

Robust Optimization-Based Resilient Distribution Network Planning Against Natural Disasters

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Abstract—Natural disasters such as Hurricane Sandy can seriously disrupt the power grids. To increase the resilience of an electric distribution system against natural disasters, this paper proposes a resilient distribution network planning problem (RDNP) to coordinate the hardening and distributed generation resource allocation with the objective of minimizing the system damage. The problem is formulated as a two-stage robust optimization model. Hardening and distributed generation resource placement are considered in the distribution network planning. A multi-stage and multi-zone based uncertainty set is designed to capture the spatial and temporal dynamics of an uncertain natural disaster as an extension to the traditional N - K contingency approach. The optimal solution of the RDNP yields a resilient distribution system against natural disasters. Our computational studies on the IEEE distribution test systems validate the effectiveness of the proposed model and reveal that distributed generation is critical in increasing the resilience of a distribution system against natural disasters in the form of microgrids.

Index Terms—Natural disaster, microgrid, robust optimization, distribution network planning, resilience, distributed generation.

NOMENCLATURE

Indices, Sets and Parameters

\mathbf{N}	Set of indices of nodes.
\mathbf{L}	Set of indices of branches, i.e., power lines.
\mathbf{T}	Set of indices of time periods.
n	Node index, $n \in \mathbf{N}$.
(i, j)	Power line from node i to node j , directed, $i \in \mathbf{N}$, $j \in \mathbf{N}$ and $(i, j) \in \mathbf{L}$.
t	Time period index, $t \in \mathbf{T}$.

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Z_t	Set of power lines affected in Zone t .
H	Budget for hardening power lines.
G	Budget for installing distributed generation units.
C_d	Monetary network planning budget.
x_{ij}, r_{ij}	Reactance and resistance of the power line (i, j) .
P_{nt}, Q_{nt}	Active and reactive power demand at node n in period t .
G_n^p	Capacity of distributed generation unit n .
\underline{v}, \bar{v}	Lower and upper bounds on voltage levels.

Decision Variables

y_{ij}	Binary, 1 if line (i, j) is hardened, 0 otherwise.
δ_n	Binary, 1 if distributed generator is placed at node n , 0 otherwise.
$u_{ij,t}$	Binary, 0 if power line (i, j) is damaged during a natural disaster in period t , 1 otherwise.
v_{nt}	Voltage magnitude at node n in period t .
g_{nt}^p	Active power generation of distributed generation unit at node n in period t .
g_{nt}^q	Reactive power supply at node n in period t .
$P_{ij,t}, Q_{ij,t}$	Active and reactive power flow on power line (i, j) in period t .
p_{nt}^{ld}	Load shedding at node n in period t .
$\mathbf{p}, \mathbf{q}, \mathbf{v}$	Vectors of active power flow variables $p_{ij,t}$, reactive power flow variables $q_{ij,t}$, and voltage variables v_{nt} .
\mathbf{h}	Concatenation of vector $\{y_{ij}\}$ and vector $\{\delta_n\}$, i.e., a network planning scenario.
\mathbf{u}	Vector of $u_{ij,t}$, a natural disaster scenario.
\mathbf{z}	Vector of power flow variables including active power flow, reactive power flow, and voltage levels.

I. INTRODUCTION

IMPROVING the resilience of power grids against natural disasters has been a fundamental issue for the whole society. In recent years, hurricanes and extreme weather conditions have caused enormous economic losses and even human casualties. Since the mission of power industry is to keep the lights on, it is important to increase the resilience of existing power grids against the uncertain natural disasters. According to the report [1] prepared by the President's Council of Economic Advisers and the U.S. Department of Energy, power outages that occurred in the United States due to severe

weather contributed to 58% of U.S. grid outages and cost the economy an annual average of 18 to 33 billion dollars between 2003 and 2012. Unfortunately, the impacts and financial costs of natural disasters related to floods, drought, and other weather events are expected to increase in significance as what are historically considered to be rare events are becoming more common and intense due to the climate change [2]. It is further emphasized that continued investment in grid modernization and resilience will mitigate these costs over time, saving the economy billions of dollars and reducing the hardship suffered by millions of Americans [1]. In fact, hardening and modernizing the whole grids is prohibitively expensive. Hence, it is fundamental to answer how to effectively allocate budget limited resources to design a resilient power grid against natural disasters. Due to the complex nature of this problem, various optimization models are proposed to facilitate the decision making process. These models range from mixed-integer programs and quadratic programs to more sophisticated stochastic programs and robust optimization that can take account of the inherent randomness and uncertainties.

A. Literature Review

As emphasized in [1], hardening is considered to be one of the most effective approaches that can increase the resilience of a power grid through undergrounding power lines, vegetation management, pole reinforcing, stockpiling power lines, etc. Many previous studies on power grid hardening [3]–[8] focuses on the hardening of transmission networks considering natural disasters or terrorist attacks. A fundamental problem is to identify the critical components for hardening, which is often formulated using the $N - K$ contingency criterion, i.e., the bi-level network interdiction model (also known as attacker-defender model) with up to K simultaneous failures or attacks on the non-hardened components [9], [10]. Then, the hardening planning problem on transmission system is formulated as a defender-attacker-defender sequential game model, a formulation mathematically equivalent to tri-level two-stage robust optimization model. Various heuristics and exact solutions have been proposed to solve this complicated game theoretical problem [3]–[5], [7], [10] and it is observed that an optimal solution of this model guarantees the effectiveness of hardening under the worst-case attack by alleviating system damage. Nevertheless, we note that the $N - K$ cardinality description on the attack set ignores the spatial and temporal dynamics of the occurrence of a natural disaster. Hence, adopting such static set causes us to consider unrealistic attack scenarios and lead to less effective hardening plans.

