North American Journal of Economics and Finance 42 (2017) 421–432

Contents lists available at ScienceDirect

North American Journal of Economics and Finance

journal homepage: www.elsevier.com/locate/ecofin

Forecasting broad money velocity

Alexander Jung [1]

European Central Bank, Sonnemannstr. 20, 60314, Germany

ARTICLE INFO

Article history:
Received 6 December 2016
Received in revised form 7 August 2017
Accepted 14 August 2017

JEL classification:
C51
C53
E47

Keywords:
Money velocity
Mixed frequency
United States
Euro area

ABSTRACT

This paper applies traditional approaches and mixed-data sampling (MIDAS) to explain and forecast velocity of broad money in the euro area and the United States. Our results show that despite financial innovations, over the last two decades broad money velocity followed a declining trend with one break around the start of the financial crisis in both economies. A new result is that applying mixed-frequency techniques, we find improvements in velocity forecasts for the euro area at all horizons considered (one to eight quarters ahead), whereas for the US possible gains only refer to shorter-term forecasts.

1. Introduction

Monetary developments can help to guide monetary policy decisions of central banks. The literature provides two principal motivations for monitoring the velocity of broad money. One is that the velocity of circulation of money is a key concept in monetary theory and an important element of monetary analysis (Brunner & Meltzer, 1963; Fisher, 1911; Laidler, 1990). It can provide policymakers with additional insights about inflationary and deflationary risks. If velocity is not constant, as assumed in the quantity theory of money, this can have a bearing on the relationship between money and prices. Moreover, the velocity of money provides a complementary perspective on money demand, thereby allowing policy-makers to cross-check the information gained from money demand models. However, unexpected shifts in the velocity of money may cause the long-run correlation between inflation and money growth to deteriorate (Lucas, 1988; Reynard, 2006) and thereby distort the leading indicator properties of monetary aggregates.

A second motivation is that understanding short-run fluctuations of velocity is important for understanding the role of money in business cycles. According to the monetarist perspective, changes in the money stock are important sources of output fluctuations. Central to this view is the assumption that velocity is a stable function of a few macro variables, such as interest rates. A large variability of velocity at the business cycle frequency presents a challenge to this assumption, espe-

[1] The author benefitted from a course on mixed-frequency regressions by E. Ghysels. I would like to thank M. Falagiarda for comments and discussion. The views expressed by the author are his own and do not necessarily reflect those of the Eurosystem. The author remains responsible for any errors or omissions.

E-mail address: alexander.jung@ecb.europa.eu

http://dx.doi.org/10.1016/j.najef.2017.08.005
1062-9408/

cially when most of it cannot be explained by variations in the macro variables. Hence, it is important for a central bank to understand changes in velocity.

In empirical research broad money has received considerable attention, because the Federal Reserve announced monitoring ranges for M2 and the ECB has monitored a reference value for broad money M3. The aim of this paper is to explain and forecast velocity for broad money in the United States and in the euro area. When examining the relationship between money and prices, a central debate refers to whether velocity is constant or not and whether there are structural changes owing to factors such as financial innovation. The present empirical analysis considers that money velocity can be time-varying. Though, we do not treat shocks to potential output and equilibrium velocity as latent variables as in El-Shagi, Giesen, and Logan (2015).

When explaining and forecasting money velocity an issue is that velocity and its explanatory variables are not sampled at the same frequency. For example, velocity data have a quarterly frequency, whereas opportunity costs have a monthly (or even daily) frequency. The neglect of higher frequency information in standard approaches can have a bearing on both regressions and forecast performance, because latest information at the higher frequency is not incorporated in traditional approaches. Against this background, the contribution of the paper to the literature is threefold. First, it provides new results on the stability of broad money velocity covering the financial crisis period. Second, it estimates velocity for broad money for two major economies applying an approach that uses mixed frequency techniques (MIDAS), as proposed by Gyhsels, Santa-Clara, and Valkanov (2004) and Ghysels, Sinko, and Valkanov (2007). Third, by comparing the results obtained by MIDAS with traditional approaches, it informs the debate on whether mixed frequency regressions outperform conventional approaches.

The paper is organised as follows. Section 2 explains the data used for the study. Section 3 describes velocity trends of broad money and introduces traditional approaches and the MIDAS approach to modelling money velocity. Section 4 discusses the results of the velocity estimates and compares velocity forecasts from the two approaches. Section 5 concludes.

2. Data and descriptive analysis

In this section, we describe the data used in the study. For the euro area, which was created in 1999, the time series are relatively short, but previous studies have demonstrated that velocity can be explained by a trend and some standard macro variables (Brand, Gerdesmeier, & Roffia, 2002; Dreger & Wolters, 2009; Faruqee, 2005). In the case of the US, the history is longer and while previous papers explain velocity by a trend and macro variables, these studies also identify sizeable structural breaks, which cannot be explained by these factors (Anderson, Bordo, & Duca, 2016; Judson, Schlusche, & Wong, 2014; Orphanides & Porter, 2000). In this respect, Orphanides and Porter (2001) argue that a lesson from the policy failure during the Great Depression of the 1930s and the Great Inflation of the 1970s was the importance of monitoring monetary developments.

Stock and Watson (2007) argue that inflation has become harder to forecast owing to instabilities in the Phillips curve and despite the fact that inflation has become less volatile over recent decades. At the same time, it has been shown in the literature that a combination of monetary and economic indicators (Dreger & Wolters, 2014; Falagiarda & Sousa, 2017; Fischer, Lenza, Pill, & Reichlin, 2009; Hofmann, 2009) can improve the forecast performance using monetary data. Unexpected shifts in the velocity of money, which distort the leading indicator properties of monetary aggregates, could be an explanation why the performance of money as a leading indicator of inflation on times deteriorated.

