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Empirical justification of the elementary model of money circulation

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HIGHLIGHTS

- The set of equation for money circulation in the production system is discussed.
- The contribution to GDP from the money system is defined and discussed.
- The applicability of the model to the money circulation in Russia is confirmed.

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ABSTRACT

This paper proposes an elementary model describing the money circulation for a system, composed by a production system, the government, a central bank, commercial banks and their customers. A set of equations for the system determines the main features of interaction between the production and the money circulation. It is shown, that the money system can evolve independently of the evolution of production. The model can be applied to any national economy but we will illustrate our claim in the context of the Russian monetary system.

1. Introduction

In the previous article [1], considering a national economic system and following the macroeconomic lines defined by Godley and Lavoie [2], we succeeded in designing a simple model of money circulation that incorporated the government, the central and commercial banks, and many producers and consumers—customers of commercial banks. To catch the main peculiarities of money circulation, we have developed a simple macroeconomic scheme, each actor of which, apart of the central bank, is characterized by its deposit and debt, and the actors are interacting via fluxes of products and money. The model allows us to discuss some features of the relationship between the money and real-production layers of the system, which was the main aim of the previous article [1]. This lays in the context of appeal of some authors [3,4] calling for the development of new tools to study and better understands the relationship between the real economy and GDP.

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The developed model describes money circulation disregarding its mechanisms, resulting eventually from money exchange between separate agents, that was described and investigated in many modern work [5–10]. Physicists suggested several potential solution in the study of the monetary circulation. Wang and Zhang [5] developed agent model to show that the velocity of the money is mainly based on consumers' income and debts. There also exists works using a macro perspective to characterize the evolution of wealth [6,7,10], emphasizing an existing gap between monetary system and productive sphere. In line with these studies, this paper uses a descriptive macro-model to show that the evolution of the monetary system is mainly based on financial fluxes meaning that it can therefore move independently of the evolution of production. We think that some characteristic peculiarities of money circulation could be better understood under a general look at the system, considering the balance relation for each economic actor. This article offers an empirical justification of the circuitist approach of money circulation in a Russian context.

We shall remind the main aim of our research. The economic system takes the form of a productive process likely to generate real wealth for society. This created value is usually estimated through the Gross Domestic Product (GDP), which is the sum of outputs of all production sectors. In line with [1] and earlier work one of the authors [11], it is convenient to assume that the production layer consists of three sectors: the first sector creates basic production equipment, the second one creates non-material intermediate products (consumed by the other two sectors and stored in warehouses and depositories for the future production and non-production consumption) and the third sector creates products for direct consumption by humans. This Keynesian way of structuring the national economy is common in the economic literature, and the conventional expression for GDP takes the following form

$$Y = I + G + C, \tag{1}$$

where I, G and C are the final outputs of the sectors, that are considered as components of the GDP, estimated (in money units) by value of created commodities: the quantity I is estimation of the value of investment, distributed over the three sectors; the quantity G is estimation (in money units) of results of all long-lasting projects (investment in human capital, R&D, infrastructure and so on); we consider this quantity is equal to the government expenditures. For simplicity, it is assumed that the product of the third sector in the amount C is completely consumed.

The exchange of products between sectors, as we know, is accompanied with circulation of money. Globally then, the finance layer generates the amount of money corresponding to the value created by the productive layer. In this context, there is a correspondence between fluxes of money and fluxes of products. However, one can note that there is no indication of the activity of finance layer in Eq. (1). To introduce money into this frame, we investigated [1] the motion of products and money in the three-sector model, which has allowed us to generalize the expression for the Gross Domestic Product, that will be discussed in Section 2.4. This article aims to confirm the previous results and, in line with the circuitist approach to money [12–16], suggest a potential research by providing in this way a solid base to understand interactions between the real-production and money sides of the economy.

In the next section, we review the balance relations for the actors of the model: the government, a central bank, commercial banks and many customer of the commercial banks. In accordance with the works of Keen [15,16], we will treat debt as a data record rather than a negative money. This specific assumptions is very important since it implies that money used to repay debts goes into a debt account which negates an equivalent sum of debt.[1]

2. The actors of the elementary model of money circulation

The production system, which is at the heart of the human society, is considered here to be immersed in the money system [1]. This monetary system is organized and managed by a central bank and commercial banks. The situation is shown schematically in Fig. 1 wherein we use two characteristic quantities: paper money M_0 and the sum of paper money and all deposits in commercial banks D, that can be written as follows, $M_2 = M_0 + D$. This quantity is called the *monetary mass*. Economic actors interact with each other by using money as an exchange means. This money is used for purchasing of resources, both for consumption and for production.

