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Risk-adjusted performance of portfolio insurance and investors' preferences

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ABSTRACT

This paper draws a clear line between investors' risk preferences and their choice of either Option Based Portfolio Insurance (OBPI) or Constant Proportion Portfolio Insurance (CPPI). For this purpose, OBPI and CPPI are compared using partial-moments-based risk-adjusted performance measures, which are adequate for comparing asymmetric return distributions and can be easily tailored to reflect investors' preferences. The analysis covers expected utility and prospect utility investors, among others, and the results show investors' risk preferences in the gain domain are the key determinants of the choice between OBPI and CPPI.

1. Introduction

Whether investors' risk preferences determine their choice between Option Based Portfolio Insurance (OBPI) and Constant Proportion Portfolio Insurance (CPPI) remains a controversial question among researchers, who, in the quest for an answer, have employed a wide range of criteria.

The most popular criterion, expected utility maximization,[1] does not give a clear ranking of OBPI and CPPI[2]. Bernard and Kwak (2016) and Pézier and Scheller (2013) showed that CPPI is the optimal strategy for hyperbolic absolute risk aversion (HARA) investors, while El Karoui et al. (2005) demonstrated that OBPI is optimal for constant relative risk aversion (CRRA) investors. Furthermore, Grossman and Vila (1992) proved that CPPI is optimal in the framework of long-term risk-sensitive portfolio optimization. Nevertheless, when the guarantee constraint is exogenous, the optimal strategy has been shown to be OBPI.[3]

Stochastic dominance (SD) criterion, which is linked to investors' risk preferences through the order of SD[4], does not discriminate between OBPI and CPPI. Bertrand and Prigent (2005) documented no first-order SD between OBPI and CPPI,

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[1] Expected utility theory assumes investors are risk averse at all wealth levels.
[2] The ranking produced by this criterion crucially depends on utility specification.
[3] These sutdies assumed the risky asset follows a geometric Brownian motion.
[4] The first-order SD corresponds to increasing utility, the second-order to a concave utility, meaning risk aversion and the third order to a convex utility, meaning risk seeking. See Levy (1992) for more details.

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and Annaert et al. (2009) noted no second-order SD; however, Maalej and Prigent (2016) showed that CPPI can dominate OBPI at the third-order, but only when the implied volatility is high relatively to the empirical volatility.

Dierkes et al. (2010) and Dichtl and Drobetz (2011) adopted a different comparative approach to portfolio insurance (PI) strategies that assumes investors behave according to Tversky and Kahneman's (1992)) cumulative prospect theory (CPT).[5] However, although their results explained the popularity of PI strategies versus uninsured investment strategies, they did not define a dominant PI strategy for CPT investors.

This paper contributes to the previous literature in the following ways. First, it provides a simple unifying approach to identifying the portfolio insurance strategy (OBPI or CPPI) that corresponds to the type of investor preferences without using formal preference functions to describe the investor's behavior. It is sufficient that the investor defines her risk preferences, including risk aversion, risk seeking, or risk neutrality in general terms, and adapts her comparison criterion accordingly. Second, this paper considers different types of preferences, and covers a broader set of investors than in any previous work on this subject.

Partial-moments-based risk-adjusted performance measures and, in particular, the Farinelli and Tibiletti (2008) ratio are used to compare OBPI and CPPI. This ratio uses the upper partial moment (UPM) of order p as a measure of return and the lower partial moment (LPM) of order q as a measure of downside risk. Two reasons lie behind choosing this ratio: first, it is a ratio of the expected upside gain above a certain reference point L relatively to the risk measured by the expected loss below L. It is therefore suitable for describing both the upside participation and the downside protection profiles of PI. Second, this ratio can be easily adapted to suit investors' risk preferences through the orders of partial moments (p and q) as well as the reference point L.[6] A wide range of preferences, including expected utility and prospect utility investors' preferences, can be modeled. This method can be quite noteworthy, especially for individual investors who may prefer the simplicity and the flexibility it provides and who are the main buyers of PI in practice (See Fodor et al. (2013)).