The velocity of money is the frequency at which one unit of currency is used to purchase domestically-produced goods and services within a given time period. Money velocity (V) is defined using the quantity identity:

$$V_t = \frac{P_t \cdot YR_t}{M_t} \tag{1}$$

with P is the price level, YR is real income and M the money stock. Velocity is thus the ratio of the current value of total nominal transactions to the stock of money. It can be used to determine the velocity of a given component of the money supply, providing some insight into whether consumers and firms are saving or spending their money. Rewriting (1) in terms of growth rates yields:

$$\Delta v = \Delta yr + \Delta p - \Delta m \tag{2}$$

For the euro area, the empirical exercise of this paper covers the period since the start of monetary union (i.e., 1999–2016). For monetary aggregates in the euro area and inflation monthly series are available, whereas for (nominal and real) GDP only quarterly series can be used.[2] Measures of end-of-month outstanding amounts denominated in euro (source: ECB) are used for the broad monetary aggregates M3 for the euro area. The data is working day and seasonally adjusted. Nominal GDP for the euro area denominated in euro is the series reported by Eurostat which is compliant with ESA95 National Accounts and has been seasonally adjusted. For real housing wealth (WR), we use a series on households' non-financial assets (fixed assets and land underlying dwellings) from the flow-of-funds statistics (source: ECB), which is quarterly and has been deflated by the HICP and has been seasonally adjusted. We construct the opportunity cost of M3 as the difference between the yield on

[2] While using monthly data of industrial production would be a way out, it is known from the literature that these data are quite volatile.

short-term interest rates (EURIBOR three month rates) (source: BIS) and the average rate paid on M3 balances (source: ECB). Alternatively, opportunity costs have been computed as difference between the long-term interest rate on government bonds and the own rate of return of M3. While the indications are broadly similar, Levy, Calza, and Gerdesmeier (2001) show that the short-term spread as measure of opportunity cost normally behaves better than a long-term measure. When using daily data, we should calculate a component weighted average of returns of the components of M3 with daily interest rate data and monthly monetary weights. Since these data are not available, we included daily data on the EURIBOR (three months) as proxy for daily opportunity cost.

For the United States, the data were obtained from FRED Economic Data. GDP series, real and nominal are those reported by the US Bureau of Economic Analysis. Inflation refers to the consumer price deflator (source: US Bureau of Economic Analysis), which is available at the monthly frequency. We construct the opportunity cost of M2 as the difference between the yield on three-month treasury bills and the average rate paid on M2 balances (source: Federal Reserve). When analysing daily data we use the difference between three-month treasury rates and the fed funds rate (source: Alfred) as proxy for opportunity cost of M2. Moreover, household wealth data at the quarterly frequency were obtained from the financial accounts of the United States (series household net worth; source: Federal Reserve).

Before estimating the empirical functions, it may be useful to briefly review some basic time series properties of the data. Conventional unit root tests serve to check the integration properties of the variables. For the euro area, the results of the ADF-tests (see Table 1a) on the monetary aggregates, real GDP, velocity (all in logs), inflation and opportunity cost (measured in percentages per annum) indicate that these variables are all integrated of order one. In line with previous studies for the euro area, the ADF-tests indicate that broad money M3 and housing wealth are integrated of order two.

A weakness of the conventional unit root tests is their potential confusion of structural breaks in the series as evidence of non-stationarity. They may be biased towards a false unit root null when the data are trend stationary with a structural break, i.e., for the series that are found to be I(1), there may be a possibility that they are in fact stationary around the structural break(s), I(0), but are erroneously classified as I(1). We therefore conducted additional unit root tests with a breakpoint. In these tests, the null hypothesis is that the series has a unit root with a structural break against the alternative hypothesis that they are stationary with break. Regarding M3, the breakpoint unit root test point confirms the result from the Phillips-Perron test and suggests it to be an I(1) process.[3] For the US (see Table 1b), the results from both types of unit root tests fully match. They suggest that all variables considered including M2 and wealth are integrated of order one while annual inflation is stationary I(0).[4]

The unit root tests with breakpoints also indicate that the structural break related to the financial crisis may have affected several variables (interest rates, broad money and velocity). In the econometric analysis, we therefore include a financial crisis dummy (FINDUM) related to the collapse of Lehman Brothers on September 15, 2008. It captures the break owing to the financial crisis, and is zero until 2008Q3 and 1 otherwise. Furthermore, money demand models for the euro area display a break in the slope of its downward long-run trend around the period of the cash changeover (in 2002). We specify a dummy for the cash changeover (CASHDUM), which is 1 between 2001Q and 2002Q1 and zero otherwise.

3. Approaches to modelling money velocity

3.1. Velocity trends

A conventional approach is to describe money income velocity by a linear time trend for specific sample periods (5 year, 10 year, or even longer). Before applying this simple approach one has to examine whether the log level of velocity (v) is stationary around a linear trend (t):

$$v_t = \alpha + \beta t + \varepsilon_t \tag{3}$$

where ε_t is some mean-zero trend stationary process. A shortcoming of the linear trend is that velocity is not always stationary. There may be structural changes in the transmission mechanism and there can be statistical breaks in the series (for example when a large shock hits the money demand). Such factors may imply large forecast errors when forecasting velocity using linear trends.

Fig. 1 shows velocity trends for broad monetary aggregates in the US (M2) and for the euro area (M3) relative to a linear trend. In both cases, velocity displays a declining trend, but there have been substantial deviations from a linear trend, in the euro area in particular coinciding with the introduction of euro banknotes and coins in 2002 but also with the financial crisis in 2008. In the euro area, as a result of the heightened economic and financial uncertainty prevailing at that time, significant portfolio shifts into monetary assets took place between late 2000 and mid-2003 (Fischer et al., 2009). In recognition of this special factor, the ECB computed a series corrected for the estimated impact of these portfolio shifts and published it in real time. Though, in retrospective, M3 velocity adjusted for portfolio shifts exhibited a trend that was broadly similar to the

[3] Taking into account that the power of unit root tests is generally low, we made checks for robustness of the results by applying the Phillips-Perron tests on these variables for the identical sample. These tests show that the results are generally robust, but M3 appears to be a borderline case, since they suggest M3 to be integrated of order one.

[4] Note that the results of the unit root tests may be different, if longer runs of data are employed (e.g. annual inflation in the US would then be an I(1) process).

Table 1a
Unit root tests for euro area variables.

Variables	Level of series	Series in first differences	Series in second differences	Order of integration
ADF test statistics				
Monetary aggregates				
M3	−1.34	−2.69	−14.04**	I(2)
Output and prices				
YR	−2.42	−5.01**		I(1)
π	−1.48	−6.36**		I(1)
Opportunity costs and other variables				
OC	−2.83	−5.55**		I(1)
WR	−2.16	−1.61	−3.54**	I(2)
W	−2.36	−1.61	−9.56**	I(2)
Velocity				
VM3	−0.53	−5.56**		I(1)
Unit root tests with breakpoint				
M3	−2.79	−5.84**		I(1)
Output and prices				
YR	−1.15	−4.82**		I(1)
π	−2.51	−11.01**		I(1)
Opportunity costs and other variables				
OC	−3.99	−7.18**		I(1)
WR	−3.09	−4.06	−19.70**	I(2)
W	−3.13	−3.93	−10.55**	I(2)
Velocity				
VM3	−2.84	−6.44**		I(1)

Notes: the null hypothesis is that the series contains a unit root versus the alternative of no unit root; tests performed over full sample of each variable (1996Q1–2016Q2). **Indicates rejection of null at 1% significance; *Indicates rejection of null at 5% significance. VM3 denotes velocity of broad money M3. The other abbreviations of the variables are explained in the data section of the main text.

