Metaheuristic Strategies for Solving the Optimal Reactive Power Dispatch with Discrete Variables

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***Abstract*— Optimal power flow (OPF) is a classical nonlinear, large-scale, non-convex problem. It is used as a tool to determine the optimal operating point ofthe electric power system, improving its performance. Usually its resolution complexity is rather increased in the presence of discrete control variables, which poses as a challenge for the application of general classical nonlinear optimization algorithms. In this paper, the Evolutionary Particle Swarm Metaheuristic algorithm that combines features of classical optimization methods and non-deterministic methods is proposed to solve a specific class of OPF problems, the Optimal Reactive Power Dispatch with discrete control variables such as in-phase transformer taps. Two strategies have been used to initialize the control variables. In order to validate the proposed method, tests have been performed on IEEE 14, 30, 57, 118 and 300 bus systems.**

I. Introduction

The deregulation on electric power systems (EPS) brought new paradigms for planning and operation. In this new scenario, optimization techniques to aid decision-making plays a key role in EPS planning and operation, specially when considering the limited natural resources. The decision-making is, in general, guided by economic efficiency criteria to save generation and transmission costs.

Due to the increase of electric power demand in Brazil, several studies in the field of EPS has been made in order to achieve a safer and more economical operation. While some authors have used optimization algorithms packages, with specific methods for solving real problems [1], [2], others use metaheuristics [3], [4] to achieve a balance between the solution quality and computational cost.

A special optimization problem in EPS is known as Optimal Power Flow (OPF). The OPF is a nonlinear, large-scale, constrained and non-convex problem. It is used as a tool to determine the best operation point, improving its performance. The first OPF model was proposed by Carpentier [5], and since then several methods have been proposed to solve this problem. One specific OPF is the Optimal Reactive Power Dispatch problem. The difference to general OPF is due to the fact that control variables are related to reactive power control.

The formulation of real problems as nonlinear optimization problems, such as the OPF, is found in various areas of knowledge such as engineering, mathematics, agronomy, economics, among others. These problems are often difficult to solve because of its nonlinearity. Different mathematical approaches, such as the Gradient Method [6]; Interior Point Method [7] and Newton-Raphson Method [8] have been applied to find a solution to the OPF problem. These approaches are named as classical or deterministic methods.

Besides the classical approaches, heuristic (or non-deterministic) methods have also been used to solve the OPF problem, such as Evolutionary Programming [9], Genetic Algorithm [10] and Particle Swarm Optimization (PSO) [11].

The PSO was originally developed by a social psychologist named James Kennedy and an electrical engineer, Russell Eberhart, in 1995. It emerged from experiments with algorithms that modeled the behavior of many bird species in their search for food. Over the years, new research addressed variations of the original PSO algorithm. Miranda and Fonseca [12] presented a new metaheuristic known as Evolutionary Particle Swarm Optimization (EPSO), which put together the best features of Evolutionary Programming and PSO.

According to Frank et al. [13], deterministic approaches can be computationally much faster than non-deterministic approaches. However, the deterministic methods are limited to provide optimal solutions and the quality of the solution is sensitive to the starting point. To overcome the fact the algorithm can get stuck in a local optimum, local search techniques can be combined with the global search procedures provided by non-deterministic methods. Taking this fact into account, in this paper a hybrid nonlinear programming method, which is a combination of deterministic and non-deterministic methods is proposed.

The kernel of the method is the EPSO metaheuristic to solve the Optimal Reactive Power Dispatch, which is a variation of the Optimal Power Flow. In this problem, the controls are mainly related to reactive power such as voltage regulation, transformer taps, shunt elements. The objective function is the minimization of system losses and the discrete control variables associated with in-phase transformers taps are modeled accordingly to provide the best solution performance.

To evaluate the performance of the proposed method, tests were carried out using the IEEE test cases of 14, 30, 57, 118 and 300 buses [14].