As a first step, I derive analytical solutions for the UPM and the LPM of OBPI and CPPI under the geometric Brownian motion (GBM). Then, I analyze the relation between investors' risk preferences (described by the parameters p, q, and L) and their choice between OBPI and CPPI. I also analyze the sensitivity of the results to market conditions, and using real market data, I provide a robustness check. The results demonstrate that investors preferences in the gain domain, rather than in the loss domain, play a determinant role in defining their choice of a PI strategy.

This paper' s results include a generalization of Bertrand and Prigent' s (2011)) comparison of OBPI and CPPI, which they performed using the Omega ratio introduced by Keating and Shadwick (2002) and Kazemi et al. (2004). The Omega ratio is, in fact, a special case of the Farinelli and Tibiletti (2008) ratio, where ($p = q = 1$).[7] This set of parameters implies that all deviations from the reference point are equally weighted to their original value and reflects a risk-neutral behavior in gains and losses.

The reminder of this paper is organized as follows: in Section 2, the properties of the Farinelli & Tibiletti ratio and those of OBPI and CPPI are recalled, and analytical solutions for the LPM and the UPM of OBPI and CPPI under the GBM are derived. In Section 3, the impact of the risk preferences of investors on their choice of a specific PI strategy is analyzed, and a sensitivity analysis of the results to market conditions is also provided. A robustness check using real market data is presented in Section 4, and Section 5 offers a general conclusion.

2. Risk-adjusted performance measurement of PI strategies

2.1. *The Farinelli and Tibiletti (2008) Ratio*

Denoted hereafter by($\Phi_{p,q}$), the Farinelli and Tibiletti (2008) ratio is defined as follows:

$$\Phi_{p,q} = \frac{\sqrt[p]{UPM_p(L)}}{\sqrt[q]{LPM_q(L)}} = \frac{\sqrt[p]{\mathbb{E}[x-L]_+^p}}{\sqrt[q]{\mathbb{E}[L-x]_+^q}}. \tag{1}$$

Where x is a series of portfolio returns, L is a reference point that separates the gains above L from the losses below L. The order of the UPM and the LPM is $p > 0$ and $q > 0$, respectively. The ratio $\Phi_{p,q}$ can be adapted to fit the risk preferences of investors through two main elements: the reference point L and the orders of partial moments (p and q).

The ratio $\Phi_{p,q}$ is a decreasing function w.r.t L which is usually defined exogenuously. Unser (2000) demonstrated that the level of L varies according to investors' preferences and increases with risk aversion.[8]

The parameters p and q, on the other hand, respectively reflect the desired relevance given to the magnitude of positive and negative deviations from L. The higher the value of q (with $q > 1$), the higher the investor's risk aversion to large losses and lower values $q < 1$ reflect a risk-seeking behavior in the loss domain. Similarly, $p < 1$ reflects risk aversion in the gain domain, while $p > 1$ reflects risk-seeking. Consequently, risk-averse expected utility investors can be modeled by $q > 1$ and

[5] CPT states that investors are risk averse in the gain domain, above a specific reference point and risk seeking in the loss domain below the reference point.

[6] Zakamouline (2014) established an explicit link between partial-moments-based performance measures and investor's utility function.

[7] For $p = 1$ and $q = 2, 3,$ we get the Kappa measures introduced by Kaplan and Knowles (2004).

[8] For an investor buying PI, it is logical to think that the minimum value of L corresponds to the insured capital at maturity.

$p < 1$. Prospect theory investors, who are risk-seeking below and risk-averse above L, can be modeled by $q < 1$ and $p < 1$. The type of investors analyzed in Markowitz (1952), risk-averse below and risk-seeking above L,[9] can be modeled by $q > 1$ and $p > 1$, and risk-seeking investors can be modeled by $q < 1$ and $p > 1$ (See e.g. Zakamouline (2011)).