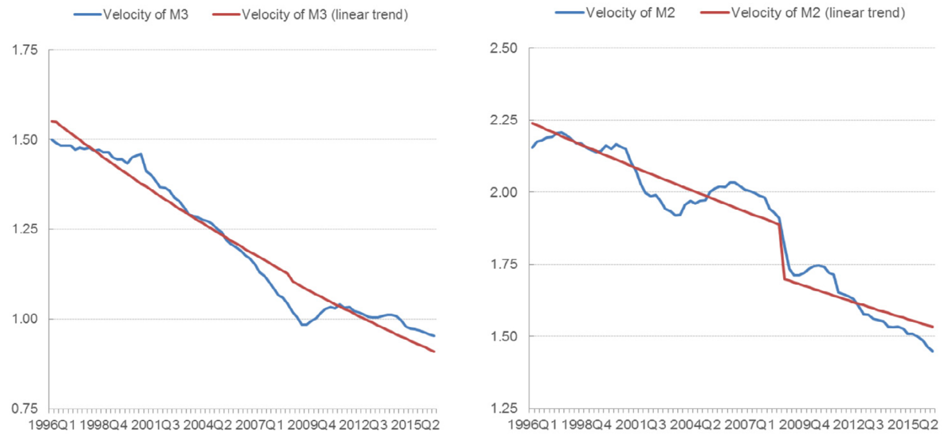
Table 1b
Unit root tests for US variables.

Variables	Level of series	Series in first differences	Series in second differences	Order of integration
ADF test statistics				
Monetary aggregates				
M2	−0.10	−6.15**		I(1)
Output and prices				
YR	−2.75	−5.72**		I(1)
π	−6.67**			I(0)
Opportunity costs and other variables				
OC	−2.44	−6.69**		I(1)
WR	−1.19	−6.83**		I(1)
W	−1.11	−6.76**		I(1)
Velocity				
VM2	0.13	−5.97**		I(1)
Unit root tests with breakpoint				
M2	−1.48	−7.29**		I(1)
Output and prices				
YR	−4.13	−6.72**		I(1)
π	−8.61**			I(0)
Opportunity costs and other variables				
OC	−4.19	−7.61**		I(1)
WR	−2.48	−7.18**		I(1)
W	−2.45	−7.64**		I(1)
Velocity				
VM2	−1.99	−6.66**		I(1)

Notes: the null hypothesis is that the series contains a unit root versus the alternative of no unit root; tests performed over full sample of each variable (1996Q1–2016Q2). **Indicates rejection of null at 1% significance; *Indicates rejection of null at 5% significance. VM2 denotes velocity of broad money M2. The other abbreviations of the variables are explained in the data section of the main text.

unadjusted series (Bordes, Clerc, & Marimoutou, 2007). In the US, the decrease in broad money velocity during the financial crisis has been attributed to heightened risk premia and to exceptionally low opportunity cost of broad money in an environment of low interest rates (Anderson et al., 2016).

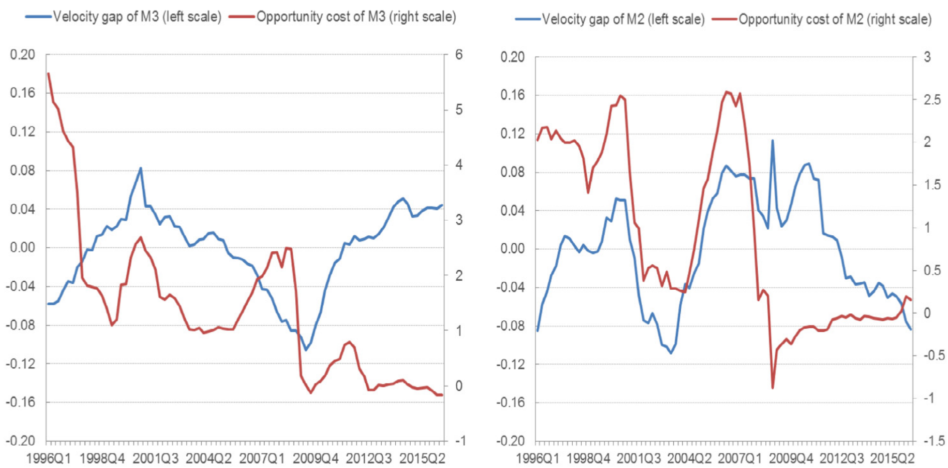
Fig. 2 shows the velocity gap (computed as the difference between actual velocity and its time trend) against the opportunity costs. As expected, both variables display a strong correlation. Until the start of the financial crisis in 2008, opportunity costs of money balances explain well velocity once accounted for the underlying equilibrium trend. However, thereafter



Source: ECB, Federal Reserve.

Note: The linear trend is estimated applying equation (3) that includes FINDUM.

Fig. 1. Velocity in the euro area (LHS) and the US (RHS).



Source: ECB, Federal Reserve.

Note: Velocity gaps are shown in logs and opportunity costs are shown as percentage points.

Fig. 2. Velocity gap and opportunity costs in the euro area (LHS) and the US (RHS).

there is evidence of a decoupling of the two variables. While this change could be attributable to a fundamental shift in the relationship between broad money and opportunity cost, it may be the by-product of both regulatory changes and the non-standard monetary policy measures adopted by the central banks (Judson et al., 2014).

3.2. Traditional approaches

When modelling money velocity trends, the literature suggests (e.g., Brand, Gerdesmeier, & Roffia, 2002) that a money demand framework is better than a simple trend (3), because these models also include macro variables. The static model is the most basic model, and is based on the assumption that money velocity moves with changes in the opportunity cost of money (Moore, Porter, & Small, 1990). This approach assumes that money demand has a unitary elasticity with respect to nominal income, conditional on the level of the opportunity cost.

The specification is as follows:[5]

$$v_t = \alpha + \beta \cdot OP_t + \gamma \cdot t + \delta \cdot D_t + \omega_t \tag{4}$$

where OP denotes opportunity cost of money, optionally a linear time trend (t) can be included and D is a dummy variable which can capture known velocity shifts. For example, a structural break can be owing to financial innovation, portfolio shifts or to a financial crisis.[6]

Beyer (2006) demonstrates that wealth, especially housing wealth, as additional variable in a money demand specification might be able to explain that behaviour without having recourse to a dummy variable. Due to multicollinearity problems, income and wealth effects cannot be easily separated unless it is assumed that the income elasticity can be restricted to unity. Then, Dreger and Wolters (2009) formulate a velocity equation which includes real wealth (WR) and no linear trend, which is as follows:

$$v_t = \alpha + \beta \cdot wr_t + \gamma \cdot OP_t + \delta \cdot \pi_t + \varepsilon \cdot D_t + \omega_t \tag{5}$$

where π is inflation and price homogeneity is assumed to be valid as a long-run condition.