II. Optimal Reactive Power Dispatch (ORPD)

This section presents the problem formulation with continuous and discrete variables and the use of a discretization function to deal with the discrete variables.

A. ORPD with continuous and discrete variables

The transformer taps and the equivalent susceptances of capacitor banks and shunt reactors are modeled as control variables in the problem formulation with continuous and discrete variables. Therefore, the model is defined as:

$$\min \sum_{k,m \in L \cup T} g_{km} \left(\frac{1}{t_{km}^2} V_k^2 + V_m^2 - 2 \frac{1}{t_{km}} V_k V_m \cos\theta_{km} \right) \quad (1)$$

subject to:

$$P_k - \sum_{m \in v_k} P_{km}(V, \theta, t) = 0 \qquad \forall k \in G' \cup C \quad (2)$$

$$Q_k + Q_k^{sh}(V_k, b_k^{sh}) - \sum_{m \in v_k} Q_{km}(V, \theta, t) = 0 \quad \forall k \in C \quad (3)$$

$$Q_{G_k}^{\min} \le Q_{G_k}(V_k, b_k^{sh}) \le Q_{G_k}^{\max} \qquad \forall k \in G'' \quad (4)$$

$$V_k^{\min} \le V_k \le V_k^{\max} \qquad \forall k \in B \quad (5)$$

$$b_k^{sh} \in D_k^{sh} \qquad \forall k \in B^{sh} \quad (6)$$

$$t_{km} \in D_{km}^{tap} \qquad \forall k, m \in T \quad (7)$$

where V and θ are vectors representing the magnitude and phase angle of voltages in all system buses; t is the vector of in-phase transformers taps; b^{sh} is the vector of capacitor banks and shunt reactors equivalent susceptances; P_k and Q_k represent, respectively, the net active and reactive power injections at bus k; Q_{G_k} is the generated reactive power at bus k; Q_k^{sh} is the reactive power injected by the shunt elements at bus k; P_{km} and Q_{km} are the active and reactive power flow at the branch $k-m$; θ_{km} is the phase angle difference between the voltages at buses k and m; B is the set of all system buses; G'' is the set of all generation buses; G' is the set of all generation buses except the slack bus; C is the set of all load buses; B^{sh} is the set of all buses with voltage magnitude controled by capacitor banks and shunt reactors; L is the set of branches $k-m$ representing the transmission lines; T is the set of branches $k-m$ representing the in-phase transformers with variable tap; v_k is the set of buses connected to the bus k; D_k^{sh} is the set of all discrete values that the capacitor banks and shunt reactors equivalent susceptances may assume; D_{km}^{tap} is the set of all discrete values that the transformer taps may assume.

B. Discretization Function to deal with discrete variables

Soler et al. [2], [15] proposed an approach in which the discrete variables are treated as continuous variables by incorporating sinusoidal functions into the objective function of the problem, turning the nonlinear continuous and discrete programming problem into a nonlinear programming problem with only continuous control variables.

Consider the following nonlinear programming problem:

$$\begin{aligned}
&\min && f(x) && && (8)\\
&\text{subject to:} && g_i(x) = 0 && i = 1, \ldots, p && (9)\\
& && h_i(x) \le 0 && i = 1, \ldots, q && (10)\\
& && x_{1_i}^{\min} \le x_{1_i} \le x_{1_i}^{\max} && i = 1, \ldots, m_1 && (11)\\
& && x_{2_i} \in D_{x_i} && i = 1, \ldots, m_2 && (12)
\end{aligned}$$

where $x_{1_i} \in \mathbb{R}^{m_1}$ is the vector of continuous variables and $x_{2_i} \in D_x \subset \mathbb{R}^{m_2}$ is the vector of discrete variables; with $x = (x_1, x_2)$; D_{x_i} is the set of discrete values for each variable x_{2_i}; $f : \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \to \mathbb{R}$; $g : \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \to \mathbb{R}^p$, with $p < m_1 + m_2$; and $h : \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \to \mathbb{R}^q$.