On the other hand, very limited work has been done to support the hardening planning on distribution networks, although storm-related outages often occur on distribution systems. According to [1], about 90% of outages during the storm event occur on distribution systems. Nevertheless, the models for hardening transmission systems are not directly applicable to the hardening of distribution systems because distribution systems mostly possess a radial tree-like network topology while transmission networks are often more connected meshed networks. The relatively simple linear program

based DC power flow models, which ignore reactive power and voltage profiles, are widely used in transmission systems to approximate the power flow. To be practical and realistic, distribution systems require the consideration of reactive power and voltage profiles in the power flow calculation as demonstrated in [11].

In [12], a two-stage stochastic mixed-integer program for designing a resilient distribution network against natural disaster is presented, where damage scenarios from natural disasters are modeled as a set of stochastic events. A multi-commodity network flow model is used to approximate the power flow in the distribution network. The stochastic events on a distribution network are predetermined with certain components at fault. A two-stage robust optimization model for the distribution network reconfiguration considering load uncertainty is proposed in [13]. A mitigation method for electric distribution networks after natural disasters by sectionalizing a distribution network into microgrids with distributed generation (DG) units is presented in [14].

In the meantime, distributed generation resources impact critically the operations of a distribution system. DG can improve power quality, enhance the reliability of supply, and reduce system losses [15], [16]. The DG placement problem has therefore attracted the interest of many research efforts in the last two decades since it can provide the distribution system operators, regulators and policy makers useful input for the derivation of incentives and regulatory measures. A common use of DG is serving as generation backup in case of main supply interruption [15] or natural disasters [14]. In [17], a robust optimization based model is proposed for placing DG units in microgrids with the consideration of load uncertainty over the planning horizon. It is meaningful to plan the investment of DG units in view of uncertain natural disasters since one of the main purposes of DG is to backup the system during natural disasters.

B. Our Approach

This paper proposes a robust optimization based decision support tool for the planning of a resilient distribution network. The optimal solution provides a network planning decision that coordinates the hardening and DG resource placement and improves the resilience of the distribution system against natural disasters.

The key contributions of the paper include:

- (i) We extend the traditional attacker-defender game based worst-case network interdiction model to a more practical network interdiction model with a multi-stage and multi-zone based uncertainty set to capture the spatial and temporal dynamics of natural disasters such as hurricanes.
- (ii) A robust optimization based framework that considers uncertain natural disaster occurrence is proposed to coordinate the planning of distribution systems using hardening and DG resource placement. A computational algorithm is developed for solving the model. The empirical studies validate the effectiveness of the proposed model.

(iii) Results reveal the importance of DG, which transforms a distribution network into several microgrids, in improving the distribution system's resilience under natural disasters.

The remainder of the paper is organized as follows. Section II describes the two-stage robust optimization formulation for distribution network planning, network planning decisions, natural disaster modeling, and distribution network power flow model. Section III provides the column-and-constraint based decomposition algorithm for the optimization model. Section IV presents the empirical results and discusses the effectiveness of the proposed model. Finally, a conclusion and discussion on future research is given in Section V.

II. MATHEMATICAL FORMULATION

In this section, the overall robust optimization model for resilient distribution network planning and the essence of the model are provided, followed with details on each component of the model. A brief description of the networking planning decisions is given. In addition, the natural disaster occurrence model is proposed as an extension to the network interdiction problem. Finally, we will introduce the power flow formulations for distribution system power flow with given network planning and natural disaster events.

A. Robust Optimization Based Resilient Distribution Network Planning Model

The overall resilient distribution network planning problem is formulated as a defender-attacker-defender game model [3]–[5], [7], which also takes the form of two-stage robust optimization [18]. We note that even though the two-stage robust optimization and the defender-attacker-defender game model have different origins, they share an identical tri-level optimization structure.

In this sequential game, the defender deploys a distribution network design plan in the first stage. In the second stage of the game, a natural disaster which is an enemy to the distribution system disrupts the system aiming at the maximum damage by attacking the power lines. Finally, the distribution system operator, as the defender, reacts to the disruption by adjusting power flow on the remaining distribution network to minimize the damage. In the sense of two-stage robust optimization, the network planning decision maker determines the “here-and-now” network planning decisions before the realization of the uncertain natural disaster is known. In the second stage, after the natural disaster is observed, an immediate recourse action with “wait-and-see” decisions is deployed to mitigate the system damage. The abstract mathematical formulation of the model is given in (1).

$$\min_{\mathbf{h} \in \mathbb{Y}} \max_{\mathbf{u} \in \mathbb{U}} \min_{\mathbf{z} \in \mathbb{F}(\mathbf{h}, \mathbf{u})} \sum_{n,t} p_{nt}^{ld} \quad (1)$$

In (1), \mathbb{Y} is the feasibility set for distribution network planning decisions which consists of budget constraints for hardening and distributed generation placement. \mathbb{U} is the uncertainty set of a natural disaster which occurs after the deployment of a network design plan. The disaster will cause

a worst-case scenario attack with the objective of maximizing the damage through a max-min bi-level game after the network planning decision. Finally, after the natural disaster is realized and observed, the distribution system immediately respond to the disruption with feasible power flow decisions as defined by $\mathbb{F}(\mathbf{h}, \mathbf{u})$ to minimize the load shedding.