2.1. The customer of the commercial banks

In our model, we deal with the aggregated behavior of customers and producers whose deposit and credit are respectively characterized by D and B. Customers of commercial banks create a demand for credit money, and they appear to be the major sources of the progress of the economic system. At any time, when the customers need money, they determine whether their own money can be a possible source for expenses or if their situation requires a loan from a commercial bank. The motion of money in the three-sector production system was described in details in a previous paper [1], which determined the balance equation for the aggregate quantities D (deposits) and B (debts), recorded as

$$\frac{\mathrm{d}D}{\mathrm{d}t} = rD - qB + Y - I - C + W_{\mathrm{G}} - T + A_0 - \kappa(D + M_0) + \frac{\mathrm{d}B}{\mathrm{d}t} \tag{2}$$

[1] A number of authors in the circuitist literature often assume that the repayment of debts destroys money, see Graziani [17] or Keen [15] for further information on that point.

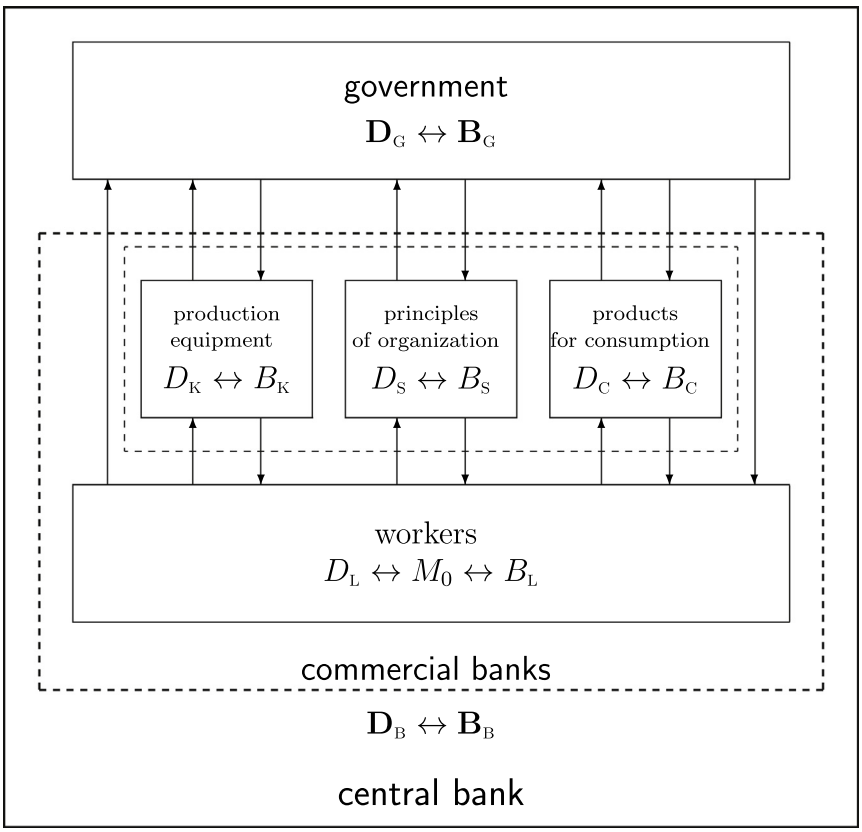

Fig. 1. The scheme of money fluxes. The central bank and the commercial banks create money which is a means of exchange for the economic actors and are characterized with their deposits and credits (symbols D and B with corresponding indexes). The three sectors identified in the text create all products and originate the fluxes of money between the sectors (not shown here), which determines components of Gross Domestic Product Y. Workers receive money in the form of wages W. The money is partly returning to the producers, when workers are buying products. The government receives its part of produced value in the form of taxes T, which in different amounts is returning to economic subjects via government spending. For each arrow representing a flux of money, there is one in the opposite directions representing fluxes related to labor force and products.

where Y is the GDP that is assumed here, in contrast to Eq. (1), to include the contribution from the finance layer; W_{G} is a flux of money to workers in the form of wages, which are received from the government; T is the total amount of taxes, received by the government from the producers and consumers,

$$T = \theta_{\mathrm{P}} Y + \theta_{\mathrm{L}} W, \tag{3}$$

where W is the total amount of wages paid to workers, including the wage payments to the civil servants W_{G}. The government fixes the taxation rate θ_{P} and θ_{L} to provide expenses of the government G, which represents investments in various national projects.

The right-hand side of Eq. (2) contains payments to and from the commercial banks. The banks ask the interest rate q for debts and gives the interest rate r to customers for their deposits. This context leads to a payment for debts, customers try to reduce quantity of debts as far as possible by keeping some money on the depositary accounts in commercial banks.

To take into account the commission fees for the transactions between actors and various gathering for carrying out of operations, the quantity proportional to the amount of money $D + M_0$ is installed into Eq. (2). The factor of proportionality κ represents an assessment of efforts on maintenance of one monetary unit under circulation and is a characteristic of the system. This equation also shows a quantity A_0 representing a possible flux of money into the accounts of clients from external sources. This flux includes foreign debts/credits, money coming from export/import etc. and this amount can be positive (inflow money) or negative (outflow money).

2.2. The commercial bank

Commercial banks are considered as intermediaries in the interactions between the production sectors and consumers of the economy. More specifically, money deposits and loans in commercial banks are to facilitate the economic activity. Commercial banks are supported by central bank and they aimed at getting profit from the operations with the customers.