The function $\phi : \mathbb{R} \to \mathbb{R}$ to deal with discrete variables x_2 is, according to Soler et al. [15], defined as:

$$\phi(y_i) = \left[\sin\left(\frac{y_i}{y_i^{\sup} - y_i^{\inf}} \pi + \alpha_i \right) \right]^{2\beta} \quad (13)$$

where $\beta > 0$ is an integer parameter that determines the shape of (13); $y_i^{\sup}$ and $y_i^{\inf}$ are discrete values immediately above and below y_i, respectively; $y_i^{\min}$ and $y_i^{\max}$ are the minimum and maximum quantities from the set of discrete values D_{x_i}, respectively; and α_i is a constant defined in the interval $[0; \pi]$ such that (13) is canceled in $y_i = x_{2_i} \in D_{x_i}$.

The function given in (13) is formulated as:

$$\phi(y_i) = \begin{cases} 0 & y_i \in D_{x_i} \\ \rho > 0 & \text{otherwise} \end{cases} \quad (14)$$

That is, equation (13) is null if and only if y_i assumes discrete values. By incorporating (13) in the objective function (8) for each discrete variable, the following modified problem is obtained:

$$\begin{aligned}
&\min && f(x) + \gamma \sum_{i=1}^{m_2} \phi(x_{2_i}) && && (15)\\
&\text{subject to:} && g_i(x) = 0 && i = 1, \ldots, p && (16)\\
& && h_i(x) \le 0 && i = 1, \ldots, q && (17)\\
& && x_{1_i}^{\min} \le x_{1_i} \le x_{1_i}^{\max} && i = 1, \ldots, m_1 && (18)\\
& && x_{2_i} \in D_{x_i} && i = 1, \ldots, m_2 && (19)
\end{aligned}$$

where $\gamma > 0$ is a penalty parameter that controls the sinusoidal function amplitude; $x_1 \in \mathbb{R}^{m_1}$ and $x_2 \in \mathbb{R}^{m_2}$ are the vectors of continuous variables, with $x = (x_1, x_2)$ and $x \in \mathbb{R}^n$; $f : \mathbb{R}^n \to \mathbb{R}$; $g : \mathbb{R}^n \to \mathbb{R}^p$, with $p < n$; and $h : \mathbb{R}^n \to \mathbb{R}^q$. In the above modified problem, the functions f, g and h are class C^2 and $x_2^{\min}$, $x_2^{\max} \in \mathbb{R}^{m_2}$ are the vectors of lower and upper limits of continuous variables, respectively, with $x_{2_i}^{\min} = \min\{D_{x_i}\}$ and $x_{2_i}^{\max} = \max\{D_{x_i}\}$ for $i = 1, \ldots, m_2$.

As $\sin(y) = 0$ only when $y = n\pi$, where $n \in \mathbb{Z}$, the constant α in the penalty function (13) is determined in respect of $x_2 \in D_x$ as follows:

$$\frac{x_2}{x_2^{\sup} - x_2^{\inf}}\pi + \alpha = \left(n - \frac{x_2}{x_2^{\sup} - x_2^{\inf}}\right)\pi \tag{20}$$

As $\alpha \in [0;\pi]$, n must be equal to the next higher integer closest to:

$$\frac{x_2}{x_2^{\sup} - x_2^{\inf}} \tag{21}$$

The parameter α is the same for every interval of D_x and the penalty process consists in solving a sequence of modified problems (15) until all variables associated with the original problem discrete variables assumes discrete values. The initialization of the parameter γ is critical to the solution of the problem, and it is given by:

$$\gamma^{k+1} = c\gamma^k \tag{22}$$

where c is the growth factor of γ and k represents the k^{th} iteration of the algorithm. The parameter γ is updated in the next iteration if the absolute difference between a variable x_2^k and its closer discrete value is less then a given tolerance. This condition is given by:

$$\left|\left|x_2^k - x_2'\right|\right|_\infty \leq \varepsilon \tag{23}$$

where $x_2' \in D_{x_i}$ is the closest discrete value from the variable x_2^k, for $i = 1, \ldots, m_2$, and $\varepsilon = 10^{-3}$ is the discretization tolerance of the algorithm.