2.2. the ratio $\Phi_{p,q}$ for the OBPI strategy

2.2.1. The OBPI strategy

Leland and Rubinstein (1976) introduced the OBPI strategy, which consists of purchasing j shares of a risky asset S covered by j shares of a European put option written on it. The option' s strike price K corresponds to the insured capital at the maturity date T whatever the value of S_T. The value of the OBPI portfolio at maturity, V_T^{OBPI}, is given by [10]:

$$V_T^{OBPI} = j\big(S_T + (K - S_T)^+\big) = j\big(K + (S_T - K)^+\big). \tag{2}$$

And the value the OBPI portfolio at any time $t \le T$, V_t^{OBPI}, is:

$$V_t^{OBPI} = j(S_t + P(t, S_t, K)) = j\big(Ke^{-r(T-t)} + C(t, S_t, K)\big). \tag{3}$$

Where $C(t, S_t, K)$ and $P(t, S_t, K)$ are respectively the time t Black-Scholes prices of European call and put options written on S with a maturity T and a strike price K. Eq. (3) shows for all dates $t \le T$, the value of the OBPI portfolio can not fall below the deterministic level $jKe^{-r(T-t)}$.

In general, investors seek to recover a percentage α of their initial investment V_0 at maturity. In this case, the following relation must be verified:

$$\alpha V_0 = \alpha j\big[K.e^{-rT} + C(0, S_0, K)\big] = jK. \tag{4}$$

2.2.2. Computation of the LPM and the UPM of the OBPI strategy

Proposition 1. *The lower partial moment of order n of the OBPI strategy, LPM_n^{OBPI}, is given by:*

$$LPM_n^{OBPI} = \sum_{i=0}^{n} \binom{n}{i} (-1)^i L^{n-i} * \Big[(jK)^i \mathrm{N}(-d_{0,K}) + \beta (jS_0)^i \Big[\mathrm{N}\big(-d_{i,L/j}\big) - \mathrm{N}(-d_{i,K})\Big]\Big]. \tag{5}$$

And the upper partial moment of order n of the OBPI strategy, UPM_n^{OBPI}, is given by:

$$UPM_n^{OBPI} = \sum_{i=0}^{n} \binom{n}{i} (-1)^{n-i} L^{n-i} \beta (jS_0)^i \mathrm{N}\big(d_{i,L/j}\big). \tag{6}$$

Where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$, $\beta = exp\big(i\mu T + \frac{1}{2}(i^2 - i)\sigma^2 T\big)$ and $d_{i,X} = \frac{ln(S_0/X) + (\mu - \sigma^2/2)T + i\sigma^2 T}{\sigma\sqrt{t}}$.

Proof. The expression of the LPM_n^{OBPI} can be written as follows:

$$\begin{aligned} LPM_n^{OBPI} &= \mathbb{E}\big(L - V_T^{OBPI}\big)_+^n, \\ &= \sum_{i=0}^{n} \binom{n}{i} (-1)^i L^{n-i} \mathbb{E}\big(V_T^{OBPI}\big)^i \mathbb{I}_{\{V_T^{OBPI} \le L\}}. \end{aligned} \tag{7}$$

Where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$ and [11]:

$$\begin{aligned} \mathbb{E}\big(V_T^{OBPI}\big)^i \mathbb{I}_{\{V_T^{OBPI} \le L\}} &= \mathbb{E}\big((jK)^i \mathbb{I}_{\{S_T \le K\}} * \mathbb{I}_{\{jK \le L\}}\big) + \mathbb{E}\big((jS_T)^i \mathbb{I}_{\{S_T > K\}} * \mathbb{I}_{\{jS_T \le L\}}\big), \\ &= \mathbb{E}(jK)^i \mathbb{I}_{\{S_T \le K\}} + \mathbb{E}(jS_T)^i \mathbb{I}_{\{S_T \le L/j\}} - \mathbb{E}(jS_T)^i \mathbb{I}_{\{S_T \le K\}}. \end{aligned} \tag{8}$$

Knowing that, under the GBM:

$$\mathbb{E}(S_T)^i \mathbb{I}_{\{S_T \le X\}} = S_0^i \, exp\Big(i\mu T + \frac{1}{2}\big(i^2 - i\big)\sigma^2 T\Big)\mathrm{N}(-d_{i,X}), \tag{9}$$

Where:

$$d_{i,X} = \frac{ln(S_0/X) + (\mu - \sigma^2/2)T + i\sigma T}{\sigma\sqrt{T}}. \tag{10}$$

[9] Markowitz (1952) introduced this utility function to explain why people simultaneously buy insurance and lottery tickets.

[10] Due to the call-put parity, this strategy is equivalent to purchasing, at $t = 0$, j shares of a European call option written on S with a strike price K and a maturity T and investing an amount jKe^{-rT} in a risk-free asset.