3.3. MIDAS approaches

One common approach to integrating the information of higher frequency variables in the above regressions is to apply time averaging to the higher frequency variable. While conventional time averaging is also parsimonious, a disadvantage of it is that it can lead to an omitted variable bias and asymptotic inefficiency. An advantage of the MIDAS approach developed by Gyhsels, Santa-Clara, and Valkanov (2004) is that it incorporates the information contained in the higher frequency variable into the lower frequency regression in a parsimonious, and flexible manner. Relative to other methods dealing with mixed frequencies, such as data interpolation and State Space models, which make use of the Kalman filter, MIDAS has the advantage that it is more parsimonious and less sensitive to specification errors due to the use of non-linear lag polynomials. Moreover, this mixed data sampling approach has been already used in forecasting macroeconomic variables (see the survey by Foroni & Marcellino, 2013).

We specify a MIDAS regression model of (5) combining the information available from monthly and quarterly indicators with multiple lags of the predictors.[7] Initially, we run a regression where two higher frequency variables (OP and π) can be directly related to the lower frequency variable (v). However, the tests showed that a regression with nominal wealth (W) and one higher frequency variable (OP) displayed slightly better forecasting results. Therefore, we specify the regression:

$$v_t^Q = \alpha + \beta \cdot w_t^Q + B(L^{\frac{1}{3}};\theta) \cdot (\gamma \cdot OP_t^M) + \delta \cdot D_t + \omega_t \tag{6}$$

with the notations as above; superscripts "Q" and "M" denote quarterly and monthly frequency respectively; the function B (.) is a lag polynomial that determines the weights of the temporal aggregation:

$$B(L^{\frac{1}{3}};\theta) = \sum_{k=0}^{K} b(k;\theta) \cdot L^{\frac{k}{3}} \tag{7}$$

where the high-frequency (monthly) lag operator of a monthly regressor x is defined as $x_{t-1/3}^M = L_t^{1/3} \cdot x_t^M$. The polynomial is used to avoid parameter proliferation for long high frequency lags K. In the literature, several functional polynomial lag functions have been used in MIDAS regressions, e.g. step weighting, (exponential) Almon lag weighting and beta polynomial weighting (for details see Ghysels, 2016).[8]

As an alternative benchmark, the unrestricted U-MIDAS regression model, which does not use a polynomial, is considered. Its advantage is that it does not require making any assumption about the weights that should be attached to each higher frequency observation. In the case of U-MIDAS the number of the estimated parameters is considerably higher compared to MIDAS applications that apply polynomial weighting. But, when estimating a model with small differences in frequencies (such as the mix of quarterly and monthly data) and few lags, Foroni, Marcellino, and Schumacher (2015) show the parameter proliferation issue is not so much of a concern and regressions can still have a good performance.

[5] Note Judson et al. (2014) use a further variant of the model, namely the conference aggregate model. It assumes that velocity has a long-run cointegrating relationship with the natural log of the opportunity cost of holding money, with stationary errors.

[6] Note, for the euro area Bordes et al. (2007) provide evidence of structural changes around the start of monetary union, coinciding with the cash changeover (in 2000 and 2001). For the United States, Orphanides and Porter (2001) document a structural break at the beginning of the 1990s, which in part can be attributed to the improvement of financial services and to financial innovation.

[7] For reasons of comparability with (5), this specification does not include lags of the dependent variable. In an ADL-MIDAS model, such lags could be easily added.

[8] For an illustration of the polynomials, see Armesto, Engemann, and Owyang (2010).

4. A comparison of traditional and MIDAS models

Over recent years, the financial crisis had a strong impact on the global economy and on the transmission of monetary policy. Several economies experienced sharp falls in interest rates and even tested the zero lower bound. In response to the massive shocks, central banks lowered policy rates and maintained close to their historical lows. They adopted non-standard measures supporting the growth of the domestic money supply and implicitly reducing velocity. As a result, the velocity of money declined considerably during the financial crisis. But, given that money velocity depends on a number of factors (such as financial innovations, opportunity costs, wealth and risk premia), the observed decline in the velocity of money could be either owing to a structural break or to the behaviour of firms and households hoarding cash instead of spending it.

First, we discuss the results of the velocity estimates for the euro area and the US. Second, we make a horserace and compare the ability of traditional and MIDAS models to forecast euro area velocity. Third, we provide results using MIDAS models with daily financial data.

4.1. Results of the velocity estimates

Tables 2a and 2b show that in Eq. (3) both the constant and the trend coefficient are significant at the 1% level and that these regressions have a high explanatory power. Once annualised this estimate gives a trend of −2.6% per year for euro area M3, whereas the annualised trend is −1.2% per year for M2 in the US. Given that the validity of Eq. (3) requires that log velocity is trend stationary, we repeat the unit root tests for that variable and this time include a constant and a trend. The results confirm that log of velocity is an I(1) variable (i.e. non-stationary). This implies that deviations of velocity from trend are persistent and that velocity shocks are permanent and not transitory. It could mean that the standard model (3) fares poorly compared with other alternatives, but it could also mean that the trend is not linear. Indeed, there was a shift in velocity coinciding with the outbreak of the financial crisis and the move to a regime with very low interest rates. While the present analysis does not dismiss a link between money and prices, it suggests that the link has changed over time (see also Falagiarda & Sousa, 2017).

Moreover, in order to check for time-variation in the trend, we follow Bai and Perron (1998) and run the regressions again checking for the presence of multiple breakpoints. The test results are somewhat sensitive to the choice of the information criteria applied to determine the lag specification.[9] If a higher weight is given on the AIC and the Hannan-Quinn criteria, the tests robustly detect a break in trend velocity in 2008 around the start of the financial crisis, both in the euro area and in the US. That is, we obtain this result if we test sequentially the hypothesis of m + 1 breaks given m breaks and if we test the hypothesis on m break dates given m breaks that are global minimisers of the sum of squared residuals.

The results of Eq. (4) confirm that, in addition to a constant, opportunity cost drive the behaviour of broad money. Given that these regressions have a lower explanatory power than those using a simple linear trend, we augment the model and include real (housing) wealth and inflation as additional explanatory variables. The results show that in comparison to the previous approaches, regressions (5) display the best fit during the sample 1996–2016. In line with the literature on money demand (e.g., Beyer, 2006; Friedman, 1988), which finds financial wealth to be an important driver of money demand, this result mirrors the significant influence from the wealth variable, but does not find (annual) inflation to be a significant driver of broad money velocity.