III. Evolutionary Particle Swarm Optimization

Particle Swarm Optimization (PSO), according to Kennedy and Eberhart [16], simulates the migration and aggregation of a flock of birds looking for food. By analogy, the flock of birds represents the set of solutions, each bird represents a particle or a candidate solution, and the food is the objective function of the optimization problem. The particles p run through the search space with a velocity following the best position found by the particles themselves and by all the flock in the search of a goal. These best positions are called $pbest$ and $gbest$, where $pbest$ is the best position found by the particle; and $gbest$ is the best position found by all the particle set. At the end of the process, the best solutions related to the objective function, called as fitness solutions, are presented as the result of the optimization problem.

The Evolutionary PSO (EPSO) algorithm was proposed by Miranda and Fonseca [12] and it is an optimization method based on evolution strategy [17], [18] and the PSO method. The EPSO puts together the best features of both algorithms. This is because there is an exchange of information between the solutions while they move in the search space. The EPSO is also an evolutionary computing method, wherein the solution characteristics are mutated and passed to future generations by a selection mechanism.

The particle motion rule in the EPSO generates as descendant a new particle according to the following transformation process:

$$x_p^{k+1} = x_p^k + v_p^{k+1} \tag{24}$$

The velocity parameter, $v_{p_{k+1}}$, is obtained as follows:

$$v_p^{k+1} = w_{0p}^{k*} v_p^k + w_{1p}^{k*}(pbest_p - x_p^k) + w_{2p}^{k*}(gbest^{k*} - x_p^k) \tag{25}$$

where the weighting parameters w_{0p}^*, w_{1p}^* and w_{2p}^* are named as inercia, memory and cooperation, respectively. These parameters mutate as follows:

$$w_{[0,1,2]p}^{k*} = w_p^k + \tau N(0;1) \tag{26}$$

where $N(0,1)$ is a normally distributed random variable with zero mean and unit variance and τ is a learning parameter.

The global best is randomly distributed as follows:

$$gbest^{k*} = gbest_p^k + \tau' N(0;1) \tag{27}$$

Both τ e τ' can be treated either as fixed numbers or as parameters subject to mutation.

IV. Proposed Methodology

In this section we present the proposed methodology, which used two different ways to initialize the control variables. The first one uses the result obtained from the third iteration of the OPF problem solution through the Gradient Method [6] and the other one initializes the control variables using the values found in the IEEE database. The final solution of the problem is then given by the EPSO metaheuristic proposed by Miranda and Fonseca [12]. The algorithm steps of the proposed methodology are as follow:

Algorithm 1: Proposed Algorithm Steps.

Initialize $k \leftarrow 0$;
Initialize the particles using either the result of the third iteration of the Gradient Method or the values in the IEEE database;
Define $c \leftarrow 10$, $\beta \leftarrow 1$, $\varepsilon \leftarrow 10^{-3}$ and $\gamma_p \leftarrow 10^{-6}$;
while $k < 100$ **do**
    Solve the power flow;
    Obtain $pbest_p$;
    Obtain $gbest$: $gbest^{k*} \leftarrow gbest_p^k + \tau' N(0;1)$;
    Obtain the velocity: $v_p^{k+1} \leftarrow w_{0p}^{k*} v_p^k + w_{1p}^{k*}(pbest_p - x_p^k) + w_{2p}^{k*}(gbest^* - x_p^k)$;
    Obtain the inercia: $w_{[0,1,2]p}^{k*} \leftarrow w_p^k + \tau N(0;1)$;
    Obtain the augmented function (15) by defining the sinusoidal functions for each variable tap;
    Solve the augmented problem (15);
    if $\left|\left|x_2^k - x_2'\right|\right|_\infty > \varepsilon$ **then**
        $\gamma_p^{k+1} \leftarrow c\gamma_p^k$;
    end
    Update the control variables: $x_p^{k+1} \leftarrow x_p^k + v_p^{k+1}$;
    $k \leftarrow k + 1$;
end

