Brand loyalties in designer luxury and fast fashion co-branding alliances

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\textbf{A B S T R A C T}

Fast fashion brands, such as H & M, have co-branding projects with designer luxury brands. However, how the brand loyalties of the associated brands theoretically affect the co-branding’s performance is largely unknown. Motivated by the observed industrial practices, we build a formal analytical model to examine the impacts of brand loyalty on revenues in luxury and fast fashion co-branding. The commonly adopted schemes in industry such as the profit sharing scheme, fixed-royalty scheme and mergers scheme are examined to investigate the brand performance. It is analytically found that the associated brands would perform best under the mergers scheme. This implies that the internal cooperation within a big group is the most desirable strategy for co-branding. Moreover, we provide the analytical evidence that fast fashion brands should work with well-known luxury fashion brands for brand alliance.

1. Introduction

Co-branding is a brand alliance strategy in which two or more brands collaborate and launch their co-brand (Blackett & Board, 1999). In markets across the globe, we are now witnessing a growing interest in establishing strategic partnership in co-branding (Bengtsson & Servais, 2005). For example, the fast fashion co-branding strategy, which is formed between a designer-label fashion brand and a fast fashion brand, such as H & M, is quite popular. H & M first started its co-branded collection with the designer luxury fashion brand Chanel’s chief designer Karl Lagerfeld in 2004. It appears that H & M has shown a unique formula of selecting an appropriate luxury fashion partner and co-branding cooperation is beneficial to both involved brands (Labbrand, 2011). The history of H & M collaboration is shown in Table 1. As we can see from Table 1, most of the participating designer luxury fashion brands are famous brands. This observation motivates our study. As we will prove theoretically later on, this is in fact a wise and “optimal” mode of co-branding alliance.

The purpose of launching co-brands is to respond to the fast changing marketplace and increasing customers’ brand loyalty towards the involved brands (Voss & Mohan, 2016). The benefits of co-branding partnership in terms of brand loyalty in the designer luxury and fast fashion co-branding have been widely documented in both academic literature (Ahn, Kim, & Forney, 2010; Jang, 2006; Oeppen & Jamal, 2014; Shen, Jung, Chow, & Wong, 2014) and industrial reports (as shown in Table 2). In co-branding, companies can work with other companies to integrate resources and leverage individual core competencies, or they can use current resources within one company to promote multiple products. There are many different forms of co-branding adopted in the fashion industry. They include ingredient co-branding, joint venture co-branding and same-company co-branding. Ingredient co-branding is formed when the ingredient brand is contained within the manufacturer brand. The well-known example of ingredient co-branding in fashion is the use of YKK zippers. Joint venture co-branding is established when two or more brands/Companies enter a partnership to launch their co-brand. Joint venture co-branding includes many cases such as the fashion co-branding we mentioned above. In this paper, we call the co-branding between two brands of the same company/enterprise the “intra-company co-branding”. This is relatively rarely seen in the fashion industry. However, it is do-able as the big fast fashion corporations, such as Inditex, own many fashion brands of different kinds and even tiers. For example, the Uniqlo Group owns the brands Theory (a designer luxury fashion brand) and Uniqlo (fast fashion). Thus, a co-branding alliance can be formed between Uniqlo and Theory if the Uniqlo Group wishes. A natural question hence arises: Is intra-company co-branding a wise co-branding strategy? This
question will be answered later on in this paper.

According to the observed market practices, each specific co-branding project of the designer luxury brand and the fast fashion brand is usually a one-shot project, i.e., for the limited edition products. However, from a fast fashion brand’s perspective (e.g., H & M), it does launch co-branding projects frequently and how it selects the designer luxury label partner is an interesting question and important issue. As the brand loyalty is a critical element in the co-branding projects (Kim, Lee, & Lee, 2007), in this paper, we examine the impacts brought by “brand loyalties” on the revenues of the participating brands in co-branding. Our approach of incorporating brand loyalty into the market demand function is analogous to previous works (Raju, 2004; Srinivasan, & Lal, 1990; Villas-Boas, 2004; Wernerfelt, 1991). We study the brand alliance performance by using the commonly adopted co-operation schemes such as profit sharing, fixed royalties, and mergers in the co-branding partnership. With the neat analytical results, this model allows marketers to gain a better understanding of the effects of co-branding on business performance.

The rest of the paper is organized as follows. Section 2 shows a concise literature review. Section 3 discusses the basic model. Section 4 explores the strategic alliance schemes in co-branding. Section 5 concludes with a discussion of the remarkable insights and managerial implications. To improve presentation technical proofs are presented in the appendix.

2. Literature review

In the following review, we first examine some related co-branding studies. The spillover effect in co-branding has been greatly examined by empirical approaches. For example, Simonin and Ruth (1998) indicate that consumer attitudes towards co-branding can positively influence the subsequent consumers’ attitudes towards the individual brands that comprise the alliance. Blackett and Board (1999) find that co-branding is a way to mutually enhance the associated brands. Desai and Keller (2002) compare different ingredient branding strategies. The authors show that co-branded ingredients facilitate the initial acceptance of expansions. Washburn et al. (2004) establish a direct link between brand equity and co-branded products, and show that the high brand equity of the partner brands improves the perceived brand equity of the co-branded product and thereby generates positive spill-over effects. However, if the partnership is not appropriate in co-branding, it will have a negative effect on the associated brands. Helimg, Huber, and Leeflang (2008) argue that the co-branding strategy might have negative effects if either the combination of the two brands does not fit or the negative value perceptions about one brand hurts the partner brand. Observe that many studies in the empirical literature have explored the positive or negative spillover effect on co-branding. D’Allo Olmo Riley, Pina, and Bravo (2013) discuss the value of co-branding on the position of luxury brand by conducting experiments. Mazodier and Merunka (2014) examine the impact of self-congruity and need for uniqueness on symbolic co-branding purchases. They use the co-brands of mobile phones and fashion brands as experiments. They identify that managers should pay more attention to self-congruity, rather than the attitude towards the secondary brand, when selecting a strategic partner in co-branding alliance. Voss and Mohan (2016) empirically verify the role of corporate brand as a parent of its product brands in brand alliance by experiments. They find that the corporate brand is more diagnostic for customer evaluation of a co-branded product if the brand portfolio is more consistent. The work of Voss and Mohan (2016) provides important insights on the relationship between the parent brand and the participated brands in co-branding. In this paper, we examine the co-branding between fast fashion and luxury fashion. As a matter of fact, fast fashion brands such as H & M are more likely to be a parent of the co-brand while luxury fashion brands

