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Highlights

- This research studies the optimal pricing strategy when the firm sequentially introduces products with trade-in program to strategic consumers.
- The key to the optimal pricing strategy under trade-in is the balance between flexibility and ex-ante commitment on pricing.
- The optimal pricing strategy depends on product innovation level, salvage value, and how strategic the consumers are.
- The findings are robust and provide managerial insights for sequentially innovating firms in industries like high-tech and fashion.

Optimal Pricing Strategy under Trade-in Program in the Presence of Strategic Consumers

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Abstract: Many innovating firms use trade-in programs to encourage consumers' repeat purchasing. They can choose between dynamic pricing and preannounced pricing strategies to mitigate the impacts of consumers' strategic behavior. This paper develops a dynamic game framework to explore the optimal pricing strategy when the firm sequentially introduces new generations of products to a market populated by strategic consumers with trade-in option offered. Results show that under either pricing strategy, the firm has an incentive to sell the old generation products to new consumers in the second period if the salvage value of the old generation product is high enough. When consumers are sufficiently strategic, if both the innovation incremental value of the new generation product and the salvage value of the old generation product are low enough, the firm is better off following the preannounced pricing strategy. Besides, as the firm becomes more farsighted, the comparatively dominant position of preannounced pricing over dynamic pricing disappears gradually.

Keywords: pricing strategy; trade-in program; strategic consumer; sequential innovation; market heterogeneity

1. Introduction

Due to rapid technological and economic development, manufacturers launch new products more frequently than before to compete in the market, especially in the high-tech industry (e.g., smartphones, tablets and wearable devices). On the other hand, consumers have a variety of ways to obtain price and

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function predictions on new generations of products. Therefore, they are prone to wait strategically rather than to purchase the current generation, because of precise expectations for future alternatives [3, 50]. In addition, patrons owning the old-generation products are sometimes deterred from switching to the latest generation [34, 51]. To encourage consumers' repeat purchasing, many firms offer *trade-in program* in different forms, under which patrons can return the old-generation product and then obtain a rebate when purchasing a new-generation of the product [50]. For instance, in February 2016, Apple Inc. relaunched the trade-in program for their products, officially known as the "Apple Reuse and Recycling Program" [1]. Other manufactures also provide similar trade-in programs, such as Samsung[1] and Huawei[2].

Pricing is a critical decision in product updating. Extant literature studies pricing strategy from different perspectives. With respect to the timing of price announcement and pricing commitment, *dynamic pricing strategy* and *preannounced pricing strategy* are well studied [2, 6, 10, 11]. Dynamic pricing strategy (also called *responsive pricing* or *contingent pricing*) means that the firm decides the prices for each period contingently, based on the realized consumer demand and other information on market and product. Dynamic pricing is widely used in various industries, such as movie, airline, apparel, and household electronics [5]. For instance, Amazon.com is known to adopt complex dynamic-pricing algorithms [35]. In fast consumer electronics industry, manufacturers like Apple, Samsung, and Huawei make pricing decisions for each generation of products dynamically. Preannounced pricing strategy (also called *price commitment* or *posted pricing*) means that the firm determines and commits to both the current and future market prices before the selling season starts. Along the line of industry practices, there are many applications of preannounced pricing, such as Lands' End overstocks, Dress for Less [10], Sam's Club [14], and Pricetack.com [41]. For example, Sam's Club used to follow a markdown mechanism with preannounced prices such that the future prices at different time were listed in the website and consumers could choose to buy later at a specific mark-down price. In the fast consumer electronics industry, Sony

[1] See http://www.samsung.com/us/trade-in/ for more information.
[2] See https://www.vmall.com/recycle/ for more information.

made announcements in advance that the price cut for PS4 "could be some way off" [41]. In addition, some manufacturers preannounce the retail price of the new product to consumers through pre-ordering, such as the advance-selling of Kindle by Amazon[3]. Besides, Apple presells the iPhone X and commits its price to consumers when launching the iPhone 8. Under the assumption that consumers are myopic, dynamic pricing strategy has more revenue potential than price commitment strategy because of its flexibility and ability to capture consumer surplus [10, 11, 41]. Along the line of academic research, scholars in marketing and operations areas conclude that preannounced pricing is a good way to mitigate consumers' strategic waiting behavior, thus can even outperform dynamic pricing under some conditions [2, 6, 10, 11, 14, 15, 35, 41].

With trade-in program offered, which pricing strategy is better for the firm when consumers are getting more strategic? In current business practices, trade-in programs co-exist with dynamic pricing more often than preannounced pricing. In the aforementioned examples on trade-in programs, Apple, Samsung, and Huawei determine both the retail prices and trade-in rebate dynamically. However, firms start to use pre-commitment in different forms as a response for more strategic consumers to encourage repurchasing. For example, Apple launched "iPhone Upgrade Program" online since 2016[4], allowing the iPhone 7 users to upgrade to the iPhone 8 in the future with a 50% discount by trading in the iPhone 7[5]. Note that when Apple launched this program, the iPhone 8 has not been developed yet. Thus, Apple preannounced the trade-in rebate. Given the relatively infrequent adoption of preannounced pricing, we are interested in finding out under what circumstances the firm would be better off following preannounced pricing/dynamic pricing strategy.

To be more specific, this research explores (1) With trade-in program and strategic consumers, how should a firm make price decisions for each generation of products under different pricing strategies? (2)

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[3] See https://techcrunch.com/2013/09/03/amazons-next-kindle-paperwhite-outed-ahead-of-official-launch-via-amazons-own-leak/, http://www.businessinsider.com/amazon-kindle-paperwhite-2015-6 for more information.

[4] See https://www.apple.com/shop/iphone/iphone-upgrade-program/ for more information.

[5] The specific trade-in rebate may differ in various countries. We use the data in China as an example (see https://www.apple.com/cn/shop/iphone/iphone-upgrade-program/).

Under what conditions is a firm better off following dynamic pricing/preannounced pricing strategy? (3) When offering a trade-in program to strategic consumers, how do other factors impact the optimal pricing strategy? To answer these questions, we develop a stylized model to analyze the interaction between a monopolist firm and a population of strategic consumers in a two-period dynamic game. The monopolist firm determines its pricing strategy before the first period, and then decides the products' prices and trade-in rebate in each following period. We analyze the problem from a game theoretic perspective and find the subgame perfect equilibrium (SPE) strategy for both the consumers and the firm.

The main findings of this research are summarized as follows. First, we characterize the subgame perfect equilibrium between the firm and the strategic consumers. We find that under either pricing strategy, the firm has an incentive to sell the old generation product to new consumers in the second period if the salvage value of the old generation product is sufficiently high; otherwise, the firm prefers to sell the new generation product. However, it is noteworthy that the intrinsic reasons are opposite for the two pricing strategies. Specifically, under dynamic pricing, consumers may purchase the old generation products instead of the new generation products if the price discount is big enough. However, under preannounced pricing, consumers may wait for the price mark-down of the old generation product if and only if the introductory price of the new generation product is relatively high. Second, we find that neither pricing strategy dominates the other when the firm sells two sequential generations of products to strategic consumers. Specifically, when consumers are strategic enough, if both the innovation incremental value of the new generation product and the salvage value of the old generation product are low enough, the firm is better off using the preannounced pricing strategy, because commitment can reduce the cannibalization effect. Otherwise, dynamic pricing is more profitable, as the flexibility to adjust price dynamically gives the firm an opportunity to better discriminate consumers when they are less strategic and there is a sufficiently high level of innovation. Third, in launching new product, the optimal pricing strategy remains almost invariant between dual and single rollover approaches. In addition, as the firm becomes more farsighted, the comparatively dominant position of preannounced pricing over dynamic pricing decreases.

This research contributes to the existing literature and business practices in many ways. On the one hand, this study brings together the literature from streams on pricing strategy, trade-in program and strategic consumers. To the best of our knowledge, none of the existing literature has studied the value of commitment and pricing flexibility in the situation where the firm sequentially introduces new generations of products to strategic consumers. On the other hand, in terms of managerial implications, this paper provides reasonable explanations for Apple's attempt on the new form of the trade-in program with preannounced trade-in rebate, namely, the "iPhone Upgrade Program". As consumers becomes more strategic, the innovation level of new generations of products might not be significant enough to boost new sales. According to our research, when trade-in program is offered, the continuous adoption of dynamic pricing strategy may hurt the firm's revenue. Since an increasing number of firms in various industries are offering trade-in programs for consumers, it is worthwhile to explore the choice of pricing strategy and hence to further improve the firms' revenue performance.

The remainder of this paper is organized as follows. We review the related literature in Section 2. Section 3 introduces the model. Sections 4 derives the equilibrium under the dynamic pricing and the preannounced pricing. In Section 5, we compare the optimal profits for the two pricing strategies to illustrate the situations in which the firm will be better off under either pricing strategy. Section 6 extends the model and Section 7 concludes the paper.

2. Literature Review

This study is closely related to research on pricing strategy, trade-in program, and strategic consumers.

2.1. Research on Pricing Strategy

Before the progress of studies on strategic consumers, literature on pricing strategy in operations management and revenue management was largely based on dynamic pricing. Nevertheless, when consumers are strategic, dynamic pricing may deter them from purchasing early, as the strategic consumers may incorporate their anticipation of possible future price discounts into current purchasing decisions [2, 10]. Thus, researchers have paid attention to the selection between different pricing strategies. Some have pointed out that preannounced pricing is a good way to mitigate consumers' strategic waiting behavior and can even outperform dynamic pricing under some circumstances [2, 6, 10, 11, 14, 15, 35, 41]. However, by failing to adjust prices in response to information updates on demand and products, the firm may incur more short-term revenue loss [6]. Aviv and Pazgal [2] compare dynamic pricing strategy and fixed-preannounced pricing strategy for a monopolist firm selling a finite inventory of product during two periods to strategic consumers and conclude that fixed-preannounced pricing is profitable when the initial inventory and valuation heterogeneity are high. Papanastasiou and Savva [34] highlight the relative effectiveness of preannounced pricing and responsive pricing when the firm and the consumers face quality uncertainty and social learning. Shum et al. [41] examine the impact of cost reduction under dynamic pricing, price commitment, and price matching strategies when cost reduction originates from production learning or technology advances. Correa et al. [10] propose a special case of preannounced pricing called contingent preannounced pricing policy, in which the firm commits to a full menu of prices corresponding to each inventory level, and find that it is more competitive than other pricing policies. Haruvy et al. [19] analyze the impact of price commitment on custom-made product. Cachon and Feldman [6], Liu and van Ryzin [28], and Su and Zhang [44] take into account decisions concerning inventory capacity and ordering quantity when exploring price commitment. Their research shows that a preceding commitment to a price path can alleviate consumers' strategic waiting behavior when the availability of product in the future is dependent on both the capacity of the firm and the purchasing behavior of other consumers. Several papers explore different pricing strategies, including mark-down pricing with and without reservations [15], subscription versus per-use pricing [8], retail-fixed-markup contract versus price-protection contract between supplier and retailer [29,30], and price matching [23]. All the aforementioned papers focus on the effect of different pricing strategies in the situation of one generation of product in single or multiple periods. Our study differs from the existing literature by exploring the situation where the firm sells sequential generations of products, with the trade-in program mitigating the impacts of strategic waiting behavior.

2.2. Research on Trade-in Program

Trade-in programs are becoming more popular in practice, and researchers are thus paying more attention. Van Ackere and Reyniers [47] construct a two-period model in which a monopolist firm sells the same durable products in two periods, thus dividing consumers into two categories, holders and non-holders of the product, in the second period. Under different assumptions of consumers' rationality, they study the impact of trade-in discounts. Ray et al. [38] study the case in which the monopolist sells the same product. They develop a single-period model in which the monopolist firm adopts price discrimination by offering trade-in rebates to replacement consumers and charge a higher price to first-time buyers, and study three pricing schemes. A few researchers have considered the case of a firm selling two generations of products where the upgraded version has a higher level of quality or innovation [17, 24]. Bala and Carr [3] investigate the role of product improvement and user upgrade costs on firms' pricing decisions in the computer software industry. Ferrer and Swaminathan [16], Heese et al. [20], and Shi et al. [40] examine the effect of remanufacturing the take-back product under monopoly and duopoly competition. In view of market heterogeneity and uncertainty, Yin and Tang [51], and Yin et al. [50] analyze a two-period dynamic game to determine the optimal pricing and trade-in decisions by strategic consumers, given the presence/absence of an up-front fee. However, existing literature considering trade-in program mainly focuses on single rollover strategy, which is not consistent with reality. As a matter of fact, after the introduction of a new generation product, many firms (e.g., Apple, Samsung, and Huawei) not only provide trade-in price for the new generation products to their patrons, but also continue to sell older generations of products to consumers. Therefore, given the more realistic situation with the trade-in program and dual rollover, the research question is: what would be the optimal pricing strategy for firms in the presence of strategic consumers? To the best of our knowledge, this paper is the first attempt to explore optimal pricing strategy under trade-in program in the presence of strategic consumers, sequential product innovation, and market heterogeneity.

2.3. Research on Strategic Consumers

The term “strategic consumers” has been widely used in economics, marketing, and operations to describe the rational consumers who make purchasing decisions considering not only current but also future purchasing options. In contrast with myopic consumers, strategic consumers will delay purchasing in anticipation of future price mark-downs or new product launches. Nair [33] and Li et al. [25] provide empirical evidence of strategic consumer behavior in video game and airline industries. Moreover, ignorance of strategic consumer behavior in operations management, including price and product rollover decisions, can bring about immense losses for firms [33, 36]. A growing amount of research on strategic consumer behavior and its influence on firms’ decisions regarding pricing, inventory, new product launching and timing has stemmed from seminal work by Su [42] and Su and Zhang [44]. Shen and Su [39] provide a prominent review of strategic consumer behavior in revenue management and suggest future research directions. Cachon and Swinney [8, 9] examine the impact of quick response capability and enhanced product design in the fast fashion industry when facing uncertain demand and strategic consumers. Swinney [45] and Yu et al. [52] examine the optimal pricing strategy by taking consumers’ strategic waiting behavior into consideration when product value is ex-ante uncertain and social learning about product quality is possible. Yin et al. [49] and Whang [48] examine the effect of different inventory display format strategies and demand learning, respectively, by using an upfront commitment pricing policy. Mersereau and Zhang [32] study firms’ pricing decisions when facing an unknown proportion of strategic consumers. Besbes and Lobel [5] and Lobel et al. [31] investigate new product development and introduction decisions under pre-commitment and no-commitment strategies. Although existing literature examines the impact of strategic consumer behavior on operational decisions from a great variety of perspectives, none has examined the impact of consumers’ strategic purchasing behavior on trade-in program under different pricing strategies, which is the core research question in this study.

3. Model Description

3.1. The Firm's Problem

In the two-period model, the monopolistic firm with continuous innovation sells product V_1 in period 1 and an improved version V_2 in period 2[6]. The firm follows dual rollover strategy, where V_2 is available for sale at price p_2 in period 2 and V_1 is available for sale at price p_1 in period 1 at a discounted price $p_d (0 \leq p_d \leq p_1)$ in period 2. Under the trade-in program, consumers who purchased V_1 in period 1 are allowed to buy V_2 at $p_2 - p_t$, where $p_t (0 \leq p_t \leq p_2)$ is the trade-in rebate. The firm chooses either *dynamic pricing* or *preannounced pricing* at the beginning of period 1. Under dynamic pricing strategy, the firm decides p_1 in period 1 and p_2, p_d, p_t in period 2. Under preannounced pricing strategy, the firm determines and reveals p_1, p_2, p_d, p_t in period 1 simultaneously.