[11] The indicator $\mathbb{I}_{\{jK \le L\}}$ is equal to 1 as L is always set to a higher value than the insured capital at maturity (jK).

The expression of the LPM_n^{OBPI} in Eq. (7) can be written as follows:

$$LPM_n^{OBPI} = \sum_{i=0}^{n} \binom{n}{i} (-1)^i L^{n-i} * \left[(jK)^i \mathrm{N}(-d_{0,K}) + \beta (jS_0)^i \left[\mathrm{N}\left(-d_{i,L/j}\right) - \mathrm{N}(-d_{i,K}) \right] \right].$$

Where $\beta = exp\left(i\mu T + \frac{1}{2}\left(i^2 - i\right)\sigma^2 T\right)$.

Following a similar procedure, we derive the expression of the UPM_n^{OBPI} as follows[12]:

$$\begin{aligned} UPM_n^{OBPI} &= \mathbb{E}\left(V_T^{OBPI} - L\right)_+^n, \\ &= \sum_{i=0}^{n} \binom{n}{i} (-1)^{n-i} L^{n-i} \beta (jS_0)^i \mathrm{N}\left(d_{i,L/j}\right). \end{aligned} \tag{11}$$

□

2.3. the ratio $\Phi_{p,q}$ for the CPPI strategy

2.3.1. The CPPI strategy

Black and Jones (1987) introduced the CPPI strategy,[13] which consists in managing a dynamic portfolio of a risky and risk-free assets over time. To implement the CPPI, the investor selects a floor below which she does not want the portfolio value to fall at maturity. The floor is defined as a percentage α of the initial investmentV_0 and grows at a constant rate r:

$$F_T = \alpha V_0, \tag{12}$$

$$F_t = F_T e^{-r(T-t)} = \alpha V_0 e^{-r(T-t)}. \tag{13}$$

To achieve this guarantee, the investor allocates an amount called the exposure to a risky asset, such as a market index, and the remaining amount to a risk-free asset that evolves according to the constant rate r. The exposure must be continuously adjusted to a constant multiple m of the difference between the portfolio value and floor. This difference is called the cushion C_t

$$C_t = max\left\{V_t^{CPPI} - F_t, 0\right\}. \tag{14}$$

If the cushion becomes very small, the exposure becomes very small too preventing the portfolio value from falling below the floor. For all $m > 1$, the payoff function of the CPPI portfolio is a convex function w.r.t S_T. The higher the multiple, the more convex the payoff function is.[14]

If the underlying risky asset follows the GBM, the time t value of the cushion is given by:

$$\begin{aligned} C_t &= C_0\, exp\left[\left(m(\mu - r) + r - \frac{m^2\sigma^2}{2}\right)t + m\sigma W_t\right], \\ &= C_0\, exp(\mu_r t + \sigma_r W_t). \end{aligned} \tag{15}$$

Where μ is a constant drift, σ is the volatility of the underlying risky asset, W_t is the standard Brownian motion, $\mu_r = m(\mu - r) + r - \frac{m^2\sigma^2}{2}$ and $\sigma_r = m\sigma$.

2.3.2. Computation of the LPM and the UPM of the CPPI strategy

Proposition 2. *The lower partial moment of order n of the CPPI strategy, LPM_n^{CPPI}, is given by:*

$$LPM_n^{CPPI} = \sum_{i=0}^{n} \binom{n}{i} (-1)^i a^{n-i} \beta' C_0^i \mathrm{N}(-d_{i,a}). \tag{16}$$

And the upper partial moment of order n of the CPPI strategy, UPM_n^{CPPI},is given by:

$$UPM_n^{CPPI} = \sum_{i=0}^{n} \binom{n}{i} (-1)^{n-i} a^{n-i} \beta' C\, \mathrm{N}(d_{i,a}). \tag{17}$$

Where $a = L - F_T,\ \beta' = exp\left(i\mu_r T + \frac{1}{2}\left(i^2 - i\right)\sigma_r^2 T\right)$ and $d_{i,a} = \frac{ln(C_0/a) + \mu_r T + i\sigma_r T}{\sigma_r\sqrt{T}}$.