Tables 2a and 2b also shows that the results obtained from the mixed frequency approach (6) are similar to what has been obtained with the traditional approach based on (5), i.e. the wealth variable and opportunity cost have a significant effect on broad money velocity, both in the US and the euro area. In the mixed-frequency regressions, the standard deviations are slightly lower, and the coefficients are of comparable magnitude. For comparability, we only report the results of the more general U-MIDAS regression (with one lag). We also made tests with alternative lag polynomials (step weighting, Almon lag, exponential Almon lag, beta polynomial), confirming these results. Still, the forecast performance in both approaches could be different.

4.2. Relative forecast performance

In this section, we address the question, whether the forecasting performance applying mixed-frequency approaches is superior to traditional approaches. To investigate the forecasting performance of the mixed-frequency MIDAS models, we compare them with some benchmark models that are estimated versions of the traditional approaches (models (3), (4) and (5)). For each equation, we generate out-of-sample forecasts applying both a rolling and a recursive forecasting scheme. Relative to the estimated specifications in the previous section, additional lags have been included in the mixed-frequency regressions for the purpose of forecasting. While MIDAS allows intra-period forecasting (nowcasting), in this study we focus on end-of-period forecasting.

[9] For brevity of the analysis the test results are not reported here, but are available from the author upon request.

Table 2a
Velocity estimation for euro area M3.

Velocity	Regressors					
	Constant	Housing wealth	Opportunity costs	Inflation	Trend	R^2
Traditional approaches:						
(3)	1.40^{**}				-0.006^{**}	*0.92*
	(0.18)				(0.00)	
(4)	0.21^{**}		3.28^{**}			*0.73*
	(0.03)		(1.10)			
(5)	5.76^{**}	-0.45^{**}	$2.97^{**,1)}$	−0.02		*0.98*
	(0.37)	(0.03)	(0.72)	(0.61)		
Mixed frequency approach:						
(6) U-MIDAS$^{2)}$	5.80^{**}	-0.45^{**}	$2.71^{**,1)}$	0.07		*0.97*
	(0.31)	(0.02)	(0.54)	(0.44)		

Notes: Sample 1999Q1–2016Q2. *Indicates significance at 5%; **at 1%; standard error in parenthesis; t-value in square brackets; the regressions include FINDUM. HAC consistent errors are reported. 1) This equation includes the long-term measure for opportunity cost instead of the short-term measure. 2) For better comparability, we report the results of version (6) that decomposes nominal wealth into real wealth and inflation; U-MIDAS results are reported with one lag.

Table 2b
Velocity estimation for US M2.

Velocity	Regressors					
	Constant	Housing wealth	Opportunity costs	Inflation	Trend	R^2
Traditional approaches:						
(3)	1.31^{**}				-0.003^{**}	*0.95*
	(0.04)				(0.00)	
(4)	0.67^{**}		3.11^{**}			*0.86*
	(0.01)		(1.00)			
(5)	2.19^{**}	-0.27^{*}	3.35^{**}	−0.3		*0.95*
	(0.73)	(0.13)	(1.08)	(1.65)		
Mixed frequency approach:						
(6) U-MIDAS$^{1)}$	2.18^{**}	-0.27^{**}	3.15^{**}	−0.28		*0.95*
	(0.13)	(0.02)	(0.48)	(0.49)		

Notes: Sample 1996Q1–2016Q2. *Indicates significance at 5%; **at 1%; standard error in parenthesis; t-value in square brackets; the regressions include FINDUM. HAC consistent errors are reported. 1) For better comparability, we report the results of version (6) that decomposes nominal wealth into real wealth and inflation; U-MIDAS results are reported with one lag.

We report the forecasting performance for out-of-sample forecasts of one to eight quarters ahead ($h = \{1, \ldots, 8\}$).[10] As is standard in the macroeconomics literature, we conduct a recursive forecasting exercise by which the model is specified and re-estimated with an observation window that increases one-to-one with the sample size. In addition, we conduct a rolling window forecasting exercise by which the model is specified and re-estimated with a fixed estimation sample size. In this case, the test statistics are computed using rolling (out-of-sample) windows of a given size. We choose an observation window size of 44 quarters, which leaves about one third of the sample for forecast evaluation purposes. A potential issue is that the results could be sensitive to the choice of the window size. This concern we address by cross-checking our results with those obtained from the recursive approach. For both forecasting approaches, the evaluation sample is between 2010Q1 and 2015Q4, providing 6 years for comparison and covering the financial crisis episode.

4.3. Comparisons based on relative mean-squared prediction errors (MSPEs)

To compare forecasting performance across models, it is common in the literature on mixed frequency analysis to choose a benchmark model and to compute relative mean-squared prediction errors (MSPEs) and to compare them across models and forecast horizons.[11] As a benchmark, we choose the linear trend model (3). We compute the pairwise relative MSPE of each model to the benchmark and average over all models within a class. The relative MSPE is defined as the ratio of the square root of the mean square forecast error of model m and that of the benchmark model b (t_1 and t_2 refer to the first and the last period of the forecast evaluation sample):

[10] Note: In the case of the mixed-frequency models, we could also compute intra-period forecasting results, which extend the quarter by one or two months ($h = 1/3$ and $2/3$ respectively).
[11] Such comparisons are sometimes also based on other measures such as the mean absolute error (MAE) or Theil's inequality coefficient (U).

$$relative\ MSPE = \frac{\frac{1}{t_2-t_1+1}\cdot\sum_{t=t_1}^{t_2}\left(\pi_{t+h}^{h}-\hat{\pi}_{m,t+h|t}\right)^2}{\frac{1}{t_2-t_1+1}\cdot\sum_{t=t_1}^{t_2}\left(\pi_{t+h}^{h}-\hat{\pi}_{b,t+h|t}\right)^2} \tag{8}$$

Forecasts are evaluated against final observations. In principle, it could be argued that it would be better to use real-time data instead. For the euro area such data have been collected for a set of key variables (for a description see Giannone, Henry, Lalik, & Modugno, 2010), for the US these data are available from the Alfred database of the Federal Reserve of St. Louis. While in the euro area differences between macro variables and final data tend to be small, in the US these revisions may matter, in particular for GDP data (Orphanides, 2001). We checked that for both inflation data and monetary aggregates there is not much of a difference between real-time data and final releases. Moreover, since we are using wealth and not GDP as explanatory variable in the comparisons, we conduct the exercise with final data.

