Coordinated Charging of Plug-In Hybrid Electric Vehicles to Minimize Distribution System Losses

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Abstract—As the number of plug-in hybrid vehicles (PHEVs) increases, so might the impacts on the power system performance, such as overloading, reduced efficiency, power quality, and voltage regulation particularly at the distribution level. Coordinated charging of PHEVs is a possible solution to these problems. In this work, the relationship between feeder losses, load factor, and load variance is explored in the context of coordinated PHEV charging. From these relationships, three optimal charging algorithms are developed which minimize the impacts of PHEV charging on the connected distribution system. The application of the algorithms to two test systems verifies these relationships approximately hold independent of system topology. They also show the additional benefits of reduced computation time and problem convexity when using load factor or load variance as the objective function rather than system losses. This is important for real-time dispatching of PHEVs.

Index Terms—Distribution systems, load factor, load management, load variance, losses minimization, plug-in hybrid vehicle (PHEV), smart charging.

I. INTRODUCTION

R EDUCING dependence on foreign oil and emissions of CO_2 and particulates are among the leading reasons that plug-in hybrid electric vehicles (PHEVs) are increasing in popularity. Most PHEVs are planned to have a fully electric range between 10–40 mi, which is within the daily commute distance of the average driver [1]. The existing U.S. generation and transmission structure could support approximately 70% of the existing U.S. light duty vehicle fleet under coordinated charging [1]. It has been shown, however, that serious problems can arise under uncoordinated opportunistic charging scenarios [2].

Several recent studies show that the distribution grid could be significantly impacted by high penetration levels of uncoordinated PHEV charging [3]–[10]. These impacts include increased system peak load, losses, and decreases in voltage and system load factor. It has been shown that these impacts can be

Digital Object Identifier 10.1109/TSG.2010.2090913

largely mitigated by coordinated charging [1], [4], [5]. In previous studies, coordinated charging has been performed using sequential quadratic optimization [4], [5], [11], dynamic programming [12], and heuristic methods [1], [13].

In this work the relationship between losses, load factor, and load variance is explored in the context of coordinated charging of PHEVs. From these results, load factor and variance based objective functions for coordinated PHEV charging are formulated, which in effect minimize system losses and improve voltage regulation. These formulations are convex and therefore have two advantages over previous formulations. The first advantage of convexity is that they can be solved more quickly and efficiently using commercial solvers, such as CPLEX [14], which is essential for real-time dispatch. The second is that these objectives can be easily integrated as loss constraints in other PHEV charging objective functions such as those in [12], [13] which minimize system operating costs to, or maximize profits to, an aggregator. Another advantage of these formulations is that they are topology independent and are thus equally effective on looped structure or other advanced distribution system topologies. In summary, the contributions of this paper are:

- identifying the relationships between feeder losses, load factor, and load variance;
- formulating load factor and load variance objectives for optimization, which have the advantages of:
 - convexity for faster and more robust solving;
 - topology independence;
 - ability to be integrated as constraints in other optimization objectives.

The proposed objective functions are simulated on two test systems in Matlab to evaluate their effectiveness. While only PHEVs are emphasized in this work, the formulations and results are equally applicable to pure electric vehicles. The only difference is the battery capacity.

II. RELATIONSHIP BETWEEN LOSSES, LOAD FACTOR, AND LOAD VARIANCE

It has been shown that minimizing distribution system losses with the addition of PHEVs also minimizes the voltage impacts of their integration [4], [5]. Therefore, this work focuses on loss minimization under the knowledge that mitigating losses will improve voltage profile. The relationships between losses and load factor, and losses and load variance are topology specific and are only exact if the feeder is a single line from the substation with all loads at the end of the line. Though this is almost never the case, simulations on practical distribution systems show that these relationships are close approximations as

Manuscript received September 02, 2010; accepted October 27, 2010. Date of publication December 10, 2010; date of current version February 18, 2011. Paper no. TSG-00121-2010.

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shown in Section V. The relationship between load factor and load variance however, is topology independent.

A. Maximum Load Factor

The first relationship examined is that between losses and load factor. Considering the loss factor which is most commonly determined by the Buller and Woodrow formula [15], [16]

$$LSF = (LF)^{2}(1 - C) + (LF)C$$
 (1)

where

LSF is the loss factor;

- LF is the load factor as seen at the substation;
- *C* is an empirically determined constant usually between 0.15 and 0.30 [16].