Although the money system can contain many commercial banks, we consider here, for simplicity, a representative commercial bank as an aggregate actor of the sector.

One can assume that the commercial bank has the only account with the central bank D_B, on which it holds all its reserve, including the amount of mandatory deposit of the commercial bank ξD (where ξ is a norm of the mandatory reserve deposit that is set up by the central bank). The commercial banks have also a debt B_B to the central bank.

This deposit D_B changes due to the changes of the debt to the central bank B_B and operations with customers of commercial banks, so that the balance equation characterizing this dynamics can be written as

$$\frac{dD_B}{dt} = r_B(D_B - \xi D) - q_B B_B + qB - rD + \frac{dB_B}{dt} + \frac{d(D-B)}{dt} + A_C \tag{4}$$

where D and B are customers' deposits and debts. Here we neglect the incomes of the bank from operations with the central bank. In other words, we assume that the only income of the commercial bank is coming from operations with its customers (commission payments and other gathering, $\kappa(D + M_0)$). In addition to this assumption, we also consider that this inflow is completely spent for employees wages and current maintenance of the bank. The quantity A_C refers to the stream of money into the account of commercial bank from external sources. This amount can be positive, when the bank gets a foreign loan, or negative, when the bank send money abroad.

The central bank fixes the interest rates r_B and q_B asked for deposits and debts made by the commercial bank. Similarly, the commercial bank sets the interest rate r given (to customers) for their deposits and the interest rates q asked by banks for debts of its clients. In any case, it is expected, that value of q with any index will appear greater, than value of r with an appropriating index. Usually the central bank does not pay for mandatory deposits of commercial banks but it sets up a high level of the refinancing rate q_B. In this context, the norm of the mandatory reserve deposit ξ and the refinancing rate q_B are considered as main regulators of the amount of non-paper money.

By virtue of the Eq. (2), the Eq. (4) can be rewritten in the form

$$\frac{dD_B}{dt} = r_B(D_B - \xi D) - q_B B_B + Y - I - C + W_G - T - \kappa(D + M_0) + A_0 + E_C. \tag{5}$$

Here we introduce a symbol for the emission E_C of credit money by taking into account the external fluxes of money, more precisely,

$$E_C = A_C + \frac{dB_B}{dt}. \tag{6}$$

Note that usually the aggregate amounts of loans B and credits D appear to be greater than available banks' reserves $D_B - \xi D$. In this perspective, commercial banks create credit money, though the amount of credit is restricted. The detailed mechanism of money creation is vividly discussed [18]; investigators agree that the ultimate constraint for money creation (as, for example, the researchers [19] assert on p. 4) is monetary policy of the central bank. Whatever mechanism of money creation is assumed, there is a restricting condition, which can be written [1] in the form of equation for aggregate variables

$$\frac{dB}{dt} = \frac{1}{\xi^*}\frac{dD_B}{dt} - \frac{dD}{dt}. \tag{7}$$

The effective quantity $\xi^* > \xi$ is introduced to show a possible increase of credit money. The quantity $1/\xi^*$, echoing to the monetary multiplicator $1/\xi$ used in the economic literature [2], appears the real multiplicator.

At the fixed value ξ^*, the Eq. (7) defines a restrictive condition on a possibility of banks to increase credits. Apparently, the mechanism of multiplication works at large number of commercial banks and Eqs. (7) is valid for economic system, not for an individual bank.

Within the limits of these restrictions, commercial banks define amount of the loans B_B from the central bank, and the clients of commercial banks define quantities of their deposits D and debts B.

2.3. The central bank and the government

Institution at the heart of the money circulation, the central bank acts as a bank for commercial banks and the bank for the government. The role of the central bank is to organize, together with the government, circulation of money in the system. Beyond this role, the central bank also accounts the incomes and expenses of the government, which can roughly be summarized by two quantities: the amount of available money D_G and debts B_G. The central bank does not intend to get any profit from the service, so that we assume that the central bank's profit $q_G B_G + q_B B_B - r_G D_G - r_B(D_B - \xi D)$ is equal to zero. The rate of interest for debts and deposits of the government, q_G and r_G are specified through agreement of the central bank with the government.

The main account of the government with the central bank affects the motion of money to and from the government. The incoming fluxes of money include taxes (and other incomes) T, defined, for example, by Eq. (3). These incomes are needed to provide investments in various national projects G but also to pay wage payments to the civil servants W_G. In this context, the dynamics of the amount of money on the government's disposal D_G obeys the balance equation

$$\frac{dD_G}{dt} = q_B B_B - r_B(D_B - \xi D) + T - G - W_G + \frac{dB_G}{dt} + A_G + E_0. \tag{8}$$

For financing the activity, the government can let out paper money $E_0 = \frac{dM_0}{dt}$ and (or) address to internal creditors $\frac{dB_G}{dt}$. It is supposed also, that loans – a flux of money A_G – from external sources is possible, so that, alongside with a designation for emission of paper money, a new designation is introduced

$$E_0 = \frac{dM_0}{dt}, \quad E_G = A_G + \frac{dB_G}{dt}. \tag{9}$$

The quantity A_G includes foreign debts/credits and money for selling/buying obligations and bonds. The quantity can be positive or negative.

