incorporate the chain members' anticipated profits:

$$\chi\pi_J^c \geqslant \pi_s^{od}, \quad (1-\chi)\pi_J^c \geqslant \pi_m^d. \tag{22}$$

The following lemma establishes the condition under which there exists a set of the Pareto-optimal values of the sharing factor χ with which coordination leads to the optimal supply chain profit π_J^{o*}.

Lemma 3. *When $\underline{\rho} < \rho < \rho_4$, there exists a set of the Pareto-optimal values of the sharing factor χ that satisfies $\chi\pi_J^c \geqslant \pi_s^{od}$ and $(1-\chi)\pi_J^c \geqslant \pi_m^d$, and leads to the optimal supply chain profit π_J^{o*}, if $\rho_5 < \rho < \rho_4$, where*

$$\rho_5 = \frac{-2\alpha(c_H - c_H^o) + (2c_H^o c_R - c_H^{o^2}(1-p) + 2c_H(c_L - p(c_L + c_R)))(1-q)}{2c_H^o(c_L + c_R)(1-p)(1-q)} \tag{23}$$

and $\rho_5 < \rho_4$.

Lemma 3 indicates that the set of the Pareto-optimal values of the sharing factor χ exists if $\rho_5 < \rho < \rho_4$. The coordinating contract that can lead to the optimal supply chain profit π_J^{o*} is similar to that in Appendix B and has two contractual parameters, the purchase price w and the supplier's share τ of the manufacturer's handling cost c_H^o.

On the other hand, if on-site rework is increasingly cost effective (i.e. $\rho < \rho_5$), the supplier anticipates a higher profit of π_s^{od} which leaves no room for both parties to improve their profits via a coordinating contract. When this situation occurs, there are two alternatives for the manufacturer to coordinate the supply chain. The first is that the manufacturer coordinates the decentralized supply chain through in-house rework only, aiming to achieve the supply chain profit π_J^*. With this alternative, the set of the Pareto-optimal values of χ that satisfies $\chi\pi_J^c \geqslant \pi_s^d$ and $(1-\chi)\pi_J^c \geqslant \pi_m^d$ always exists, because $\pi_J^* > \pi_s^d + \pi_m^d$.[6] However, this alternative does not yield maximum chain performance, because $\pi_J^* < \pi_J^{o*}$. The second alternative is to negotiate lowering the supplier's anticipated profit. For instance, if the supplier agrees to lower its anticipated profit to π_s^d, then the manufacturer's coordinating contract shall be able to yield the optimal supply chain profit π_J^{o*}, because $\pi_J^{o*} > \pi_J^* > \pi_s^d + \pi_m^d$ ensures the existence of the set of the Pareto-optimal values of χ that satisfies $\chi\pi_J^c \geqslant \pi_s^d$ and $(1-\chi)\pi_J^c \geqslant \pi_m^d$.

3.3.2. Case (b) of $\rho_4 < \rho < 1$

Consider next Case (b) of $\rho_4 < \rho < 1$. If $\rho_4 < \underline{\rho}$, the range of ρ in this case is revised to $\underline{\rho} < \rho < 1$. Similar to Case (a), on-site rework benefits the supply chain because $\pi_J^{o*} > \pi_J^*$; however, unlike Case (a), both the supplier and the manufacturer prefer on-site rework in the decentralized and uncoordinated setting in Case (b). Hence, the Pareto-optimal values of the sharing factor χ shall meet the following constraints:

$$\chi\pi_J^c \geqslant \pi_s^{od}, \quad (1-\chi)\pi_J^c \geqslant \pi_m^{od}. \tag{24}$$

Because coordination can lead to the optimal supply chain profit π_J^{o*}, as discussed in Section 3.3.1 for $\rho_5 < \rho < \rho_4$, and, because $\pi_J^{o*} > \pi_s^{od} + \pi_m^{od}$, we conclude that in Case (b) there always exists a set of the Pareto-optimal values of the sharing factor χ that satisfies (24) and leads to the optimal supply chain profit π_J^{o*}.