We assume that V_1 and V_2 are of the same marginal production cost, which is a common assumption for high-tech products (e.g., Liang et al. [26], Ray et al. [38]). Without loss of generality, we set the marginal cost to be zero (e.g., Bala and Carr [3], Kornish [21]). The innovation level of V_1 and V_2 are 1 and $1 + \theta$, respectively, where $\theta\ (0 \leq \theta \leq 1)$ is the innovation incremental value of V_2 compared to V_1 [26]. Each returned old generation product has a salvage value $s\ (0 \leq s \leq \theta)$ to the firm. We assume $s \leq \theta$ to guarantee the reasonability of our results and to eliminate trivial results[7]. Actually, $s \leq \theta$ implies that the salvage value of the old generation product (i.e., s) is lower than the maximum absolute

[6] In the context of multiple generations of products, some researchers use the multi-period model to study the game between the firm and consumers. However, the two-period theoretical model is still widely adopted in studying product launching of multiple generations and/or repeated purchasing, such as Liang et al. [26], Zhou et al. [53], Yin and Tang [51], and Yin et al. [50]. Following the literature, we take two continuous generations of products as an example to investigate the intertemporal pricing decisions facing strategic consumers. If the multi-period model is adopted, the problem may become intractable. For example, in the multi-period model, we need to consider the price discount of new version in the third period and consumers' rational expectation on future generations of products before purchasing the current generation, which complicate the problem significantly. On the other hand, a two-period model is consistent with industry practice to some extent. For example, Apple keeps only the two latest generations of products available. After releasing the iPhone 7, only the iPhone 7 and iPhone 6s can be purchased on Apple's official website. The older versions are not available for sale any more. In this sense, each generation of iPhone is sold for approximately two years, corresponding to our two-period model in this research.

[7] "$s \leq \theta$" is reasonable from the perspective of business practice. In order to limit the research scope on the comparison of different pricing strategies, we do not consider the refurbishment and resale of each returned old generation. Therefore, the salvage value of V_1 to the firm depends mainly on the condition of the recycled core components, e.g., mainboard, screen, battery. Since only a part of the recycled devices are of good condition, the average salvage value is much lower than the unit production cost of a new product, which is also quite low compared with the value for sale perceived by consumers in the market.

incremental value of the new generation product (i.e., $\theta \cdot 1$). This assumption guarantees all prices and sales volumes in equilibrium to be both positive and reasonable. Otherwise, if $s > \theta$, selling V_2 through the trade-in program to V_1-holders will not bring any monetary value to the firm, thus, the firm has no incentive to sell V_2 through the trade-in program and will not offer the trade-in program. The firm is risk-neutral and its objective is to maximize the two-period total profit from two generations of products. To simplify the analysis, we assume the firm has ample capacity[8] to meet consumers' demand for both generations of products, which is reasonable for software and electronics industries [3, 50, 51]. In addition, similar to some recent literature [50-53], this assumption also helps us to focus on the impact of different pricing strategy.

3.2. The Consumer's Problem

Consumers are heterogeneous in valuation. In period 1 and 2, the consumer's valuation for using V_1 is v_1, which follows a [0, 1] uniform distribution [3, 53]. In period 2, the consumer's valuation for V_2 depends on not only its valuation for V_1 but also the innovation level of V_2, which is $(1+\theta)v_1$. All consumers arrive in period 1. Without loss of generality, we normalize the potential market size to 1.

Consumers are strategic in making purchasing decisions to maximize the two-period total surplus. Consumer's surplus in period 2 will be discounted by $\delta\ (0 \le \delta \le 1)$. In our model, δ captures not only the opportunity cost of postponed purchasing [21, 47], but also how strategic consumers are, i.e., $\delta = 0$ represents myopic consumers (e.g., Cachon and Swinney [8], Papanastasiou and Savva [35], Shum et al. [41], Swinney [45]). Note that even when $\delta = 0$ (i.e., all customers are myopic), there will still be some consumers who purchase the new generation product in the second period. For instance, the high-end

[8] Limited inventory and price uncertainty are both important factors when modeling strategic consumers. According to a review by Gönsch et al. [18] on strategic consumers, nearly 30% of research papers along this line assume infinite capacity, among which some quite recent papers, including Shum et al. [41], Papanastasiou and Savva [35], Yu et al. [52], Yin et al. [50], Zhou et al. [53], and Yin and Tang [51]. In this research, we focus on the impact of different pricing strategies by minimizing the influence of other factors, thus we assume the firm has ample capacity for both generations of products. This simplification is consistent with business practice in Apple, Samsung, and Huawei, etc., as inventory scarcity of old generation products after the introduction of new product is rarely seen. In addition, the assumption of ample capacity makes our optimization problems tractable. It is worth noting that in the extant literature considering limited inventory and strategic consumers, generally, the consumers' heterogeneity in valuation is presumed to follow a two-point distribution, i.e., either V_H or V_L. However, since we assume consumers' valuation follows a uniform distribution, the combination with limited inventory significantly increases the complexity of the problem and leads to intractability (e.g., Krishnan and Ramachandran [22]).

consumers may find replacing V_1 to V_2 more attractive due to the innovation improvement. Our results are applicable for any value of δ within [0, 1], including two extreme cases $\delta = 0$ and $\delta = 1$.

In this study, each consumer can purchase at most one unit of the same generation of product at any given time and no secondary market is available. We model a two-period dynamic game between the firm and a potential consumer with complete information, i.e., all of the parameters above are common knowledge, including the innovation incremental value of V_2 over V_1, salvage value of V_1, how strategic consumers are, market size and marginal production cost of both V_1 and V_2. The reason behind the assumption of complete information is threefold. First, from the perspective of business practice, after the rollover of several generations of products, consumers get more familiar with product innovation level. Meanwhile, websites like MacRumors.com[9] and Decide.com[10] offer helpful forecasting information on new products. For example, on September 12 2017, Apple officially launched the iPhone 8 and iPhone X. However, long before this launch, consumers are able to find out the new properties and functions of the iPhone 8 and iPhone X through "Buyer's Guide" on MacRumors.com[11]. Similar things happen for other electronic products such as MacBook, iPod and Apple Watch. Second, from the literature perspective, the assumption of complete information on the innovation level of new generation product in the first period is adopted by some recent papers in exploring operational strategy in the context of sequential innovation, such as Liang et al. [26], Krishnan and Ramachandran [22]. Third, we incorporate the complete information assumption to better focus on the impact of different pricing strategies. The notation used in this paper is summarized in Table 1, where superscripts '*D*' and '*P*' denote dynamic pricing and preannounced pricing, respectively.

Table 1 Summary of Notations

Notation	Meaning
$\boldsymbol{\theta}$	Innovation incremental value of V_2 over V_1

[9] See https://www.macrumors.com/for more information.
[10] See http://www.decide.com/for more information. While, it has been acquired by eBay.
[11] See https://buyersguide.macrumors.com/#iPhone for more information.

$\boldsymbol{s}$	Salvage value of V_1
$\boldsymbol{\delta}$	How strategic consumers are
$\boldsymbol{v_1}$	Consumer's valuation of using V_1
$\boldsymbol{p_1^D},\ \boldsymbol{p_1^P}$	Price of V_1 in period 1
$\boldsymbol{p_2^D},\ \boldsymbol{p_2^P}$	Price of V_2 in period 2
$\boldsymbol{p_t^D},\ \boldsymbol{p_t^P}$	Trade-in rebate in period 2
$\boldsymbol{p_d^D},\ \boldsymbol{p_d^P}$	Mark-down price of V_1 in period 2
$\boldsymbol{q_1^D},\ \boldsymbol{q_1^P}$	Demand of V_1 in period 1
$\boldsymbol{q_2^D},\ \boldsymbol{q_2^P}$	Demand of V_2 from new consumer in period 2
$\boldsymbol{q_t^D},\ \boldsymbol{q_t^P}$	Demand of V_2 from trade-in consumer in period 2
$\boldsymbol{q_d^D},\ \boldsymbol{q_d^P}$	Demand of V_1 in period 2

4. Equilibrium Analysis under Different Pricing Strategies

In this section, we analyze the game between the firm and strategic consumers under both dynamic pricing strategy and preannounced pricing strategy.

4.1. Dynamic Pricing Strategy

The sequence of events under dynamic pricing is summarized in Figure 1. Following Dhebar [12], consumers are capable of developing an expectation of the price in the second period when making their purchase decisions. Before period 1 starts, the firm announces its pricing strategy and then specifies p_1 in period 1 and $p_2,\ p_d,\ p_t$ in period 2. Because consumers with higher valuation will purchase earlier, we suppose that a consumer with a valuation higher than the threshold value τ_1^D purchases V_1 in period 1, and a consumer with a valuation lower than τ_1^D chooses to wait. Then, in period 2, consumers are divided into two categories, V_1-holders and non-V_1-holders. The V_1-holders determine whether to trade-in V_1 for V_2 (the corresponding second-period surplus is $(1+\theta)v_1 - p_2 + p_t$) or to keep using V_1 (the corresponding second-period surplus is v_1). The non-V_1-holders determine whether to purchase V_1 at a discounted price (the corresponding second-period surplus is $v_1 - p_d$) or to purchase V_2 at the regular price (the corresponding second-period surplus is $(1+\theta)v_1 - p_2$). Note that we do not consider the discount for the second-period surplus here, as the consumers are facing only the second-period decision now. Since the first-period choice has already been made, the first-period surplus can be regarded as a "sunk" surplus for consumers, which will not influence the second-period decision. Take the V_1-holders for example. In the second period, they are facing trade-in-or-keeping decision. Since all V_1-holders share a same form of first-period surplus, i.e., $v_1 - p_1$, their overall surpluses are either $v_1 - p_1 + \delta((1+\theta)v_1 - p_2 + p_t)$ for trade-in for V_2 or $v_1 - p_1 + \delta(v_1)$ for keeping V_1. Thereby, whether or not to consider the first-period surplus and the discount will not change the second-period decision of the V_1-holders. Things are similar for the non-V_1-holders.

Comparing the second-period consumer surplus, we can obtain their purchasing decision in period 2 straightforwardly. Specifically, the V_1-holders with a valuation higher than the threshold value τ_2^D choose to trade-in V_1 for V_2, and the others will keep using V_1. As for the non-V_1-holders, there are two cases with respect to their choice. In case I(II), they prefer V_2(V_1) much more, hence the non-V_1-holders with a valuation between τ_3^D and τ_1^D will purchase V_2(V_1) and the non-V_1-holders with a valuation between τ_4^D and τ_3^D will purchase V_1(V_2), while those with a valuation lower than τ_4^D will buy nothing. After formulating the consumer's second-period indifferent conditions, we obtain the thresholds in Table 2.

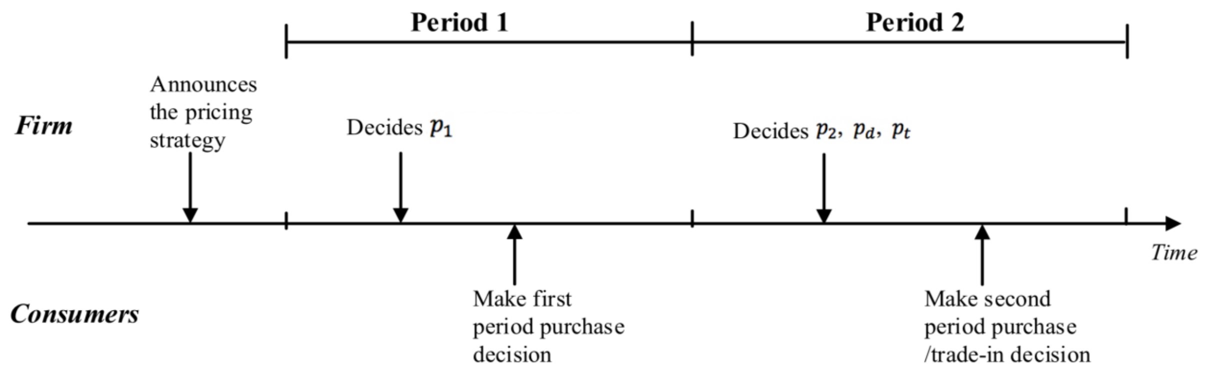


Figure 1 Sequence of Events under Dynamic Pricing

Table 2 Definition and Value of Thresholds under Dynamic Pricing

	Case I			Case II		
	Indifference point between	In which period	Value	Indifference point between	In which period	Value
$\boldsymbol{\tau_2^D}$	Trade-in purchasing V_2 and using V_1 for V_1-holders	Period 2	$\frac{p_2 - p_t}{\theta}$	Trade-in purchasing V_2 and using V_1 for V_1-holders	Period 2	$\frac{p_2 - p_t}{\theta}$
$\boldsymbol{\tau_3^D}$	Purchasing V_2 and V_1 for non-V_1-holders	Period 2	$\frac{p_2 - p_d}{\theta}$	Purchasing V_1 and V_2 for non-V_1-holders	Period 2	$\frac{p_2 - p_d}{\theta}$
$\boldsymbol{\tau_4^D}$	Purchasing V_1 and nothing for non-V_1-holders	Period 2	p_d	Purchasing V_2 and nothing for non-V_1-holders	Period 2	$\frac{p_2}{1+\theta}$

We follow backward induction to obtain the subgame perfect equilibrium. Denote Π_2 as the firm's second-period profit. The firm's second-period pricing problem is

$$max_{p_2,p_t,p_d}\,\Pi_2 = p_2 q_2 + (p_2 - p_t + s)q_t + p_d q_d \tag{1}$$

$$s.t.\;\; 0 \le q_2 \le 1 - q_1 \tag{2}$$

$$0 \le q_d \le 1 - q_1 \tag{3}$$

$$q_2 + q_d \le 1 - q_1 \tag{4}$$

$$0 \le q_t \le q_1 \tag{5}$$

$$0 \le p_t \le p_2 \tag{6}$$

$$0 \le p_d \le p_1 \tag{7}$$

Constraints (2)-(4) guarantee that only the non-V_1-holders buy V_2 at p_2 or buy V_1 at p_d in period 2. Constraint (5) guarantees only the V_1-holders can trade-in V_1 for V_2. Since the firm's objective function is jointly concave in p_2, p_t, and p_d, the optimal pricing decisions are guaranteed, which are the same under case I and II with $p_2^* = \frac{(1+\theta)\tau_1^D}{2}$, $p_t^* = \frac{(1+\theta)\tau_1^D - (\theta - s)}{2}$, and $p_d^* = \frac{\tau_1^D}{2}$.

In period 1, the strategic consumer determines whether to purchase or wait in anticipation of the firm's second-period best response function. In equilibrium, the consumer with valuation τ_1^D is indifferent between purchasing V$_1$ and waiting in period 1. That is to say, in case I, the total consumer surplus for both periods satisfies $0 + \delta((1+\theta)\tau_1^D - p_2^*) = (\tau_1^D - p_1) + \delta(\tau_1^D)$; in case II, the total consumer surplus for both periods satisfies $0 + \delta(\tau_1^D - p_d^*) = (\tau_1^D - p_1) + \delta(\tau_1^D)$. One may wonder whether the high-valuation customers have the incentive to wait in order to earn a higher surplus when the innovation incremental value θ is sufficiently high. In fact, no matter what value θ takes, a high-valuation customer will never wait instead of purchasing immediately in the first period. Consider a consumer with any valuation $v_1 \in [p_1, 1]$, the total surplus of waiting for V$_2$ is $0 + \delta((1+\theta)v_1 - p_2)$, where the first part 0 is the surplus in the first period due to no purchasing, and the second part $(1+\theta)v_1 - p_2$ is the surplus of purchasing V$_2$, which is discounted by δ. However, the customer can easily improve her total surplus by purchasing V$_1$ in the first period, and then trade-in for V$_2$ in the second period with the total surplus $v_1 - p_1 + \delta((1+\theta)v_1 - p_2 + p_t)$, since $v_1 - p_1 + \delta\big((1+\theta)v_1 - p_2 + p_t\big) > 0 + \delta((1+\theta)v_1 - p_2)$ always holds.