Tables 3a and 3b show the results of the forecast comparison based on relative MSPEs applying recursive estimation and rolling-window estimation, respectively. In line with the notion that forecast uncertainty is higher for longer horizons, this exercise suggests that longer forecast horizons coincide with higher RMSEs. The results applying traditional approaches show that in the sample considered model (5), which includes a wealth variable, was systematically better than model (4), which contains a linear trend, opportunity cost of money and a break owing to the outbreak of the financial crisis. Regarding mixed-frequency analysis, the results show that the relative forecast performance among three distinct weighting methods of the higher frequency variable (step weighting, Almon polynomial lag, beta polynomial) is broadly similar, but may vary slightly over the forecast horizon. We find that the results are typically robust when using alternative lag polynomials. For the euro area improvements in the forecast performance relative to traditional approaches can be observed for all forecast horizons considered, i.e. one to eight quarters ahead. By contrast, for the US the results are similar to those obtained with traditional approaches and we detect no gains in terms lower mean square prediction errors by applying mixed frequency techniques. This finding seems to reflect differences in the adjustment of money velocity to the shock originating from the financial crisis in both economic areas (see also Fig. 2).

4.4. Tests for relative predictive ability

In order to compare the forecasts from model (5) and (6), we make a further test on predictive ability. In the literature the Diebold-Mariano (DM) test for the equality of forecast accuracy of two forecasts (Diebold & Mariano, 1995) is widely used to test for systematic differences in the forecasting errors.[12] In our case, an issue could be that the forecasting models are nested. Two models are nested if one model contains all the terms of the other, and at least one additional term. If the two models involved are nested, the standard asymptotic theory for the Diebold-Mariano test statistics is invalid (e.g., Clark & McCracken, 2001). Because models (5) and (6) use the same variables, but model (6) adds additional terms at the monthly frequency, they cannot be considered to be strictly non-nested, implying that the standard Diebold-Mariano test does not suffice.

We therefore apply a variant of the DM test, as proposed by Clark and West (2007), which accounts for nested models and employs a non-standard limiting distribution. In finite samples, under the assumption that model 1 is the correctly specified model, the sample mean square prediction error (MSPE) from the parsimonious model will generally be lower than the sample MSPE from the alternative model 2. The significance of the differences is tested through the computation of the MSPE-adjusted statistic (Clark & West, 2007):

$$\hat{f}_{t+h} = (y_{t+h} - \hat{y}_{1,t+h})^2 - \left[(y_{t+h} - \hat{y}_{2,t+h})^2 - (\hat{y}_{1,t+h} - \hat{y}_{2,t+h})^2\right] \tag{9}$$

The null hypothesis is that both models are equally accurate in predicting a variable y. The MSPE-adjusted statistic is computed by regressing $\hat{f}_{t+h}$ on a constant, and using the resulting t-statistic as a check for the significance of a zero coefficient. When the forecast is calculated using rolling regressions, the limiting distribution of the test statistics under the null hypothesis is standard normal. Despite the distribution is not normal for the recursive regressions, based on simulation evidence Clark and West (2007) show that one can still use a one-sided test. The null hypothesis of equal forecasting power should be rejected when the test statistics is greater than +1.645 (at the 5%-significance level, one-sided test). In this case, the model 2 would be relatively better. Moreover, in these tests we use heteroscedasticity-autocorrelation consistent standard forecast errors.

Table 4 shows that in the case of the euro area the mixed-frequency models are usually statistically significantly better than their respective single-frequency benchmark (5). We find improvements in the accuracy of velocity forecasts at all horizons considered (one to eight quarters ahead) for the euro area. For the US possible gains only refer to shorter-term forecast horizons.[13] The latter behaviour is consistent with our finding that in the sample considered the money demand model forecasts of velocity for the US did not outperform a linear trend with one break around the start of the financial crisis in terms of the MSPE.

[12] Diebold has clarified that the DM test was not intended for comparing models.

[13] Note that in the case of the US gains in terms of relative forecasting accuracy refer to the rolling windows exercise only.

Table 3a
Relative MSPEs of velocity forecasts (euro area).

Variable	Forecast horizon (in quarters)								Forecast method
	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6	h = 7	h = 8	
Broad money velocity (VM3)									
Traditional approaches									
Eq. (4)	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	Recursive
Eq. (5)	0.96	0.96	0.96	0.95	0.95	0.95	0.94	0.94	Recursive
Eq. (4)	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.97	Rolling window
Eq. (5)	0.97	0.97	0.96	0.96	0.95	0.95	0.95	0.94	Rolling window
Mixed frequency approaches									
Step weighting	0.95	0.95	0.94	0.94	0.93	0.93	0.92	0.92	Recursive
Almon polynomial	0.95	0.95	0.94	0.94	0.93	0.93	0.92	0.92	Recursive
Beta weighting	0.95	0.95	0.94	0.94	0.93	0.93	0.92	0.91	Recursive
Step weighting	0.96	0.95	0.94	0.94	0.93	0.92	0.91	0.91	Rolling window
Almon polynomial	0.96	0.95	0.95	0.94	0.93	0.92	0.91	0.91	Rolling window
Beta weighting	0.96	0.95	0.94	0.94	0.93	0.92	0.91	0.91	Rolling window

Notes: Sample: 1999Q1–2015Q4. Evaluation sample 2010Q1–2015Q4. Forecasting performance for quarterly velocity of selected individual traditional and mixed-frequency models measured by MSPE of the corresponding indicator relative to MSPE of the benchmark (3).

Table 3b
Relative MSPEs of velocity forecasts (United States).

Variable	Forecast horizon (in quarters)								Forecast method
	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6	h = 7	h = 8	
Broad money velocity (VM2)									
Traditional approaches									
Eq. (4)	1.03	1.03	1.03	1.03	1.04	1.04	1.04	1.04	Recursive
Eq. (5)	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.02	Recursive
Eq. (4)	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	Rolling window
Eq. (5)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	Rolling window
Mixed frequency approaches									
Step weighting	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	Recursive
Almon polynomial	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	Recursive
Beta weighting	1.04	1.03	1.03	1.02	1.02	1.02	1.00	1.01	Recursive
Step weighting	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	Rolling window
Almon polynomial	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	Rolling window
Beta weighting	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	Rolling window

Notes: Sample: 1999Q1–2015Q4. Evaluation sample 2010Q1–2015Q4. Forecasting performance for quarterly velocity of selected individual traditional and mixed-frequency models measured by MSPE of the corresponding indicator relative to MSPE of the benchmark (3).

Table 4
Tests for relative accuracy for alternative forecast horizons.