| Table 2 | Collaborated designers/designer luxury labels with H & M. |
| Designer | Collaborative brand | Mother company (merging year) | Co-branding year | Co-brand name |
| Stella McCartney | Stella McCartney | Stella McCartney (Kering 2001) | 2005 | Stella McCartney × H & M |
| Viktor & Rolf | Viktor & Rolf | DIESEL (2008) | 2006 | Viktor & Rolf × H & M |
| Rei Kawakubo | Comme des Garçons | Comme des Garçons | 2008 | Comme des Garçons × H & M |
| Matthew Williamson | Matthew Williamson | Matthew Williamson | 2009 | Matthew Williamson for H & M |
| Sonia Rykiel | Sonia Rykiel | Li & Fung Group (2012) | 2009 | Sonia Rykiel × H & M |
| Donatella Versace (1997–now) | Versace | Gianni Versace S.P.A | 2011 | Versace for H & M |
| Isabel Marant | Isabel Marant | Isabel Marant | 2013 | Isabel Marant POUR × H & M |
| Pierre Balmain | BALMAIN | BALMAIN | 2015 | BALMAIN × H & M |

| Rivkin (2009) | “As its aesthetic evolved he found himself ripe for a mass-market collaboration. H & M is a line in the sand for me in that is an opportunity to collaborate and gather everything that everyone thinks of when they think of Matthew Williamson.” |
| Labbrand (2011) | “Co-branding helped consumers to become more knowledgeable about and familiar with luxury brands.” |
| Hutzler (2011) | “Luxury consumers won’t be confused by the brand or think less of a limited-time lower priced line extension being offered by H & M.” |
| Kong (2013) | “H & M have been trying their best to provide the greatest joy of high-end fashion for their loyal consumers by collaborating with famous designers’ brand.” |
| Paul (2014) | “When the average customer of today could become the luxury consumer of tomorrow, that’s just no way to build brand loyalty.” |
| Edmonds (2014) | “Alexander Wang is my absolute favourite designer. It got really cold about 3 am. But I was brave and didn’t get any warm drinks or anything as I didn’t want to lose my spot.” |
| Carreon (2014) | “This will be a great way for a wider audience to experience elements of the Alexander Wang brand and lifestyle.” |
| Machube (2015) | “This makes us love H & M even, more and I suspect this was all part of their plan to yet again increase our brand loyalty.” |
| Trenquet (2015) | “For designers, they can reach a new audience and seize an opportunity to build loyalty with shoppers at an early age.” |
| Teather (2015) | “The collection has caused an online frenzy due to the popularity of some of Balmain’s biggest fans.” |
are the participated one (Labbrand, 2011). The market size of fast fashion is larger than luxury fashion as the target consumers of former brand is mass consumers whereas the latter one is the rich group of consumers. Newmeyer et al. (2014) classify the structure of co-branding strategies into three dimensions: co-branding integration, co-branding exclusivity, and co-branding duration. They find that the above three dimensions significantly affect the value of focal brands in partner selection. In this paper, we examine the designer luxury and fast fashion co-branding, which is characterized by having a high integration level, a high exclusivity level, and short duration. Recently, Oeppen and Jamal (2014) examine the co-branding strategies in the fashion industry. They argue that the collaboration with limited availability in the fast fashion co-branding protects the brand from dilution or cannibalization of sales for the partner brand, and generates consumers' interest in a new market through the mass-market retailer. Observe that the above reviewed studies are all empirical in nature. The use of an analytical modeling approach to study co-branding is relatively rare. One example is the work by Geylani, Inman, and Ter Hofstede (2008) which reports an analytical modeling study on how co-branding strategies affect the brand image reinforcement and reveal that co-branding reinforces the partners’ images.

Brand performance is largely determined by brand loyalty (from consumers) which is a fundamental concept in strategic marketing (Mazodier & Merunka, 2012; Colicic, O’Connor, & Vinzi, 2016). Much of the related literature has defined brand loyalty. Aaker (1991) defines brand loyalty as the attachment that a customer has. Later, the definition is updated by Oliver (1999) who states that brand loyalty is a deeply held commitment to repurchase product or service consistently in the future by consumers. In this paper, brand loyalty refers to consumers’ repeated purchasing behaviors and consumer satisfaction (Kim et al., 2007). As a behavior, customer loyalty has been measured as the long-term choice probability for a brand (Colombo & Morrison, 1989; Wernerfelt, 1991). Kim et al. (2007) indicate that brand loyalty has emerged as a significant marketing concept for many consumer driven businesses and consumers with a high level of loyalty would spend more money on the products or services. Thus the level of loyalty is closely related to the consumers’ purchase behaviors. In an early book from Jacoby and Chestnut (1978), the authors develop models to measure the degree of consumer’s brand loyalty. Danaher, Wilson, and Davis (2003) consider the stochastic loyalty by using the online and offline sales data for over 100 brands in 19 grocery product categories. They yield an interesting finding that consumers have a high (low) degree of brand loyalty for the brand with a high (low) market share online. In this paper, we consider the situation in which the degree of brand loyalty can be measured and optimized. He et al. (2012) examine brand loyalty from a social identity perspective by collecting the survey data via mall intercept interviews. The authors integrate brand identity and identification with value, trust and satisfaction in predicting brand loyalty. Moreover, brand loyalty has been examined by analytical models. Rajju et al. (1990) analytically investigate the impact of brand loyalty and find that brands with larger brand loyalty promote less often. Agrawal (1996) examines the impact of advertising and price promotions on brand loyalty by a game theoretic analysis. Notice that one important finding in the literature on brand loyalty is: The brand loyal consumers are willing to pay higher prices and are less price-sensitive (Villas-Boas, 2004). Based on the above literature, we build a formal analytical model to conduct analysis on fashion co-branding, between a designer luxury fashion brand and a fast fashion brand, with the consideration of brand loyalty. Notice that there is still much controversy over how brand loyalty can be enhanced as it may be compensated and potentially neutralized by its rival (Shugan, 2005). In this paper, we consider the degree of brand loyalty significantly affects market demand and our focal point is to explore how the degree of brand loyalty for the participated brands in fashion co-branding influence firms’ performance.