2.4. The Gross Domestic Product

The important characteristic of the system—the Gross Domestic Product, Y, which is included in Eq. (2), has to be presented as $I + G + C$ (see Eq. (1)) plus contribution of finance system. As explained earlier [1], the form of the last contribution can be found after aggregation of Eqs. (2), (4) and (8), which gives

$$Y = I + G + C - (A_0 + A_C + A_G) + \kappa(D + M_0) - \frac{dM_0}{dt} + \frac{d(D_B - B_B)}{dt} + \frac{d(D_G - B_G)}{dt}. \tag{10}$$

This formula is, apparently, a generalization of conventional expression (1) of the Gross Domestic Product as the sums of assessments of investments I, the governmental expenses G and immediate consumption C. In addition to these quantities, the expression (10) contains export of money by customers, commercial banks and the government, $-(A_0 + A_C + A_G)$, accordingly. The quantity $\kappa(D + M_0)$ represents an assessment of efforts on maintenance of the circulation of money in amount $M_0 + D$ (precisely, the coefficient κ represents an assessment of efforts on maintenance of the circulation of one monetary unit). The last three terms in expression (10) show, that the part of the added value is preserved (with a sign plus) on accounts of the central bank; this is in correspondence with suggestion of Steve Keen [20] to introduce debt into the expression for the GDP.

Calculated in this way, the expression for the Gross Domestic Product is highly pulsating quantity. The relation (10) can be presented [1] in simplified form, if one introduces a symbol R for the ratio of an assessment of average services of bank system to the pure output of production system,

$$Y = (1 + R)(I + G + C) - (A_0 + A_C + A_G). \tag{11}$$

It is possible to propose, that the quantity R can be considered as a characteristic of the efficiency of a social production: the greater is the quantity R, the more expensive is the maintenance of monetary circulation and the lower the efficiency of the system.

In this way, we have integrated the banks profit into the national income and, therefore, into the money credit process. The presence of banking sector in GDP generated many debates in the circuitist literature, which often fail to take properly into account the banks profit: "interest payments on loans made from firms to banks are not accounted as part of national income and simply disappear, instead of being treated as a possible source of demand for goods and\or financial assets" ([13], p. 1). According to Rochon ([14], p. 125), "The existence of monetary profits at the macroeconomic (aggregate) level has always been a conundrum for theoreticians of the monetary circuit…"

3. Application to the money system of Russia

We believe that the model of money circulation developed in the previous section represents the most general features of functioning of any national economic system. To validate the adequacy of our model of the money system, in this section, we use real data provided by the Russian National central bank.

3.1. The choice of variables

Further as an illustrative example we consider dynamics of the money system of Russia since 2000, using the data provided by the Central bank of Russia [21]. We are considering seven variables: D, D_B, D_G, B, B_B, B_G and M_0 , whose evolution is shown on Fig. 2.

In the basis of an assessment of variables of the model are put numbers for cash money M_0, monetary base $M_B = M_0 + D_B$ and a money mass $M_2 = M_0 + D$, estimated by the Central Bank of the Russia. Values of deposit D in commercial banks and the deposit of commercial bank in the central bank D_B are found as $D = M_2 - M_0$ and $D_B = M_B - M_0$. Calculated in this way values conform to direct assessments of these quantities by the Central Bank of Russia [21]. Values of credits in commercial banks B and the credit of commercial bank in the central bank B_B, as well as the credit and the deposit of the government in the central bank, B_G and D_G, are estimated directly according to the Central bank of Russia.

Our description, as all models, is inevitably incomplete, but we aim here to verify our model by confronting its major mathematical structure with a concrete case. It is worth mentioning that the available analysis of the Russian bank system in 2004–2007 [22] confirms importance of the variables used in our model, though the actual lists of the aggregated variables used in that work only partly coincided with ours.

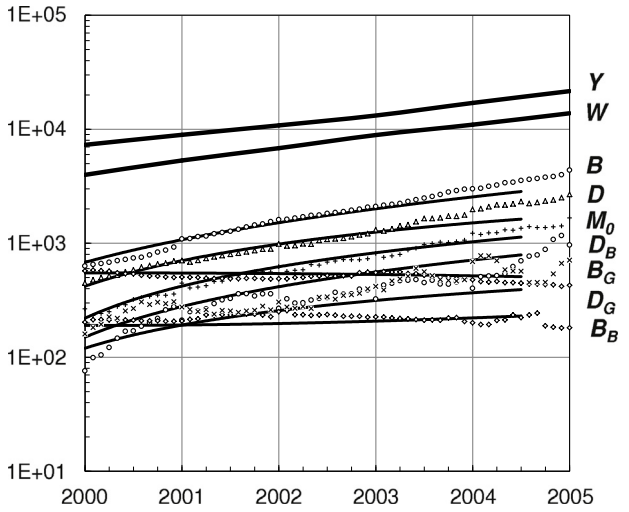

Fig. 2. The monetary situation of Russia. The two top curves represent respectively values of the Gross Domestic Product Y and income of working people W. Below, the continuous lines, noted by symbols of variables, show the calculated according to Eqs. (15) trajectories of evolution of variables. By points, empirical values of variables – cash money, credits and deposits of the central and commercial banks – are presented. All quantities are estimated in 10^9 of circulating in corresponding year rubles.