B. Network Planning Decisions

In this subsection, we will discuss the feasibility set of the “here-and-now” network planning decisions.

With a limited budget, a utility makes a plan to allocate budget limited resources in order to enhance the resilience of a distribution system. In this paper, we consider hardening power lines and DG resource placement. Other measures can be accommodated by reformulating the planning decision feasibility set accordingly. Hardening is a preventive measure that will increase the resilience of a power grid under malicious terrorist attacks or natural disasters [1]. It is assumed that the hardened lines will survive the disasters [3]–[7]. Here we use a cardinality budget set similar to the budget sets used in hardening transmission networks [3], [4], [6], [7]. Additionally, DG has been gaining interests as an effective tool for reliability, losses and voltage improvements [16], and also as a reliable energy source that can start almost instantaneously when a major contingency occurs in a distribution system [19]. To study the effectiveness of DG on system resilience during natural disasters and analyze the optimal placement of DG resources, we assume that there is a cardinality budget for the available DG units. Hence, a budget set for the decision maker can be formulated as follows.

$$\mathbb{Y} = \left\{ \sum_{(i,j) \in L} y_{ij} \leq H, \sum_{n \in N} \delta_n \leq G \right\}$$

This decision set assumes that the decision maker has a budget to harden a maximum of H power lines and to place a maximum of G DG units. As a matter of fact, we can easily modify the decision set to accommodate more sophisticated decision scenarios, such as considering the cost variations of DG units and hardening different power lines. A simple example similar to [5] is to consider the hardening cost to be proportional to the length of a power line. Thus, we specify a hardening cost c_l for each line and a cost c_n for each DG unit. Assuming that the total investment cannot exceed a monetary budget C_d , an alternative network planning decision set can be formulated as \mathbb{Y}' .

$$\mathbb{Y}' = \left\{ \sum_{(i,j) \in L} c_{ij} y_{ij} + \sum_{n \in N} c_n \delta_n \leq C_d \right\}$$

C. Natural Disaster Occurrence Model

In this study, a natural disaster corresponds to the uncertainty ($\mathbf{u} \in \mathbb{U}$) in the two-stage robust optimization model. The natural disaster occurrence model corresponds to the bi-level program $\max_{\mathbf{u} \in \mathbb{U}} \min_{\mathbf{z} \in \mathbb{F}(\mathbf{h}, \mathbf{u})} \sum_{n,t} p_{nt}^{ld}$, whose optimal solution results in a worst-case disaster event for a given network planning decision. This bi-level program falls in the category

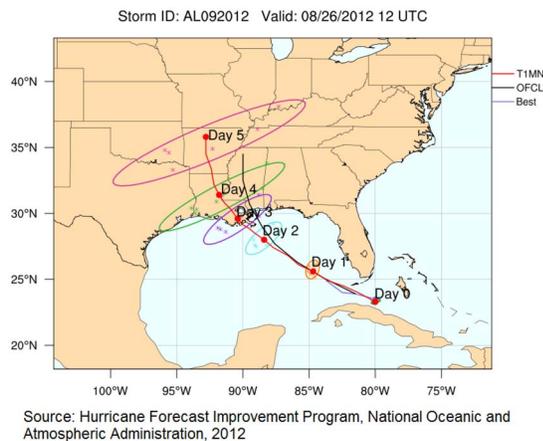


Fig. 1. A typical evolution of hurricane.

of network interdiction model, and is often used to evaluate the effectiveness of a particular network planning decision as defined in \mathbb{Y} .

Natural disasters are generally highly uncertain events that are difficult to estimate, model and predict. A lot of efforts have been made to increase our awareness of natural disasters based on historical data and the lessons we learned. The forecasting of a natural disaster is often based on statistical models or simulation models as reviewed in [20]. Predefined natural disaster scenarios are assumed in [12] and treated with an equal probability. In this paper, to capture the spatial and temporal dynamics of a natural disaster, we develop a multi-stage natural disaster occurrence model that extends the traditional $N - K$ network interdiction model. To illustrate this idea, we consider the case of hurricanes.

1) Spatial and Temporal Dynamics of Hurricanes:

As revealed by the Hurricane Forecast Improvement Program [21], a hurricane often follows a path that consists of multiple periods and several associated geographic zones (see Fig. 1). Also, the wind speed of a hurricane, which is one of the most destructive forces of a hurricane, decreases once the storm lands and drifts away from the sustaining heat and moisture provided by ocean or gulf waters. This can be seen from Fig. 2 that wind speed decays quickly over time after landfall. Geographically, the wind speed decays along its path as shown in Fig. 3 based on the inland wind model [22].