3. Basic model

In this section, we develop an analytical model to examine the strategy of designer luxury and fast fashion co-branding. We denote the fast fashion brand as “brand A” (e.g. H & M), and the designer luxury brand as “brand B” (e.g., Jimmy Choo, Lavin, Alexander Wang), their co-brand as “brand C” (e.g., Jimmy Choo × H & M, Lavin for H & M; Alexander Wang × H & M). We denote $l_i$ as the level of brand loyalty of customers towards brand $i$, where $i \in \{A, B\}$. According to the extant literature, brand loyalty refers to the consumers’ repeated purchase behavior, i.e. a higher degree of brand loyalty implies the consumers purchase the products more repeatedly (Kim et al., 2007; Oliver, 1999). Thus, we consider the situation when the brand loyalties of both collaborated brands may affect their co-brand, but we do not consider the brand loyalty of brand C as this co-brand project is usually a one shot project which vanishes after the co-branding products are sold out during a short duration of time. We have the following assumption to construct our model.

Assumption 1. Co-branding has a spillover effect, and the consumer purchase is influenced by the brand loyalty levels of both cooperated brands.

Under Assumption 1, we consider the situation when the co-brand C’s demand is related to the brand loyalty of its associated brands A and B. We consider the demand function of co-brand C as follows.

$$D_C = a + b(l_A + \mu l_B),$$

where $a$ represents the basic market demand, $b$ is the coefficient of the difference between $l_A$ and $l_B$, where $b > 0$, and $\mu$ is the coefficient of $l_B$ with respect to $l_A$, where $\mu > 0$. We consider: (i) $b$ is positive because many industrial reports show that the final consumers are the actual consumers of fast fashion and may purchase fast fashion products repeatedly (Kong, 2013; Machube, 2015; Paul, 2014); (ii) $\mu$ is the frequency level of purchasing designer luxury fashion brands. A positive and larger $\mu$ means the brand loyalty of luxury fashion positively affects the market demand of co-brand C and the group of luxury fashion consumers would purchase the luxury fashion brands more frequently, whereas a negative and smaller $\mu$ means the brand loyalty of luxury fashion negatively influences the market demand of co-brand C and the group of luxury fashion consumers purchases the luxury fashion brands less frequently. This assumption is consistent with the conspicuous luxury consumption pattern under which there are bandwagon and snob consumption of luxury products (Kastanakis & Balabanis, 2014; Shen, Qian, Chen, & Jochen, 2015; Shen, Qian, & Choi, 2017). We denote $p$ as the retail price and $c$ as the production cost. Thus, the profit function for co-brand C is

$$\pi_C(l_A, l_B) = D_C(p - c) = (a + b(l_A + \mu l_B)) (p - c).$$

In this paper, we consider the case when the retail price and the production cost are exogenously given. The brand’s profit is defined as $f_i(l_i)$. After launching the co-brand C, the profits of the associated brands, brands A and B, might be changed compared to the case without co-branding. This profit change is denoted by $\pi_i(l_i)$. As a notation, we denote $\pi_i(l_i)$ as the profit for brand $i$ excluding the profit of co-brand C, namely, $\pi_i(l_i) = f_i(l_i) + \pi_i(l_i)$. Moreover, we denote $\pi_C(l_i)$ as the profit for brand $i$ including the profit of co-brand C, namely, $\pi_C(l_i) = f_i(l_i) + \pi_i(l_i) + \pi_C(l_A, l_B)$. To avoid trivial cases, we have: $\pi_1(l_i) \geq 0$. To have analytically tractable results, we consider that $\pi_i(l_i)$ is increasingly concave in $l_i$. Notice that this assumption is mild and it might be true that when the cost of brand loyalty enhancement has an exponential or a quadratic growth in the degree of brand loyalty. In this case, when the degree of brand loyalty is relatively small, enhancing brand loyalty can increase the firm’s profit, whereas when the degree of brand loyalty is...
sufficiently large, further enhancing brand loyalty hurts firm’s net profit as the cost of brand loyalty improvement is too high; in this case, the brand will stop enhancing brand loyalty. Thus, we consider the case in which enhancing brand loyalty can increase the firm’s profit, i.e. $\pi_l(l)$ is strictly increasing in $l_l$.