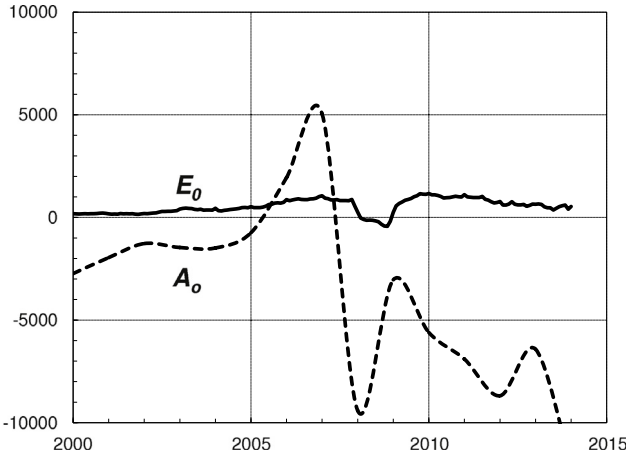

Fig. 3. Fluxes of money into accounts of clients of commercial bank. The continuous line presents quantity of paper money emitted E_0, the dashed line represents the external flux of money A_0 into accounts of clients of commercial banks, according to the Central Bank of Russia [21]. Unit of estimation of quantities is 10^9 of circulating in corresponding year rubles in a year.

3.2. *External fluxes and their approximation*

The existence of exogenous fluxes A_0, A_C and A_G in the balance equations (2), (4) and (8) characterizes the 'international' activity of considered economic actors. The quantities of these fluxes estimated directly by the Central bank of Russia are shown on Figs. 3–5. Negative values of the quantities mean, that clients and commercial banks prefer to send the reserves on storage not in the central bank, but to somewhere abroad. The fluxes include accumulation of money in securities or on accounts outside of the system. One can notice a important increase of a negative stream A_C in 2006 when the rule about mandatory sales of currency proceeds to the central bank has been canceled in Russia.

Empirical data and the balance equations (4) and (8) for seven variables D, D_B, D_G, B, B_B, B_G, M_0 combined with given data on the norms of payment of credits and deposits allow us to estimate independently the fluxes of money from external sources A_C and A_G. The calculated and averaged (the period of averaging—year) values of the external fluxes A_C and A_G are shown on Figs. 4 and 5 together with the empirical values specified by the Central Bank of Russia [21]. Let us mention however, that, the Gross Domestic Product Y in Eq. (2), defined by Eq. (10), contains values of exogenous fluxes of money (A_0 included), so that the quantity A_0 is not obviously possible to estimate from Eq. (2), this quantity is shown on Fig. 3 according to the Central Bank of Russia [21].

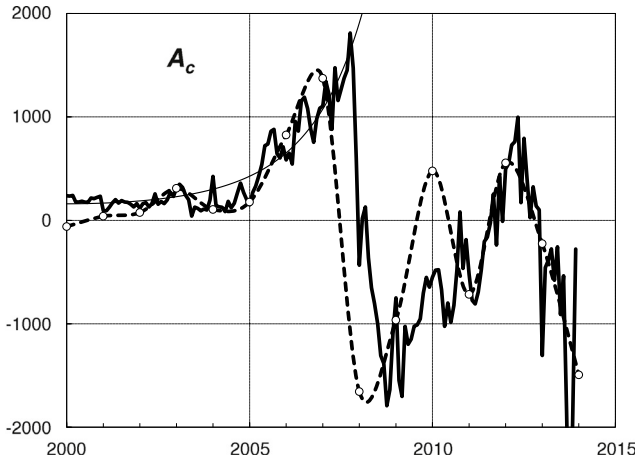

Fig. 4. The flux of external money into accounts of commercial bank. The solid line presents average values of an external flux of money A_C, calculated according to Eq. (4). By the dashed line with points, the empirical values estimated by the Central Bank of Russia [21] are shown. The quantities are estimated in 10^9 circulating in corresponding year of rubles in a year. A thin solid line shows an approximation of the flux with the exponential function.