[12] Detailed proofs of the expressions of UPM_n^{OBPI} and $UPM_n^{CPPI}$are available at request.

[13] See also Perold and Sharpe (1988) and Black and Perold (1992).

[14] Many extensions of the standard CPPI method are proposed in practice. See e.g. Lee et al. (2011), Hamidi et al. (2014) and Ameur and Prigent (2014).

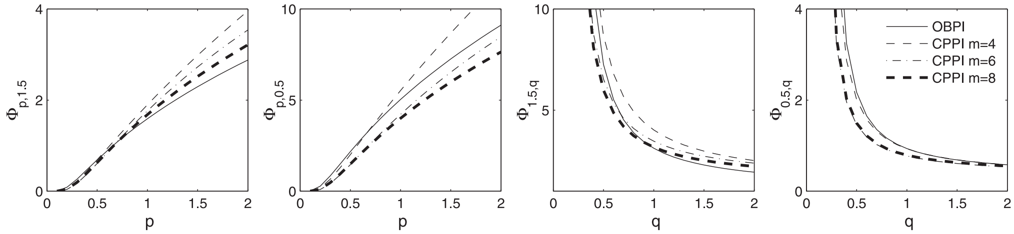

Fig. 1. $\Phi_{p,q}$ as function of p and q for $\alpha = 0.9$ and $L = 102$.

Proof. The expression of the LPM_n^{CPPI} can be written as follows:

$$\begin{aligned} LPM_n^{CPPI} &= \mathbb{E}\left(L - V_T^{CPPI}\right)_+^n, \\ &= \mathbb{E}(L - F_T - C_T)_+^n, \\ &= \sum_{i=0}^{n} \binom{n}{i} (-1)^i a^{n-i} \mathbb{E}\left(C_T^i \mathbb{I}_{\{C_T \le a\}}\right) \end{aligned} \tag{18}$$

Where $a = L - F_T$ and $\mathbb{E}\left(C_T^i \mathbb{I}_{\{C_T \le a\}}\right)$ is given by Eq. (9).
The final form of the LPM_n^{CPPI} can be written as follows:

$$LPM_n^{CPPI} = \sum_{i=0}^{n} \binom{n}{i} (-1)^i a^{n-i} \beta' C_0^i \mathrm{N}(-d_{i,a}). \tag{19}$$

Where $\beta' = exp\left(i\mu_r T + \frac{1}{2}\left(i^2 - i\right)\sigma_r^2 T\right)$.
Following a similar procedure, we derive the expression of the UPM_n^{CPPI} as follows:

$$\begin{aligned} UPM_n^{CPPI} &= \mathbb{E}\left(V_T^{CPPI} - L\right)_+^n \\ &= \sum_{i=0}^{n} \binom{n}{i} (-1)^{n-i} a^{n-i} \beta' C_0^i \,\mathrm{N}(d_{i,a}). \end{aligned} \tag{20}$$

□

3. OBPI Versus CPPI

In order to compare OBPI and CPPI, the two strategies are assumed to have the same initial portfolio value $\left(V_0^{OBPI} = V_0^{CPPI}\right)$ and to provide the same guarantee ($\alpha V_0 = jK = F_T$) at maturity T. The second assumption implies that $F_0 = jKe^{-rT}$, and thus, the initial value of the cushion C_0 is equal to the call price $C(0, S_0, K)$. The underlying risky asset of both strategies is supposed to evolve according to the GBM with the following base parameters: $T = 1$ year, $\mu = 8\%$, $\sigma = 20\%$, $S_0 = 100$, and $r = 3\%$. Different values of the insured percentage at maturity are considered, and CPPI strategies with different levels of the multiple are compared.

3.1. The impact of investors' preferences

3.1.1. The parameters p *and* q

For an insured percentage $\alpha = 0.9$ and a reference point $L = 102$, an investor with moderate gain preferences (risk aversion in the gain domain with $p < 1$) will prefer the OBPI to the CPPI, regardless of the investor's preferences in the loss domain ($q > 1$ or $q < 1$). These sets of parameters correspond to expected utility and prospect utility investors, respectively. Increasing the value p to $p > 1$ leads to an inverse ranking. CPPI strategies start to dominate the OBPI strategy when p is around 1. For $p > 1$ and $q > 1$, an investor who is risk seeking in gains and risk averse in losses will prefer the CPPI strategy. However, no absolute dominance is observed when $p > 1$ and $q < 1$, which is risk seeking in both gains and losses. In this case, the OBPI dominates the CPPI unless the value of the multiple m is relatively small ($m = 4$) (see Fig. 1).