V. Results

In this section, tests performed using the IEEE test cases of 14, 30, 57, 118 and 300 bus [14] to evaluate the efficiency of the proposed strategies are shown. The algorithms were implemented using Matlab 8.1.0.604 (R2013a). The power flow problems were solved using the Power System Analysis Toolbox (PSAT), which uses the Newton-Raphson Method. Table I shows the characteristics of each test case.

TABLE I: Characteristics of IEEE test cases

Test System	Equality Constraints	Continuous Variables	Discrete Variables
14	22	27	4
30	53	59	6
57	106	113	18
118	181	235	23
300	530	599	121

For the algorithms that consider the transformer taps as discrete variables, it is considered that these variables belong to an equally spaced set of discrete values with steps of 0.0075pu. The lower and upper limits for the taps are, respectively, 0.88 pu and 1.12 pu, with a total of 33 positions. The minimum and maximum values for the voltage magnitudes are $V^{\min} = 0.95$ and $V^{\max} = 1.10$, respectively.

In all tests, the minimized objective function was the active power losses in transmission lines. The equality constraints of the problems were represented by the power flow equations and the limits for voltage magnitudes and generated reactive power for the continuous problem. For the discrete problem, the transformer taps variables were constrained between its limits. The base power for all test cases was 100 MVA and each system was simulated using 100 particles, 100 iterations and 10 seeds.

Tables II and III present the solutions using the IEEE database as initial conditions for the continuous and discrete problem, respectively.

TABLE II: Solutions using the IEEE database as initial conditions for the control variables (continuous problem).

Test System	Mean Value [MW]	Best Value [MW]	Time [s]
14	13.36	13.35	6
30	17.52	17.51	7
57	25.65	25.64	10
118	118.55	116.85	14
300	377.36	375.91	25

TABLE III: Solution using the IEEE database as initial conditions for the control variables (discrete problem with the sinusoidal approach).

Test System	Mean Value [MW]	Best Value [MW]	Time [s]
14	13.36	13.36	7
30	17.47	17.46	8
57	26.02	25.80	11
118	120.55	120.32	15
300	382.97	381.35	25

The solutions using the results of third iteration of the Gradient Method [6] as initial conditions for all test cases are presented in Tables IV and V for the continuous and discrete problems, respectively.

TABLE IV: Solution using the Gradient Method to initialize the control variables (continuous problem).

Test System	Mean Value [MW]	Best Value [MW]	Time [s]
14	12.30	12.27	6
30	16.22	16.22	7
57	26.66	26.66	11
118	118.07	116.00	14
300	373.80	372.26	21

TABLE V: Solution using the Gradient Method to initialize the control variables (discrete problem with the sinusoidal approach).

Test System	Mean Value [MW]	Best Value [MW]	Time [s]
14	12.30	12.28	6
30	16.15	16.15	7
57	25.90	25.90	12
118	123.22	116.80	15
300	388.00	378.00	24

A. *IEEE 14 bus*

The IEEE 14 bus test case has the following characteristics: 1 slack bus, 5 buses with reactive power control, 9 load buses, 1 capacitor bank, 17 transmission lines and 3 transformers.

Table VI shows the results for the control variables of the discrete problem when using the Gradient Method to obtain the initial conditions. The discrete variables of transformer taps were processed using the sinusoidal approach.

TABLE VI: Final values of control variables for the discrete model.

Type	Position	Value
Voltage	Bus 1	1.10
Voltage	Bus 2	1.09
Voltage	Bus 3	1.05
Voltage	Bus 6	1.09
Voltage	Bus 8	1.09
Tap	Branch 4-7	0.9775
Tap	Branch 4-9	0.9700
Tap	Branch 5-6	0.9325

Figure 1 shows the discretization evolution of transformer taps in IEEE 14 bus test case using the sinusoidal approach. Although 100 iterations were performed, taps were only shown in the figure when there was a change in their values from one iteration to another.