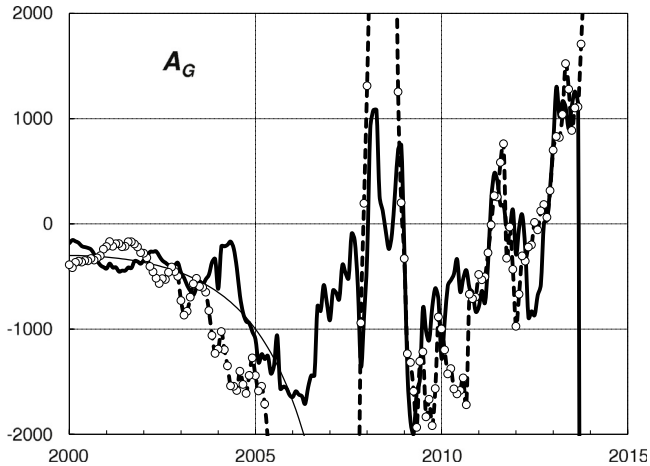

Fig. 5. The flux of external money into the account of the government of Russia. The solid line shows the values of an external flux of money A_G, calculated according to Eq. (8). The dashed line represent variation (with a sign a minus) the international reserves of the government. All quantities are estimated in 10^9 circulating in corresponding year rubles for a year. A thin line shows an optional approximation of the flux by exponential function.

Figs. 4 and 5 exhibits a conformity of two results of assessments of the flux A_C. Results confirm the assumption made in Section 2.2 according to which commission payments and other gathering completely go on a current content and a wages employee of bank and, consequently, is not included in a balance parity (10). Some exceptions appear in some years, since 2007. The situation with a flux A_G is more complex; actual variation of the international reserves of the government in 2005–2011 does not coincide with an assessment on a balance parity (10).

As it is possible to notice from Figs. 3–5, that sources of money E_0, A_C, A_G appear strongly pulsating functions, which suggests, that the parities (2), (4) and (8) should be the stochastic equations. However, in a first step we omitted the random aspect working with average quantities. Further, turning to calculation of behavior of variables, we shall use an exponential approximation for the sources of external money

$$A_C = 160\exp(0.0065t), \quad A_G = -300\exp(0.008t). \tag{12}$$

3.3. Fundamental characteristics of the system

A particularity in this modeling of the monetary circulation is the introduction and the use of fundamental characteristics, among which: coefficient of efficiency of the system R, which is the ratio of an assessment of average services of bank system to the real output of production system (see Section 2.4), expenses for production and maintenance of circulation of one ruble

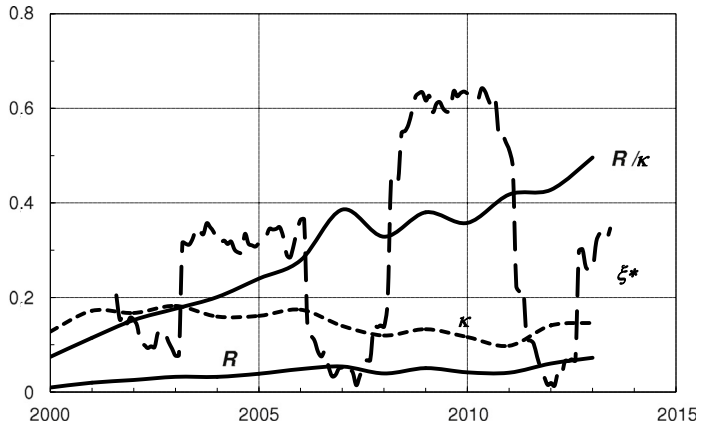

Fig. 6. Fundamental characteristics of the system. The sharply changing dotted curve represents a measure of propagation of credit money. A solid bottom line presents a measure of participation of the money system in production activity (the effectiveness ratio), the dashed line above—expenses of a society for maintenance in circulation one cash or non-cash ruble. The top solid line represents 'time of circulation of money' R/κ in 'the quantity theory of money'.

in unit of time κ and a measure of propagation of credit money ξ^*. The ratio κ/R is 'the velocity of circulation of money' in the known 'quantity theory of money' [23].

The coefficient of efficiency of system R may be calculated from values of the Gross Domestic Product Y and the contribution of bank system H; these values can be found in Year books of Rosstat (see, for example, [24]). In addition to this, the values of the fluxes A_0, A_C, A_G can also be found in the national balance of payment of the target country. As consequence of the Eqs. (11), value R is estimated according to the formula

$$R = \frac{H}{Y - H + A_0 + A_C + A_G}. \tag{13}$$

As an illustration, Fig. 6 shows calculated values of R estimated with the written equation. Values of efficiency increase from $R \approx 0.01$ in year 2000 up to $R \approx 0.04$ towards the end of the considered period. Figure also shows values of κ, which characterizes expenses for maintenance of production and circulation of one ruble. The values of the quantity in the beginning and in the end of the period are estimated as $\kappa \approx 0.1$ ruble, meaning that the costs of one circulating cash or non-cash ruble is equal to 1 cent. These numbers will be used to calculate a trajectory of evolution of the system in Section 3.5.