2) Modeling Impacts of Natural Disasters on Power Grids:

Natural disaster occurrences or terrorist attacks on a power system are often modeled and evaluated using the attacker-defender game, i.e., the bi-level worst-case network interdiction model [3]–[7], [9], [23]. The outer level represents the attacker's decision with limited attack resources and the lower level is a defender response decision with a re-dispatch of power flow based on the damage caused by the attacker. To capture non-trivial attack decisions made by the attacker, an attack set is defined with a limited budget on attack resources, which reflects the system operator's estimation on the number of disruptions caused by possible attacks. When that budget is set to K , it nicely matches the $N - K$ criterion adopted in power industry. So, it rather becomes a

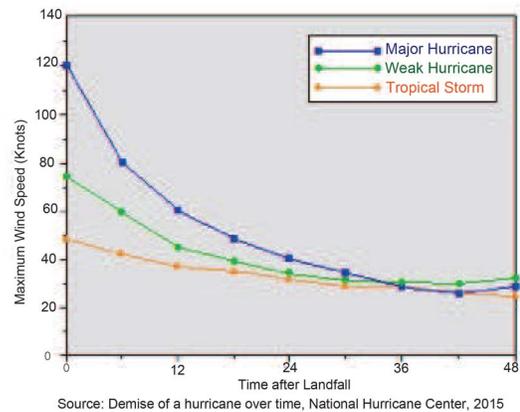


Fig. 2. Decay of hurricane attack.

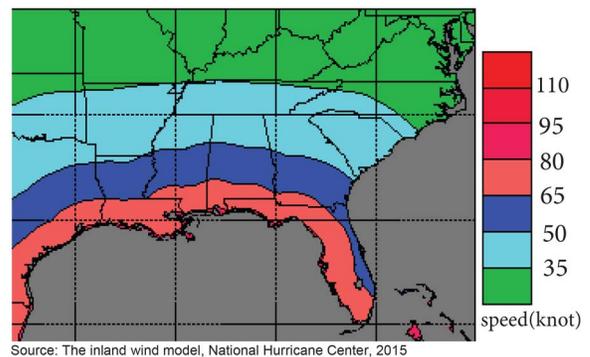


Fig. 3. Extent of inland winds from category 3 hurricanes with forward speed 20 knots.

convention in power system research to use an attack set to model system contingency [24], [25], terrorists attacks, and natural disasters [4], [7], [23]. In this paper, noting that such modeling scheme compactly includes worst-case scenarios of all possible natural disasters in the feasible set of attacker, we also use the attacker-defender game based network interdiction model in this paper to capture the impacts of natural disasters. However, different from the traditional attacker-defender models [4], [9], [10], where a single cardinality budget constraint on the whole system damage is assumed, the multi-stage uncertainty set proposed in this paper takes account of the spatial and temporal dynamics of a natural disaster by constructing a sophisticated but more realistic uncertainty set.

Based on the spatial and temporal dynamics of a hurricane, we assume that when a hurricane moves into an area, it will land on the zone that is close to the coastline and the flood and strong rotating wind will impact the power lines within the zone whereas the far-away power lines will stay intact. Within the affected zone, an $N - K$ contingency constraint is used to estimate impacts of the hurricane. As the hurricane pushes towards the inland area, it will affect the regions from one zone to another based on the geographic locations.

An illustrative case based on the IEEE 33-node distribution system is provided in Fig. 4. First of all, the distribution system is divided into several zones based on the path of the hurricane movement and the geographic locations of the power lines.

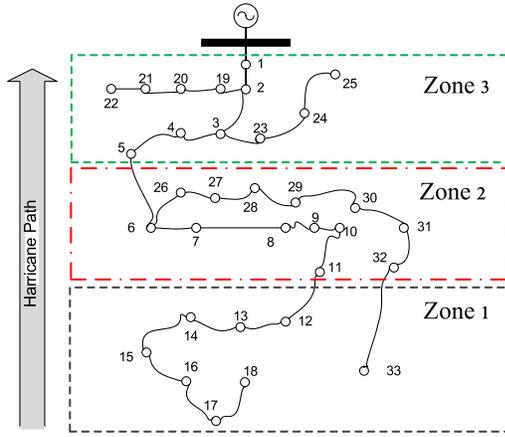


Fig. 4. A hurricane occurrence model on the IEEE 33-node distribution system.

Zone 1 is close to the coastline. So, the power lines of this zone will suffer seriously from the hurricane impact during the initial stage. In the second period, the hurricane will move to the inland area and cause damage to the power lines in Zone 2. Finally, the hurricane will reach Zone 3 and affect, generally to a less extent, the associated power lines. Given such dynamics and observations, we can divide the hurricane occurrence into multiple periods and zones based on the path of the hurricane. Then, this type of natural disaster events can be inclusively captured using the following set \mathbb{U} , which is referred to as the multi-stage and multi-zone based uncertainty set.

$$\mathbb{U} = \left\{ \begin{array}{l} \sum_{(i,j) \in Z_1} (1 - u_{ij,1}) \leq B_1, \\ u_{ij,1} = 1, \forall (i,j) \in \{\mathbf{L} \setminus Z_1\}, \\ \sum_{(i,j) \in Z_2} (1 - u_{ij,2}) \leq B_2, \\ u_{ij,2} = u_{ij,1}, \forall (i,j) \in \{\mathbf{L} \setminus Z_2\}, \\ \dots \\ \sum_{(i,j) \in Z_T} (1 - u_{ij,T}) \leq B_T, \\ u_{ij,t} = u_{ij,t-1}, \forall (i,j) \in \{\mathbf{L} \setminus Z_t, t = T\}. \end{array} \right\} \quad (2)$$

$$u_{ij,1} = 1, \forall (i,j) \in \{\mathbf{L} \setminus Z_1\}, \quad (3)$$

$$\sum_{(i,j) \in Z_2} (1 - u_{ij,2}) \leq B_2, \quad (4)$$

$$u_{ij,2} = u_{ij,1}, \forall (i,j) \in \{\mathbf{L} \setminus Z_2\}, \quad (5)$$