3.3.3. Case (c) of $1 < \rho < \rho_1$

In Case (c) of $1 < \rho < \rho_1$, on-site rework is also beneficial to the supply chain, because $\pi_J^* < \pi_J^{o*}$. However, in the decentralized and uncoordinated setting, the supplier does not prefer on-site rework whereas the manufacturer does. With coordination benefiting both parties, the sharing parameter χ under coordination shall satisfy the following constraints that incorporate the chain members' anticipated profits:

$$\chi\pi_J^c \geqslant \pi_s^d, \quad (1-\chi)\pi_J^c \geqslant \pi_m^{od}. \tag{25}$$

We define

$$\rho_6 = \frac{1}{(c_L + c_R)(1-p)(1-q)}\Big[\alpha + c_R(1-q) - \sqrt{(\alpha - (c_L - c_H^o - p(c_L + c_R - c_H^o)))(\alpha - (c_L + c_H^o - p(c_L + c_R + c_H^o)))}\Big], \tag{26}$$

and establish the condition under which there exists a set of the Pareto-optimal values of the sharing factor χ in the following lemma.

Lemma 4. *When* $1 < \rho < \rho_1$, *there exists a set of the Pareto-optimal values of the sharing factor* χ *that satisfies* $\chi\pi_J^c \geqslant \pi_s^d$ *and* $(1-\chi)\pi_J^c \geqslant \pi_m^{od}$ *and leads to the optimal supply chain profit* π_J^{o*} *if* $1 < \rho < \min\{\rho_1, \rho_6\}$, *where* ρ_6 *is given in (26).*

Lemma 4 indicates that the set of the Pareto-optimal values of the sharing parameter in Case (c) may not exist, depending on the value of ρ_6 in (26). Specifically, this set always exists for $1 < \rho < \min\{\rho_1, \rho_6\}$, but does not exist for $\rho_6 < \rho < \rho_1$. Therefore, when $\rho_6 < \rho < \rho_1$ takes place, the manufacturer needs to consider other coordinating alternatives, similar to those considered for $\rho < \rho_5$ in Case (a). The first alternative is to coordinate the supply chain with in-house rework to achieve the supply chain profit π_J^*. The second alternative is to negotiate lowering the manufacturer's anticipated profit, for example, from π_m^{od} to π_m^d, and coordinate the supply chain with on-site rework to yield the supply chain profit π_J^{o*}. Clearly, the first alternative will not yield maximum chain performance, but the second will.

3.3.4. Case (d) of $\rho_1 < \rho < \overline{\rho}$

Finally, in Case (d) of $\rho_1 < \rho < \overline{\rho}$, on-site rework is not beneficial to the supply chain, because $\pi_J^* > \pi_J^{o*}$. Furthermore, the supplier does not prefer on-site rework but the manufacturer does. With coordination aiming to achieve the supply chain profit π_J^*, we find that there does not exist any set of the Pareto-optimal values of the sharing parameter χ such that $\chi\pi_J^c \geqslant \pi_s^d$ and $(1-\chi)\pi_J^c \geqslant \pi_m^{od}$, because $\pi_J^* < \pi_s^d + \pi_m^{od}$ for $\rho_1 < \rho < \overline{\rho}$. Coordination to achieve the supply chain profit π_J^{o*} is also infeasible because $\pi_J^{o*} < \pi_J^*$. The only plausible coordinating alternative in this case is to negotiate lowering the manufacturer's anticipated profit and then to coordinate the supply chain with in-house rework to yield the supply chain profit π_J^*.

3.4. Numerical illustration

We now illustrate the above cases with a numerical example at different values of ρ. The base setting is as follows: $q = 0.8$, $p = 0.24$, $c_S = 2$, $\alpha = 0.5$, $c_R = 3$, $c_L = 1.5$, $V = 15$, $w = 6$, $c_H = 2.2$, $c_H^o = 1.0$. Under the base setting, we obtain $\underline{\rho} = 0.877$, $\rho_4 = 0.270$, $\rho_1 = 1.267$, and $\overline{\rho} = 1.386$. Table 2 summarizes the supply chain profits for $\rho = 0.9$, 1.1, 1.3 and the corresponding cases in Table 1. Note that because $\rho_4 < \underline{\rho}$, we ignore case (a) of Table 1 and change the range $\rho_4 < \rho < 1$ for case (b) to $\underline{\rho} < \rho < 1$.