Figures 2a and 2b show the convergence of the objective function for the IEEE 14 bus test case when using, respectively, IEEE database and the Gradient Method to obtain the initial conditions.

When using the Gradient Method to obtain the initial conditions, the objective function converged to an optimum of better quality compared to the local optimum found using IEEE database.

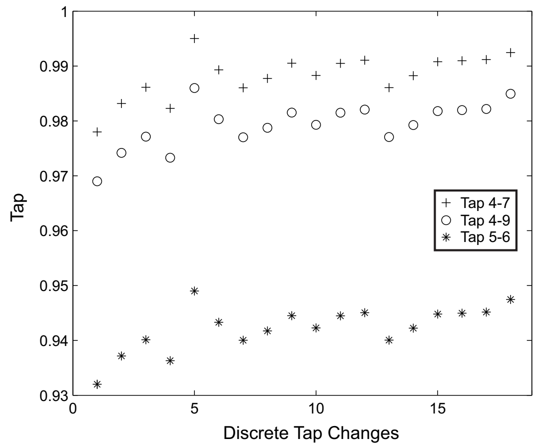
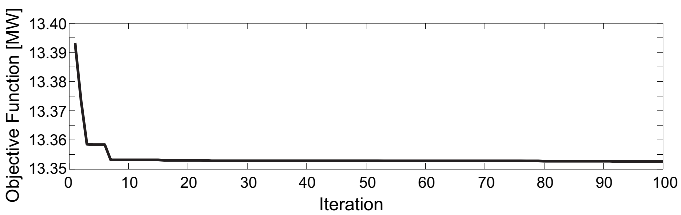
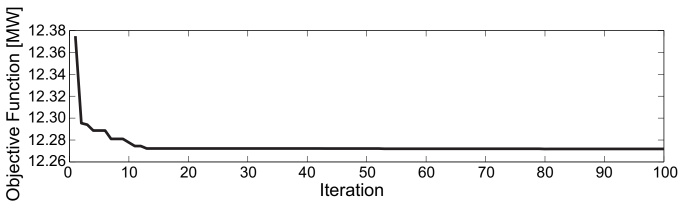


Fig. 1: Discretization evolution for transformer taps in IEEE 14 bus.



(a) Objective function convergence using IEEE database as initial conditions.



(b) Objective function convergence using the Gradient Method to initialize the control variables.

Fig. 2: Objective function convergence for IEEE 14 bus.

B. IEEE 300 bus

The IEEE 300 bus test case has the following characteristics: 1 slack bus; 69 buses with reactive power control, 231 load buses; 8 capacitor banks; 6 shunt reactors; 302 transmission lines and 107 transformers.

Figure 3 shows the final values for the discrete variables of transformer taps for the IEEE 300 bus system using the Gradient Method to obtain the initial conditions and the sinusoidal approach to discretize variables. The horizontal lines in the figure represent the discrete values that transformer taps may assume.

Figure 4 shows the discretization evolution of some transformer taps in IEEE 300 bus test case discrete problem using the sinusoidal approach. Although 100 iterations were performed, taps were only shown in the figure when there was a change in their values from one iteration to another.

Figures 5a and 5b show the convergence of the objective function for the IEEE 300 bus test case when using, respectively, IEEE database and the Gradient Method to obtain the initial conditions.

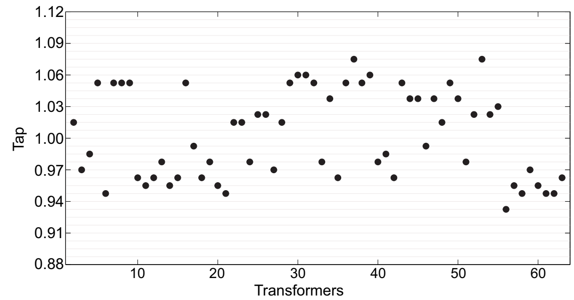


Fig. 3: Transformer taps for IEEE 300 bus.