$$\dots \quad (6)$$

$$\sum_{(i,j) \in Z_T} (1 - u_{ij,T}) \leq B_T, \quad (7)$$

where u_{ijt} is the interdiction decision variable for the network interdiction model which represents whether a power line (i, j) is affected (i.e., $u_{ijt} = 0$) or not-affected in period t . B_t is the cardinality budget for the number of damaged lines in the affected zone Z_t in period t . For example, constraint (2) specifies that the number of affected power lines (i.e., $u_{ij,1} = 0$) in Zone 1 cannot exceed budget B_1 . $\{\mathbf{L} \setminus Z_t\}$ is the set of power lines that are not located in Zone t . Hence, constraint (3) means that the power lines not located in Zone 1 will not be attacked (i.e., $u_{ijt} = 1$). Constraint (7) represents that power lines not in the currently affected area Zone t will remain in the same state (on or off) in period $t - 1$.

With this multi-stage and multi-zone uncertainty set \mathbb{U} , the hurricane impact within each zone is formulated as an

$N - K$ worst-case network interdiction problem. Note that the uncertainty set becomes the traditional $N - K$ worst-case contingency analysis if the whole distribution system is considered as one zone that is affected at the same time. We mention that a few recent papers, including [26]–[29], have proposed to define more sophisticated uncertainty sets when accurate and sufficient data are available. Given that natural disasters occur in a less frequent fashion and the fact that the $N - K$ criterion is widely accepted by various stakeholder, we believe that such spatially and temporally extended $N - K$ uncertainty set is more appropriate for power systems. Hence, in the remaining of this paper, the above uncertainty set is adopted to define the feasible set of the attacker in the attack-defender model.

D. Distribution Network Power Flow

In this subsection, the distribution network operational model, i.e., distribution network power flow model, is presented as the “wait-and-see” recourse decisions to mitigate the aftermath of a natural disaster. Even though a distribution system would eventually be restored to normal operating conditions through a system restoration process, this restoration process, first of all, requires a running communication system to discover the states of the assets. Even if a robust communication system survives the natural disaster and discovers the states of all the assets in the distribution system, a sophisticated restoration plan often requires an advanced solution based on artificial intelligence such as expert systems [30] or optimization methods [14] that are not intuitive for implementation. If the distribution system operator is fortunate enough to achieve a smart restoration plan, the operator might also have to deal with the hazards caused by the disaster which often prevent the engineers or electricians from accessing a system component for safety considerations. Hence, to model the whole restoration process as the recourse action is not a reasonable option in this study. In the previous literature on electric transmission system vulnerability analysis and transmission network hardening planning [3]–[5], [10], solving a DC optimal power flow model, or economic dispatch, is often used as a post-contingency mitigation method with the objective of minimizing system load shedding. In this study, we use the distribution network power flow model with the objective of minimizing system load shedding as the operator’s mitigation model since this approach can be easily automated without a robust communication system, a smart restoration plan, and engineer or electrician’s efforts. We believe that a system with higher level of satisfied demand immediately after a disaster is a good starting point for the future restoration process as well.

1) *DG Operations*: When a natural disaster occurs to a distribution system, backup or standby DG units such as fossil fueled combustion generators will pick up certain lost load. Previous literature on DG placement assumes that if a DG unit is placed at a node n , this DG unit can supply power to node n and the child branches of node n in the radial tree network [17]. No power flow is allowed from any node to its parent node. Even though system reconfiguration, through which a power system still maintains a radial topology [11]

III. SOLUTION METHODOLOGY

The column-and-constraint generation (CCG) is a generic decomposition algorithm framework for two-stage robust optimization and is proven to be an efficient and finite convergent exact solution [32]. In this section, we describe how to formulate the corresponding master problem and subproblem for the proposed model (1).

A. CCG Master Problem

The CCG master problem contains a set of worst-case disaster scenarios $\hat{\mathcal{U}} = \{\hat{\mathbf{u}}^s, s = 1, 2, \dots, k\}$,

$$\min \alpha \quad (26)$$

$$\text{st. } \mathbf{h} \in \mathbb{Y} \quad (27)$$

$$\alpha \geq \sum_{n,t} p_{nt}^{ld,s}, \quad \forall s = 1, 2, \dots, k \quad (28)$$

$$\mathbf{z}^s \in \mathbb{F}(\mathbf{h}, \hat{\mathbf{u}}^s), \quad \forall s = 1, 2, \dots, k. \quad (29)$$

α is a scalar variable introduced in constraint (28), which makes sure that the optimal solution of the master problem dominates the included worst-case scenarios. The worst-case disaster scenarios are obtained from the CCG subproblem over the iterations. Note that solving the CCG master problem yields a network planning decision $\hat{\mathbf{h}}$ and a lower bound of the original model since the CCG master problem is a relaxation of the original model. Indeed, if $\hat{\mathcal{U}}$ contains all possible disaster scenarios, the master problem is equivalent to the original model.