As illustrated in Table 2, the base setting with $\rho = 0.9$ corresponds to case (b) in which $\pi_s^d = 3.323$ is less than $\pi_s^{od} = 3.385$, $\pi_m^d = 8.722$ is less than $\pi_m^{od} = 8.852$, and $\pi_J^* = 12.157$ is less than $\pi_J^{o*} = 12.261$. In this case, both the supplier and the manufacturer prefer on-site rework in the decentralized and uncoordinated setting and coordination in the presence of on-site rework is beneficial to both of them. Next, consider the base setting with $\rho = 1.1$ which corresponds to case (c) of Table 1. As depicted in Table 2, $\pi_s^d = 3.323$ is greater than $\pi_s^{od} = 3.271$, $\pi_m^d = 8.722$ is less than $\pi_m^{od} = 8.894$, and $\pi_J^* = 12.157$ is less than $\pi_J^{o*} = 12.188$. In order to apply Lemma 4, we calculate the value of ρ_6 in (26), which is 1.042. Because of $\rho > \rho_6$, Lemma 4 reveals that the set of the Pareto-optimal values of the sharing parameter does not exist. In this situation, the supply chain members shall consider the coordinating alternatives mentioned in Section 3.3.3. From the above analysis, we obtain that when $0.877 < \rho < 1.042$ (i.e. $3.947 < c_R^o < 4.689$), the supply chain members are able to coordinate to achieve the maximum chain profit. Finally, the base setting with $\rho = 1.3$ corresponds to case (d) of Table 1. In this case, there does not exist any set of the Pareto-optimal values of the sharing parameter, as discussed in Section 3.3.4.

3.5. Summary

The analysis of the four cases above reveals that the set of the Pareto-optimal values of the sharing parameter χ that leads to maximum supply chain performance exists over a certain range of the cost effectiveness ρ of on-site rework, i.e., $\max\{\underline{\rho}, \rho_5\} < \rho < \min\{\rho_1, \rho_6\}$. When

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[6] The supplier's anticipated profit is now π_s^d instead of π_s^{od}, because rework can only be performed in-house.

Table 2
Profit comparison in the absence and presence of on-site rework in the base setting by varying $\rho \in \{0.9, 1.1, 1.3\}$, where $\underline{\rho} = 0.877$, $\rho_4 = 0.270$, $\rho_1 = 1.267$, and $\bar{\rho} = 1.386$.

	Absence of on-site repair			Presence of on-site repair			Corresponding
ρ	π_s^d	π_m^d	π_J^*	π_s^{od}	π_m^{od}	π_J^{o*}	case in Table 1
0.9	3.323	8.722	12.157	3.385	8.853	12.261	case (b): $\underline{\rho} < \rho < 1$
1.1	3.323	8.722	12.157	3.271	8.894	12.188	case (c): $1 < \rho < \rho_1$
1.3	3.323	8.722	12.157	3.194	8.936	12.154	case (d): $\rho_1 < \rho < \bar{\rho}$

ρ is not within this range, the set of the Pareto-optimal values of χ that leads to maximum supply chain performance may not exist. The manufacturer can circumvent this coordination difficulty by adopting one of the following two alternatives—aiming for sub-optimal supply chain profit, or for optimal supply chain profit through negotiation of lowering anticipated profits. However, when $\rho > \rho_1$, only the second alternative is feasible.

4. Parametric analysis

We now investigate how model parameters affect the optimal inspection reliabilities and supply chain profits in the absence and presence of on-site rework, and on coordination performance, if coordination does not deliver maximum supply chain performance. And, we focus on the effects of quality specification, quality requirement, the cost parameter for improving inspection reliability (α), the manufacturer's unit handling cost (c_H), the supplier's unit external failure cost (c_L), and the supplier's unit on-site rework cost (c_R^o).

4.1. Effects of quality specification and quality requirement

We begin the analysis by exploring the effects of QS and QR. Recall that a tighter QR is equivalent to a smaller probability p with the probability q of a Type A component fixed, and a tighter QS is equivalent to a smaller probability q with the probability $(1-p)(1-q)$ fixed.

Consider first the effects of QR. The following lemma provides the analysis on the effects of QR.

Lemma 5. *As QR becomes looser, π_J^* or π_J^{o*} increases, but r^* or r^{o*} decreases. When $\rho < (>) \rho_1$, the difference $r^* - r^{o*}$ ($r^{o*} - r^*$) decreases as QR becomes looser.*

Lemma 5 reveals that a looser QR is beneficial to the supply chain and leads to lower inspection reliability, regardless of whether on-site rework is present. Furthermore, the absolute difference between r^* and r^{o*} narrows as p increases, indicating that when more components from the supplier meet the manufacturer's quality requirement, the optimal inspection reliabilities in the absence and presence of on-site rework will converge.