Models	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6	h = 7	h = 8	Forecast method
Euro area									
Eq. (6a)	2.46	2.62	2.83	3.14	3.44	3.91	4.41	5.09	Recursive
Eq. (6b)	2.30	2.37	2.51	2.69	3.17	3.60	4.14	4.45	Recursive
Eq. (6c)	2.36	2.52	2.73	3.01	3.34	3.87	4.48	5.22	Recursive
Eq. (6a)	2.42	2.53	2.65	2.89	3.11	3.47	3.92	4.59	Rolling window
Eq. (6b)	2.14	2.26	2.45	2.66	3.10	3.40	3.74	4.04	Rolling window
Eq. (6c)	2.30	2.41	2.56	2.75	3.08	3.48	4.05	4.71	Rolling window
United States									
Eq. (6a)	0.28	0.48	0.39	0.37	0.32	0.34	0.68	0.79	Recursive
Eq. (6b)	−1.57	−1.55	−1.59	−1.74	−1.92	−2.05	−2.37	−2.88	Recursive
Eq. (6c)	0.61	1.25	1.04	1.49	1.42	1.21	1.50	1.12	Recursive
Eq. (6a)	1.64	1.65	1.80	1.76	1.68	1.51	1.46	1.45	Rolling window
Eq. (6b)	1.98	2.10	2.27	2.30	2.18	1.82	1.55	1.38	Rolling window
Eq. (6c)	0.41	0.44	0.39	0.30	0.33	0.44	0.60	0.68	Rolling window

Notes: Sample: 1999Q1–2015Q4. Evaluation sample 2010Q1–2015Q4. This table reports the t-statistics of the test of equal MSPE of two models proposed by Clark and West (2007), where the benchmark model is the corresponding model with single frequency (5). A t-statistic greater than +1.645 (for a one sided 0.05 test) indicates that Model 2 (rows) has a significant smaller MSPE than Model 1 (columns) and vice versa. HAC errors have been computed. (6a) refers to step weighting, (6b) refers to Almon polynomial and (6c) refers to a beta polynomial.

Table 5a
Relative MSPEs of velocity forecasts with daily interest rate data (euro area).

Variable	Forecast horizon (in quarters)								Forecast method
	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6	h = 7	h = 8	
Broad money velocity (VM3)									
Mixed frequency approaches									
Step weighting	0.97	0.97	0.97	0.97	0.97	0.96	0.96	0.96	Recursive
Almon polynomial	0.97	0.97	0.97	0.97	0.97	0.96	0.96	0.96	Recursive
Beta weighting	0.97	0.97	0.97	0.97	0.96	0.96	0.96	0.96	Recursive
Step weighting	0.98	0.98	0.98	0.97	0.97	0.97	0.96	0.96	Rolling window
Almon polynomial	0.98	0.98	0.98	0.97	0.97	0.97	0.96	0.96	Rolling window
Beta weighting	0.97	0.96	0.96	0.95	0.94	0.94	0.93	0.93	Rolling window

Notes: Sample: 1999Q1–2015Q4. Evaluation sample 2010Q1–2015Q4. Forecasting performance for quarterly velocity of selected individual traditional and mixed-frequency models measured by MSPE of the corresponding indicator relative to MSPE of the benchmark (3).

Table 5 b
Relative MSPEs of velocity forecasts with daily interest rate data (United States).

Variable	Forecast horizon (in quarters)								Forecast method
	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6	h = 7	h = 8	
Broad money velocity (VM2)									
Mixed frequency approaches									
Step weighting	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.02	Recursive
Almon polynomial	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	Recursive
Beta weighting	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	Recursive
Step weighting	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.02	Rolling window
Almon polynomial	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.02	Rolling window
Beta weighting	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.02	Rolling window

Notes: Sample: 1999Q1–2015Q4. Evaluation sample 2010Q1–2015Q4. Forecasting performance for quarterly velocity of selected individual traditional and mixed-frequency models measured by MSPE of the corresponding indicator relative to MSPE of the benchmark (3).

4.5. Velocity estimates with daily financial data

Since macroeconomic data are typically sampled at quarterly or monthly frequency, the standard approach is to match macro data with monthly or quarterly aggregates of financial series to build prediction models. In a robustness check (Eqs. (10) and (11)), we replace the monthly opportunity cost measure by the corresponding daily measure and include it in Eq. (6). This is important because using monthly financial data may imply that information through temporal aggregation is lost when forecasting quarterly data series, if financial data are observable at the daily frequency (Andreou, Ghysels, & Kourtellos, 2010). In the robustness check, the higher frequency variable (OP) can be directly related to the lower frequency variable (v):

$$v_t^Q = \alpha + \beta \cdot wr_t^Q + \gamma \cdot \pi_t^Q + B(L^{\frac{1}{n}};\theta) \cdot \delta \cdot OP_t^D + \varepsilon \cdot D_t + \omega_t \tag{10}$$

with the notations as above; n denotes the number of days in a month; superscripts "Q" and "D" denote quarterly and daily frequency respectively; the function B(.) is a lag polynomial that determines the weights of the temporal aggregation:

$$B\left(L^{\frac{1}{3}};\theta\right) = \sum_{k=0}^{K} b(k;\theta) \cdot L^{\frac{k}{n}} \tag{11}$$

where the high-frequency (daily) lag operator of a daily regressor x is defined as $x_{t-1/n}^M = L_t^{1/n} \cdot x_t^D$.

Tables 5a and 5b show the results for the velocity estimates when daily opportunity cost data are included. For the euro area, the results are better than the benchmark model, but including daily financial data does not help to improve velocity forecasting relative to a mixed frequency model which includes monthly data. A possible explanation for this finding is that financial data can be quite noisy and that the temporal aggregation made with monthly data helps to filter out the noisy component. For the US the results are largely similar. With the inclusion of daily opportunity cost data, the velocity model did not outperform the forecast obtained from a linear trend that takes into account a break around the start of the financial crisis. These findings are typically robust to the chosen specification of lag polynomials.

5. Conclusions

This paper contributes to the literature by explaining and forecasting velocity of broad money in the euro area and the United States. In the framework of money demand models, it considers MIDAS as alternative forecasting method suitable for forecasting with mixed-frequency data.

Our results show that despite financial innovations, over the last two decades broad money velocity has followed a declining trend both in the euro area and the US with a break around the start of the financial crisis in both economies. While the present analysis detects one significant shift in equilibrium velocity around the start of the financial crisis, instabilities in the link between broad money and prices appear to have been temporary. Then, we show that a money demand model that includes housing wealth and opportunity cost as explanatory variables tracks well velocity of broad money even in the presence of large-scale asset purchase programmes and an environment of very low interest rates. The results applying traditional approaches show that in the sample considered a model, which includes a wealth variable, was systematically better than a model, which contains a linear trend, opportunity cost of money and a break owing to the outbreak of the financial crisis.