There are several scenarios for launching co-brand $C$ on the associated brand $i$:

**Scenario 1** $\sigma_l(l_l) > 0$, and $\pi_l(l_l) = f_l(l_l) + \pi_l(l_l) > 0$;

**Scenario 2** $\sigma_l(l_l) = 0$, and $\pi_l(l_l) = f_l(l_l) + \pi_l(l_l) > 0$;

**Scenario 3** $\sigma_l(l_l) < 0$, and $\pi_l(l_l) = f_l(l_l) + \pi_l(l_l) > 0$.

Notice that the scenarios “$\sigma_l(l_l) > 0$ and $f_l(l_l) + \sigma_l(l_l) < 0$” and “$\sigma_l(l_l) = 0$ and $f_l(l_l) + \sigma_l(l_l) < 0$” will not happen because $f_l(l_l)$ is sufficiently large. Here, $\sigma_l(l_l)$ is defined as the spillover effect of launching co-brand $C$ for the corresponding brand. Thus, $\sigma_l(l_l) > 0$ implies that launching co-brand $C$ is beneficial to the corresponding brand, $\sigma_l(l_l) < 0$ refers to the scenario under which launching co-brand $C$ hurts the corresponding brand, and $\sigma_l(l_l) = 0$ means that there is no co-brand partnership or launching co-brand $C$ has no effect on the participated brands. Without loss of generality, we consider $\sigma_n(l_{n,x}) = \delta_{i,n} l_{n,x}$ where $n \in \{1,2,3\}$, $\delta_{i,1} > 0$, $\delta_{i,2} = 0$ and $\delta_{i,3} < 0$ (i.e. $\delta_{i,1} > \delta_{i,2} > \delta_{i,3}$). We define that a larger $\sigma_n(l)$ implies the spillover effect of launching co-brand $C$ for the corresponding brand is larger.

**Lemma 1.** When there is no partnership between brands $A$ and $B$, namely, $\sigma_n(l_l) = 0$, the optimal brand loyalty of brand $i$ is $l_l^* = \arg\{f_l(l_l) = 0\}$.

**Lemma 1** gives a simple expression for finding the unique optimal brand loyalty for each parent brand in the absence of the co-branding alliance.

4. Strategic alliance schemes in fashion co-branding

In this section, we investigate the effectiveness of strategic alliance schemes (profit sharing scheme, fixed royalties scheme, and mergers scheme) between the designer luxury fashion and the fast fashion brands. We propose the mergers scheme in the partnership of co-branding as it is widely observed in the real world and also being examined in the literature.

4.1. Profit sharing scheme

We first examine the profit sharing (PS) scheme between brand $A$ and brand $B$, under which brand $A$ obtains $(1 - \mu)$ and brand $B$ takes the remaining $\mu$ of the co-brand profit, where $0 < \mu < 1$. The profit maximization problems of brands $A$ and $B$ are shown below.

\[
\max_{l_{A,B}} \pi_{A,B}(l_{A,B}) = \pi_{A,B}(l_{A,B}) + (1 - \mu)\pi_C(l_{A,B}, l_{B})
\]

\[
\max_{l_{A,B}} \pi_{B}(l_{B}) = \pi_{B}(l_{B}) + \lambda\pi_C(l_{A,B}, l_{B})
\]

s.t. $l_{A,B} > 0 \text{ and } 0 < \lambda < 1$.

We now denote the optimal brand loyalty for Eqs. (1) and (2), respectively. We denote the optimal solution by $l_{A,B}^*$, where $i \in (A,B)$. We have Proposition 1.

**Proposition 1.** (i) The optimal brand loyalty for brands $A$ and $B$ exist and are unique under all the scenarios; (ii) $l_{A,PS,1}^* > l_{A,PS,2}^* > l_{A,PS,3}^*$ and $l_{B,PS,1}^* > l_{B,PS,2}^* > l_{B,PS,3}^*$.

**Proposition 1 (i)** indicates that the optimal brand loyalty for both fast fashion and designer fashion brands can be found. Proposition 1(ii) implies that the high levels of brand loyalty towards the collaborated brands in a co-brand have a significant spillover effect, i.e. a high level of brand loyalty towards the collaborated brand will have a higher impact on the brand’s profit after co-branding collaboration. These results are some important managerial findings for luxury and fast fashion co-branding.

Further, we consider the situation when brand $A$, brand $B$ and their co-brand $C$ constitute an alliance system. We use the subscript $SC$ to denote the alliance cases. The corresponding profit of the alliance is defined by $\pi_{SC}(l_{A,n}, l_{B})$. We denote the optimal brand loyalty of alliance for brand $i$ as $l_{i,SC,n}^*$, and it maximizes the alliance profit. When the brand loyalties of brands $A$ and $B$ achieve $l_{A,SC,n}^* = l_{A,n}^*$ and $l_{B,SC,n}^* = l_{B,n}^*$ simultaneously, both brands $A$ and $B$ maximize not only the respective individual brand profits but also the centralized brand alliance profit. Thus, we have the following definition:

**Definition 1.** When the brand loyalties for brands $A$ and $B$ simultaneously satisfy $l_{A,SC,n}^* = l_{A,n}^*$ and $l_{B,SC,n}^* = l_{B,n}^*$, the alliance coordination is said to be achieved.

Since the alliance profit includes the profits from brands $A$ and $B$ as well as the profit from co-brand $C$, its function can be expressed below

\[
\pi_{SC}(l_{A,n}, l_{B}) = \pi_{A,B}(l_{A,n}) + \pi_{B}(l_{B}) + f_{A,B}(l_{A,n}) + \pi_C(l_{A,n}, l_{B}) = f_{A,B}(l_{A,n}) + \pi_{A,B}(l_{A,n}) + f_{A,B}(l_{B}) + \pi_C(l_{A,n}, l_{B})
\]

(3)

To optimize the alliance profit $\pi_{SC}(l_{A,n}, l_{B})$, we can derive the first and second order derivatives of $\pi_{SC}(l_{A,n}, l_{B})$ with respect to $l_{A,n}$ and $l_{B,n}$, and find Proposition 2 by using the first order conditions.

**Proposition 2.** (i) The alliance coordination cannot be achieved under the PS scheme; (ii) $l_{A,SC,n}^* < l_{A,PS,n}^*$ and $l_{B,SC,n}^* > l_{B,PS,n}^*$.

**Proposition 2** indicates that the PS scheme cannot help coordinate the alliance. This is a surprising result as the PS scheme is known to be very useful in coordinating alliance systems. In addition, the centralized optimal brand loyalty for brand $A$ is higher than its decentralized counterpart under Scenarios 1, 2, and 3.