Increasing the level of guarantee favors CPPI strategies (see Fig. 2). For $\alpha = 1$, CPPI strategies dominate OBPI for almost all values of p and q; this result also holds true for all values of the multiple m. For this numerical example, CPPI strategies with a lower value of m dominate those with higher values of this parameter.

3.1.2. The reference point L

Investors with different reference points L (with $L \geq \alpha V_0$) are considered. The values of L are incorporated with selected sets of the parameters p and q in order to reflect investors' preferences. $\Phi_{0.5,\,0.5}$ is used for prospect utility investors, $\Phi_{1.5,\,0.5}$ for risk seeking investors, $\Phi_{0.5,\,1.5}$ for risk-averse expected utility investors and $\Phi_{1.5,\,1.5}$ for Markowitz (1952) investors.

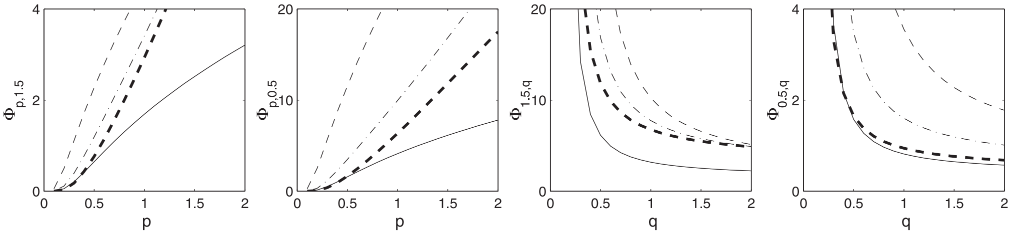

Fig. 2. $\Phi_{p,q}$ as function of p and q for $\alpha = 1$ and $L = 102$.

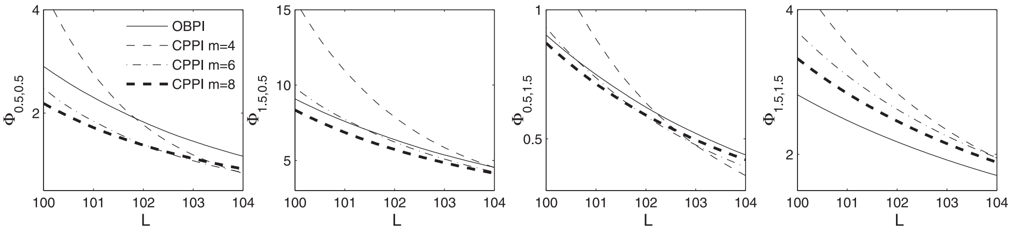

Fig. 3. $\Phi_{p,q}$ as function of the reference point L for $\alpha = 0.9$.

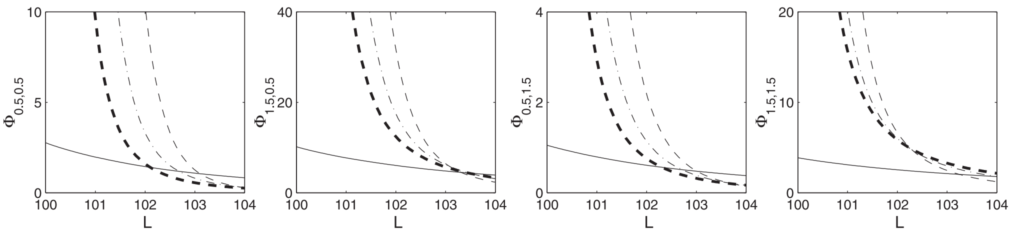

Fig. 4. $\Phi_{p,q}$ as function of the reference point L for $\alpha = 1$.