In light of the effect of QR on the difference $\pi_J^{o*} - \pi_J^*$, we are interested in the range of $\rho < \rho_1$, because coordination in Case (d) of $\rho_1 < \rho < \bar{\rho}$ always aims to achieve the chain profit π_J^*, as discussed in Section 3.3.4. We can establish that the difference $\pi_J^{o*} - \pi_J^*$ is concave in p for $\rho < \rho_1$, and it increases in p, if $p < p_1$, and decreases in p, if $p > p_1$, where $p_1 = 1 - \frac{\alpha + c_R(1-q)}{(1-q)(c_H + c_H^o + (c_L + c_R)(1+\rho))}$. This means that p has the opposite effects on the difference $\pi_J^{o*} - \pi_J^*$ and that the maximal value of $\pi_J^{o*} - \pi_J^*$ takes place at $p = p_1$, and is given by $\frac{(\alpha + c_R(1-q))^2 (c_H - c_H^o + (c_L + c_R)(1-\rho))}{2\alpha(c_H + c_H^o + (c_L + c_R)(1+\rho))}$. Recall that when $\rho < \rho_5$ or $\rho_6 < \rho < \rho_1$, negotiation between the supplier and manufacturer may lead the manufacturer to choose a coordinating contract that yields the supply chain profit π_J^* rather than π_J^{o*}, and fails to achieve maximum supply chain performance. If the manufacturer did choose such a coordinating contract, then the above analysis demonstrates that this coordinating contract would have the worse chain performance at $p = p_1$.

We proceed to examine the effects of QS in Lemma 6 below.

Lemma 6. *As QS becomes looser, π_J^* or π_J^{o*} increases, as does r^* or r^{o*}. Varying QS has no effect on the difference $r^{o*} - r^*$. When $\rho < (>) \rho_1$, the difference $\pi_J^{o*} - \pi_J^*$ ($\pi_J^* - \pi_J^{o*}$) decreases as QS becomes looser.*

Lemma 6 shows that a looser QS is beneficial to the supply chain regardless of whether on-site rework is present, similar to the effect of QR on the supply chain profits. However, a looser QS leads to higher inspection reliability, a trend opposite to the effect of QR on inspection reliability. This is because the reduction in the rework cost of the Type B components identified at the manufacturer's production site outweighs the increase in the inspection cost, resulting in higher inspection reliability. With regard to the effect of QS on the difference $r^{o*} - r^*$, we find that varying QS has no effect on $r^{o*} - r^*$, because QS has the same effect on both r^{o*} and r^*. Finally, the absolute value of the difference $\pi_J^{o*} - \pi_J^*$ narrows as QS becomes looser. This suggests that if coordination does not achieve maximum chain performance in $\rho < \rho_5$ or $\rho_6 < \rho < \rho_1$, the chain performance worsens as QS becomes tighter.

4.2. Effects of α, c_H, c_L, and c_R^o

Regarding the effects of α, we can establish that π_J^*, r^*, π_J^{o*}, and r^{o*} decrease with α. We can further establish that if $\rho > (<) \rho_1$, $r^{o*} - r^*$ ($r^* - r^{o*}$) decreases with α. The latter trend is similar to the effect of QR on the difference between r^{o*} and r^*. In light of the effects of α on $\pi_J^{o*} - \pi_J^*$, we are also interested in the range of $\rho < \rho_1$. We continue using the numerical example given in Section 3.4 for illustrative purposes. Fig. 2 depicts the effects of α on $\pi_J^{o*} - \pi_J^*$ at $\rho = 0.9$, 1.0, 1.1, 1.2. The increasing trends in Fig. 2 indicate that the difference $\pi_J^{o*} - \pi_J^*$ widens as α increases.

Consider next the effects of c_H and c_L. Because r^* increases with c_H or c_L and r^{o*} stays unchanged as c_H or c_L varies, the difference $r^{o*} - r^*$ decreases with c_H or c_L. Furthermore, because π_J^* decreases with c_H or c_L and π_J^{o*} stays unchanged as c_H or c_L varies, the difference $\pi_J^{o*} - \pi_J^*$ increases with c_H or c_L. Fig. 3 illustrates the increasing trends of $\pi_J^{o*} - \pi_J^*$ in c_H at $\rho = 0.9$, 1.0, 1.1, 1.2.