The value of $\mu$ affects the optimal brand loyalty for brand $B$ (i.e. the luxury brand).

**Proposition 3.** $l_{B,PS,n}^*$ and $l_{B,SC,n}^*$ are increasing in $\mu$.

**Proposition 3** reveals the impacts of $\mu$ on the brand loyalty. Recall that $\mu$ is the coefficient of luxury fashion brand loyalty on co-branding market demand. A positive and larger $\mu$ means the level of brand loyalty for the designer luxury fashion brand positively affects the market demand of co-brand $C$ and the group of luxury fashion consumers would purchase the luxury fashion brands more frequently, whereas a negative and smaller $\mu$ means the brand loyalty of luxury fashion consumers negatively influences the market demand of co-brand $C$ and the group of luxury fashion consumers would purchase the luxury fashion brands infrequently. The co-brand would be more successful (i.e. $\mu$ is positive and larger) when the brand loyalty of designer luxury fashion is strong (i.e. consumers have a high frequency of repeated purchase). Notice that this spillover effect is common in co-branding (Desai & Keller, 2002; Helming et al., 2008). It is an important result for collaboration partner selection. If the partnership is not appropriate in co-branding, a brand might have a negative impact on the market demand of its co-brand.
4.2. Fixed royalties scheme

After exploring the PS scheme, we now examine the fixed-royalty (FR) scheme under which the profit gained from co-brand C is not shared between brands A and B; instead, one party (e.g. brand A) is responsible for running co-brand C and collecting its revenue, and then a fixed royalty \( w \) is paid to the other party (e.g. brand B). Here, we separate this scheme into 2 cases, where Case 1 considers the situation when brand A plays the role of the leader and pays \( w \) to brand B, whereas the brand B plays the role of the leader and pays \( w \) to brand A in Case 2. The profit functions of brand A, brand B and the co-brand alliance under the fixed-royalty scheme are shown below.

**Case 1 (Brand A is the leader):**

\[
\pi_{ai}(l_{ai}, l_{bi}) = \pi_{ai}(l_{ai}) + \pi_{c}(l_{ai}, l_{bi}) - w, \tag{4}
\]

\[
\pi_{bi}(l_{bi}) = \pi_{bi}(l_{bi}) + w, \tag{5}
\]

and

\[
\pi_{SC, i}(l_{ai}, l_{bi}) = \pi_{ai}(l_{ai}) + \pi_{bi}(l_{bi}) + \pi_{c}(l_{ai}, l_{bi}). \tag{6}
\]

**Case 2 (Brand B is the leader):**

\[
\pi_{bi}(l_{bi}) = \pi_{bi}(l_{bi}) + w, \tag{7}
\]

\[
\pi_{ai}(l_{ai}) = \pi_{ai}(l_{ai}) + \pi_{c}(l_{ai}, l_{bi}) - w, \tag{8}
\]

and

\[
\pi_{SC, i}(l_{ai}, l_{bi}) = \pi_{ai}(l_{ai}) + \pi_{bi}(l_{bi}) + \pi_{c}(l_{ai}, l_{bi}). \tag{9}
\]

We use the subscript \( L \) to represent the leader and subscript \( F \) to denote the follower in the alliance. The leader implies the party who is the host, i.e. who runs and operates the co-brand. The follower then receives the fixed royalty fee. By maximizing the various parties’ profits, we have two propositions.

**Proposition 4.** (i) The optimal brand loyalties for brands A and B are unique under the FR scheme; the optimal brand loyalties satisfy \( l_{a, FR, l}^* > l_{b, FR, f}^* > l_{a, FR, c}^* \) and \( l_{b, FR, l}^* > l_{b, FR, f}^* > l_{b, FR, c}^* \).

**Proposition 5.** The alliance coordination cannot be achieved under the FR scheme.

**Proposition 4** indicates the optimal brand loyalties for both luxury and fast fashion brands after launching their co-brand under the FR scheme. The explicit analytical relationships are found. One important insight is: The brand leadership in establishing the co-brand significantly affects the relationships among the optimal brand loyalties. This is a critical issue because it relates to the proper matching of the fast fashion and designer luxury brands. For different desirable situations, different leaderships should be adopted. **Proposition 5** further shows that the alliance cannot be coordinated under the FR scheme.

4.3. Mergers scheme

As discussed above, both the PS and the FR schemes fail to achieve alliance coordination. To develop a proper coordination measure, we now examine the mergers (MG) scheme under which brands A and B share the profit from the whole alliance with the launching of co-brand C. To be specific, brand B obtains \( \eta \) of the whole alliance profit and brand A gets the remaining \((1 - \eta)\), where \( 0 < \eta < 1 \). In this case, the profit functions of brands A and B are shown below.

\[
\pi_{ai}(l_{ai}) = (1 - \eta)\pi_{SC, i}(l_{ai}, l_{bi}), \tag{10}
\]

\[
\pi_{bi}(l_{bi}) = \eta\pi_{SC, i}(l_{ai}, l_{bi}). \tag{11}
\]

\[
\pi_{SC, i}(l_{ai}, l_{bi}) = \pi_{ai}(l_{ai}) + \pi_{bi}(l_{bi}) + \pi_{c}(l_{ai}, l_{bi}). \tag{12}
\]

By maximizing various parties’ profits, we have **Propositions 6 and 7**.

**Proposition 6.** (i) The optimal brand loyalties for brands A and B uniquely exist under the MG scheme; (ii) \( l_{MG, 1}^* > l_{MG, 2}^* > l_{MG, 3}^* \) and \( l_{MG, 1}^* > l_{MG, 2}^* > l_{MG, 3}^* \), (iii) When \( \mu \) is positive, \( l_{MG, 1}^* \) is increasing in \( \mu \).