The total energy losses on a feeder over a time interval T is given by

$$losses_{tot} = LSF * losses_{max} * T.$$
(2)

Theorem 1: For a fixed energy consumed E_{tot} in a period T, minimizing energy losses in a distribution feeder is equivalent to maximizing the load factor.

Proof: Let the coincident demand as seen by the substation at a given time t be $D(t) = D_T$ where $t \in [0, T]$. Also, assume that voltage is constant or near constant, then the current I_t is related to the demand D_t by

$$I_t = k_1 D_t \tag{3}$$

where k_1 is a constant greater than 0.

Then the relationships for the maximum and average load currents are given by (4) and (5) respectively

$$I_{\max} = k_1 D_{\max} \tag{4}$$

$$I_{\rm avg} = k_1 D_{\rm avg}. \tag{5}$$

Now assume that maximum losses are given by

$$losses_{max} = k_2 I_{max}^2 \tag{6}$$

where k_2 is a constant greater than 0. Then LF is given by

$$LF = \frac{\left(\sum_{t} D_{t}/T\right)}{D_{\max}}$$
$$= \frac{k_{1}\left(\sum_{t} I_{t}/k_{1}T\right)}{I_{\max}}$$
$$= \frac{I_{avg}}{I_{\max}}$$
(7)

and

$$E_{\text{tot}} = D_{\text{avg}}T$$

= $I_{\text{avg}}T/k_1$
= constant. (8)

Therefore, I_{avg} is constant. Now total losses can be written as a function of I_{max} as

losses_{tot} =
$$[(I_{\text{avg}}/I_{\text{max}})^2(1-C) + (I_{\text{avg}}/I_{\text{max}})C] k_2 I_{\text{max}}^2 T.$$
(9)

Equation (9) simplifies to

$$losses_{tot} = I_{max}A + B \tag{10}$$

where

$$4 = I_{\text{avg}} k_2 C T \tag{11}$$

$$B = I_{\rm avg}^2 k_2 (1 - C)T.$$
 (12)

Also, A and B are independent of I_{max} . Since A > 0, when losses are minimized I_{max} is also minimized. Since

$$LF = \frac{I_{avg}}{I_{max}}.$$
 (13)

Thus, minimizing I_{max} maximizes LF. As a consequence minimizing losses also maximizes LF.

As a check, the minimum value of I_{max} cannot be less than I_{avg} therefore for $I_{\text{max}} = I_{\text{avg}}$ the LF = 1, the maximum load factor. While this proof relies on the Buller and Woodrow formula other formulations for LSF could be equally considered.

B. Minimizing Load Variance

The next relationship examined is that between losses and load variance.

Theorem 2: For a fixed energy E_{tot} in period T, minimizing the feeder energy losses is equivalent to minimizing load variance if the following hold:

Condition I: load is proportional to current as in (3). This is approximately true under small voltage fluctuations.

Condition II: total feeder losses are proportional to the average of I_t^2 .

Theorem 2 is exactly true if the feeder is a single branch with resistance R. This condition is expressed analytically as

$$losses_{tot} = \mu_{I^2} R \tag{14}$$

where

$$\mu_{I^2} \equiv \frac{1}{T} \sum_{t=1}^{t_{\text{max}}} I_t^2 \tag{15}$$

and R is a constant. *Proof:* Define

$$\mu_I \equiv \frac{1}{T} \sum_{t=1}^{t_{\text{max}}} I_t = I_{\text{avg}}.$$
 (16)

Then

$$\mu_I = \text{constant}$$
 (17)

8) as implied by (8).

The load variance seen by the substation, σ_I^2 , is defined as

$$\sigma_I^2 \equiv \frac{1}{T} \sum_t (I_t - \mu_I)^2.$$
(18)

Equation (18) can be expanded to become

$$\sigma_I^2 = \frac{1}{T} \sum_t I_t^2 - 2\mu_I \frac{1}{T} \sum_t I_t + \frac{1}{T} \sum_t \mu_I^2.$$
(19)

Substituting (15) and (16) into (19) gives

$$\mu_{I^2} = \mu_I^2 + \sigma_I^2 \tag{20}$$

Since μ_I is constant, minimizing σ_I^2 minimizes μ_{I^2} according to (20). Minimizing μ_I^2 minimizes losses according to (14). Thus, minimizing σ_I^2 minimizes losses. Finally, according to (3), minimizing the variance of D_t is equivalent to minimizing the variance of I_t . So minimizing load variance minimizes losses.