B. CCG Subproblem

The CCG subproblem seeks the worst-case natural disaster scenario with a given network planning decision $\hat{\mathbf{h}}$ from the CCG master problem. Let $\boldsymbol{\pi}$ be the vector of the dual variables for constraints (17)-(25) and $\boldsymbol{\Omega}(\hat{\mathbf{h}}, \mathbf{u})$ denotes the feasible set of the dual inner linear program,

$$\boldsymbol{\Omega}(\hat{\mathbf{h}}, \mathbf{u}) = \{\boldsymbol{\pi} : C^T \boldsymbol{\pi} \leq I\},$$

with a given hardening plan $\hat{\mathbf{h}}$. Then the bi-level subproblem can be transformed to a bilinear program as follows.

$$\begin{aligned} \max_{\mathbf{u} \in \mathcal{U}} \min_{\mathbf{z} \in \mathbb{F}(\hat{\mathbf{h}}, \mathbf{u})} \sum_{n,t} p_{nt}^{ld} &= \max_{\mathbf{u} \in \mathcal{U}} \max_{\boldsymbol{\pi} \in \boldsymbol{\Omega}(\hat{\mathbf{h}}, \mathbf{u})} (\mathbf{e} - A\hat{\mathbf{h}} - B\mathbf{u})\boldsymbol{\pi} \\ &= \max_{\mathbf{u} \in \mathcal{U}, \boldsymbol{\pi} \in \boldsymbol{\Omega}(\hat{\mathbf{h}}, \mathbf{u})} (\mathbf{e} - A\hat{\mathbf{h}} - B\mathbf{u})\boldsymbol{\pi} \end{aligned} \quad (30)$$

The above bilinear program can be linearized using the big-M method and then solved by an MIP solver. The solution from CCG subproblem together with the corresponding network planning solution $(\hat{\mathbf{h}}, \hat{\mathbf{u}}, \hat{\mathbf{z}})$ form a feasible solution for the original tri-level program and thus provide an upper bound for the original model.

C. Algorithm Implementation

The detailed algorithm implementation procedures are described in Algorithm 1. The tolerance gap of optimality for the algorithm is ϵ .

Algorithm 1 CCG Decomposition Algorithm for RDNP

Initialization: set $LB \leftarrow -\infty$, $UB \leftarrow \infty$, $\hat{\mathcal{U}} \leftarrow \emptyset$, iteration index $k \leftarrow 0$, $\text{gap} \leftarrow \infty$, $\hat{\mathbf{h}} \leftarrow \mathbf{0}$

while $\text{gap} \geq \epsilon$ **do**

 solve CCG subproblem with given plan $\hat{\mathbf{h}}$, obtain objective value $objSP$ and disaster scenario \mathbf{u}^* , $UB \leftarrow \min\{UB, objSP\}$, update gap and $k \leftarrow k + 1$;

 add \mathbf{u}^* to $\hat{\mathcal{U}}$, create dispatch variables (\mathbf{z}^k) , and add these variables (i.e., columns) with corresponding constraints $\mathbf{z}^k \in \mathbb{F}(\mathbf{h}, \mathbf{u}^k)$ to CCG master problem;

 solve CCG master problem, update LB with optimal value $objMP$, update network planning plan $\hat{\mathbf{h}}$ and gap ;

end while

return $\mathbf{h}^* \leftarrow \hat{\mathbf{h}}$.

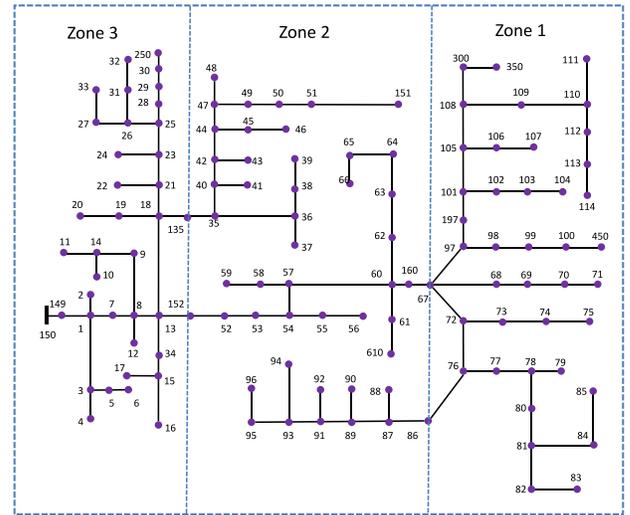


Fig. 6. The three-period hurricane occurrence model for the IEEE 123-node distribution system.

IV. NUMERICAL RESULTS

In this section, we perform computational experiments on the proposed model and algorithm. The IEEE 33-node distribution system and the IEEE 123-node distribution system [33] are used in this study. The solution algorithm is implemented in C++ with CPLEX 12.6 on a dual-core PC with 6GB RAM.

A. Hurricane Occurrence

In the RDNP, hurricane occurrence model is the bi-level multi-stage and multi-zone network interdiction model, which is part of the robust optimization framework. The natural disaster occurrence model for IEEE 33-node distribution system is based on the uncertainty set \mathcal{U} defined for hurricane attacks in Section II. A similar three-period natural disaster occurrence model is designed for the IEEE 123-node distribution system by modifying the uncertainty set accordingly. The hurricane also affects this IEEE 123-node distribution system chronologically from Zone 1 to Zone 3 as demonstrated in Fig. 6.