**Proposition 7.** The alliance coordination can be achieved under the MG scheme.

**Proposition 7** shows the relationship of optimal brand loyalties under different scenarios for the fast fashion brand and the designer luxury fashion brand under the MG scheme. Under the MG scheme, the brand loyalty for designer luxury fashion brand is similar to that under the PS scheme. The co-brand would be successful (i.e. \( \mu \) is positive and large) when the brand loyalty for designer luxury fashion is large (e.g., consumers have a high frequency of repeated purchase). **Proposition 7** is interesting and important. It implies that under the MG scheme, both fast and luxury fashion brands would simultaneously build their levels of brand loyalty equal to the centralized optimal brand loyalty for profit maximization. Thus, the alliance can achieve the best outcome with respect to profit. As a result, the MG scheme is a useful measure to yield an optimal alliance.

5. Conclusion and managerial implications

In this study, we have proposed a general model to examine the impacts of brand loyalty on revenue management in co-branding of designer luxury fashion and fast fashion. We have considered a co-brand alliance which is consisted of two associated brands (i.e. a designer luxury fashion brand and a fast fashion brand) and their co-brand. As we have proven analytically in this paper, such an alliance is beneficial to both parties: both fast fashion and designer fashion brands can increase customers’ brand loyalties and expand their consumer base with profit maximization. We have explored the commonly adopted schemes in the industry such as the PS scheme, the FR scheme and the MG scheme and reveal whether the “brand alliance” can be coordinated. We have identified the optimal brand loyalty levels with respect to the different financial outcomes of launching a co-brand and different types of brand alliance. Based on the results we derived from our analytical models, we have yielded the following managerial implications.

5.1. A. Impacts brought by brand loyalties of participating brands.

Among the three explored schemes, if the fast fashion wants to increase its profit after launching the co-brand, the brand loyalty for fast fashion should be enhanced; if the designer luxury fashion brands want to increase their profit after launching the co-brand, when \( \mu \) is positive (negative), the brand loyalty for luxury fashion should be improved (reduced). Moreover, we find that if the co-brand is successful, the brand loyalty for the fast fashion brand should be high. This insight is consistent with the industrial observation that most consumers purchasing the co-brands are the customers interested in fast fashion brands and they are only the potential customers of designer luxury fashion (e.g. Carreon, 2014; Kong, 2013; Paul, 2014; Rivkin, 2009; Tronquet, 2015).

5.2. B. Brand alliance coordination.

We find that the alliance can only be coordinated by the MG scheme, but not the PS and FR schemes. Brand alliance coordination guarantees the attainment of the optimal outcome among the parent brands and their co-brand. In other words, the total profit of the fast fashion brand, the designer luxury fashion brand, and the co-brand are maximized. Our results imply that both the PS and FR schemes are not able to achieve alliance coordination. In other words, many current cooperation strategies adopted in practice are not yet the most efficient (e.g. H & M when working with the designer luxury fashion brands).
5.3. C. Intra-company co-branding.

Our analytical results show that the brand alliance can only be coordinated by the MG scheme. In other words, adopting the MG scheme is the most efficient partnership strategy for launching co-brands among the three strategies. The MG scheme can be adopted for the brands which belong to the same company. This shows that internal cooperation within the group is the most efficient scheme for co-branding. This also means, e.g., the big fashion groups, such as Uniqlo Corporation and Inditex Group (which have multiple fashion brands) can organize some fashion brands within themselves to launch some co-brands and apply the MG scheme for overall profit maximization. Moreover, this finding further suggests that fast fashion brands like H&M may consider merging with some designer luxury fashion brands for co-branding and adopt the MG scheme.

5.4. D. Co-branding partner selection for fast fashion brand (e.g. H&M).

From our analytical result under the PS, FR, and MG schemes, the co-brand would be more successful when \( \mu \) is positive and large, i.e., when the brand loyalty for the designer luxury fashion brand is strong (i.e. consumers have a high frequency of repeated purchase). This result implies that fast fashion brands such as H&M should cooperate with the designer luxury fashion brand which possesses a high degree of brand loyalty. This result is consistent with the real industrial practices according to the summary of H&M co-brands, as shown in Table 1. H&M works with the well-known fashion brands which have a large group of consumers with a strong level of brand loyalty. This insight could be interpreted from the other side: If the designer fashion brands are willing to work with the fast fashion brands for co-branding, they should work hard to attract a larger group of consumers with a strong level of brand loyalty. Therefore, selecting the right partners is important to join hands in strength, realize the win-win cooperation, and achieve both firms’ business sustainability in the co-branding venture.

Our study is subject to two main limitations that point towards fruitful directions for future research. First, to focus on brand loyalties in luxury and fast fashion, we simplify the analytical model and consider the case when the retail price is given. In future research, it would be interesting to examine optimal pricing issues and consider how the pricing issues of the participating brands and co-brand would affect the optimal brand loyalties and firms’ performance. Second, the brand loyalty towards the participated brands in the co-brand partnership may be changed after launching the co-brand. Thus it would be important to evaluate the brand loyalty changes in the short and long terms after launching the co-brand by a longitudinal study (Dawes, Meyer-Waardenb, & Driesener, 2015; Fatma & Rahman, 2017). This may lead to a future extension of the analytical model. Last but not least, information asymmetry (Yue, Mukhopadhyay, & Zhu, 2006) is a critical issue which can also be explored in the analytical model in the future.

Appendix A. all proofs and tables

A.1. Proof of Lemma 1:

We take the first and the second order derivatives of \( \pi_i \) with respect to \( l_i \) when \( \pi_i(l_i) = 0 \). As \( f_i(l_i) \) is defined as the concave function of \( l_i \), we can find that when \( \pi_i(l_i) = 0 \), the optimal brand loyalty of brand \( i \) is \( l_i^* = \text{arg} \left( f_i'(l_i) = 0 \right) \). (Q.E.D.)

A.2. Proof of Proposition 1

Based on Eqs. (1) and (2), in order to obtain the optimal brand loyalties for brands A and B, we take the first and the second order derivatives of \( \pi_i \) with respect to \( l_i \).