C. Load Variance and Load Factor Relationship

If the maximum base load energy $E_{tot,base}$ is small enough and the sum of the energy required for all PHEVs, $E_{tot,PHEV}$ is large enough, then maximizing load factor is equivalent to minimizing variance.

Theorem 3: If

$$E_{\text{tot,PHEV}} + E_{\text{tot,base}} \ge I_{\text{max,base}} VT$$
 (21)

where $I_{\text{max,base}}$ is the maximum current of the base load as seen by the substation, then maximizing load factor is equivalent to minimizing load variance.

Proof: The left side of (21) represents the total energy consumed by the system in time T. The right side of (21) represents the maximum total energy that the base load plus the PHEVs can consume without requiring the maximum total current to increase above the maximum base load current even if load variance is minimized. If (21) is true then minimizing the load variance leads to

$$\mu_I = I_{\max} \tag{22}$$

Given (22), then LF = 1 and $\sigma_I^2 = 0$ so load variance is also minimized. Thus, given (21), maximizing load factor is equivalent to minimizing load variance.

These three relationships therefore imply one another in a conditional "triangle equivalence." This is shown in Fig. 1.

III. PROBLEM FORMULATION

Since maximizing load factor and minimizing load variance are equivalent under (21), and approximately minimize losses, these three optimization objectives are now formulated for coordinated PHEV charging. For each objective function it is assumed that the load profile at each node of the distribution system is known with some degree of certainty and the only load that is controllable is that of the PHEVs connected to the system. This is a reasonable assumption given distribution system load forecasting. If there are other controllable loads on the system they can be optimized with the PHEVs using the



Fig. 1. Triangle equivalence of losses, load factor, and load variance.

same objectives with additional energy constraints specific to those loads.

A. Minimizing Losses Formulation

The loss minimization as given in [5] is

$$\begin{array}{l} \underset{I}{\text{minimize}} \sum_{t=1}^{T} \sum_{l=1}^{lines} R_l I_{l,t}^2 \\ \text{subject to :} \end{array} \tag{23}$$

$$S_{m,t} = V_{m,t} (I_{m,t})^*$$
(24)

$$I_{m,t} = I_{l,t} - I_{l+1,t}$$
(25)

$$S_{m,t} \ge S_{\min,m,t} \tag{26}$$

$$S_{m,t} \le S_{\max,m,t} \tag{27}$$

$$\sum_{t} S_{m,t} = E_{\text{tot},m} \tag{28}$$

where

R_l	is the resistance of line l ;
$I_{l,t}$	is the current of line l at time t ;
$S_{m,t}$	is the load at node m at time t ;
$S_{\min,m,t}$	is minimum allowable load at node m at time t ;
$S_{\max,m,t}$	is maximum allowable load at node m at time t ;
$V_{m,t}$	is the voltage at node m at time t ;
E_{i-1}	is the total energy delivered to node m over the

 $E_{\text{tot},m}$ is the total energy delivered to node *m* over the period.

In the above formulation, the maximum allowable load at each node is given by

$$S_{\max,m,t} = S_{\min,m,t} + (\text{EVnode}_{m,t})MP_{m,t}$$
(29)

where

MP is the maximum power draw of the PHEV at the node.

This formulation assumes that the PHEVs are not capable of delivering power back to the grid which is the case for the current PHEVs in production. While the objective function is convex, the constraints given by (24) and (25) are not convex. Therefore, this optimization must be solved by either a heuristic method or the sequential method described in [5].

B. Maximizing Load Factor Formulation

Since minimizing the losses maximizes the load factor this is the next objective to be formulated. The maximization of load factor is given by

$$\begin{array}{l} \underset{S}{\text{maximize}} \frac{\mu_D}{\max\left(\sum\limits_{m=1}^{\text{nodes}} S_{m,t}\right)}\\ \text{subject to: (26)-(28)} \end{array}$$
(30)

where μ_D is the average distribution system load during T.

Though this formulation is not convex, it is equivalent to

$$\underset{S}{\text{minimize}} \frac{\max\left(\sum_{m=1}^{\text{nodes}} S_{m,t}\right)}{\mu_D}$$

subject to: (26)-(28) (31)

Equation (31) is a convex linear program because μ_D is constant with respect to the charging PHEVs due to the constraint in (28).