The firm chooses p_1 to maximize its total profit Π over two periods. The firm's first-period pricing problem is

$$max_{p_1}\ \Pi = p_1 q_1 + \Pi_2^* \tag{8}$$

$$s.t.\ \ 0 \le q_1 \le 1 \tag{9}$$

$$0 \le p_1 \le 1 \tag{10}$$

where Π_2^* is the firm's equilibrium profit in period 2. Constraints (9) and (10) ensure the first-period quantity and price are nonnegative and reasonable.

Proposition 1. Under dynamic pricing strategy, with dual rollover and trade-in program, there exists a unique subgame perfect equilibrium.

(1) In case I, $p_1^D = \frac{(\theta-s)(2+\delta-\delta\theta)}{4\theta}$, $p_2^D = \frac{(\theta-s)(1+\theta)}{4\theta}$, $p_t^D = \frac{(\theta-s)(1-\theta)}{4\theta}$, $p_d^D = \frac{\theta-s}{4\theta}$, $q_1^D = \frac{\theta+s}{2\theta}$, $q_2^D = \frac{\theta-s}{2\theta}$, $q_t^D = \frac{\theta+s}{2\theta}$, and $q_d^D = 0$.

(2) In case II, $p_1^D = \frac{(\theta-s)(2+\delta)}{4\theta}$, $p_2^D = \frac{(\theta-s)(1+\theta)}{4\theta}$, $p_t^D = \frac{(\theta-s)(1-\theta)}{4\theta}$, $p_d^D = \frac{\theta-s}{4\theta}$, $q_1^D = \frac{\theta+s}{2\theta}$, $q_2^D = 0$, $q_t^D = \frac{\theta+s}{2\theta}$, and $q_d^D = \frac{\theta-s}{4\theta}$.

(3) It is optimal for the firm to choose case I if and only if $s \le \frac{(1-2\delta)\theta}{1+2\delta}$; otherwise, case II is optimal.

Because $q_1^D = q_t^D$ in both cases, Proposition 1 implies that the V_1-holders will purchase V_2 through trade-in program in period 2 in either case. The non-V_1-holders purchase V_2 at p_2^D in case I while purchase V_1 at p_d^D in case II. The difference between two cases lies in the relative price position of V_1 and V_2 (i.e., which generation is charged at a higher price). Although $p_1^D \ge p_2^D$ holds for both cases, in case II, the difference between p_1^D and p_2^D decreases at a slower pace with the increase of θ, which makes the price discount in V_1 more attractive in period 2. The firm is better off in case I if and only if the salvage value of V_1 is small enough. The intuition behind this is that a high salvage value means that selling more V_1 generates more profit. The firm can thus obtain a higher profit in case II, because it sells more V_1 in total.

Table 3 Sensitivity Analysis of Equilibrium Demand, Profit, and Price under Dynamic Pricing

		q_1^D	q_2^D	q_t^D	q_d^D	Π^D	p_1^D	p_2^D	p_t^D	p_d^D
Case I	θ	↓	↑	↓	–	↑	↑ then ↓	↑	↑ then ↓	↑
	s	↑	↓	↑	–	↑ then ↓	↓	↓	↓	↓
	δ	–	–	–	–	↑	↑	–	–	–
Case II	θ	↓	–	↓	↑	↑	↑	↑	↑ then ↓	↑
	s	↑	–	↑	↓	↑ then ↓	↓	↓	↓	↓
	δ	–	–	–	–	↑	↑	–	–	–

Table 3 summarizes the results of a sensitivity analysis on equilibrium demand, profit, and price with respect to θ, s and δ under dynamic pricing strategy. From Table 3, we see that the impacts of parameters on demands q_1^D and q_t^D are the same in Cases I and II. Specifically, in both cases, demands q_1^D and q_t^D decrease in the innovation incremental value θ, increase in the salvage value s, and are independent of how strategic the consumers are. The intuition behind this is that all the V_1-holders choose to trade-in V_1 for V_2. Note that demand q_d^D in Case I and demand q_2^D in Case II do not change in any of these parameters. That is because in Case I(II), the price discount of V_1(the introduction of V_2) is less attractive to non-V_1-holders, leading to no purchase of V_1(V_2) from the non-V_1-holders. As for the total profit Π^D, in both cases, the more innovative the new product is, and/or the more strategic consumers are, the higher profit the firm obtains. However, the impact of the salvage value s on the total profit Π^D is bell-shaped. Thus, the excessive salvage value for the old product may hurt the firm's total profit, since the firm has a strong incentive to lower the price in order to encourage the repeat purchasing consumer base through trade-in, resulting in a low profit margin. It is also important to investigate the impacts of parameters on prices, because strategic consumers take the firm's second-period pricing decisions into consideration when making decisions (see Table 3). It is interesting to note that as the salvage value s increases, all the prices p_1^D, p_2^D, p_t^D, and p_d^D decrease in both cases. The intuition behind this is that the high salvage value makes the trade-in program more attractive for the firm to lower the price, especially when all the V_1-holders will trade-in V_1 for V_2 in equilibria.

4.2. Preannounced Pricing Strategy

The sequence of events under preannounced pricing is summarized in Figure 2. Before the beginning of period 1, the firm announces its pricing strategy, and then reveals all price decisions including p_1, p_2, p_d, p_t in period 1.The strategic consumer makes a purchasing decision in period 1 to maximize her total surplus in two periods considering future options, i.e., $u = max\{u_1, u_2, u_3, u_4, u_5\}$, where $u_i (i = 1, \dots, 5)$ represents consumer surplus corresponding to the five purchasing options for strategic consumer summarized in Table 4.

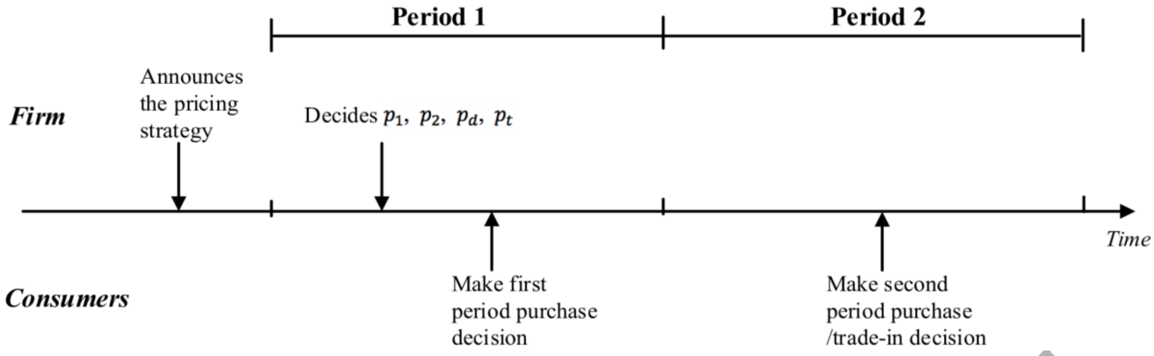


Figure 2 Sequence of Events under Preannounced Pricing

Table 4 Purchasing Options for Consumers and the Corresponding Total Surplus

Notation	Purchase Option	Consumer Surplus
NN	Buy nothing in either period	$u_1 = 0$
NV_1	Buy nothing in period 1 and buy V_1 in period 2	$u_2 = 0 + \delta(v_1 - p_d)$
NV_2	Buy nothing in period 1 and buy V_2 in period 2	$u_3 = 0 + \delta((1+\theta)v_1 - p_2)$
V_1N	Buy V_1 in period 1 and nothing in period 2	$u_4 = (v_1 - p_1) + \delta(v_1)$
V_1V_2	Buy V_1 in period 1 and trade-in for V_2 in period 2	$u_5 = (v_1 - p_1) + \delta((1+\theta)v_1 - p_2 + p_t)$

Lemma 1 summarizes the consumer's strategic optimal purchasing decisions by segmentation.

Lemma 1. Under preannounced pricing strategy, with dual rollover and trade-in program, the consumer's optimal purchasing strategies with respect to different values of v_1 are as follows.

(1) When $v_1 \in [0, \tau_1^P]$, it is optimal for the consumer to follow the NN strategy.

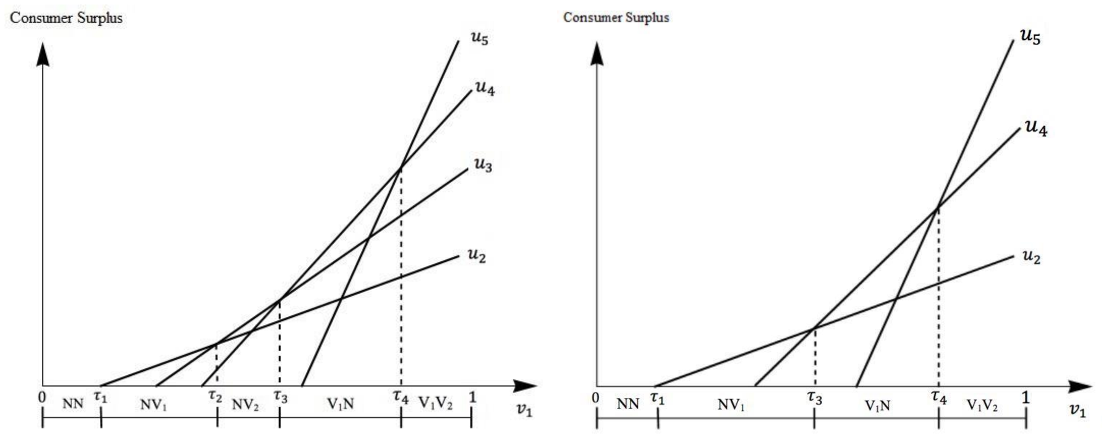
(2) When $v_1 \in [\tau_1^P, \tau_2^P]$, it is optimal for the consumer to follow the NV_1 strategy.

(3) When $v_1 \in [\tau_2^P, \tau_3^P]$, it is optimal for the consumer to follow the NV_2 strategy.

(4) When $v_1 \in [\tau_3^P, \tau_4^P]$, it is optimal for the consumer to follow the V_1N strategy.

(5) When $v_1 \in [\tau_4^P, 1]$, it is optimal for the consumer to follow the V_1V_2 strategy.

The five consumer segments constitute a mutually exclusive and complete interval of v_1. According to Figure 3, the above four thresholds τ_1^P, τ_2^P, τ_3^P, and τ_4^P depend on the relative price position of two generations of products in price skimming and penetration pricing. In price skimming, the firm charges a higher price for V_1, which is oriented to high-end consumers, than V_2, which is sold to relatively low-end consumers. In penetration pricing, the firm sets V_1 aiming to appeal to a wider consumer base with a more attractive price than V_2 (Besanko and Winston [4], Krishnan and Ramachandran [22]). Specifically, if the firm chooses price skimming, all the five consumer segments exist, if the firm chooses penetrating pricing, only four of the five segments exist, because NV_2 is weakly dominated by V_1N. We solve the above four thresholds using the consumer's indifferent conditions and summarize the results in Table 5.



(a) Price Skimming ($p_1 > \delta p_2$) (b) Penetration Pricing ($p_1 \leq \delta p_2$)

Figure 3 Consumer Segmentation under Preannounced Pricing

Table 5 Definition and Value of Thresholds under Preannounced Pricing

	Price Skimming			**Penetration Pricing**		
	Indifference point between	In which period	Value	Indifference point between	In which period	Value
$\boldsymbol{\tau_1^P}$	Purchasing V_1 and nothing for non-V_1-holders	Period 2	p_d	Purchasing V_1 and nothing for non-V_1-holders	Period 2	p_d
$\boldsymbol{\tau_2^P}$	Purchasing V_1 and V_2 for non-V_1-holders	Period 2	$\frac{p_2 - p_d}{\theta}$	N.A.	N.A.	N.A.
$\boldsymbol{\tau_3^P}$	Purchasing V_1 and waiting for the introduction of V_2	Period 1	$\frac{p_1 - \delta p_2}{1 - \delta\theta}$	Purchasing V_1 and waiting for the discount of V_1	Period 1	$p_1 - \delta p_d$
$\boldsymbol{\tau_4^P}$	Trade-in purchasing V_2 and	Period 2	$\frac{p_2 - p_t}{\theta}$	Trade-in purchasing V_2 and	Period 2	$\frac{p_2 - p_t}{\theta}$

	using V_1 for V_1-holders	using V_1 for V_1-holders

Under preannounced pricing strategy, the firm's problem is

$$max_{p_1,p_2,p_t,p_d}\,\Pi = p_1q_1 + p_2q_2 + (p_2 - p_t + s)q_t + p_dq_d \quad (11)$$

$$s.t.\ 0 \le q_1 \le 1 \quad (12)$$

$$0 \le p_1 \le 1 \quad (13)$$

$$0 \le q_2 \le 1 - q_1 \quad (14)$$

$$0 \le q_d \le 1 - q_1 \quad (15)$$

$$q_2 + q_d \le 1 - q_1 \quad (16)$$

$$0 \le q_t \le q_1 \quad (17)$$

$$0 \le p_t \le p_2 \quad (18)$$

$$0 \le p_d \le p_1 \quad (19)$$

$$p_1 > \delta p_2 \text{ or } p_1 \le \delta p_2 \quad (20)$$

Constraints (14)-(16) guarantee that only those who do not own V_1 can buy V_2 at p_2 or buy V_1 at p_d in period 2. Constraint (17) guarantees that only those who purchased V_1 in period 1 can trade-in V_1 for V_2. Constraint (20) is the inequality defining price skimming or penetration pricing. Specifically, $p_1 > \delta p_2$ represents price skimming, while $p_1 \le \delta p_2$ represents penetration pricing.

Proposition 2. Under preannounced pricing strategy, with dual rollover and trade-in program, there exists a unique subgame perfect equilibrium.

(1) In price skimming, $p_1^P = \frac{2(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_2^P = \frac{(1+\theta)(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_t^P = \frac{2-(1+\delta)\theta+(1-\delta)\theta^2+s(3-\delta-\theta-\delta\theta)}{2(3-\delta-\theta-\delta\theta)}$, $p_d^P = \frac{1-\delta\theta}{3-\delta-\theta-\delta\theta}$, $q_1^P = \frac{1-\theta}{3-\delta-\theta-\delta\theta}$, $q_2^P = \frac{1-\delta}{3-\delta-\theta-\delta\theta}$, $q_t^P = \frac{1-\theta}{3-\delta-\theta-\delta\theta}$, and $q_d^P = 0$.

(2) In penetration pricing, $p_1^P = \frac{2}{3-\delta}$, $p_2^P = \frac{2}{(3-\delta)\delta}$, $p_t^P = \frac{2}{(3-\delta)\delta} - \frac{1}{2}(\theta - s)$, $p_d^P = \frac{1}{3-\delta}$, $q_1^P = \frac{1}{3-\delta}$, $q_2^P = 0$, $q_t^P = \frac{1}{3-\delta}$, and $q_d^P = \frac{1-\delta}{3-\delta}$.

(3) Price skimming is optimal for the firm if and only if $s \le 1 - \delta - \theta$; otherwise, penetration pricing is optimal.