For brand A,

\[
\frac{\partial \pi_{iA}}{\partial l_{iA}} = f_{iA} (l_{iA}) + \delta_{iA} + \frac{\partial (1 - \lambda) \pi_C (l_{iA}, l_{iB})}{\partial l_{iA}} = f_{iA} (l_{iA}) + \delta_{iA} + (1 - \lambda)(p - c)b,
\]

(A1)

and

\[
\frac{\partial^2 \pi_{iA}}{\partial l_{iA}^2} = f_{iA}' (l_{iA}) < 0.
\]

(A2)

For brand B,

\[
\frac{\partial \pi_{iB}}{\partial l_{iB}} = f_{iB} (l_{iB}) + \delta_{iB} + \frac{\partial (1 - \lambda) \pi_C (l_{iA}, l_{iB})}{\partial l_{iB}} = f_{iB} (l_{iB}) + \delta_{iB} + \lambda \mu (p - c)b,
\]

(A3)

and

\[
\frac{\partial^2 \pi_{iB}}{\partial l_{iB}^2} = f_{iB}' (l_{iB}) < 0.
\]

(A4)

As the second order derivatives of \( \pi_i \) with respect to \( l_i \) are all negative, we can obtain the result of Proposition 1(i) that the optimal brand loyalties exist and are unique. Then, we can compare the optimal brand loyalties.

For Proposition 1(ii), we let the Eqs. (A1) and (A3) be zero and obtain the optimal brand loyalty level. As we assume \( \delta_{i,1} > \delta_{i,2} > \delta_{i,3} > 0 \), we have \( l_{iA,PS,1} > l_{iA,PS,2} > l_{iA,PS,3} \) and \( l_{iB,PS,1} > l_{iB,PS,2} > l_{iB,PS,3} \). (Q.E.D.)

A.3. Proof of Proposition 2

Based on Eq. (3), in order to obtain the optimal brand loyalties for alliance with respect to brand A or B, we take the first and the second order derivatives of \( \pi_{iSC} \) with respect to \( l_i \), respectively. Then we can find

\[
l_{iSC}^* = \text{arg} \left( f_i'(l_{iA}) = -\delta_{iA} - (p - c)b \right)
\]

and

following the summary of H&M co-brands, as shown in Table 1. H&M works with the well-known fashion brands which have a large group of consumers with a strong level of brand loyalty. This insight could be interpreted from the other side: If the designer fashion brands are willing to work with the fast fashion brands for co-branding, they should work hard to attract a larger group of consumers with a strong level of brand loyalty. Therefore, selecting the right partners is important to join hands in strength, realize the win-win cooperation, and achieve both firms’ business sustainability in the co-branding venture.

Our study is subject to two main limitations that point towards fruitful directions for future research. First, to focus on brand loyalties in luxury and fast fashion, we simplify the analytical model and consider the case when the retail price is given. In future research, it would be interesting to examine optimal pricing issues and consider how the pricing issues of the participating brands and co-brand would affect the optimal brand loyalties and firms’ performance. Second, the brand loyalty towards the participated brands in the co-brand partnership may be changed after launching the co-brand. Thus it would be important to evaluate the brand loyalty changes in the short and long terms after launching the co-brand by a longitudinal study (Dawes, Meyer-Waardenb, & Driesener, 2015; Fatma & Rahman, 2017). This may lead to a future extension of the analytical model. Last but not least, information asymmetry (Yue, Mukhopadhyay, & Zhu, 2006) is a critical issue which can also be explored in the analytical model in the future.

Appendix A. all proofs and tables

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A.2. Proof of Proposition 1

Based on Eqs. (1) and (2), in order to obtain the optimal brand loyalties for brands A and B, we take the first and the second order derivatives of \( \pi_i \) with respect to \( l_i \).

For brand A,

\[
\frac{\partial \pi_{iA}}{\partial l_{iA}} = f_{iA} (l_{iA}) + \delta_{iA} + \frac{\partial (1 - \lambda) \pi_C (l_{iA}, l_{iB})}{\partial l_{iA}} = f_{iA} (l_{iA}) + \delta_{iA} + (1 - \lambda)(p - c)b,
\]

(A1)

and

\[
\frac{\partial^2 \pi_{iA}}{\partial l_{iA}^2} = f_{iA}' (l_{iA}) < 0.
\]

(A2)

For brand B,

\[
\frac{\partial \pi_{iB}}{\partial l_{iB}} = f_{iB} (l_{iB}) + \delta_{iB} + \frac{\partial (1 - \lambda) \pi_C (l_{iA}, l_{iB})}{\partial l_{iB}} = f_{iB} (l_{iB}) + \delta_{iB} + \lambda \mu (p - c)b,
\]

(A3)

and

\[
\frac{\partial^2 \pi_{iB}}{\partial l_{iB}^2} = f_{iB}' (l_{iB}) < 0.
\]

(A4)

As the second order derivatives of \( \pi_i \) with respect to \( l_i \) are all negative, we can obtain the result of Proposition 1(i) that the optimal brand loyalties exist and are unique. Then, we can compare the optimal brand loyalties.

For Proposition 1(ii), we let the Eqs. (A1) and (A3) be zero and obtain the optimal brand loyalty level. As we assume \( \delta_{i,1} > \delta_{i,2} > \delta_{i,3} > 0 \), we have \( l_{iA,PS,1} > l_{iA,PS,2} > l_{iA,PS,3} \) and \( l_{iB,PS,1} > l_{iB,PS,2} > l_{iB,PS,3} \). (Q.E.D.)