Finally, consider the effects of c_R^o. We can establish that r^{o*} increases with c_R^o and π_J^{o*} decreases with c_R^o. Because both r^* and π_J^* stay

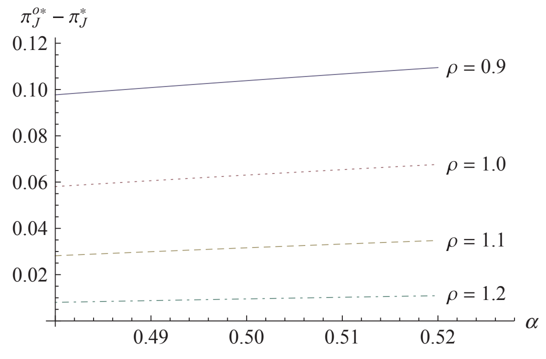

Fig. 2. Effect of α on $\pi_J^{o*} - \pi_J^*$ at $\rho = 0.9$, 1.0, 1.1, 1.2.

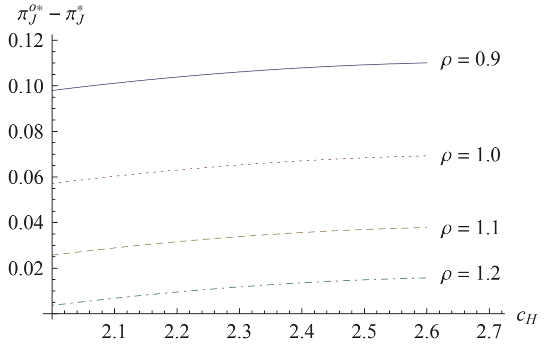

Fig. 3. Effect of c_H on $\pi_J^{o*}–\pi_J^*$ at $\rho = 0.9, 1.0, 1.1, 1.2$.

unchanged as c_R^o varies, the difference $r^{o*}–r^*$ increases but the difference $\pi_J^{o*}–\pi_J^*$ decreases with c_R^o. The above analysis reveals that if coordination fails to achieve maximum chain performance, the chain performance worsens as α, c_H, c_L becomes larger or c_R^o becomes smaller.

5. Summary and conclusions

In this study we investigate coordinated inspection and rework policies in a two-stage supply chain with the option of on-site rework. On-site rework could be beneficial to the supply chain even if it is more costly than in-house rework. We find that coordinating the decentralized supply chain with the option of on-site rework leads to maximum supply chain performance over a certain cost range of on-site rework. Coordinating the supply chain outside this range can still yield maximum supply chain performance if negotiation with regard to lowering the supply chain members' anticipated profits is achievable. Finally, we find that the quality requirement (for the supplier's components, as demanded by the manufacturer's production process) and quality specification (that the supplier's components need to meet in outbound inspection) have different effects on supply chain profit and inspection reliability. Specifically, regardless of whether on-site rework is present, a looser quality requirement or quality specification is beneficial to the supply chain, and a looser quality requirement leads to lower inspection reliability whereas a looser quality specification leads to higher inspection reliability. In cases where the supply chain profit in the presence of on-site rework is greater than the supply chain profit in the absence of on-site rework, quality requirement has the opposite effects on the difference between these supply chain profits. However, a looser quality specification always lead to a smaller difference of these supply chain profits. The above insights suggest that when the option of on-site rework is available, the supply chain members should not forego this option simply because it is more costly than in-house rework. Instead, they should base their decisions on the benefits derived from supply chain coordination with or without the option of on-site rework. Furthermore, with the knowledge of whether coordination yields optimal or sub-optimal supply chain profit over a certain cost range of on-site rework, the supplier and the manufacturer can prepare themselves in the negotiation process of the coordinating contract. Finally, a tighter QS is detrimental to the supply chain. If the supplier's QS was set by the manufacturer and the current QS is much tighter than the current QR, the manufacturer should ask the supplier to set a looser QS. On the other hand, if the supplier's QS was set by its own commitment, the manufacturer should discuss with the supplier on the possibility of a looser QS.