Since $q_1^P = q_t^P$ in both case I and II, Proposition 2 implies that the V_1-holders will trade-in to purchase V_1 for V_2 in period 2 in either case. The main difference between price skimming and penetration pricing lies in the second-period choice of non-V_1-holders. In price skimming, the firm charges a higher price for V_1 compared with V_2; thus, non-V_1-holders would like to purchase V_2 instead of V_1. In contrast, in penetration pricing, non-V_1-holders would like to purchase V_1 at the discounted price p_d^P. The firm is better off following price skimming if and only if the salvage value of V_1 is sufficiently small.

No matter which pricing strategy is adopted, the firm can segment consumers in the market by manipulating the price of two generations of products. Under either pricing strategy, the firm has an incentive to sell the old generation products instead of the new to the non-V_1-holders in the second period if the salvage value is sufficiently high; otherwise, the firm prefers selling the new generation products. It is noteworthy that the intrinsic reasons are opposite for the two pricing strategies. Under dynamic pricing, $p_1^D \ge p_2^D$ always holds, while under preannounced pricing, $p_1^P \ge p_2^P$ holds when the firm chooses to sell V_2 in the second period, i.e., the price skimming. In addition, under dynamic pricing, if the introductory price of the old generation product is not that high, no consumers would purchase the old generation products in the second period because the price discount seems less attractive. However, under preannounced pricing, because all prices are determined simultaneously prior to the selling season, consumers make their purchase-or-wait decision in the first period. As a result, they wait for the price mark-down of the old generation product if and only if the introductory price of the new generation product is high.

Table 6 Sensitivity Analysis of Equilibrium Demand, Profit, and Price under Preannounced Pricing

		q_1^P	q_2^P	q_t^P	q_d^P	Π^P	p_1^P	p_2^P	p_t^P	p_d^P
Price	θ	↓	↑	↓	–	↑ then ↓	↑	↑	↓ then ↑	↑

Skimming	s	–	–	–	–	↑	–	–	↑	–
	δ	↑	↓	↑	–	↑	↑	↑	↑	↑
Penetration Pricing	θ	–	–	–	–	↑	–	–	↓	–
	s	–	–	–	–	↑	–	–	↑	–
	δ	↑	–	↑	↓	↑	↑	↓	↓	↑

Table 6 summarizes the results of a sensitivity analysis of equilibrium demand, profit, and price with respect to θ, s and δ under preannounced pricing strategy. It's noteworthy that the impacts of parameters are quite different in two cases, i.e., price skimming and penetration pricing. Recall that in the above analysis for dynamic pricing strategy, we found that the impacts of parameters are similar in Cases I and II, which illustrates a significant distinction between two pricing strategies. Take the innovation incremental value as an example. Note that the demands do not change with θ in the case of penetration pricing. That is because in penetration pricing, the new product V_2 is out of the consideration for the non-V_1-holders, while the V_1-holders always choose to trade-in V_1 for V_2 regardless of the innovation incremental value. Besides, given the coexistence of the trade-in program and price pre-commitment, the relative price of the two generations of products remain unchanged (i.e., which generation is charged at a higher price), hence the salvage value s has no impact on the equilibrium demands. As for the prices, it is interesting to note that in penetration pricing θ has no impact on p_1^P, p_2^P, or p_d^P, but has a positive impact on p_t^P. In addition, the firm only needs to guarantee all the V_1-holders are willing to trade-in V_1 for V_2 in the second period as the change of s, which is why p_t^P increases in s. Besides, concerning the consumers' strategic behavior, as δ increases, p_2^P and p_t^P increase in price skimming while decrease in penetration pricing.

5. Comparison of Different Pricing Strategies

In this section, we compare the difference between dynamic pricing and preannounced pricing.

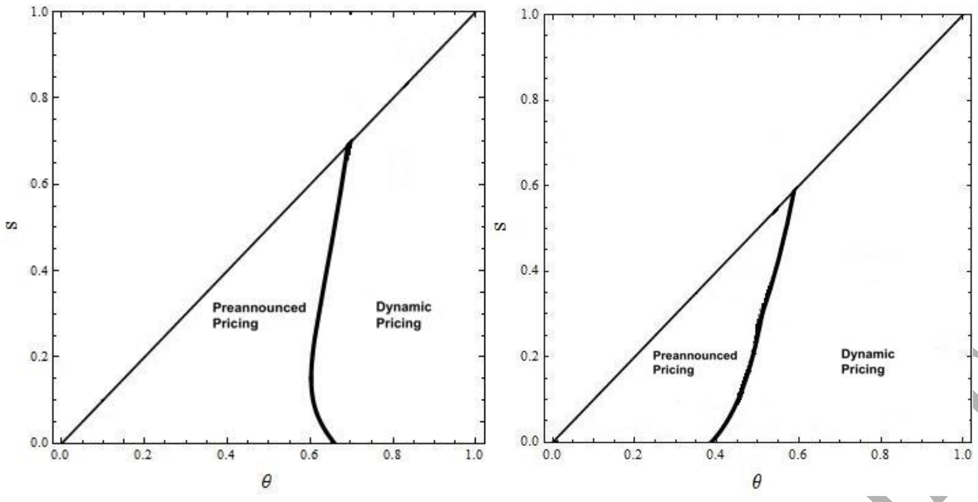
Table 7 Comparison of Dynamic Pricing and Preannounced Pricing

	V_1 Introduction	V_2 Introduction	V_2 Trade-in	V_1 Discount
Price	$p_1^D < p_1^P$	$p_2^D < p_2^P$	$p_t^D < p_t^P$	$p_d^D < p_d^P$
Quantity	$q_1^D > q_1^P$	$q_2^D < q_2^P$	$q_t^D > q_t^P$	$q_d^D < q_d^P$
		if $\frac{\theta(1-3\delta+\theta+\delta\theta)}{-3+\delta+\theta+\delta\theta} < s < 1-\delta-\theta$		if $s > max\{\frac{\theta(3\delta-1)}{3-\delta}, 1-\delta-\theta\}$

Table 7 summarizes the comparison results on price and quantity between dynamic pricing and preannounced pricing. Compared with dynamic pricing, preannounced pricing strategy always leads to higher prices (and higher trade-in rebate) for both generations of products in both periods. With respect to the volumes of sale, things are much more complicated. Surprisingly, compared with dynamic pricing, though the price is even higher (i.e., $p_2^D < p_2^P$), the second period demand generated from the non-V_1-holders under preannounced pricing is sometimes higher than that under dynamic pricing (i.e., $q_2^D < q_2^P$). Moreover, the demand for V_1 in period 2 from the non-V_1-holders is also higher under preannounced pricing (i.e., $q_d^D < q_d^P$), even though the discounted price is higher (i.e., $p_d^D < p_d^P$). The dynamic pricing strategy, in contrast, brings about higher demands for V_1 in period 1 (i.e., $q_1^D > q_1^P$) and more demands for trade-in to buy V_2 (i.e., $q_t^D > q_t^P$) in period 2, because there is more flexibility to attract consumers to the main product in each period.

We compare the firm's expected profits to study the optimal pricing strategy for the firm when selling sequential generations of products with the trade-in program. Proposition 3 and Figure 4 elaborate the optimal pricing strategy, where the θ-axis is the innovation incremental value of V_2 over V_1 and the s-axis is the salvage value of V_1.

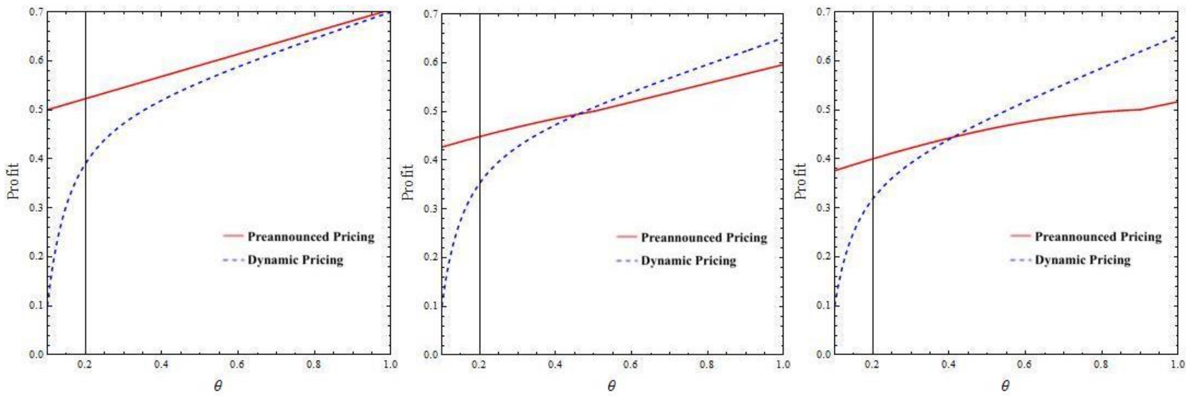
Proposition 3. In the case of dual rollover with trade-in program, there exists a threshold $T_1(\theta) \in (0,1)$ such that if $s < T_1(\theta)$, the firm will be better off following dynamic pricing strategy; otherwise, the firm is better off following preannounced pricing strategy.



(a) More Strategic Consumer ($\delta = 0.8$) (b) Less Strategic Consumer ($\delta = 0.4$)

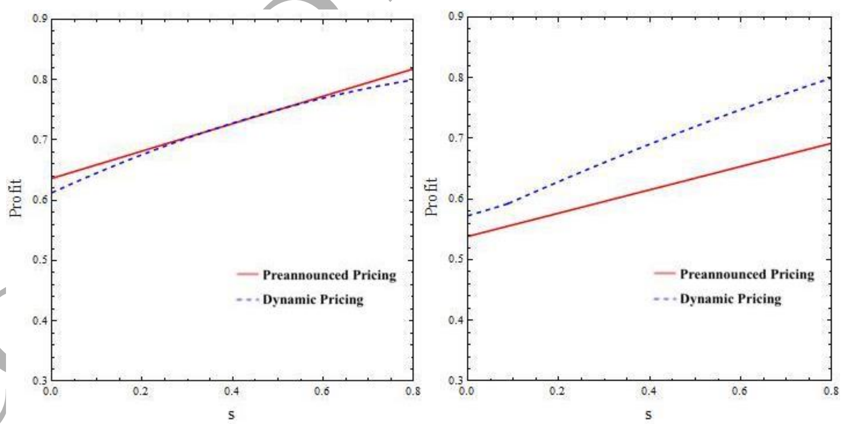
Figure 4 Optimal Pricing Strategy under Dual Rollover with Trade-in Program

Figure 4 demonstrates the optimal pricing strategy under dual rollover with the trade-in program for different levels of strategic consumers Interestingly, neither preannounced pricing nor dynamic pricing dominates the other. In particular, when both the innovation incremental value and salvage value are sufficiently high, it is more profitable for the firm to choose dynamic pricing; otherwise, it is better for the firm to choose preannounced pricing. It is worth noting that the optimal region of preannounced pricing becomes larger as consumers become more farsighted. Intuitively, with a high-level innovation, the new generation products entice not only low-end consumers to buy V_2 in period 2, but also high-end consumers to buy V_1 in period 1, because they can purchase V_2 with a trade-in rebate. Moreover, the flexibility provided by dynamic pricing can offer significant advantages to the firm, enabling it to adjust second-period prices to obtain higher demand. That is why dynamic pricing can bring more profit in such situations. In contrast, with a low-level innovation, the new generation products become less attractive to consumers. Under dynamic pricing, consumers may strategically wait for V_1 to be discounted instead of purchasing V_2, which decreases not only the consumer base in period 1, i.e., the V_1-holders, but also the demand for V_2 in period 2 from the non-V_1-holders. Therefore, the firm cannot attract consumers to compensate for the low profit margin in dynamic pricing because of the competition within the two generations of products. However, because the firm can commit to not offering a deep price cut for V_1 in period 2 under preannounced pricing, V_2 becomes more attractive in the second period. Furthermore, the firm can also extract more surplus through the high margin from a larger consumer base because of consumers' delay in purchasing. Consequently, preannounced pricing is optimal for the high profit margin in both periods and the relative attractiveness of product in the second period as well.

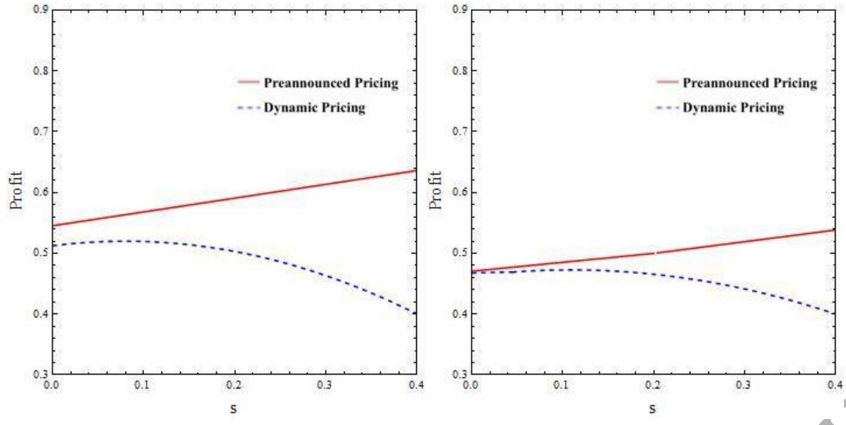


(a) More Strategic Consumer ($\delta = 0.8$) (b) Less Strategic Consumer ($\delta = 0.4$) (c) Myopic Consumer ($\delta = 0$)

Figure 5 Impacts of Product Innovation Incremental Value on Profit under Dynamic Pricing and Preannounced Pricing



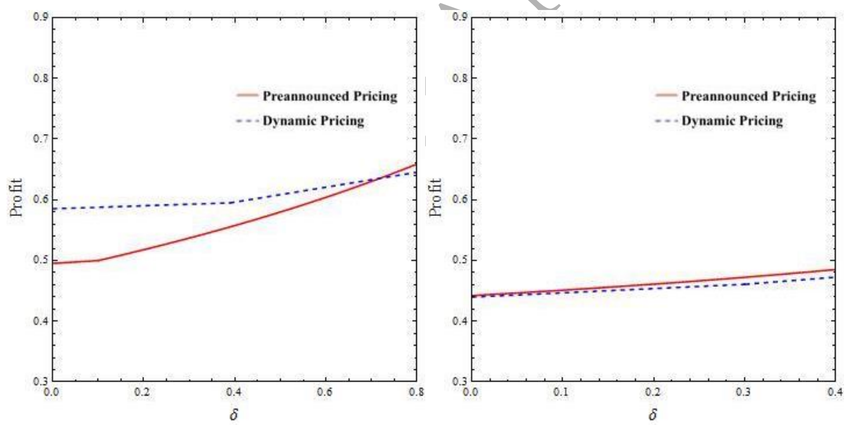
(a) High Innovation + More Strategic Consumer ($\theta = 0.8;\ \delta = 0.8$) (b) High Innovation + Less Strategic Consumer ($\theta = 0.8;\ \delta = 0.4$)



(c) Low Innovation + More Strategic Consumer (d) Low Innovation + Less Strategic Consumer

($\theta = 0.4;\ \delta = 0.8$) ($\theta = 0.4;\ \delta = 0.4$)

Figure 6 Impacts of Product Salvage Value on Profit under Dynamic Pricing and Preannounced Pricing



(a) High Innovation ($\theta = 0.8$) (b) Low Innovation ($\theta = 0.4$)

Figure 7 Impacts of Strategic Consumers on Profit under Dynamic Pricing and Preannounced Pricing

It is also important to discuss how the firm's profit changes as the three important parameters (i.e., the innovation incremental value of V_2 over V_1, the salvage value of V_1, and how strategic consumers are) change (Figures 5-7). We use $\delta = 0.8,\ 0.4,\ 0$ to represent that consumers are high strategic, less strategic and myopic, respectively. $\theta = 0.8$ (0.4) indicates that the new version has a high (low) level of innovation. To make the numerical analysis reasonable, we set $s = 0.2$, which satisfies our assumption $s \leq \theta$. As shown in Figure 5(a), when consumers are strategic enough, preannounced pricing dominates dynamic pricing regardless of the other two parameters. However, from Figures 5(b)-(c), we can see that as consumers become less strategic and the firm is better off under dynamic pricing when the innovation incremental value of the new generation product is high enough. In addition, compared with preannounced pricing, the firm's profit under dynamic pricing is more sensitive to the change of new product's innovation incremental value. According to Figure 6, there is an interaction between the innovation incremental value and the salvage value, such that the relationship between the firm's total profit and the salvage value completely reverses with the change in the level of innovation increment under dynamic pricing. Under preannounced pricing, however, the firm's profit continuously increases as the salvage value increases, regardless of the innovation incremental value. Moreover, the optimal pricing strategy changes significantly with the level of innovation for the new generation product, such that either pricing strategy can be optimal under high-level innovation, while preannounced pricing is much better than dynamic pricing under low-level innovation. Figure 7 reveals that the firm can benefit more from consumers' increasingly strategic behavior under high-level innovation than under low-level innovation.