C. Minimizing Load Variance Formulation

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Minimizing the load variance is formulated as

$$\underset{S}{\text{minimize}} \sum_{t=1}^{T} \left(\frac{1}{T} \left(\sum_{m=1}^{\text{nodes}} (S_{m,t} - \mu_D) \right)^2 \right)$$

subject to: (26)-(28) (32)

This formulation is a convex quadratic program.

The formulations above give the optimal charging profile for PHEVs during the time period, usually one day, for the given distribution system load forecast. There are several advantages of the linear and quadratic formulations (31) and (32) over the minimal losses formulation. The first is that they can be solved without having to compute a power flow or having to iterate the optimization as is required to solve for minimal losses. This allows them to be solved easily on looped configuration and other advanced distribution topologies that may be prevalent in the future for increased reliability [17]. The second advantage is that linear and quadratic programs can be solved very quickly on many commercial optimization packages. The third is that the objective functions themselves can be used as either linear or quadratic constraints to other optimization functions involving PHEVs. Such optimizations may focus on charging cost minimization or some vehicle-to-grid profit maximizations [12], [13].

IV. APPLICATIONS TO TWO TEST SYSTEMS

The three different algorithms; loss minimization, load factor maximization, and load variance minimization, are applied on two test residential distribution systems to compare their performance. These simulations are run on a 2.4 GHz Intel Core 2 Duo CPU with 2.00 GB of RAM computer. The optimization functions are solved in Matlab using convex optimization package CVX [18]. Monte Carlo simulations are run for each algorithm with PHEVs randomly placed at different nodes at penetration



Fig. 2. The nine-bus distribution system used in simulation. Load buses are 2, 3, 6–9.



Fig. 3. Single house base load profile.

levels of 10%, 20%, 50%, and 100%. All power flows are solved using the forward-backward sweep method.

The first system used is a nine-bus, radial, three-phase unbalanced primary distribution system shown in Fig. 2. The phase conductors are overhead ACSR #4 with ACSR #6 for the neutral operating at 12.47-kV line-to-line voltage. Buses 2, 3, and 6–9 are load buses. Each load bus has two houses connected to each phase transformer for a total of 36 houses. The daily base load profile starting at midnight for each house is based on that shown in Fig. 3. This is an hourly load profile. From this profile, two other profiles are generated by time shifting ± 2 h. Each house is then randomly assigned one of these three load profiles for their base load.

The second test system used is an adjusted version of the unbalanced 18 bus system examined in [17], [19], and [20]. This system is adjusted by replacing all conductors with overhead ACSR #4 with ACSR #6 for the neutral, replacing all single-phase laterals with three-phase, and adding two house loads to each phase of each bus. This gives a total of 102 houses with randomly assigned load profiles as described above. This system is shown in Fig. 4.

Each PHEV load is modeled as a constant real power during each time step as in [5]. Reactive power can also be included in the formulation in which case it will be more complex. It should also be emphasized that the real losses could be different for the same systems. When charging, the charging power may or may not vary for coordinated charging, and does not vary for uncoordinated charging. A fully charged PHEV holds close to 10 kWh



Fig. 4. The 18-bus distribution system used in simulation. Load buses are 2-18.



Fig. 5. Average losses for 10% EV penetration as a function of Monte Carlo runs.

[1], [5] which is sufficient for the average daily driving distance of 33 miles (53 km) established by EPRI [1]. It is assumed that the PHEV is plugged into a standard 120-V/15-A wall outlet and has a corresponding maximum charge rate of 1800 W. It is assumed charging is not limited by the maximum ramp rate of the PHEV battery. In all cases, PHEVs will plug into the grid with a fully discharged battery at 18:00 h, and will be connected until 06:00 h the next day.

V. RESULTS

For each of the three algorithms formulated above, the average losses, PHEV load profile, and run time of the Monte Carlo runs are compared for different penetration levels. The first step is to determine the number of Monte Carlo runs to achieve the steady state solution. As seen in Fig. 5, the solution stabilizes around 400 runs; therefore, this number is selected as the upper limit for the simulation purposes.

Demand (load) profiles for the different charging algorithms on the nine-bus system can be seen in Figs. 6–9. Since the profiles are very similar in the 18-bus case, only those in the



Fig. 6. Load profiles for the different charging algorithms at 10% PHEV penetration for the nine-bus system.