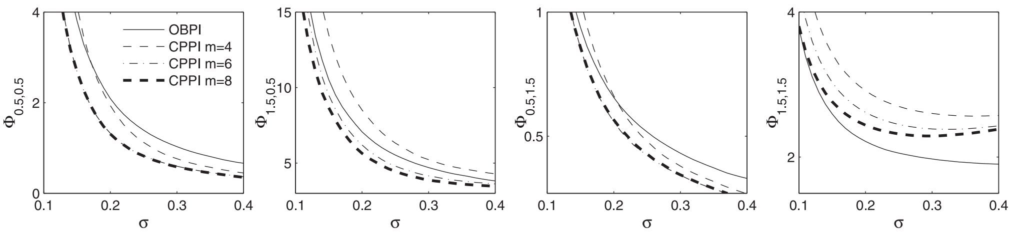

Fig. 5. $\Phi_{p,q}$ as function of the volatility σ for $\alpha = 0.9$ and $L = 102$.

For $\alpha = 0.9$, increasing the value of L, which indicates higher risk aversion, favors the OBPI strategy, see Fig. 3. For all ratios $\Phi_{p,q}$, the gap between the curves of OBPI and CPPI diminishes when L increases. Neither strategy dominates the other for all values of L except for $\Phi_{1.5,\,1.5}$ where the CPPI dominates.[15] The multiple m plays a determinant role in the ranking of OBPI and CPPI. CPPI strategies with high values of m are dominated by OBPI for almost all values of L. This result holds for $\Phi_{0.5,\,0.5}$, $\Phi_{1.5,\,0.5}$ and $\Phi_{0.5,\,1.5}$. CPPI strategies intersect each other at high values of L (around 103). Higher values of m are preferred when high reference points, relatively to the initial investment, are considered. Increasing the insured percentage to $\alpha = 1$ favors the CPPI strategies, which strongly dominate the OBPI, especially for small values of L. However, the OBPI strategy still dominates for high values of L, see Fig. 4.

3.2. The impact of market conditions

In order to analyze the impact of market conditions on the relative risk-adjusted performance of OBPI and CPPI, different levels of the volatility of the underlying (between 10% and 40%) are considered. Fig. 5 shows that all $\Phi_{p,q}$ ratios are decreasing functions w.r.t the volatility σ. For $\alpha = 0.9$ and $L = 102$, neither strategy dominates the other for all volatility levels except for $\Phi_{1.5,\,1.5}$ where the CPPI dominates. In all other cases, the OBPI dominates CPPI strategies with high values of the

[15] This result is in line with the results of Bertrand and Prigent (2011).

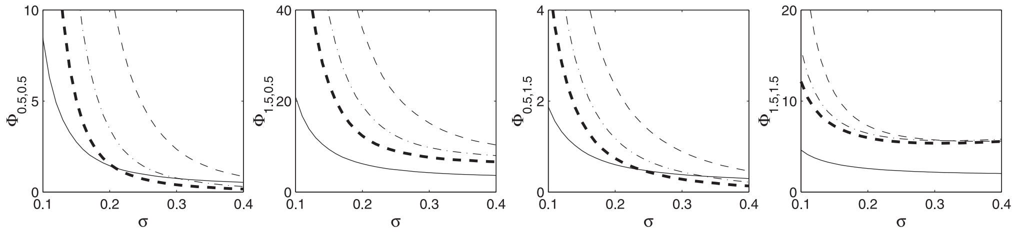

Fig. 6. $\Phi_{p,q}$ as function of the volatility σ for $\alpha = 1$ and $L = 102$.

Table 1
Summery statistics of CAC40 returns for the period from 3/1/2000 to 30/12/2016.

CAC40 statistics	
Mean (Annualized)	−1.12%
Standard deviation (Annualized)	23.48%
Skewness	−0.034
Kurtosis	7.8033
ρ-value autocorrelation (Ljung-Box test)	0.000
ρ-value autocorrelation (Ljung-Box test squared returns)	0.000
ρ-value heteroscedasticity (Engle's ARCH test)	0.000

multiple ($m = 6$ and $m = 9$). Regardless of the value of q, the OBPI strategy dominates for moderate to high volatility levels (between 20% and 40%) when $p < 1$. This shows that, under all market conditions, the preferences in the gain domain (risk aversion when $p < 1$) have a more significant impact on the ranking of OBPI and CPPI than those in the loss domain.