6. Extensions

In this section, we examine how the results change if some of the assumptions in our base model are relaxed. We follow the notation used in the base model and do not differentiate the symbols across different extensions. First, we extend our analysis to the case of single rollover, to test if the optimal pricing strategy retains invariant. We, then, study the case when the firm discounts the second-period profit in making decisions.

6.1. Single Rollover with Trade-in Program

In launching new products, some firms follow a dual rollover strategy, i.e., keeping the old products on the shelf at a mark-down price after launching the new ones, like Apple and Samsung. Others follow a single rollover strategy, i.e., phasing out the old products as soon as the new ones are introduced, like Zara and H&M [27, 53]. Our above analyses provide insights into the profitability of preannounced pricing under dual rollover strategy with trade-in program in some conditions. Nevertheless, if a firm follows single rollover strategy, would this finding still hold? We thus derive the equilibria under both pricing strategies under single rollover with trade-in program. Since the analysis are almost the same as in Section 4, we omit them for the sake of brevity (refer to the detailed proof in Appendix I).

Proposition 4. Under dynamic pricing strategy, with single rollover and trade-in program, there exists a unique subgame perfect equilibrium with $p_1^D = \frac{(\theta-s)(2+\delta-\delta\theta)}{4\theta}$, $p_2^D = \frac{(\theta-s)(1+\theta)}{4\theta}$, $p_t^D = \frac{(\theta-s)(1-\theta)}{4\theta}$, $q_1^D = \frac{\theta+s}{2\theta}$, $q_2^D = \frac{\theta-s}{2\theta}$, and $q_t^D = \frac{\theta+s}{2\theta}$.

Proposition 5. Under preannounced pricing strategy, with single rollover and trade-in program, there exists a unique subgame perfect equilibrium.

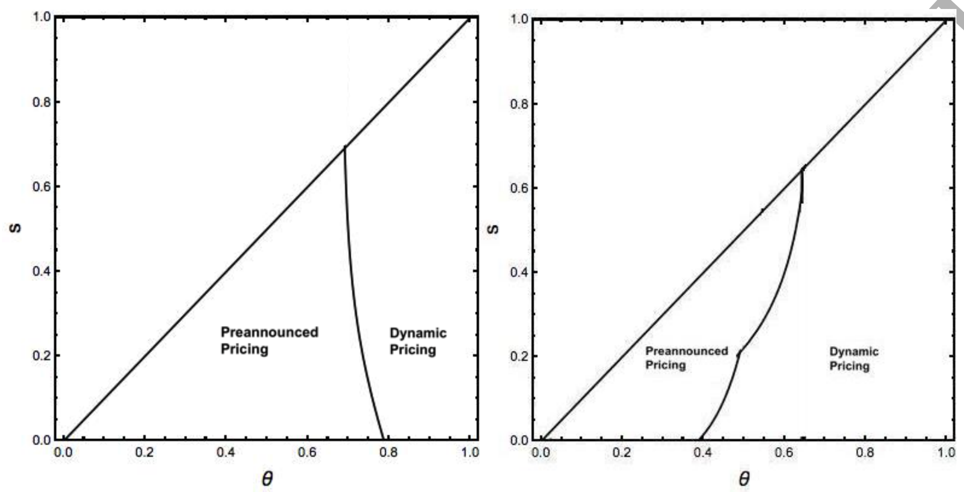
(1) In price skimming, $p_1^P = \frac{2(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_2^P = \frac{(1+\theta)(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_t^P = \frac{2-(1+\delta)\theta+(1-\delta)\theta^2+s(3-\delta-\theta-\delta\theta)}{2(3-\delta-\theta-\delta\theta)}$, $q_1^P = \frac{1-\theta}{3-\delta-\theta-\delta\theta}$, $q_2^P = \frac{1-\delta}{3-\delta-\theta-\delta\theta}$, and $q_t^P = \frac{1-\theta}{3-\delta-\theta-\delta\theta}$.

(2) In penetration pricing, $p_1^P = \frac{1+\delta}{2}$, $p_2^P = \frac{1+\delta}{2\delta}$, $p_t^P = \frac{1+\delta}{2\delta} - \frac{1}{2}(\theta - s)$, $q_1^P = \frac{1}{2}$, $q_2^P = 0$, and $q_t^P = \frac{1}{2}$.

(3) Price skimming is optimal for the firm if and only if $s \leq 1 - \delta - \theta$; otherwise, penetration pricing is optimal.

Given propositions 4 and 5, we see that the equilibria under single rollover with trade-in program is quite different compared with those under dual rollover, except for the case of price skimming, in which dual rollover degenerates into single rollover. Additionally, if the firm chooses dynamic pricing strategy, there is only one case under single rollover, whereas there are two cases under dual rollover. This difference is due to the fact that the relationship $p_1^D > p_2^D$ holds for both cases in the equilibrium of dynamic pricing. While, note that $p_1^P > p_2^P$ only holds in part of the cases in the equilibrium of preannounced pricing (i.e., price skimming). Next, we compare the firm's expected profits to obtain the optimal pricing strategy under single rollover with trade-in program. Proposition 6 describes the optimal pricing strategy, which is also illustrated in Figure 8.

Proposition 6. In the case of single rollover with trade-in program, there exists a threshold $T_2(\theta) \in (0,1)$ such that if $s < T_2(\theta)$, the firm is better off following dynamic pricing strategy; otherwise, the firm is better off following preannounced pricing strategy.



(a) More Strategic Consumer ($\delta = 0.8$) (b) Less Strategic Consumer ($\delta = 0.3$)

Figure 8 Optimal Pricing Strategy under Single Rollover with Trade-in Program

Comparing Figure 8 with Figure 4, it is easy to find that the optimal pricing strategy stays almost invariant, which is also a threshold policy for the firm, depending on the relative innovation increment of the new generation product and the salvage value of the old generation product. Dynamic pricing is more profitable for the firm when the innovation incremental value is sufficiently high. However, there is little difference between the boundary values. The optimal zone for preannounced pricing under single rollover is slightly larger than that under dual rollover.

6.2. Firm's Discount Factor

We now investigate the case when the firm discounts the second-period profit when making decisions. Denote β $(0 \leq \beta \leq 1)$ as the firm's discount factor, which can also be regarded as how farsighted the firm is [41]. Note that in this case, there are many exogenous parameters that it is hard to obtain the rule of closed-form comparison between dynamic pricing and preannounced pricing. Therefore, we investigate the impact of firm's discounted factor via numerical study (refer to Appendix II for the Equilibrium Analysis for this section). For the basic settings, we consider the situations with $\beta \in [0, 0.2, 0.4, 0.6, 0.8, 1.0]$, $\delta \in [0, 0.2, 0.4, 0.6, 0.8, 1.0]$, $\theta \in [0.4, 0.7, 1.0]$, and $s \in [0.1, 0.2, 0.3]$. Hence, there are 324 scenarios in total. Note that $\beta = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ represent the firm's shift from being myopic to less farsighted and then to more farsighted. Table 8 demonstrates the percentage of scenarios under which either pricing strategy is optimal.

Table 8 Percentage of Scenarios under which either Pricing Strategy is Optimal

	Dynamic Pricing (%)	Preannounced Pricing (%)	Total (%)
$\boldsymbol{\beta = 0}$	0.00	100.00	100.00
$\boldsymbol{\beta = 0.2}$	16.67	83.33	100.00
$\boldsymbol{\beta = 0.4}$	27.78	72.22	100.00
$\boldsymbol{\beta = 0.6}$	44.44	55.56	100.00
$\boldsymbol{\beta = 0.8}$	62.96	37.04	100.00
$\boldsymbol{\beta = 1.0}$	85.19	14.81	100.00
Total	39.51	60.49	100.00

Table 8 shows the firm's rationality also influence the choice of the optimal pricing strategy significantly. Generally speaking, as β increases, i.e., as the firm becomes more farsighted, the percentage of scenarios where dynamic pricing is optimal increases, while that of preannounced pricing decreases. To be specific, when $\beta = 0$, the firm will absolutely choose preannounced pricing, although in reality it is almost impossible for a firm to ignore future revenue. We include this special case in our analysis for not only the completeness of description, but also to understand why the percentage of scenarios where dynamic pricing is optimal increases when the firm becomes more farsighted. In this case, the preannounced pricing is more profitable since the firm sets all prices in advance in order to optimize the expected profits of two periods. By contrast, under dynamic pricing, since the firm sets the prices for each period sequentially in the corresponding period, the price optimization in the second period will inevitably influence the firm's revenue in the first period, leading to an inferior performance of dynamic pricing. When $\beta = 0.2, 0.4, 0.6$, dynamic pricing becomes optimal in some scenarios, even though the proportion is less than that of preannounced pricing. The intuition behind that is as the firm becomes more farsighted, the second-period profit weighs more to the firm. Thus, the firm needs to balance the profitability of the flexibility of dynamic pricing and the ex-ante commitment of preannounced pricing, especially in response to strategic consumers. While, when $\beta = 0.8, 1.0$, compared with preannounced pricing, dynamic pricing is more often to be optimal for the firm. The intuition is that as the firm weighs more on future profit, the flexibility provided by dynamic pricing looks more attractive and brings in more potential future profits to the firm.

7. Conclusion

In this paper, we study the impacts of the trade-in program and strategic consumer behavior on firms' pricing strategy and profit when the firm sells two sequential generations of products. To answer this question, we develop a stylized model to explore the strategic interplay between a monopolist firm and a population of strategic consumers. In the two-period model, a monopolist firm determines its pricing strategy prior to period 1, and then decides the prices and trade-in rebate in each period. Following the backward induction, we derive the optimal pricing strategy and the firm's corresponding profit. The trade-off lies in the selection of pricing flexibility or an ex-ante commitment toward consumers. In addition to the optimal pricing strategy obtained under dual rollover, we also extend our model by incorporating single rollover and firm's discount factor to test whether our main findings are robust.

Our results are summarized as follows. First of all, no matter which pricing strategy is adopted, the firm is able to realize different consumer segmentations by setting the relative price position of two generations of products (i.e., which generation is charged at a higher price). In the second period, the non-V_1-holders can be forced to purchase either the old or the new product by the firm's pricing decision. Specifically, under either pricing strategy, the firm has an incentive to sell the old generation products instead of the new to the non-V_1-holders in the second period if the salvage value is sufficiently high;

otherwise, the firm prefers to sell the new product. It is noteworthy that the intrinsic reasons are opposite for the two pricing strategies. Specifically, under dynamic pricing, if the introductory price of the old generation product is not too high, no consumers would purchase the old generation products in the second period because the price discount seems to be less attractive. In other words, the non-V_1-holders may purchase the old instead of the new generation product if the price discount is large enough. However, under preannounced pricing, since all prices are determined simultaneously before the selling season, consumers make a purchase-or-wait decision in the first period. As a result, consumers will wait for the price mark-down of the old generation product if and only if the introductory price of the new generation product is relatively high.

Second, neither pricing strategy consistently dominates the other when the firm sells two sequential generations of products to strategic consumers under trade-in program. When consumers are sufficiently strategic, if both the innovation incremental value of the new generation product and the salvage value of the old generation product are low enough, the firm is better off following the preannounced pricing strategy, because the commitment power can significantly reduce the cannibalization effect, hence inducing more sales for the new generation products. Otherwise, dynamic pricing is more profitable, because the firm enjoys the flexibility of adjusting price dynamically to better discriminate consumers when they are less strategic and the innovation improvement is high enough. Preannounced pricing is preferable in more situations as consumers become more strategic.

Third, when single rollover is adopted instead of dual rollover, even though the equilibria under single rollover are apparently different, the optimal pricing strategy stays almost invariant, which is also a threshold policy depending on the relative innovation increment of the new and salvage value of the old. However, preannounced pricing under single rollover is slightly more attractive to the firm than under dual rollover. In addition, as the firm becomes more farsighted, the dominant position of preannounced pricing over dynamic pricing decreases.

Certainly, there are several future research directions that might be explored based on the findings in this paper. First, aside from price commitment, firms can also make an ex-ante quantity commitment or ex-post availability guarantee to alleviate strategic consumer behavior. It would be valuable to examine the impact of other forms of commitment on firms' profits when selling sequential generations of products. Second, we assume that the incremental innovation value of new versions of products over old versions is exogenous and determinant. However, the firm can decide the amount of investment in research and development (R&D) in order to determine the innovation level. This direction is of particular concern in studying the interplay between investment decisions and rollover decisions. Third, our model is based on a two-period setting, which is common in the existing literature on strategic consumers and new product launching. It would be interesting to expand our model to a multiple-period setting, which is more realistic. Finally, we assume the firm has ample capacity for both generations of products to focus on the impact of different pricing strategies. However, it's interesting to incorporate the possible stockout in the second period, since the limited inventory and the price uncertainty are both important factors when modeling strategic consumers. Note that limited inventory may have different impact on the equilibrium on two pricing strategies.

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References

[1] Apple Website. 2016. Recycling an Apple product should be as easy as using one, accessed July 28, 2016, available at http://www.apple.com/recycling/.

[2] Aviv, Y., Pazgal, A., 2008. Optimal pricing of seasonal products in the presence of forward-looking consumers. *Manufacturing & Service Operations Management*, 10(3), 339-359.

[3] Bala, R., Carr, S., 2009. Pricing software upgrades: The role of product improvement and user costs. *Production and Operations Management*, 18(5), 560-580.

[4] Besanko, D., Winston, W.L., 1990. Optimal price skimming by a monopolist facing rational consumers. *Management Science*, 36(5), 555-567.

[5] Besbes, O., Lobel, I., 2015. Intertemporal price discrimination: Structure and computation of optimal policies. *Management Science*, 61(1), 92-110.

[6] Cachon, G.P., Feldman, P., 2015. Price commitments with strategic consumers: Why it can be optimal to discount more frequently than optimal. *Manufacturing & Service Operations Management*, 17(3), 399-410.

[7] Cachon, G.P., Swinney, R., 2009. Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science*, 55(3), 497-511.

[8] Cachon, G.P., Swinney, R., 2011. The value of fast fashion: Quick response, enhanced design, and strategic consumer behavior. *Management Science*, 57(4), 778-795.

[9] Chen, M., Chen, Z., 2015. Recent developments in dynamic pricing research: Multiple products, competition, and limited demand information. *Production and Operations Management*, 24(5), 704-731.