The optimal solution of the worst-case hurricane occurrence model is obtained by solving the bi-level

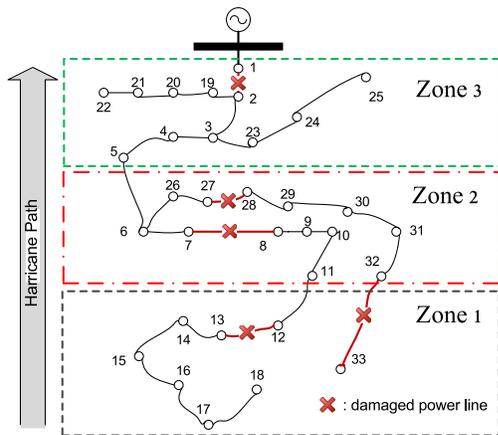


Fig. 7. A worst-case hurricane scenario on the IEEE 33-node distribution system.

attacker-defender model, i.e., CCG subproblem. As shown in Fig. 4, the whole dynamic evolution process of the hurricane on the IEEE 33-node distribution system is divided into three periods. During each period, the hurricane attack will cause power line failures with a cardinality budget. The power lines that failed in the previous time period will remain at fault in the next stage. Fig. 7 gives a worst case hurricane attack plan with the natural disaster uncertainty set ($B_1 = 2, B_2 = 2, B_3 = 1$), which means there are at most two power lines out for Zone 1 and Zone 2, and one power line out for Zone 3. The optimal solution of the hurricane occurrence model gives a worst-case hurricane scenario within a given uncertainty set. To be specific, in the first time period, lines 12-13 and 32-33 will be damaged as the hurricane lands in Zone 1. In the second period, the hurricane moves along its path and damages lines 7-8 and 27-28 in Zone 2. Finally, when the hurricane arrives in Zone 3, it cuts off line 1-2. This attack will cause a power outage of 5520 KW out of the total demand of 11435 KW. Note that this hurricane scenario dominates any other possible hurricane scenarios in the defined uncertainty set.

B. Effectiveness of Hardening

To validate the effectiveness of hardening on distribution networks, we study the system load shedding with various hardening budgets under hurricane occurrence in the distribution system. For the IEEE 33-node distribution system, the hurricane scenarios considered is ($B_1 = B_2 = B_3 = 1$), which means that there will be exactly one power line damaged in the affected zone during each period. The optimal hardening plans, corresponding worst-case hurricane scenarios and load shedding are given in Table I for hardening budget from $H = 0$ to $H = 6$. The power lines damaged under the worst-case hurricane scenarios are listed in the order of occurrence. Without hardening the system, this worst-case hurricane scenario will cause 4730 KW load shedding in the grid. However, if we can harden one line (1-2), the load shedding will be reduced to 4270 KW. Each time we add one more line to the hardening budget, the load shedding for the uncertain natural disaster decreases. By hardening six power lines in the IEEE 33-node system, the load shedding will be reduced from 4730 KW to 2830 KW, which is a reduction of more than 40%.

TABLE I
DETAILED HARDENING PLANS FOR IEEE 33-NODE
DISTRIBUTION SYSTEM

H	hardened lines	load shedding (KW)	worst-case attack
0	None	4730	(12-13;27-28;1-2) ¹
1	1-2	4270	(12-13;27-28;2-3)
2	1-2,2-3	3880	(12-13;27-28;3-23)
3	1-2,2-3,12-13	3700	(13-14;27-28;3-23)
4	1-2,2-3,12-13,13-14	3340	(12-13;6-26;3-23)
5	1-2,2-3,27-28,28-29,29-30	3120	(12-13;30-31;3-23)
6	1-2,2-3,12-13,13-14,3-23,23-24	2830	(14-15;27-28;24-25)

¹(12-13;27-28;1-2) means power line 12-13 in Zone 1 is damaged in the first period, line 27-28 in Zone 2 is damaged in the second period, and finally line 1-2 is down in Zone 3.

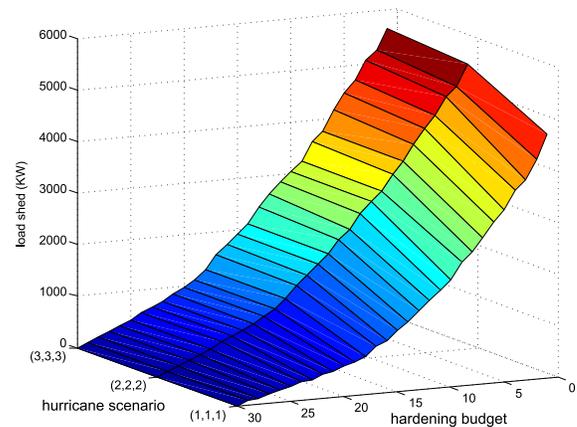


Fig. 8. Load shedding under various hardening budgets and natural disaster scenarios for the IEEE 33-node system.

A more comprehensive study of the effectiveness of hardening in terms of load shedding reduction based on the IEEE 33-node distribution system and the IEEE 123-node distribution system with various hardening budgets and hurricane uncertainty sets, ($B_1 = B_2 = B_3 = 1$), ($B_1 = B_2 = B_3 = 2$), and ($B_1 = B_2 = B_3 = 3$), is presented in Fig. 8 and Fig. 9. As shown in Fig. 8 and Fig. 9, for all natural disaster occurrence scenarios, the load shedding in the distribution system is monotonically decreasing with respect to the increase of the hardening budget. This proves the effectiveness of hardening. Moreover, for the same hardening budget, a stronger hurricane, i.e., a hurricane with larger B_1 , B_2 , and B_3 , will cause more load shedding in the system. An interesting observation is that the marginal benefit of hardening (i.e., the reduction of load shedding when increasing the hardening budget by one) generally decreases since the load shedding plot is steep when hardening budgets are small and is getting flat with the increase of hardening budget. Clearly, this observation suggests an effective way of determining the budget for hardening by comparing the marginal benefit from hardening one additional power line with respect to the cost of hardening. Moreover, this plot can serve as a decision support tool to determine the hardening budget needed with respect to the estimated natural disaster category and the desired distribution system resilience level.