The norm of mandatory reservation ξ is set by the central bank; its value can be estimated according to the known fraction of mandatory deposit and is equal $\xi \approx 0.1$ in the beginning of the considered period and decreases up to $\xi \approx 0.015$ towards year 2012. Effective values ξ^*, characterizing true expansion of credit money, are calculated specifically under the formula (7) copied in the form

$$\xi^* = \frac{\mathrm{d}D_B}{\mathrm{d}t} : \left(\frac{\mathrm{d}D}{\mathrm{d}t} + \frac{\mathrm{d}B}{\mathrm{d}t}\right). \tag{14}$$

The calculated values ξ^* are strongly pulsating; average values of quantity, shown on Fig. 6, are increasing from the beginning of the considered period towards the end, reaching after the year 2010 the average value 0.3. The stationary values ξ^*, calculated under the formula (14) at current values of variables, appear lower than actual values and they practically coincide with values of ξ.

3.4. Dynamics of the money system

We follow the previous work [1] to formulate the system of evolutionary equations. Using the relations (2), (5), (7), (8) to connect the state variable of the four interacting economic actors, we write the system of the evolutionary equations as follows,

$$\begin{aligned}
\frac{\mathrm{d}D_B}{\mathrm{d}t} &= r_B(D_B - \xi D) - q_B B_B - \Delta - A_C - A_G + E_C,\\
\frac{\mathrm{d}D}{\mathrm{d}t} &= \frac{1}{2}(rD - qB) + \frac{1}{2\xi^*}[r_B(D_B - \xi D) - q_B B_B] -\\
&\quad - \frac{1+\xi^*}{2\xi^*}[\Delta + A_C + A_G] + \frac{1}{2\xi^*}E_C,\\
\frac{\mathrm{d}D_G}{\mathrm{d}t} &= q_B B_B - r_B(D_B - \xi D) + \Delta + E_G + E_0.
\end{aligned}$$

$$\frac{dB}{dt} = -\frac{1}{2}(rD - qB) + \frac{1}{2\xi^*}[r_B(D_B - \xi D) - q_B B_B] - \\ - \frac{1-\xi^*}{2\xi^*}[\Delta + A_C + A_G] + \frac{1}{2\xi^*}E_C,$$

$$\frac{dM_0}{dt} = E_0, \quad \frac{dB_B}{dt} = E_C - A_C, \quad \frac{dB_G}{dt} = E_G - A_G. \tag{15}$$

The definition (11) for output Y is used while we introduce a symbol Δ to characterize the government's surplus over expenses that can be associated with the surplus the budget

$$\Delta = T - G - W_G.$$

The quantities on the right hand part of the equation are connected with the activity of production system.

The central bank establishes norm of mandatory deposit ξ that, as we know (see a discussion at the end of Section 2.2 and, in more details, in Section 2.2 of the previous paper [1]), does not coincide with its effective value ξ^*, determining the expansion of credit money. These quantities should be set. The central bank establishes also norms of payments r_B, q_B for deposit and credit of the commercial bank, which, in its turn, defines norms of payments for deposits and credits, r and q, of the clients. The parameters, established by the government, the central bank and commercial bank, are not constant, but depend on the situation and the economic actors' behavior.

The system of Eqs. (15) describes the behavior of seven variables D, D_B, D_G, B, B_B, B_G and M_0 at preset values of the listed parameters. In this context, the evolution of financial variables is determined by the sources of money E_0, E_C, E_G, which includes the external fluxes A_C and A_G. It worth noticing, that the external stream A_0, also as the real production, are not present in the system of Eqs. (15).

In this context, the money system looks to evolve autonomously, the correspondence with the real production is being established by hands of workers of the central bank after some analysis. The money authority should determine a program of credit and issuing paper money (i.e. emission) E_0, so that the sources of money are E_0, E_C, E_G. A global appreciation of appropriateness of authority's decision is behavior of the price index, which can be calculated according to the framework developed in [1].

3.5. Trajectory of evolution

To show the consistency and adequacy of our concepts, we shall test the system of Eq. (15) for the Russian monetary system within several years after 2000. We believe, that the monetary system accompanies the production program with the growth rates, which for sectoral output is equal to the rate of growth of the GDP. According to official data [24], the average nominal value of the growth rate of GDP (in the current prices) for first three years is equal $\sigma = 0.24$ per a year. The income of the government depends on the GDP and income of the population (see Eq. (3)), but for characterizing the evolution of the monetary system in line with Eq. (15), only the budget proficit Δ is required. In the initial years the income of the government exceed expenses; for calculation, the constant value $\Delta = 180 \cdot 10^9$ rubles per a year is accepted.

The evolution of the money system, described by the system of Eqs. (15), is essentially determined by the values related to emitted paper and credit money E_C, E_0, E_G, which include the fluxes of money into the system A_0, A_C and A_G. The quantity E_0 and E_G are defined by the government and the central bank, proceeding from the necessity to provide additional financing the budget area and government projects. The emission of credit money E_C is defined by joint actions of the central and commercial banks.