Fig. 7. Load profiles for the different charging algorithms at 20% PHEV penetration for the nine-bus system.

nine-bus case are shown. It is clear that uncoordinated charging significantly adds to the peak load in all cases. Also in all cases, minimizing the load variance produces almost exactly same profile as minimizing the losses, so much so that the two profiles are nearly indistinguishable when plotted on the same graph. The only reason for the difference is that the topology of the distribution system is not a single line with all loads connected to the end, as is required in the proof. In Figs. 6-8, where the condition in (21) is not satisfied, maximizing the load factor does not minimize the variance or the losses. In this case the maximum load factor profile can be easily distinguished from the other two optimal charging algorithms. However, the maximum load factor profile also does not add to the peak load. As seen in Fig. 9, where the condition in (21) is met, maximizing the load factor produces an identical result to minimizing the load variance and therefore the curves overlap throughout the graph. Also in this case, minimizing the losses produces a slight increase in the peak load. This is because voltage effects and the system topology are taken into account.

The main metrics for comparison of the methods are total losses and time required to optimize. Total losses can be seen for the nine-bus system in Fig. 10, and the 18-bus system in Fig. 11.



Fig. 8. Load profiles for the different charging algorithms at 50% PHEV penetration for the nine-bus system.



Fig. 9. Load profiles for the different charging algorithms at 100% PHEV penetration for the nine-bus system.

It is clear that uncoordinated charging is by far the worst. Also, minimizing losses and load variance are almost identical. The percentage reduction in losses from the uncoordinated case by the three algorithms for the nine-bus system is shown in Fig. 12, and the 18-bus system is shown in Fig. 13. Three conclusions can be drawn from these figures. First, the difference between minimizing losses and minimizing load variance is less than 0.1%. Second, maximizing load factor underperforms the other two by less than 2% and the gap reduces as the penetration level increases. The third conclusion is that these previous two results are not dependent on the system size or topology since two very different systems produce similar results.

Computation time is an important aspect, especially if optimization is to be performed in real-time. It can be seen in for the nine-bus system in Fig. 14, and the 18-bus system in Fig. 15, that minimizing losses takes significantly longer to compute than the other two. It takes 20 times longer to compute than maximizing



Fig. 10. Total losses for each charging profile over a 24 h period for the nine-bus system.



Fig. 11. Total losses for each charging profile over a 24 h period for the 18-bus system.



Fig. 12. Reduction in losses over uncoordinated charging for the nine-bus system.

the load factor, and 10 times longer than minimizing load variance for the nine-bus system. It takes over six times longer than maximizing load factor and over three times as long as minimizing load variance for the 18-bus system. This difference is a function of the ratio of the number of lines to the number load



Fig. 13. Reduction in losses over uncoordinated charging for the 18-bus system.



Fig. 14. Time required for each Monte Carlo run for the nine-bus system.



Fig. 15. Time required for each Monte Carlo run for the 18-bus system.

points, since minimizing losses uses line current as a decision variable while the other two algorithms use demand at the nodes. The nine-bus system has a high ratio of the number of lines to the number of loads while the 18-bus system has a lower ratio. Also, the time required to minimize losses will increase nonlinearly with the size of the system since several iterations of the ac power flow are required. Advanced distribution system topologies such as looped or meshed topologies will increase the solving time even more.

VI. CONCLUSION

In this paper, the problem of PHEV impacts on certain aspects of distribution system performance is explored. It is shown that if the distribution system is a single feeder from the substation with all loads connected at the end, then minimizing losses maximizes the load factor and minimizing load variance minimizes losses exactly. For practical systems, minimizing load variance will minimize losses approximately. For the two test systems studied here, the difference in losses is less than 0.1% even for different system sizes and topologies. Additionally, when the system peak load increase is unavoidable as given by the condition in (21), maximizing system load factor is equivalent to minimizing load variance independent of the system topology. Simulations on two test systems confirm the effectiveness of load factor maximization and load variance minimization.

For a given daily load profile forecast, the load variance method is found to be more versatile than minimizing the losses because it produces an almost identical result in a fraction of the time. This is important for real-time dispatch of PHEVs. It also solves the problem independent of system topology, and can be applied to looped or meshed distribution systems easily. Since load variance is quadratic, it has the additional utility of being able to be used as a constraint which addresses distribution system losses in a cost or profit optimization function. In the condition of unavoidable peak load increase, maximizing load factor outperforms the other formulations. This is because it gives the same result as load variance with half the computation time. It is linear as well and can be more easily integrated as a distribution system constraint to another optimization function. If computation time is essential, the load factor maximization still gives excellent results faster than the other two even if they are suboptimal.

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