C. Influence of Distributed Generation and Microgrids

The DG units can continue supplying power to connected loads in the form of microgrids when the distribution system is at fault [14], [19]. To investigate the importance of DG

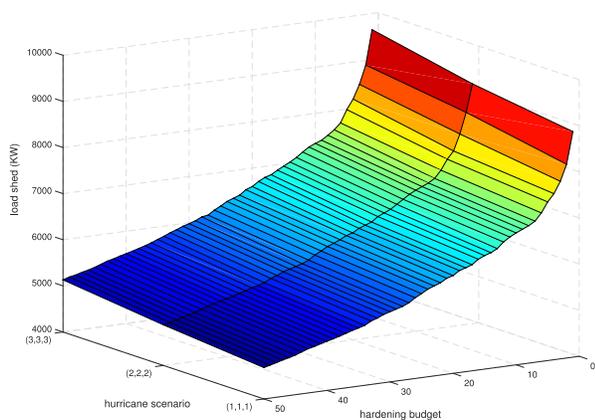


Fig. 9. Load shedding under various hardening budgets and natural disaster scenarios for the IEEE 123-node system.

in the distribution system under natural disasters, a comparison among network planning “without DG”, “with random DG placement” and “with optimal DG placement” is studied on the IEEE 33-node distribution system. No DG units are placed in the distribution system for the “without DG” scenario. The corresponding plan is obtained through solving the robust optimization based RDNP. The distribution system “with random DG placement” scenario is generated by randomly placing DG units to the distribution system and then solving the RDNP with only hardening available in the network planning decision set. The distribution system “with optimal DG placement” scenario is obtained through solving the RDNP with both hardening and DG placement in the network planning decision set. To be specific, for the “without DG” scenario, no DG units are placed in the distribution system. For the “with random DG placement” scenario, three DG units are initially located at nodes 7, 12, and 27, each with a capacity of 10 MW. For the “with optimal DG placement” scenario, a budget of three DG units (each with a capacity of 10MW) is given in the network planning decision set \mathbb{Y} .

As can be seen from Fig. 10, the effectiveness of hardening in terms of reduced load shedding in a hurricane is improved by DG placement as the load shedding of “with random DG placement” and “with optimal DG placement” are generally less than that of “without DG”, except for the case where every power line is hardened. Moreover, with a coordinated network planning solution of hardening and DG placement, i.e., the “with optimal DG placement” scenario, the distribution system is the most resilient among the three scenarios as it results in the least load shedding. In fact, when a distribution system is damaged by a natural disaster, the loads in branches that are disconnected from the main grid will be picked up by a DG unit if available. The DG units with connected branches will form microgrids where the power can be supplied by the DG within the microgrid. This interesting observation points to the importance of placing DG units and forming microgrids to increase the distribution network resilience under natural disasters as elaborated in [12] and [14] and the necessity to coordinate the placement of DG resource placement with hardening in the distribution network planning process.

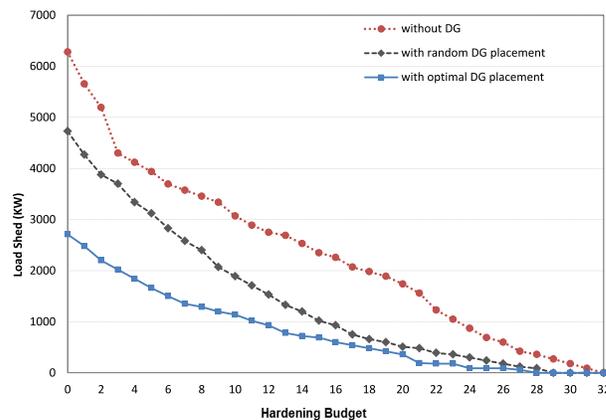


Fig. 10. Impacts of DG on distribution system resilience.

V. CONCLUSION

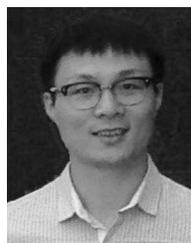
This paper proposes a novel model for the planning of a resilient distribution system with hardening and distributed generation based on two-stage robust optimization to minimize the total load shedding under natural disasters. The proposed model coordinates the optimal planning of hardening and DG resource placement. A multi-stage and multi-zone based uncertainty set is used to capture the uncertainty of natural disaster as an extension of the traditional $N - K$ interdiction model. A decomposition algorithm is designed and implemented to solve the tri-level program. Numerical results validate the effectiveness of model. Studies also point to the importance of placing distributed generation to increase the resilience of a distribution system against natural disasters.

In the future, we will study other natural disasters types and evaluate the impacts of hardening and DG on the system resilience to these extreme events. More distribution network planning resources such as storage and electrical vehicles will be explored and coordinated through the robust optimization based model.

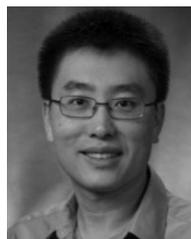
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