A.3. Proof of Proposition 2

Based on Eq. (3), in order to obtain the optimal brand loyalties for alliance with respect to brand A or B, we take the first and the second order derivatives of \( \pi_{iSC} \) with respect to \( l_i \), respectively. Then we can find

\[
l_{iSC}^* = \text{arg} \left( f_i'(l_{iA}) = -\delta_{iA} - (p - c)b \right)
\]

and
\[ l_{SC,n}^* = \arg \left\{ f'_{B_n} (l_{B_n}) = -\delta_{Bn} - b\nu (p - c) \right\} \]  
\[ (A6) \]

We can easily find that \( l_{SC,n}^* \neq l_{PS,n}^* \). As a result, the alliance does not achieve coordination as shown in Proposition 2(i). Moreover, we compare the optimal brand loyalty of centralized with the decentralized one, we can obtain Proposition 2(ii) and (iii). (Q.E.D.)

### A.4. Proof of Proposition 3

According to the results of Propositions 1 and 2, we can find that \( l_{A,SC,n}^* < l_{A,PS,n}^* \) and \( l_{B,SC,n}^* > l_{B,PS,n}^* \). (Q.E.D.)

### A.5. Proofs of Propositions 4 and 5

To obtain the optimal brand loyalties for the collaborated brands \( A \) and \( B \) as well as the alliance, we take the first and second order derivatives of the profit functions with respect to the corresponding brand loyalty, respectively.

When brand \( A \) is the leader,

\[ \frac{\partial \pi_A}{\partial l_A} = f'_{A_n} (l_{A_n}) + \nu'_{A_n} (l_{A_n}) + \frac{\partial \pi_C (l_{A_n}, l_{B_n})}{\partial l_A} = f'_{A_n} (l_{A_n}) + \delta_{Bn} + (p - c)b \]  
\[ (A7) \]

and

\[ \frac{\partial^2 \pi_A}{\partial l_A^2} = f''_{A_n} (l_{A_n}) < 0 \]  
\[ (A8) \]

\[ \frac{\partial \pi_B}{\partial l_B} = f'_{B_n} (l_{B_n}) + \delta_{Bn} \]  
\[ (A9) \]

and

\[ \frac{\partial^2 \pi_B}{\partial l_B^2} = f''_{B_n} (l_{B_n}) < 0. \]  
\[ (A10) \]

When brand \( B \) is the leader,

\[ \frac{\partial \pi_A}{\partial l_A} = f'_{A_n} (l_{A_n}) + \nu'_{A_n} (l_{A_n}) = f'_{A_n} (l_{A_n}) + \delta_{Bn} \]  
\[ (A11) \]

and

\[ \frac{\partial^2 \pi_A}{\partial l_A^2} = f''_{A_n} (l_{A_n}) < 0. \]  
\[ (A12) \]

\[ \frac{\partial \pi_B}{\partial l_B} = f'_{B_n} (l_{B_n}) + \nu'_{B_n} (l_{B_n}) + \frac{\partial \pi_C (l_{A_n}, l_{B_n})}{\partial l_B} = f'_{B_n} (l_{B_n}) + \delta_{Bn} + \mu (p - c)b \]  
\[ (A13) \]

and

\[ \frac{\partial^2 \pi_B}{\partial l_B^2} = f''_{B_n} (l_{B_n}) < 0. \]  
\[ (A14) \]

We find that the second order derivatives with \( l \) are all negative. Thus, by letting the first order derivatives of profit functions with respect to the corresponding brand loyalty be zero (i.e., by the first order conditions in Eqs. (A7), (A9), (A11), and (A13)), we can obtain the optimal brand loyalties \( l_{A,FR,n}^* \) and \( l_{B,FR,n}^* \). Then we compare the results of brand loyalties, we obtain Proposition 4. Moreover, comparing the results of optimal brand loyalties in the centralized system with the ones in the decentralized system (from Eqs. (A5) and (A6)), we can easily see that the brand loyalties of fast fashion and designer luxury fashion in the decentralized case are not equal to the ones in the centralized system simultaneously, i.e., the alliance coordination cannot be achieved. Then we obtain Proposition 5. (Q.E.D.)

### A.6. Proofs of Proposition 6

To obtain the optimal brand loyalties for the collaborated brands \( A \) and \( B \), we take the first and second order derivatives of the profit functions with respect to the corresponding brand loyalty, respectively.

\[ \frac{\partial \pi_A (l_{A_n})}{\partial l_A} = \left( 1 - \eta \right) \frac{\partial \pi_{SC,n} (l_{A_n})}{\partial l_A} + \left( 1 - \eta \right) \frac{\partial \pi_C (l_{A_n}, l_{B_n})}{\partial l_A} \]  
\[ (A15) \]

\[ \frac{\partial^2 \pi_A (l_{A_n})}{\partial l_A^2} = \left( 1 - \eta \right) \frac{\partial^2 \pi_{SC,n} (l_{A_n})}{\partial l_A^2} < 0. \]  
\[ (A16) \]

\[ \frac{\partial \pi_B (l_{B_n})}{\partial l_B} = \frac{\partial \pi_{SC,n} (l_{B_n})}{\partial l_B} + \frac{\partial \pi_C (l_{A_n}, l_{B_n})}{\partial l_B} \]  
\[ (A17) \]

\[ \frac{\partial^2 \pi_B (l_{B_n})}{\partial l_B^2} = \frac{\partial^2 \pi_{SC,n} (l_{B_n})}{\partial l_B^2} < 0. \]  
\[ (A18) \]

Let the first order derivatives of the profit functions (i.e. Eqs. (A15) and (A17)) with respect to the corresponding brand loyalty be zero, we can
obtain the optimal brand loyalties. Then by comparing the results of brand loyalties, we can obtain Proposition 6. (Q.E.D.)

A.7. Proofs of Proposition 7

Moreover, comparing the result of optimal brand loyalties in the centralized system with the ones in the decentralized system, we can find that when calculating the optimal brand loyalty in brand $A$ and brand $B$, $(1 - \gamma)$ in Eq. (A15) and $\gamma$ in Eq. (A17) are cancelled out. In other words, we have $\eta_{B, SC, \gamma} = \eta_{MG, \gamma}$ and $\eta_{B, SC, \gamma} = \eta_{B, MG, \gamma}$.

References