Both the central bank and commercial banks have to take into account the development of production. One can assume [1] that the central bank considers the deposits of the government and commercial bank, D_G and D_B, should increase according to progress of production subsector; in the simplest case, with the rate of growth of production output σ. Then, assuming, for simplicity, that $E_G = 0$, the two other quantities can be estimated as

$$E_0 = \sigma D_G - q_B B_B + r_B(D_B - \xi D) - \Delta, \tag{16}$$
$$E_C = \sigma D_B - r_B(D_B - \xi D) + q_B B_B + \Delta + A_C + A_G.$$

The external fluxes A_C and A_G are considered to be set; at calculations, we use the approximations (12) shown in Figs. 4–5.

According to the central bank of Russia [21], the rates of payment for credits and debts in central and commercial banks are pulsating quantities with an average value in initial years after 2000 equal to $q = 0.16$ and $r = 0.05$. In the same years average value of the discount rate or the rate of refinancing (nowadays called the key rate) in the central bank was $q_B = 0.20$, while the payment for deposits of commercial banks was $r_B = 0.02$.

Fundamental characteristics of system R (coefficient of efficiency) and κ (factor of friction) slowly change (see Fig. 6), but for simplicity, these values, as well as the norm of mandatory deposit ξ, is assumed constant during all period of consideration. More precisely, we use value of characteristics of the system estimated in Section 3.3: value of coefficient of efficiency $R = 0.01$; factor of friction $\kappa = 0.1$ for a year, norm of the mandatory deposit $\xi = 0.04$; value of a measure of propagation of credit money $\xi^* = 0.19$.

We used the data of the year 2000 as initial variables, that is: $D = 420$, $D_B = 150$, $D_G = 120$, $B = 680$, $B_B = 190$, $B_G = 550$ and $M_0 = 220$. We shall remind, that all quantities are estimated in 10^9 rubles. The results of calculation are shown in Fig. 2 in the beginning of this Section, one can see that our model reproduces the initial behavior of data with a good accuracy.

4. Conclusion

This article provides an empirical application of a circuitist model of money circulation in which the money supply is mainly defined by quantity of credit money held by banking sector. While this paper directly contributed to the circuitist literature (by offering a coherent framework integrating banks' profit into national income, by avoiding the profit paradox and by defining a specific valuation for monetary parameters such as mandatory deposit, expenses of one paper monetary unit or the average time of rotation of the money), it also emphasizes the relevance of circuitist approach of monetary circulation in which the money price depends on the quantity of money. This article was designed to describe the main features of money circulation in the production system in accordance with the call [3,4] for the development of new tools to study and better understand the relationship between real economy and GDP. The model assumed the emission of money by the central bank and the creation of credit money[2] by the commercial bank; extra Eq. (7) restricts the emission of credit money. A specific feature of our approach to the description of monetary circulation is the introduction and the use of fundamental phenomenological characteristics of the system, among which: the ratio of system's efficiency R, the expenses for maintenance of circulation one monetary unit κ and the measure of propagation of credit money ξ^*. The ratio κ/R appears to be 'the velocity of circulation of money' in the known 'quantitative theory of money' [23]. We have used the data of the Russian Central Bank to demonstrate the adequacy of the model, and we were satisfied to be confirmed that, at the proper setting of the parameters, the model appears to describe the change of money variables.

It has appeared that there is no direct causal relationship between development of production and evolution of monetary system; such linkages is being established by hands of workers of the central bank after some analysis. To support the production development, the central organizations should define the program of issue of credit and paper money. The simple reasons stated in Section 3.5 are based on the assumptions, that the government and the central bank in their activity are guided only by the interests of creation of favorable environment for development of national production. Unfortunately, it not always true: if a national monetary system is strongly integrated into the World environment, when economic subjects can interact directly with the external agents, there appears additional reasons connected with the necessity to provide favorable (or adverse) interoperability with external agents. Additional conditions can lead to contradictions with national interests and the big art is required from the bank managers to maneuver among controversial requirements.

Let us notice once more, that our description is rather rough: we used highly aggregated variables, which, nevertheless, provides the way to describe the monetary circulation and justify the circuitist approach in the understanding of the connection between motion of real things and the motion of money in macroeconomics.

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[2] There is a discussion about creation of credit money, but, perhaps, it is a question of point of view. At every exchange between agents the sum of money is conserved, so the authors [6,9] insist on the existence of law of conservation of money; as it is noted [9, p.3329] 'the increase of money in circulation among agents is exactly equal to the increase of debt in the system'. One can say the money is conserved, if we regard the debt as negative money and take into account all records. Still one can note that commercial bank can increase the amount of credit money, and there is a distinction between credit money that is circulating in the real production system and money as a record about the debt in bank archives. It happens sometimes that the records about debts have been disappeared (it is a crime from point of view of the social law, but no violation of the laws of nature); some more moderate cases are described in [9, Sec. 2.8]. The operation does not influence on the former credit money that is already in circulation in the system of real things. Is the law of conservation of money valid in this case? It is not out of place to remind here the analogy with the particle–antiparticle pair creation [9]; somebody succeeded to eliminate all antiprotons in our universe for us to live and think about money circulation. Who knows where are the corresponding antiprotons?

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