Two extensions to this study are possible. Firstly, we considered that the supplier has an exogenous production quality characteristic. In cases where improvement of production quality is possible, factoring the supplier's production quality decision into the model is worth pursuing. Secondly, the analysis of the interactive dynamics between the supply chain members was based on the same information being available to both parties. In practice, however, certain information might be private to one supply chain member. For instance, although the supplier's quality specification is very likely to be known to the manufacturer, the manufacturer may not want to reveal the true quality requirement of the supplier's components to the supplier. It would be worthwhile to explore the strategic interactions between the supply chain members under conditions of information asymmetry in an extended framework.

Appendix A. Proofs of the lemmas and propositions

Proof of Lemma 1. Because $\mathrm{d}^2\pi_J/\mathrm{d}r^2 = -\alpha < 0$, where π_J is given in (3), solving $\mathrm{d}\pi_J/\mathrm{d}r = 0$ for r gives the optimal inspection reliability r^* in (7). Consequently, substituting $r = r^*$ into π_J yields the optimal supply chain profit π_J^* in (8). Note that $0 < r^* < 1$, because of Assumption 1. □

Proof of Lemma 2. In a similar way to the proof of Lemma 1, we solve $\mathrm{d}\pi_J^o/\mathrm{d}r = 0$ for r to obtain the optimal inspection reliability r^{o*} in (16), because $\mathrm{d}^2\pi_J^o/\mathrm{d}r^2 = -\alpha < 0$, where π_J^o is given in (12). Then, substituting $r = r^{o*}$ into π_J^o yields the optimal supply chain profit π_J^{o*} in (17). Note also that $0 < r^{o*} < 1$, because of Assumption 2. □

Proof of Proposition 1. Define $\Delta_1 = \pi_J^{o*}–\pi_J^*$. Substituting $c_R^o = \rho(c_R + c_L)$ into Δ_1 gives

$$\Delta_1 = \frac{(1–p)(1–q)(c_H–c_H^o + c_L + c_R–\rho(c_L + c_R))(2\alpha–(1–q)\omega_1)}{2\alpha},$$

where $\omega_1 = (c_H + c_H^o + c_L)(1–p)–c_R(1 + p) + \rho(c_L + c_R)(1–p)$. Because $\mathrm{d}^2\Delta_1/\mathrm{d}\rho^2 = \frac{((c_L + c_R)(1 - p)(1 - q))^2}{\alpha} > 0$,$\Delta_1$ is convex in ρ. Solving $\mathrm{d}\Delta_1/\mathrm{d}\rho = 0$ for ρ yields $\rho^* = \frac{\alpha + (c_R - c_H^o(1 - p))(1 - q)}{(c_L + c_R)(1 - p)(1 - q)}$, which is identical to $\bar{\rho}$ in (18). Thus, Δ is decreasing convex in ρ for $\rho < \bar{\rho}$. Furthermore, solving $\Delta_1 = 0$ for ρ yields two roots $\rho_1 = \frac{c_H - c_H^o + c_L + c_R}{c_L + c_R}$ and $\rho_2 = \frac{2\alpha - (c_H + c_H^o + c_L - c_R - p(c_H + c_H^o + c_L + c_R))(1 - q)}{(c_L + c_R)(1 - p)(1 - q)}$. With Assumption 1, we find $\rho_1 < \bar{\rho}$ and $\rho_2 > \bar{\rho}$, and conclude that Δ_1 is positive when $\rho < \rho_1$, and negative when $\rho_1 < \rho < \bar{\rho}$. □

Proof of Proposition 2. Define $\Delta_2 = \pi_s^{od}–\pi_s^d$ with $c_R^o = \rho(c_L + c_R)$ substituted. The second derivative of Δ_2 with respect to ρ is $\frac{(c_L + c_R)^2(1 - p)^2(1 - q)^2}{\alpha} > 0$, indicating that Δ_2 is convex in ρ. Solving the first-order condition of Δ_2 with respect to ρ gives $\rho^{**} = \frac{\alpha + c_R(1 - q)}{(c_L + c_R)(1 - p)(1 - q)}$. Because $\rho^{**} > \bar{\rho}$,Δ_2 is decreasing convex in ρ. Furthermore, solving $\Delta_2 = 0$ for ρ yields two roots: 1 and $\rho_3 = \frac{2\alpha - (c_L - c_R - p(c_L + c_R))(1 - q)}{(c_L + c_R)(1 - p)(1 - q)}$. Because the second root ρ_3 is greater than $\bar{\rho}$, we obtain that $\pi_s^d < \pi_s^{od}$, if $\rho < 1$, and $\pi_s^d > \pi_s^{od}$ if $1 < \rho < \bar{\rho}$.