[10] Correa, J.R., Montoya, R., Thraves, C., 2016. Contingent preannounced pricing policies with strategic consumers. *Operations Research*, 64(1), 251-272.

[11] Dasu, S., Tong, C., 2010. Dynamic pricing when consumers are strategic: Analysis of posted and contingent pricing schemes. *European Journal of Operational Research*, 204(3), 662-671.

[12] Dhebar, A., 1994. Durable-goods monopolists, rational consumers, and improving products. *Marketing Science*, 13(1), 100-120.

[13] Digitaltrends Website. 2016. Chinese buyers think Apple’s new trade-in program might be lowballing them, accessed September 20, 2016, available at http://www.digitaltrends.com/mobile/apple-iphone-trade-in-program-china/.

[14] Elmaghraby, W., Gulcu, A., Keskinocak, P., 2008. Designing optimal preannounced markdowns in the presence of rational customers with multiunit demands. *Manufacturing & Service Operations Management*, 10(1), 126-148.

[15] Elmaghraby, W., Lippman, S.A., Tang, C.S., Yin, R., 2009. Will more purchasing options benefit customers. *Production and Operations Management*, 18(4), 381-401.

[16] Ferrer, G., Swaminathan, J.M., 2006. Managing new and remanufactured products. *Management Science*, 52(1), 15-26.

[17] Fudenberg, D., Tirole, J., 1998. Upgrades, tradeins, and buybacks. *The RAND Journal of Economics*, 29(2), 235-258.

[18] Gönsch, J., Klein, R., Neugebauer, M., Steinhardt, C., 2013. Dynamic pricing with strategic customers. *Journal of Business Economics*, 83(5), 505-549.

[19] Haruvy, E., Li, T., Sethi, S., 2012. Two-stage pricing for custom-made products. *European Journal of Operational Research*, 219(2), 405-414.

[20] Heese, H.S., Cattani, K., Ferrer, G., Gilland, W., Roth, A.V., 2005. Competitive advantage through take-back of used products. *European Journal of Operational Research*, 164(1), 143-157.

[21] Kornish, L.J., 2001. Pricing for a durable-goods monopolist under rapid sequential innovation. *Management Science*, 47(11), 1552-1561.

[22] Krishnan, V., Ramachandran, K., 2011. Integrated product architecture and pricing for managing sequential innovation. *Management Science*, 57(11), 2040-2053.

[23] Lai, G., Debo, L.G., Sycara, K., 2010. Buy now and match later: Impact of posterior price matching on profit with strategic consumers. *Manufacturing & Service Operations Management*, 12(1), 33-55.

[24] Levinthal, D.A., Purohit, D., 1989. Durable goods and product obsolescence. *Marketing Science*, 8(1), 35-56.

[25] Li, J., Granados, N., Netessine, S., 2014. Are consumers strategic? Structural estimation from the air-travel industry. *Management Science*, 60(9), 2114-2137.

[26] Liang, C., Cakanyildirim, M., Sethi, S., 2014. Analysis of product rollover strategies in the presence of strategic customers. *Management Science*, 60(4), 1033-1056.

[27] Lim, W.S., Tang, C.S., 2006. Optimal product rollover strategies. *European Journal of Operational Research*, 174(2), 905-922.

[28] Liu, Q., Van Ryzin, G., 2008. Strategic capacity rationing to induce early purchases. *Management Science*, 54(6), 1115-1131.

[29] Liu, Y., Fry, M.J., Raturi, A.S., 2009. Retail price markup commitment in decentralized supply chains. *European Journal of Operational Research*, 192(1), 277-292.

[30] Liu, Y., Qin, F., Fry, M.J., Raturi, A.S., 2012. Multi-period modeling of two-way price commitment under price-dependent demand. *European Journal of Operational Research*, 221(3), 546-556.

[31] Lobel, I., Patel, J., Vulcano, G., Zhang, J., 2016. Optimizing product launches in the presence of strategic consumers. *Management Science*, 62(6), 1778-1799.

[32] Mersereau, A.J., Zhang, D., 2012. Markdown pricing with unknown fraction of strategic customers. *Manufacturing & Service Operations Management*, 14(3), 355-370.

[33] Nair, H., 2007. Intertemporal price discrimination with forward-looking consumers: Application to the US market for console video-games. *Quantitative Marketing and Economics*, 5(3), 239-292.

[34] Okada, E.M., 2001. Trade-ins, mental accounting, and product replacement decisions. *Journal of Consumer Research*, 27(4), 433-446.

[35] Papanastasiou, Y., Savva, N., 2017. Dynamic pricing in the presence of social learning and strategic consumers. *Management Science*, 63(4), 919-939.

[36] Parlaktürk, A.K., 2012. The value of product variety when selling to strategic consumers. *Manufacturing & Service Operations Management*, 14(3), 371-385.

[37] Pcworld Website. 2016. Apple's iPhone trade-in program in China could boost sales, accessed September 20, 2016, available at

http://www.pcworld.com/article/2904192/apples-iphone-tradein-program-in-china-could-boost-sales.html.

[38] Ray, S., Boyaci, T., Aras, N., 2005. Optimal prices and trade-in rebates for durable, remanufacturable products. *Manufacturing & Service Operations Management*, 7(3), 208-228.

[39] Shen, Z.M., Su, X., 2007. Customer behavior modeling in revenue management and auctions: A review and new research opportunities. *Production and Operations Management*, 16(6), 713-728.

[40] Shi, T., Gu, W., Chhajed, D., Petruzzi, N.C., 2016. Effects of remanufacturable product design on market segmentation and the environment. *Decision Sciences*, 47(2), 298-332.

[41] Shum, S., Tong, S., Xiao, T., 2017. On the impact of uncertain cost reduction when selling to strategic customers. *Management Science,* 62(3), 843-860.

[42] Su, X., 2007. Intertemporal pricing with strategic customer behavior. *Management Science*, 53(5), 726-741.

[43] Su, X., Zhang, F., 2008. Strategic customer behavior, commitment, and supply chain performance. *Management Science*, 54(10), 1759-1773.

[44] Su, X., Zhang, F., 2009. On the value of commitment and availability guarantees when selling to strategic consumers. *Management Science*, 55(5), 713-726.

[45] Swinney, R., 2011. Selling to strategic consumers when product value is uncertain: The value of matching supply and demand. *Management Science*, 57(10), 1737-1751.

[46] Todayonline Website. 2015. Apple upgrade program seen driving demand for new iPhones, accessed July 28, 2016, available athttp://www.todayonline.com/business/apple-upgrade-program-seen-driving-demand-new-iphones?singlepage=true.

[47] van Ackere, A., Reyniers, D.J., 1995. Trade-ins and introductory offers in a monopoly. *RAND Journal of Economics*, 26(1), 58-74.

[48] Whang, S., 2015. Demand uncertainty and the Bayesian effect in mark-down pricing with strategic customers. *Manufacturing & Service Operations Management*, 17(1), 66-77.

[49] Yin, R., Aviv, Y., Pazgal, A., Tang, C.S., 2009. Optimal mark-down pricing: Implications of inventory display formats in the presence of strategic customers. *Management Science*, 55(8), 1391-1408.

[50] Yin, R., Li, H., Tang, C.S., 2015. Optimal pricing of two successive-generations of products with trade-in options under uncertainty. *Decision Sciences*, 46(3), 565-595.

[51] Yin, R., Tang, C.S., 2014. Optimal temporal customer purchasing decisions under trade-in programs with up-front fees. *Decision Sciences*, 45(3), 373-400.

[52] Yu, M., Debo, L.G., Kapuscinski, R., 2016. Strategic waiting for consumer-generated quality information: Dynamic pricing of new experience goods. *Management Science*, 62(2), 410-435.

[53] Zhou, E., Zhang, J., Gou, Q., Liang, L., 2015. A two period pricing model for new fashion style launching strategy. *International Journal of Production Economics*, 160(2015), 144-156.

[54] Zdnet Website. 2015. Apple's trade-in program meets lukewarm response in China, accessed September 20, 2016, available at http://www.zdnet.com/article/apples-trade-in-program-meets-lukewarm-responses-in-china/#.

[55] Zdnet Website. 2016. Apple resumes trade-in program in China, accessed September 20, 2016, available at http://www.zdnet.com/article/apple-resumes-trade-in-program-in-china/.

Appendix I: Proofs

Proof of Proposition 1. Under dual rollover with trade-in program, if the firm follows dynamic pricing,

(1) In case I, the firm's second-period pricing problem is

$$\max_{p_2, p_t, p_d} \Pi_2 = p_2\left(\tau_1^D - \frac{p_2 - p_d}{\theta}\right) + (p_2 - p_t + s)\left(1 - \frac{p_2 - p_t}{\theta}\right) + p_d\left(\frac{p_2 - p_d}{\theta} - p_d\right)$$

$$s.t.\ 0 \le \tau_1^D - \frac{p_2 - p_d}{\theta} \le 1 - (1 - \tau_1^D)$$

$$0 \le \frac{p_2 - p_d}{\theta} - p_d \le 1 - (1 - \tau_1^D)$$

$$\tau_1^D - \frac{p_2 - p_d}{\theta} + \frac{p_2 - p_d}{\theta} - p_d \le 1 - (1 - \tau_1^D)$$

$$0 \le 1 - \frac{p_2 - p_t}{\theta} \le 1 - \tau_1^D$$

$$0 \le p_t \le p_2$$

$$0 \le p_d \le p_1$$

The unconstrained solution is $p_2^* = \frac{(1+\theta)\tau_1^D}{2}$, $p_t^* = \frac{(1+\theta)\tau_1^D-(\theta-s)}{2}$, and $p_d^* = \frac{\tau_1^D}{2}$. To meet the second-period constraints, p_1 should satisfy $\frac{\theta-s}{1+\theta} \le \tau_1^D(p_1) \le \frac{\theta-s}{2\theta}$. Solving the indifferent condition in case I, $0 + \delta((1+\theta)\tau_1^D - p_2^*) = (\tau_1^D - p_1) + \delta(\tau_1^D)$, we obtain $\tau_1^D = \frac{2p_1}{2+\delta-\delta\theta}$. We add those constraints to the first-period problem, which is

$$\max_{p_1} \Pi = p_1\left(1 - \frac{2p_1}{2+\delta-\delta\theta}\right) + p_2^*\left(\frac{2p_1}{2+\delta-\delta\theta} - \frac{p_2^* - p_d^*}{\theta}\right) + (p_2^* - p_t^* + s)\left(1 - \frac{p_2^* - p_t^*}{\theta}\right) + p_d^*\left(\frac{p_2^* - p_d^*}{\theta} - p_d^*\right)$$

$$s.t. 0 \le 1 - \frac{2p_1}{2+\delta-\delta\theta} \le 1$$

$$0 \le p_1 \le 1$$

$$\frac{\theta - s}{1+\theta} \le \frac{2p_1}{2+\delta-\delta\theta} \le \frac{\theta - s}{2\theta}$$

The unconstrained solution is $p_1^* = \frac{(2+\delta-\delta\theta)^2}{2(3+2\delta-\theta-2\delta\theta)}$, which means the upper constraint $\frac{2p_1}{2+\delta-\delta\theta} \le \frac{\theta-s}{2\theta}$ never holds, while the other constraints hold. Thus, we set $\frac{2p_1^*}{2+\delta-\delta\theta} = \frac{\theta-s}{2\theta}$ in equilibrium. Therefore, we have $p_1^D = \frac{(\theta-s)(2+\delta-\delta\theta)}{4\theta}$, $p_2^D = \frac{(\theta-s)(1+\theta)}{4\theta}$, $p_t^D = \frac{(\theta-s)(1-\theta)}{4\theta}$, $p_d^D = \frac{\theta-s}{4\theta}$, $q_1^D = \frac{\theta+s}{2\theta}$, $q_2^D = \frac{\theta-s}{2\theta}$, $q_t^D = \frac{\theta+s}{2\theta}$, and $q_d^D = 0$.

(2) In case II, the second-period pricing problem of the firm is

$$\max_{p_2,p_t,p_d} \Pi_2 = p_2\left(\frac{p_2 - p_d}{\theta} - \frac{p_2}{1+\theta}\right) + (p_2 - p_t + s)\left(1 - \frac{p_2 - p_t}{\theta}\right) + p_d\left(\tau_1^D - \frac{p_2 - p_d}{\theta}\right)$$

$$s.t. 0 \le \frac{p_2 - p_d}{\theta} - \frac{p_2}{1+\theta} \le 1 - (1 - \tau_1^D)$$

$$0 \le \tau_1^D - \frac{p_2 - p_d}{\theta} \le 1 - (1 - \tau_1^D)$$

$$\frac{p_2 - p_d}{\theta} - \frac{p_2}{1+\theta} + \tau_1^D - \frac{p_2 - p_d}{\theta} \le 1 - (1 - \tau_1^D)$$

$$0 \le 1 - \frac{p_2 - p_t}{\theta} \le 1 - \tau_1^D$$

$$0 \le p_t \le p_2$$

$$0 \le p_d \le p_1$$

The unconstrained solution is $p_2^* = \frac{(1+\theta)\tau_1^D}{2}$, $p_t^* = \frac{(1+\theta)\tau_1^D-(\theta-s)}{2}$, and $p_d^* = \frac{\tau_1^D}{2}$. To meet the second-period constraints, p_1 should satisfy $\frac{\theta-s}{1+\theta} \le \tau_1^D(p_1) \le \frac{\theta-s}{2\theta}$. Solving the indifferent condition in case II, $0 + \delta(\tau_1^D - p_d^*) = (\tau_1^D - p_1) + \delta(\tau_1^D)$, we obtain $\tau_1^D = \frac{2p_1}{2+\delta}$. We add those constraints to the first-period problem, which is

$$\max_{p_1} \Pi = p_1\left(1 - \frac{2p_1}{2+\delta}\right) + p_2^*\left(\frac{p_2^* - p_d^*}{\theta} - \frac{p_2^*}{1+\theta}\right) + (p_2^* - p_t^* + s)\left(1 - \frac{p_2^* - p_t^*}{\theta}\right) + p_d^*\left(\frac{2p_1}{2+\delta} - \frac{p_2^* - p_d^*}{\theta}\right)$$

$$s.t. \; 0 \le 1 - \frac{2p_1}{2+\delta} \le 1$$

$$0 \le p_1 \le 1$$

$$\frac{\theta-s}{1+\theta} \le 1 - \frac{2p_1}{2+\delta} \le \frac{\theta-s}{2\theta}$$

The unconstrained solution is $p_1^* = \frac{(2+\delta)^2}{2(3+2\delta)}$, which means the upper constraint $\frac{2p_1}{2+\delta} \le \frac{\theta-s}{2\theta}$ never holds, while the other constraints hold. Thus, we set $\frac{2p_1^*}{2+\delta} = \frac{\theta-s}{2\theta}$ in equilibrium. Therefore, we have $p_1^D = \frac{(\theta-s)(2+\delta)}{4\theta}$, $p_2^D = \frac{(\theta-s)(1+\theta)}{4\theta}$, $p_t^D = \frac{(\theta-s)(1-\theta)}{4\theta}$, $p_d^D = \frac{\theta-s}{4\theta}$, $q_1^D = \frac{\theta+s}{2\theta}$, $q_2^D = 0$, $q_t^D = \frac{\theta+s}{2\theta}$, and $q_d^D = \frac{\theta-s}{4\theta}$.