A new result is that applying mixed-frequency techniques, we find improvements in velocity forecasts for the euro area at all forecast horizons considered (i.e. one to eight quarters ahead), whereas for the US possible gains only refer to shorter-term forecasts. This finding seems to reflect differences in the adjustment of money velocity to the shock originating from the financial crisis in both economic areas. In this context, we find that the inclusion of daily financial data for opportunity cost did not help to improve velocity forecasts beyond what has been achieved in mixed-frequency models that use monthly data. While applying mixed-frequency techniques can help to improve velocity forecasts, forecasting turning points remains challenging also under this approach.

Regarding monetary policy implications, we suggest that good velocity forecasts can help policymakers in assessing the risks to price stability over longer horizons. Accounting for changes in equilibrium velocity is important to cross-check inflation forecasts from a monetary analysis perspective. Taking this information into account could help to make better money-based inflation forecasts. Whether this has actually been the case could be examined in more depth in a follow-up research using real-time data.

References

Anderson, R., Bordo, M., & Duca, J. (2016). Money and velocity during financial crises: from the Great Depression to the Great Recession. NBER Working Paper No. 22100.

Andreou, E., Ghysels, E., & Kourtellos, A. (2010). Regression models with mixed sampling frequencies. *Journal of Econometrics, 158*(2), 246–261.

Armesto, M., Engemann, K., & Owyang, M. (2010). Forecasting with mixed frequencies. *Federal Reserve of St. Louis Review, 92*(6), 521–536.

Bai, J., & Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica, 66*, 47–78.

Beyer, A. (2006). A stable model for euro area money demand revisiting the role of wealth. ECB working paper No. 1111.

Bordes, C., Clerc, L., & Marimoutou, V. (2007). Is there a structural break in equilibrium velocity in the euro area? Banque de France working paper No. 168.

Brand, C., Gerdesmeier, D., & Roffia, B. (2002). Estimating the trend of M3 income velocity underlying the reference value for monetary growth. ECB occasional paper No. 3.

Brunner, K., & Meltzer, A. (1963). Predicting velocity: Implications for theory and policy. *Journal of Finance, 18*(2), 319–354.

Clark, T., & McCracken, M. (2001). Tests of equal forecast accuracy and encompassing for nested models. *Journal of Econometrics, 105*(11), 85–110.

Clark, T., & West, K. (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of Econometrics, 138*(1), 291–311.

Diebold, F., & Mariano, R. (1995). Comparing predictive accuracy. *Journal of Business and Economic Studies, 13*(3), 253–263.

Dreger, C., & Wolters, J. (2009). Money velocity and asset prices in the euro area. *Empirica, 36*, 51–63.

Dreger, C., & Wolters, J. (2014). Money demand and the role of monetary indicators in forecasting euro area inflation. *International Journal of Forecasting, 30*, 303–312.

El-Shagi, M., Giesen, S., & Logan, K. (2015). The quantity theory revisited: A new structural approach. *Macroeconomic Dynamics, 19*, 58–75.

Falagiarda, M., & Sousa, J. (2017). Forecasting euro area inflation using targeted predictors: Is money coming back? ECB working paper No. 2015.

Faruqee, H. (2005). Declining money velocity in the euro area: implications for the ECB's monetary analysis. *IMF Country Report, 05*(266), 32–61.

Fischer, B., Lenza, M., Pill, H., & Reichlin, L. (2009). Monetary analysis and monetary policy in the euro area 1999–2006. *Journal of International Money and Finance, 28*(7), 1138–1164.

Fisher, I. (1911). *The purchasing power of money*. New York: MacMillan.

Foroni, C., & Marcellino, M. (2013). A survey of econometric methods for mixed frequency data. Norges Bank Working Paper No. 2013-06.

Foroni, C., Marcellino, M., & Schumacher, C. (2015). Unrestricted mixed data sampling (MIDAS): MIDAS regressions with unrestricted lag polynomials. *Journal of the Royal Statistical Society, 178*, 57–82.

Friedman, M. (1988). Money and the stock market. *Journal of Political Economy, 96*(2), 221–245.

Ghysels, E. (2016). *MIDAS Matlab toolbox (version 2.1). Mimeo available from Eric Ghysels website*. University of North Carolina.

Ghysels, E., Sinko, A., & Valkanov, R. (2007). MIDAS regressions: Further results and new directions. *Econometric Reviews, 26*, 53–90.

Giannone, D., Henry, J., Lalik, M., & Modugno, M. (2010). An area-wide real-time database for the euro area. ECB working paper No. 1145.

Gyhsels, E., Santa-Clara, P., & Valkanov, R. (2004). *The MIDAS touch: Mixed data sampling regression models*. University of North Carolina and UCLA Discussion Paper.

Hofmann, B. (2009). Do monetary indicators lead euro area inflation? *Journal of International Money and Finance, 28*, 1165–1181.

Judson, R., Schlusche, B., & Wong, V. (2014). Demand for M2 at the zero lower bound: the recent U.S. experience. Federal Reserve Board Finance and Economics Discussion Series No. 2014-22.

Laidler, D. (1990). Understanding velocity: New approaches and their policy relevance—introduction. *Journal of Policy Modeling, 12*(2), 141–163.

Levy, J., Calza, A., & Gerdesmeier, D. (2001). Euro area money demand: measuring the opportunity costs appropriately. IMF Working Paper No. 01/179.

Lucas, R. (1988). Money demand in the United States: A quantitative review. *Carnegie Rochester Series on Public Policy, 29*(1), 137–167.

Moore, G., Porter, R., & Small, D. (1990). Modeling the disaggregated demands for M2 and M1: The U.S. experience in the 1980s. In P. Hooper, K. Johnson, D. Kohn, D. Lindsey, R. Porter, & R. Tryon (Eds.), *Financial sectors in Open Economies: Empirical analysis and policy issues* (pp. 21–105). Federal Reserve System, Board of Governors.

Orphanides, A. (2001). Monetary policy rules based on real-rime data. *American Economic Review, 91*(4), 964–985.

Orphanides, A., & Porter, R. (2000). P∗ revisited: Money-based inflation forecasts with a changing equilibrium velocity. *Journal of Economics and Business, 52*(1/2), 87–100.

Orphanides, A., & Porter, R. (2001). Money and inflation: the role of information regarding the determinants of M2 behaviour. In H.-J. Klöckers & C. Willeke (Eds.), *Monetary analysis: Tools and applications*. Frankfurt am Main: ECB.

Reynard, S. (2006). Money and the great disinflation. Swiss National Bank Working Paper No. 2006-7.

Stock, J., & Watson, M. (2007). Why has U.S. inflation become harder to forecast? *Journal of Money, Credit and Banking, 39*(1), 3–34.