Next, define $\Delta_3 = \pi_m^{od}–\pi_m^d$ with $c_R^o = \rho(c_L + c_R)$ substituted. The first derivative of Δ_3 with respect to ρ is $\frac{c_H^o(c_L + c_R)(1 - p)^2(1 - q)^2}{\alpha} > 0$, indicating that Δ_3 increases in ρ. Solving $\Delta_3 = 0$ for ρ yields ρ_4 in (21). With Assumption 1, we obtain $\rho_4 < 1$. Hence, $\pi_m^d > \pi_m^{od}$ if $\rho < \rho_4$, and $\pi_m^d < \pi_m^{od}$, if $\rho_4 < \rho < \bar{\rho}$. □

Proof of Lemma 3. We establish the condition under which $\pi_J^{o*} > (\pi_s^{od} + \pi_m^d)$ holds in this proof, but explain the coordinating contract that leads to the optimal supply chain profit π_J^{o*} in Appendix B. Define $\Delta_4 = \pi_J^{o*}–(\pi_s^{od} + \pi_m^d)$ with $c_R^o = \rho(c_L + c_R)$ substituted. The first derivative of Δ_4 with respect to ρ is $\frac{c_H^o(c_L+c_R)(1-p)^2(1-q)^2}{\alpha}>0$. Solving $\Delta_4=0$ for ρ gives ρ_5 in (23). We further find that $\rho_5-\rho_4=\frac{-c_H^o}{2(c_L+c_R)}<0$, where ρ_4 is given in (21), and conclude that if $\rho_5<\rho<\rho_4,\pi_J^{o*}>(\pi_s^{od}+\pi_m^d)$ and the set of the Pareto-optimal values of the sharing factor χ that satisfies $\chi\pi_J^c\geqslant\pi_s^{od}$ and $(1-\chi)\pi_J^c\geqslant\pi_m^d$ exists. □

Proof of Lemma 4. Define $\Delta_5=\pi_J^{o*}-(\pi_s^d+\pi_m^{od})$ with $c_R^o=\rho(c_L+c_R)$ substituted. The second derivative of Δ_5 with respect to ρ is $\frac{(c_L+c_R)^2(1-p)^2(1-q)^2}{\alpha}>0$, indicating that Δ_5 is convex in ρ. Solving the first-order condition of Δ_5 with respect to ρ gives $\rho^{**}=\frac{\alpha+c_R(1-q)}{(c_L+c_R)(1-p)(1-q)}$. Because $\rho^{**}>\overline{\rho},\Delta_5$ is decreasing convex in ρ in Case (c) of $1<\rho<\rho_1$. Solving $\Delta_5=0$ for ρ yields two roots. Let ρ_6 denote the smaller root of $\Delta_5=0$, as given in (26). Because $\Delta_5>0$ at $\rho=1$ and ρ_6 could be greater than ρ_1, we obtain that if $1<\rho<\min\{\rho_1,\rho_6\},\pi_J^{o*}>(\pi_s^d+\pi_m^{od})$. The coordinating contract in this case can be constructed in a similar way to that in Case (a), and is omitted here. □

Proof of Lemma 5. Because a tighter (looser) QR is equivalent to a smaller (larger) probability of p, we differentiate π_J^* in (8) with respect to p and obtain

$$\frac{d\pi_J^*}{dp}=\frac{(c_H+c_L+c_R)(1-q)(\alpha-(c_H+c_L-p(c_H+c_L+c_R)))}{\alpha}$$

which is positive, because of Assumption 1. The first derivative of π_J^{o*} in (17) with respect to p is

$$\frac{d\pi_J^{o*}}{dp}=\frac{(c_H^o+c_R^o)(1-q)(\alpha-(c_H^o+c_R^o-c_R-p(c_H^o+c_R^o))(1-q))}{\alpha}$$

which is positive, because of Assumption 2. The above results indicate that π_J^* or π_J^{o*} increases as QR become looser. The first derivative of r^* in (7) with respect to p is $\frac{(c_H+c_L+c_R)(q-1)}{\alpha}<0$. And, the first derivative of r^{o*} in (16) with respect to p is $\frac{(c_H^o+c_R^o)(q-1)}{\alpha}<0$. Hence, r^* or r^{o*} decreases as QR become looser. From the above results, we establish that the first derivative of r^*-r^{o*} with respect to p is $\frac{(q-1)(c_H+c_L+c_R-c_H^o-\rho(c_L+c_R))}{\alpha}$. Therefore, if $\rho<\rho_1$, the difference r^*-r^{o*} is positive and decreases in p, and if $\rho>\rho_1,r^{o*}-r^*$ is positive and decreases in p. In other words, if $\rho<(>)\rho_1,r^*-r^{o*}$ $(r^{o*}-r^*)$ decreases as QR becomes looser. □