VI. Analysis of Results

Comparing the results shown in Tables II and IV, it was observed that the tests where the control variables have been initialized using the Gradient Method presented better performance for power losses in the IEEE 14, 30, 118 and 300 bus systems for the continuous model. The same behavior was observed for the discrete model by comparing Tables III and V. The simulation times of all test cases were considered acceptable.

Table VI shows that the control variables of voltage magnitudes are within their maximum and minimum limits and the discrete variables of transformer taps were successfully discretized.

Figure 1 shows the evolution of the transformer tap discretization and Figure 3 shows that all transformer taps from the IEEE 300 bus system successfully reached discrete values using the sinusoidal approach.

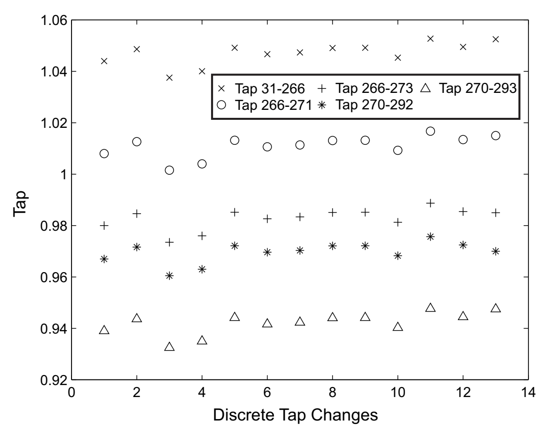
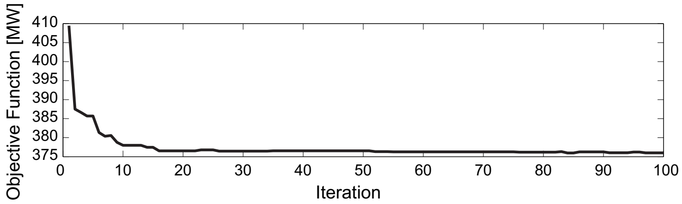


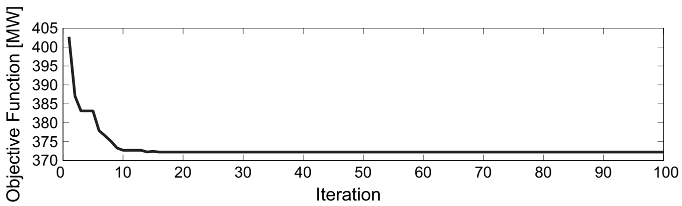
Fig. 4: Discretization evolution for transformer taps in IEEE 300 bus.

VII. Conclusion

In this study, the ROPD problem has been addressed as a nonlinear programming problem with continuous and discrete variables to minimize losses. Two different ways were presented to initialize the control variables in the EPSO metaheuristic. The approach where the variables were initialized using the Gradient Method achieved a better performance in terms of power losses if compared with the one that used



(a) Objective function convergence using IEEE database as initial conditions.



(b) Objective function convergence using the Gradient Method to initialize the control variables.

Fig. 5: Objective function convergence for IEEE 300 bus.

the IEEE database values. Thus, it can be concluded that the Gradient Method provides a better initial condition to the EPSO metaheuristic, facilitating the metaheuristic search.

A sinusoidal approach was used to deal with the discrete variables of the problem. In this approach, the discrete variables are treated as continuous variables by sinusoidal functions incorporated to the objective function of the problem. Thus, the nonlinear continuous and discrete programming problem was turned into a nonlinear programming problem with only continuous variables.

To evaluate the performance of the proposed method, tests were performed using the IEEE 14, 30, 57, 118 and 300 bus systems, that are all considered as small-scale problems. The proposed method were effective in finding good solutions to the continuous and discrete problems for all test cases. For further validation of the proposed method, large-scale systems will be also considered.

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