(3) We can obtain the condition for choosing case I is $s \le \frac{(1-2\delta)\theta}{1+2\delta}$ by comparing the firm's profits under case I and case II. □

Proof of Table 2. Based on the equilibria under dynamic pricing in Proposition 1, it is straightforward to obtain how the innovation incremental value θ, the salvage value s, and the consumer's degree of strategy δ influence the price, the sales volume, and the total profit by solving the first order derivative. Take p_1^D in case I for example, since $\frac{\partial p_1^D}{\partial\theta} = \frac{s(2+\delta)-\delta\theta^2}{4\theta^2} > 0$ when $0 \le \theta < \frac{\sqrt{s}\sqrt{2+\delta}}{\sqrt{\delta}}$, $\frac{\partial p_1^D}{\partial\theta} = \frac{s(2+\delta)-\delta\theta^2}{4\theta^2} < 0$ when $\frac{\sqrt{s}\sqrt{2+\delta}}{\sqrt{\delta}} < \theta \le 1$, $\frac{\partial p_1^D}{\partial s} = \frac{-2-\delta+\delta\theta}{4\theta} < 0$, $\frac{\partial p_1^D}{\partial\delta} = \frac{(\theta-s)(1-\theta)}{4\theta} > 0$, thus p_1^D in case I will increase first then decrease as θ increases, decrease as s increases, and increase as δ increases. □

Proof of Lemma 1. According to the comparison of slopes and intercepts of consumer surpluses corresponding to these five alternatives, we have Figure 3. It is easy to obtain the optimal purchasing decisions with respect to consumers' valuation v_1 from Figure 3. Specifically, in price skimming, there are four indifferent conditions, $u_1(\tau_1^P) = u_2(\tau_1^P)$, $u_2(\tau_2^P) = u_3(\tau_2^P)$, $u_3(\tau_3^P) = u_4(\tau_3^P)$, and $u_4(\tau_4^P) = u_5(\tau_4^P)$. While, in penetration pricing, there are three indifferent conditions for consumers, $u_1(\tau_1^P) = u_2(\tau_1^P)$, $u_2(\tau_2^P) = u_4(\tau_2^P)$, and $u_4(\tau_4^P) = u_5(\tau_4^P)$. Solving the above indifferent conditions, we can obtain the thresholds. □

Proof of Proposition 2. Under dual rollover with trade-in program, if the firm follows preannounced pricing,

(1) In price skimming, the firm's pricing problem is

$$\max_{p_1,p_2,p_t,p_d} \Pi = p_1\left(1-\frac{p_1-\delta p_2}{1-\delta\theta}\right) + p_2\left(\frac{p_1-\delta p_2}{1-\delta\theta} - \frac{p_2-p_d}{\theta}\right) + (p_2-p_t+s)\left(1-\frac{p_2-p_t}{\theta}\right) + p_d\left(\frac{p_2-p_d}{\theta} - p_d\right)$$

$$s.t.\, 0 \le 1-\frac{p_1-\delta p_2}{1-\delta\theta} \le 1$$

$$0 \le p_1 \le 1$$

$$0 \le \frac{p_1-\delta p_2}{1-\delta\theta} - \frac{p_2-p_d}{\theta} \le 1-\left(1-\frac{p_1-\delta p_2}{1-\delta\theta}\right)$$

$$0 \le \frac{p_2-p_d}{\theta} - p_d \le 1-\left(1-\frac{p_1-\delta p_2}{1-\delta\theta}\right)$$

$$\frac{p_1-\delta p_2}{1-\delta\theta} - \frac{p_2-p_d}{\theta} + \frac{p_2-p_d}{\theta} - p_d \le 1-\left(1-\frac{p_1-\delta p_2}{1-\delta\theta}\right)$$

$$0 \le 1-\frac{p_2-p_t}{\theta} \le 1-\frac{p_1-\delta p_2}{1-\delta\theta}$$

$$0 \le p_t \le p_2$$

$$0 \le p_d \le p_1$$

$$p_1 > \delta p_2$$

The unconstrained solution is $p_1^* = \frac{2(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_2^* = \frac{(1+\theta)(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_t^* = \frac{2-(1+\delta)\theta+(1-\delta)\theta^2+s(3-\delta-\theta-\delta\theta)}{2(3-\delta-\theta-\delta\theta)}$, and $p_d^* = \frac{1-\delta\theta}{3-\delta-\theta-\delta\theta}$. Because the constraint $q_t^* \le q_1^*$ never holds, while the other constraints hold, we set $q_t^* = q_1^*$ in equilibrium. Therefore, we have $p_1^P = \frac{2(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_2^P = \frac{(1+\theta)(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$,

$p_t^P = \frac{2-(1+\delta)\theta+(1-\delta)\theta^2+s(3-\delta-\theta-\delta\theta)}{2(3-\delta-\theta-\delta\theta)}$, $p_d^P = \frac{1-\delta\theta}{3-\delta-\theta-\delta\theta}$, $q_1^P = \frac{1-\theta}{3-\delta-\theta-\delta\theta}$, $q_2^P = \frac{1-\delta}{3-\delta-\theta-\delta\theta}$, $q_t^P = \frac{1-\theta}{3-\delta-\theta-\delta\theta}$, and $q_d^P = 0$.

(2) In penetration pricing, the firm's pricing problem is

$$\max_{p_1,p_2,p_t,p_d} \Pi = p_1(1-p_1+\delta p_d) + (p_2-p_t+s)\left(1-\frac{p_2-p_t}{\theta}\right) + p_d(p_1-\delta p_d-p_d)$$

$$s.t. 0 \le 1-p_1+\delta p_d \le 1$$

$$0 \le p_1 \le 1$$

$$0 \le p_1-\delta p_d-p_d \le 1-(1-p_1+\delta p_d)$$

$$0 \le 1-\frac{p_2-p_t}{\theta} \le 1-p_1+\delta p_d$$

$$0 \le p_t \le p_2$$

$$0 \le p_d \le p_1$$

$$p_1 \le \delta p_2$$

The unconstrained solution is $p_1^* = \frac{2}{3-\delta}$, $p_2^* = \frac{2}{(3-\delta)\delta}$, $p_t^* = \frac{2}{(3-\delta)\delta} - \frac{1}{2}(\theta-s)$, and $p_d^* = \frac{1}{3-\delta}$. Because the constraint $q_t^* \le q_1^*$ never holds, while the other constraints hold, we set $q_t^* = q_1^*$ in equilibrium. Therefore, we have $p_1^P = \frac{2}{3-\delta}$, $p_2^P = \frac{2}{(3-\delta)\delta}$, $p_t^P = \frac{2}{(3-\delta)\delta} - \frac{1}{2}(\theta-s)$, $p_d^P = \frac{1}{3-\delta}$, $q_1^P = \frac{1}{3-\delta}$, $q_2^P = 0$, $q_t^P = \frac{1}{3-\delta}$, and $q_d^P = \frac{1-\delta}{3-\delta}$.

(3) We obtain the condition for choosing price skimming is $s \le 1-\delta-\theta$ by comparing the firm's profits in price skimming and penetration pricing. □

Proof of Table 4. Based on the equilibria under preannounced pricing in Proposition 2, it is straightforward to obtain how the innovation incremental value θ, the salvage value s, and the consumer's degree of strategy δ influence the price, the sales volume, and the total profit by solving the first order derivative. Take p_1^P in price skimming for example, since $\frac{\partial p_1^P}{\partial \theta} = \frac{2(1-\delta)^2}{(3-\delta-\theta-\delta\theta)^2} > 0$, $\frac{\partial p_1^P}{\partial s} = 0$, $\frac{\partial p_1^P}{\partial \delta} = \frac{2(1-\theta)^2}{(3-\delta-\theta-\delta\theta)^2} > 0$, thus p_1^P in price skimming will increase in θ and δ, while remains the same in s. □

Proof of Proposition 3. Comparing the equilibria in Propositions 1 and 2, we can obtain the optimal pricing strategy under dual rollover with trade-in program. The threshold $T_1(\theta)$ is as follows.

(1) When $s \le \frac{(1-2\delta)\theta}{1+2\delta}$, $T_1(\theta) = \frac{-3\theta+\delta\theta+6\theta^2-2\delta\theta^2+\theta^3-3\delta\theta^3+2\sqrt{A_1}}{9+3\delta-2\delta^2-18\theta-6\delta\theta+5\theta^2+7\delta\theta^2+2\delta^2\theta^2}$, where $A_1 = -3\delta^2\theta^2 - 2\delta^3\theta^2 + \delta^4\theta^2 + 4\delta^2\theta^3 + 4\delta^3\theta^3 + 4\theta^4 + 2\delta^2\theta^4 - 2\delta^4\theta^4 - 8\theta^5 - 4\delta^2\theta^5 - 4\delta^3\theta^5 + 4\theta^6 + \delta^2\theta^6 + 2\delta^3\theta^6 + \delta^4\theta^6$.

(2) When $\frac{(1-2\delta)\theta}{1+2\delta} \le s < 1-\delta-\theta$, $T_1(\theta) = \frac{3\theta-\delta\theta-9\theta^2+3\delta\theta^2+4\delta\theta^3-2\sqrt{A_2}}{-9-3\delta+2\delta^2+15\theta+\delta\theta+2\delta^2\theta-4\theta^2-4\delta\theta^2}$, where $A_2 = -3\delta^2\theta^2 - 2\delta^3\theta^2 + \delta^4\theta^2 - 12\theta^3 + 16\delta\theta^3 + 6\delta^2\theta^3 - 4\delta^3\theta^3 + 2\delta^4\theta^3 + 32\theta^4 - 32\delta\theta^4 + \delta^2\theta^4 - 2\delta^3\theta^4 + \delta^4\theta^4 - 16\theta^5 + 8\delta^2\theta^5 + 4\theta^6$.

(3) When $s > 1-\delta-\theta$, $T_1(\theta) = \frac{3\theta-\delta\theta-8\theta^2+4\delta\theta^2-2\sqrt{-3\delta^2\theta^2-2\delta^3\theta^2+\delta^4\theta^2+4\theta^4}}{-9-3\delta+2\delta^2+12\theta-4\delta\theta}$. □

Proof of Proposition 4. Under single rollover with trade-in program, when the firm follows dynamic pricing, its second-period pricing problem is

$$\max_{p_2,p_t} \Pi_2 = p_2\left(\tau_1^D - \frac{p_2}{1+\theta}\right) + (p_2 - p_t + s)\left(1 - \frac{p_2 - p_t}{\theta}\right)$$

$$s.t.\ 0 \le \tau_1^D - \frac{p_2}{1+\theta} \le 1 - (1 - \tau_1^D)$$

$$0 \le 1 - \frac{p_2 - p_t}{\theta} \le 1 - \tau_1^D$$

$$0 \le p_t \le p_2$$

$$0 \le p_d \le p_1$$

The unconstrained solution is $p_2^* = \frac{(1+\theta)\tau_1^D}{2}$ and $p_t^* = \frac{(1+\theta)\tau_1^D - (\theta - s)}{2}$. To meet the second-period constraints, p_1 should satisfy $\frac{\theta-s}{1+\theta} \le \tau_1^D(p_1) \le \frac{\theta-s}{2\theta}$. Solving the indifferent condition $0 + \delta((1+\theta)\tau_1^D - p_2^*) = (\tau_1^D - p_1) + \delta(\tau_1^D)$, we obtain $\tau_1^D = \frac{2p_1}{2+\delta-\delta\theta}$. We add those constraints to the first-period problem, which is

$$\max_{p_1} \Pi = p_1\left(1 - \frac{2p_1}{2+\delta-\delta\theta}\right) + p_2^*\left(\frac{2p_1}{2+\delta-\delta\theta} - \frac{p_2^*}{1+\theta}\right) + (p_2^* - p_t^* + s)\left(1 - \frac{p_2^* - p_t^*}{\theta}\right)$$

$$s.t.\ 0 \le 1 - \frac{2p_1}{2+\delta-\delta\theta} \le 1$$

$$0 \le p_1 \le 1$$

$$\frac{\theta - s}{1+\theta} \le \frac{2p_1}{2+\delta-\delta\theta} \le \frac{\theta - s}{2\theta}$$

The unconstrained solution is $p_1^* = \frac{(2+\delta-\delta\theta)^2}{2(3+2\delta-\theta-2\delta\theta)}$, which means the upper constraint $\frac{2p_1}{2+\delta-\delta\theta} \leq \frac{\theta-s}{2\theta}$ never holds, while the other constraints hold. Thus, we set $\frac{2p_1^*}{2+\delta-\delta\theta} = \frac{\theta-s}{2\theta}$ in equilibrium. Therefore, we have $p_1^D = \frac{(\theta-s)(2+\delta-\delta\theta)}{4\theta}$, $p_2^D = \frac{(\theta-s)(1+\theta)}{4\theta}$, $p_t^D = \frac{(\theta-s)(1-\theta)}{4\theta}$, $q_1^D = \frac{\theta+s}{2\theta}$, $q_2^D = \frac{\theta-s}{2\theta}$, and $q_t^D = \frac{\theta+s}{2\theta}$. □

Proof of Proposition 5. Under single rollover with trade-in program, when the firm follows preannounced pricing,

(1) In price skimming, the firm's pricing problem is

$$\max_{p_1,p_2,p_t,p_d} \Pi = p_1\left(1-\frac{p_1-\delta p_2}{1-\delta\theta}\right) + p_2\left(\frac{p_1-\delta p_2}{1-\delta\theta} - \frac{p_2}{1+\theta}\right) + (p_2-p_t+s)\left(1-\frac{p_2-p_t}{\theta}\right)$$

$$s.t. 0 \leq 1-\frac{p_1-\delta p_2}{1-\delta\theta} \leq 1$$

$$0 \leq p_1 \leq 1$$

$$0 \leq \frac{p_1-\delta p_2}{1-\delta\theta} - \frac{p_2}{1+\theta} \leq 1-\left(1-\frac{p_1-\delta p_2}{1-\delta\theta}\right)$$

$$0 \leq 1-\frac{p_2-p_t}{\theta} \leq 1-\frac{p_1-\delta p_2}{1-\delta\theta}$$

$$0 \leq p_t \leq p_2$$

$$p_1 > \delta p_2$$

The unconstrained solution is $p_1^* = \frac{2(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_2^* = \frac{(1+\theta)(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, and $p_t^* = \frac{2-(1+\delta)\theta+(1-\delta)\theta^2+s(3-\delta-\theta-\delta\theta)}{2(3-\delta-\theta-\delta\theta)}$. Because the constraint $q_t^* \leq q_1^*$ never holds, while the other constraints hold, we set $q_t^* = q_1^*$ in equilibrium. Therefore, we have $p_1^P = \frac{2(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_2^P = \frac{(1+\theta)(1-\delta\theta)}{3-\delta-\theta-\delta\theta}$, $p_t^P = \frac{2-(1+\delta)\theta+(1-\delta)\theta^2+s(3-\delta-\theta-\delta\theta)}{2(3-\delta-\theta-\delta\theta)}$, $q_1^P = \frac{1-\theta}{3-\delta-\theta-\delta\theta}$, $q_2^P = \frac{1-\delta}{3-\delta-\theta-\delta\theta}$, and $q_t^P = \frac{1-\theta}{3-\delta-\theta-\delta\theta}$.

(2) In penetration pricing, the firm's pricing problem is

$$\max_{p_1,p_2,p_t,p_d} \Pi = p_1\left(1-\frac{p_1}{1+\delta}\right) + (p_2-p_t+s)\left(1-\frac{p_2-p_t}{\theta}\right)$$

$$s.t. 0 \leq 1-\frac{p_1}{1+\delta} \leq 1$$

$$0 \leq p_1 \leq 1$$

$$0 \leq 1-\frac{p_2-p_t}{\theta} \leq 1-\frac{p_1}{1+\delta}$$

$$0 \leq p_t \leq p_2$$

$$p_1 \leq \delta p_2$$

The unconstrained solution is $p_1^* = \frac{1+\delta}{2}$, $p_2^* = \frac{1+\delta}{2\delta}$, and $p_t^* = \frac{1+\delta}{2\delta} - \frac{1}{2}(\theta - s)$. Because the constraint $q_t^* \leq q_1^*$ never holds, while the other constraints hold, we set $q_t^* = q_1^*$ in equilibrium. Therefore, we have $p_1^P = \frac{1+\delta}{2}$, $p_2^P = \frac{1+\delta}{2\delta}$, $p_t^P = \frac{1+\delta}{2\delta} - \frac{1}{2}(\theta - s)$, $q_1^P = \frac{1}{2}$, $q_2^P = 0$, and $q_t^P = \frac{1}{2}$.