Proof of Lemma 6. Define $\phi=(1-p)(1-q)$ to be the probability that a component fails to meet the QR. For a given QR, ϕ is a constant as QS becomes tighter or looser. When we use q to characterize the effect of QS, p shall vary accordingly, and is represented by $p=1-\phi/(1-q)$. In order to examine the effect of QS on π_J^*, we substitute $p=1-\phi/(1-q)$ into π_J^* and differentiate the resulting profit with respect to q:

$$\frac{d\pi_J^*}{dq}=\frac{c_R[(c_H+c_L+c_R)\phi-(1-q)c_R]}{\alpha} \tag{A.1}$$

$$=\frac{c_R[c_H+c_L-p(c_H+c_L+c_R)](1-q)}{\alpha}$$

which is greater than zero, because of $r^*>0$. By the same token, we substitute $p=1-\phi/(1-q)$ into π_J^{o*} and differentiate the resulting term with respect to q:

$$\frac{d\pi_J^{o*}}{dq}=\frac{c_R[(c_H^o+c_R^o)\phi-(1-q)c_R]}{\alpha} \tag{A.2}$$

$$=\frac{c_R[(c_H^o+c_R^o)(1-p)-c_R](1-q)}{\alpha}$$

which is greater than zero, because of $r^{o*}>0$. Hence, both π_J^* and π_J^{o*} increase as QS becomes looser. To analyze the effect of QS on the optimal inspection reliability, we substitute $p=1-\phi/(1-q)$ into r^* and differentiate with respect to q, resulting in $dr^*/dq=c_R/\alpha>0$. Likewise, substituting $p=1-\phi/(1-q)$ into r^{o*} and differentiating with respect to q yields $dr^{o*}/dq=c_R/\alpha>0$. We then conclude that both r^* and r^{o*} increase as QS becomes looser. Furthermore, because $dr^{o*}/dq-dr^*/dq=0$, varying QS has no effect on the difference $r^{o*}-r^*$. Finally, by using $d\pi_J^*/dq$ in (A.1) and $d\pi_J^{o*}/dq$ in (A.2) and $c_R^o=\rho(c_R+c_L)$, we establish that

$$\frac{d\pi_J^{o*}}{dq}-\frac{d\pi_J^*}{dq}=\frac{c_R\phi[-c_H+c_H^o+(c_L+c_R)(\rho-1)]}{\alpha}.$$

Therefore, if $\rho<\rho_1,d\pi_J^{o*}/dq<d\pi_J^*/dq$, and if $\rho>\rho_1,d\pi_J^{o*}/dq>d\pi_J^*/dq$. In other words, if $\rho<(>)\rho_1,\pi_J^{o*}-\pi_J^*$ $(\pi_J^*-\pi_J^{o*})$ decreases as QS becomes looser. □

Appendix B. A cost sharing contract with in-house rework

To coordinate the supplier's decentralized decision via a cost sharing contract, we set the manufacturer's purchase price w and the supplier's share τ of the manufacturer's handling cost c_H as the contractual parameters. Under this contract, the supplier's coordinated profit π_s^c is the difference between π_s and $(1-p)(1-q)(1-r)\tau c_H$, and the manufacturer's coordinated profit π_m^c is the sum of π_m and $(1-p)(1-q)(1-r)\tau c_H$, where the term $(1-p)(1-q)(1-r)\tau c_H$ is the supplier's share of the manufacturer's expected handling cost. For a given value of χ, we can derive the contractual parameters (w^c,τ^c) by solving $d\pi_s^c/dr=d\pi_J^c/dr$ and $\pi_s^c=\chi\pi_J^c$ simultaneously for w and τ. Consequently, the cost sharing contract with the derived contractual parameters (w^c,τ^c) will lead the supplier to choose the optimal inspection reliability r^*, because maximizing its own profit under this contract is equivalent to maximizing the supply chain profit.

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