(3) We obtain the condition for choosing price skimming is $s \leq 1 - \delta - \theta$ by comparing the firm's profits in price skimming and penetration pricing. □

Proof of Proposition 6. Comparing the equilibria in Propositions 4 and 5, we obtain the optimal pricing strategy under single rollover with trade-in program. The threshold $T_2(\theta)$ is as follows.

(1) When $s \leq 1 - \delta - \theta, T_2(\theta) = \frac{-3\theta + \delta\theta + 6\theta^2 - 2\delta\theta^2 + \theta^3 - 3\delta\theta^3 + 2\sqrt{A_3}}{9 + 3\delta - 2\delta^2 - 18\theta - 6\delta\theta + 5\theta^2 + 7\delta\theta^2 + 2\delta^2\theta^2}$, where $A_3 = -3\delta^2\theta^2 - 2\delta^3\theta^2 + \delta^4\theta^2 + 4\delta^2\theta^3 + 4\delta^3\theta^3 + 4\theta^4 + 2\delta^2\theta^4 - 2\delta^4\theta^4 - 8\theta^5 - 4\delta^2\theta^5 - 4\delta^3\theta^5 + 4\theta^6 + \delta^2\theta^6 + 2\delta^3\theta^6 + \delta^4\theta^6$.

(2) When $s > 1 - \delta - \theta$, $T_2(\theta) = \frac{\theta - \theta^2 - 2\sqrt{\theta^2 - \delta\theta^2 - \delta^2\theta^2 - \theta^3 + \delta\theta^3 - \theta^4 + 2\delta\theta^4 + \delta^2\theta^4}}{-3 - 2\delta + 5\theta + 2\delta\theta}$. □

Appendix II: Equilibrium Analysis for Section 6.2

Part A: Dynamic Pricing Strategy. When the firm discounts future revenue, the analysis under dynamic pricing strategy is similar to the base model. There are two cases, thus two different objective functions.

(1) In case I, the firm's second-period pricing problem is

$$\max_{p_2, p_t, p_d} \Pi_2 = p_2\left(\tau_1^D - \frac{p_2 - p_d}{\theta}\right) + (p_2 - p_t + s)\left(1 - \frac{p_2 - p_t}{\theta}\right) + p_d\left(\frac{p_2 - p_d}{\theta} - p_d\right)$$

$$s.t. 0 \leq \tau_1^D - \frac{p_2 - p_d}{\theta} \leq 1 - (1 - \tau_1^D)$$

$$0 \leq \frac{p_2 - p_d}{\theta} - p_d \leq 1 - (1 - \tau_1^D)$$

$$\tau_1^D - \frac{p_2 - p_d}{\theta} + \frac{p_2 - p_d}{\theta} - p_d \leq 1 - (1 - \tau_1^D)$$

$$0 \leq 1 - \frac{p_2 - p_t}{\theta} \leq 1 - \tau_1^D$$

$$0 \leq p_t \leq p_2$$

$$0 \leq p_d \leq p_1$$

The unconstrained solution is $p_2^* = \frac{(1+\theta)\tau_1^D}{2}$, $p_t^* = \frac{(1+\theta)\tau_1^D-(\theta-s)}{2}$, and $p_d^* = \frac{\tau_1^D}{2}$. To meet the second-period constraints, p_1 should satisfy $\frac{\theta-s}{1+\theta} \le \tau_1^D(p_1) \le \frac{\theta-s}{2\theta}$. Solving the indifferent condition in case I, $0 + \delta((1+\theta)\tau_1^D - p_2^*) = (\tau_1^D - p_1) + \delta(\tau_1^D)$, we obtain $\tau_1^D = \frac{2p_1}{2+\delta-\delta\theta}$. We add those constraints to the first-period problem, which is

$$\max_{p_1} \Pi = p_1\left(1 - \frac{2p_1}{2+\delta-\delta\theta}\right)$$
$$+ \beta\left[p_2^*\left(\frac{2p_1}{2+\delta-\delta\theta} - \frac{p_2^*-p_d^*}{\theta}\right) + (p_2^* - p_t^* + s)\left(1 - \frac{p_2^*-p_t^*}{\theta}\right) + p_d^*\left(\frac{p_2^*-p_d^*}{\theta} - p_d^*\right)\right]$$
$$s.t.\, 0 \le 1 - \frac{2p_1}{2+\delta-\delta\theta} \le 1$$
$$0 \le p_1 \le 1$$
$$\frac{\theta-s}{1+\theta} \le \frac{2p_1}{2+\delta-\delta\theta} \le \frac{\theta-s}{2\theta}$$

The unconstrained solution is $p_1^* = \frac{(2+\delta-\delta\theta)^2}{4+2\delta-2\delta\theta-\beta-\beta\theta}$, which means the upper constraint $\frac{2p_1}{2+\delta-\delta\theta} \le \frac{\theta-s}{2\theta}$ never holds, while the other constraints hold. Thus, we set $\frac{2p_1^*}{2+\delta-\delta\theta} = \frac{\theta-s}{2\theta}$ in equilibrium. Therefore, we have $p_1^D = \frac{(\theta-s)(2+\delta-\delta\theta)}{4\theta}$, $p_2^D = \frac{(\theta-s)(1+\theta)}{4\theta}$, $p_t^D = \frac{(\theta-s)(1-\theta)}{4\theta}$, $p_d^D = \frac{\theta-s}{4\theta}$, $q_1^D = \frac{\theta+s}{2\theta}$, $q_2^D = \frac{\theta-s}{2\theta}$, $q_t^D = \frac{\theta+s}{2\theta}$, and $q_d^D = 0$.

(2) In case II, the firm's second-period pricing problem is

$$\max_{p_2,p_t,p_d} \Pi_2 = p_2\left(\frac{p_2-p_d}{\theta} - \frac{p_2}{1+\theta}\right) + (p_2 - p_t + s)\left(1 - \frac{p_2-p_t}{\theta}\right) + p_d\left(\tau_1^D - \frac{p_2-p_d}{\theta}\right)$$
$$s.t.\, 0 \le \frac{p_2-p_d}{\theta} - \frac{p_2}{1+\theta} \le 1 - (1-\tau_1^D)$$
$$0 \le \tau_1^D - \frac{p_2-p_d}{\theta} \le 1 - (1-\tau_1^D)$$
$$\frac{p_2-p_d}{\theta} - \frac{p_2}{1+\theta} + \tau_1^D - \frac{p_2-p_d}{\theta} \le 1 - (1-\tau_1^D)$$
$$0 \le 1 - \frac{p_2-p_t}{\theta} \le 1 - \tau_1^D$$
$$0 \le p_t \le p_2$$
$$0 \le p_d \le p_1$$

The unconstrained solution is $p_2^* = \frac{(1+\theta)\tau_1^D}{2}$, $p_t^* = \frac{(1+\theta)\tau_1^D-(\theta-s)}{2}$, and $p_d^* = \frac{\tau_1^D}{2}$. To meet the second-period constraints, p_1 should satisfy $\frac{\theta-s}{1+\theta} \le \tau_1^D(p_1) \le \frac{\theta-s}{2\theta}$. Solving the indifferent condition in case II,

$0 + \delta(\tau_1^D - p_d^*) = (\tau_1^D - p_1) + \delta(\tau_1^D)$, we obtain $\tau_1^D = \frac{2p_1}{2+\delta}$. We add those constraints to the first-period problem, which is

$$\max_{p_1} \Pi = p_1\left(1 - \frac{2p_1}{2+\delta}\right)$$
$$+ \beta\left[p_2^*\left(\frac{p_2^* - p_d^*}{\theta} - \frac{p_2^*}{1+\theta}\right) + (p_2^* - p_t^* + s)\left(1 - \frac{p_2^* - p_t^*}{\theta}\right) + p_d^*\left(\frac{2p_1}{2+\delta} - \frac{p_2^* - p_d^*}{\theta}\right)\right]$$
$$s.t. \, 0 \le 1 - \frac{2p_1}{2+\delta} \le 1$$
$$0 \le p_1 \le 1$$
$$\frac{\theta - s}{1+\theta} \le 1 - \frac{2p_1}{2+\delta} \le \frac{\theta - s}{2\theta}$$

The unconstrained solution is $p_1^* = \frac{(2+\delta)^2}{2(3+2\delta)}$, which means the upper constraint $\frac{2p_1}{2+\delta} \le \frac{\theta-s}{2\theta}$ never holds, while the other constraints hold. Thus, we set $\frac{2p_1^*}{2+\delta} = \frac{\theta-s}{2\theta}$ in equilibrium. Therefore, we have $p_1^D = \frac{(\theta-s)(2+\delta)}{4\theta}$, $p_2^D = \frac{(\theta-s)(1+\theta)}{4\theta}$, $p_t^D = \frac{(\theta-s)(1-\theta)}{4\theta}$, $p_d^D = \frac{\theta-s}{4\theta}$, $q_1^D = \frac{\theta+s}{2\theta}$, $q_2^D = 0$, $q_t^D = \frac{\theta+s}{2\theta}$, and $q_d^D = \frac{\theta-s}{4\theta}$.

□

Part B: Preannounced Pricing Strategy. When the firm discounts future revenue, the analysis under preannounced pricing strategy is similar to the base model. There are two cases, thus two different objective functions.

(1) In price skimming, the firm's pricing problem is

$$\max_{p_1, p_2, p_t, p_d} \Pi = p_1\left(1 - \frac{p_1 - \delta p_2}{1 - \delta\theta}\right)$$
$$+ \beta\left[p_2\left(\frac{p_1 - \delta p_2}{1 - \delta\theta} - \frac{p_2 - p_d}{\theta}\right) + (p_2 - p_t + s)\left(1 - \frac{p_2 - p_t}{\theta}\right) + p_d\left(\frac{p_2 - p_d}{\theta} - p_d\right)\right]$$
$$s.t. \, 0 \le 1 - \frac{p_1 - \delta p_2}{1 - \delta\theta} \le 1$$
$$0 \le p_1 \le 1$$
$$0 \le \frac{p_1 - \delta p_2}{1 - \delta\theta} - \frac{p_2 - p_d}{\theta} \le 1 - \left(1 - \frac{p_1 - \delta p_2}{1 - \delta\theta}\right)$$
$$0 \le \frac{p_2 - p_d}{\theta} - p_d \le 1 - \left(1 - \frac{p_1 - \delta p_2}{1 - \delta\theta}\right)$$
$$\frac{p_1 - \delta p_2}{1 - \delta\theta} - \frac{p_2 - p_d}{\theta} + \frac{p_2 - p_d}{\theta} - p_d \le 1 - \left(1 - \frac{p_1 - \delta p_2}{1 - \delta\theta}\right)$$
$$0 \le 1 - \frac{p_2 - p_t}{\theta} \le 1 - \frac{p_1 - \delta p_2}{1 - \delta\theta}$$

$$0 \le p_t \le p_2$$

$$0 \le p_d \le p_1$$

$$p_1 > \delta p_2$$

The unconstrained solution is $p_1^* = \frac{2\beta(1+\delta)(1-\delta\theta)}{2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)}$, $p_2^* = \frac{(\beta+\delta)(1+\theta)(1-\delta\theta)}{2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)}$, $p_t^* = \frac{\beta^2(\theta-s)(1+\theta)+\delta(1+\theta)(2-s\delta-\delta\theta)+2\beta(1+2s+\delta s-s\delta\theta-\theta-2\delta\theta)}{2[2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)]}$, and $p_d^* = \frac{(\beta+\delta)(1-\delta\theta)}{2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)}$. Because the constraint $q_t^* \le q_1^*$ never holds, while the other constraints hold, we set $q_t^* = q_1^*$ in equilibrium. Thus, we have $p_1^P = \frac{2\beta(1+\delta)(1-\delta\theta)}{2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)}$, $p_2^P = \frac{(\beta+\delta)(1+\theta)(1-\delta\theta)}{2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)}$, $p_t^P = \frac{\beta^2(\theta-s)(1+\theta)+\delta(1+\theta)(2-s\delta-\delta\theta)+2\beta(1+2s+\delta s-s\delta\theta-\theta-2\delta\theta)}{2[2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)]}$, $p_d^P = \frac{(\beta+\delta)(1-\delta\theta)}{2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)}$, $q_1^P = \frac{\beta(2+\delta-\delta\theta-\beta-\beta\theta)}{2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)}$, $q_2^P = \frac{(\beta-\delta)(1+\delta)}{2\beta(2-\delta-\delta\theta)-(1+\theta)(\beta^2+\delta^2)}$, $q_t^P = \frac{\theta+s}{2\theta}$, and $q_d^P = 0$.

(2) In penetration pricing, the firm's pricing problem is

$$\max_{p_1,p_2,p_t,p_d} \Pi = p_1(1-p_1+\delta p_d) + \beta\left[(p_2-p_t+s)\left(1-\frac{p_2-p_t}{\theta}\right) + p_d(p_1-\delta p_d-p_d)\right]$$

$$s.t. \, 0 \le 1-p_1+\delta p_d \le 1$$

$$0 \le p_1 \le 1$$

$$0 \le p_1-\delta p_d-p_d \le 1-(1-p_1+\delta p_d)$$

$$0 \le 1-\frac{p_2-p_t}{\theta} \le 1-p_1+\delta p_d$$

$$0 \le p_t \le p_2$$

$$0 \le p_d \le p_1$$

$$p_1 \le \delta p_2$$

The unconstrained solution is $p_1^* = \frac{2\beta(1+\delta)}{\beta(4-\beta)+\delta(2\beta-\delta)}$, $p_2^* = \frac{2\beta(1+\delta)}{\delta[\beta(4-\beta)+\delta(2\beta-\delta)]}$, $p_t^* = \frac{2\beta(1+\delta)}{\delta[\beta(4-\beta)+\delta(2\beta-\delta)]} - \frac{1}{2}(\theta-s)$, and $p_d^* = \frac{\beta+\delta}{\beta(4-\beta)+\delta(2\beta-\delta)}$. Because the constraint $q_t^* \le q_1^*$ never holds, while the other constraints hold, we set $q_t^* = q_1^*$ in equilibrium. Therefore, we have $p_1^P = \frac{2\beta(1+\delta)}{\beta(4-\beta)+\delta(2\beta-\delta)}$, $p_2^P = \frac{2\beta(1+\delta)}{\delta[\beta(4-\beta)+\delta(2\beta-\delta)]}$, $p_t^P = \frac{2\beta(1+\delta)}{\delta[\beta(4-\beta)+\delta(2\beta-\delta)]} - \frac{1}{2}(\theta-s)$, $p_d^P = \frac{\beta+\delta}{\beta(4-\beta)+\delta(2\beta-\delta)}$, $q_1^P = \frac{\beta(2-\beta+\delta)}{\beta(4-\beta)+\delta(2\beta-\delta)}$, $q_2^P = 0$, $q_t^P = \frac{\theta+s}{2\theta}$, and $q_d^P = \frac{(\beta-\delta)(1+\delta)}{\beta(4-\beta)+\delta(2\beta-\delta)}$. □