CPPI strategies dominate the OBPI for almost all volatility scenarios when $\alpha = 1$. However, when $p < 1$, the OBPI starts to dominate high multiple CPPI strategies at volatility levels ($\sigma \geq 20\%$), see Fig. 6.

4. Robustness check

The moving block bootstrap method was used to simulate a series of OBPI and CPPI portfolio returns using French market data. This method preserves the dependence and the autocorrelation features of the original data. To begin the simulations, a random starting date is drawn with replacement. The performance of OBPI and CPPI is simulated over the 252 days that follow the starting date. This procedure is then repeated 10000 times and the ratios $\Phi_{p,q}$ are computed on this 10000 OBPI and CPPI yearly returns.

The initial investment of the OBPI and the CPPI portfolios is allocated between a risky asset (the CAC40 index) and a risk-free asset that evolves according to the Euribor 12-month rate prevailing at the first day of the management period. The price data used for the underlying risky asset consist of CAC40 daily closing prices, realized within the period from January 3, 2000 to December 30, 2016. This period is of particular importance because it shows different market conditions, including the financial market crisis of 2008. The statistics of CAC40 log-returns, presented in Table 1, show the non-normality of these returns. For this period, the CAC40 has a negative annual mean return (−1.12%), an annualized standard deviation (23.48%), a small negative skewness of (−0.034) and a high kurtosis (7.8033). The results of Ljung-Box test detects significant serial correlation between the returns of the CAC40 index within this period, and significant heteroscedasticity is also detected. For the OBPI strategy, option prices are calculated according to Black-Scholes option pricing formula. The volatility input that is used to price the OBPI's call options corresponds to the annualized standard deviation calculated on the 252 daily index returns prior to the starting date of the management period. The CPPI strategy rebalanced on monthly basis. The initial floor value is computed with the one year Euribor rate prevailing at the starting date of the management period. The same rate is used to calculate option prices.

Simulation results, presented in Table 2, show that the investors who are risk averse in the gain domain, expected utility ($\Phi_{0.5,\,1.5}$) and prospect utility ($\Phi_{0.5,\,0.5}$) investors, shall prefer the OBPI strategy to the CPPI. This result is particularly true for higher values of the reference point L and lower insured percentages α. This result holds also for investors who show a risk-seeking behavior in gains and losses ($\Phi_{1.5,\,0.5}$). However, investors who are risk averse in losses and risk seeking in gains ($\Phi_{1.5,\,1.5}$) shall prefer the CPPI strategies especially for lower values of L and a higher values of α.

5. Conclusion

This paper analyzed the relation between the risk preferences of investors and their choices between PI strategies (OBPI and CPPI). For this purpose, the Farinelli and Tibiletti (2008) ratio, which can be easily adjusted to fit the type of investor preferences, was used as a risk-adjusted performance measure. The analysis proved that different risk preferences imply different ranking of OBPI and CPPI. This result is particularly true for low levels of guarantee α. In this case, where $\alpha = 0.9$,

Table 2
The ratios $\Phi_{p,q}$ for OBPI and CPPI strategies.

	$\Phi_{0.5,0.5}$				$\Phi_{1.5,0.5}$				$\Phi_{0.5,1.5}$				$\Phi_{1.5,1.5}$			
	OBPI	CPPI			OBPI	CPPI			OBPI	CPPI			OBPI	CPPI		
α%		m=4	m=6	m=8		m=4	m=6	m=8		m=4	m=6	m=8		m=4	m=6	m=8
	L=1%															
90	0.94	$\underline{0.68}$	$\underline{0.50}$	$\underline{0.53}$	2.27	2.32	$\underline{1.06}$	$\underline{2.10}$	0.32	0.32	$\underline{0.28}$	$\underline{0.31}$	0.66	0.93	1.00	1.06
92	0.98	$\underline{0.74}$	$\underline{0.49}$	$\underline{0.46}$	2.36	2.49	$\underline{2.08}$	$\underline{2.08}$	0.32	0.34	$\underline{0.28}$	$\underline{0.28}$	0.68	0.99	1.02	1.11
94	1.04	$\underline{0.82}$	$\underline{0.50}$	$\underline{0.41}$	2.48	2.77	$\underline{2.18}$	$\underline{2.08}$	0.34	0.37	$\underline{0.29}$	$\underline{0.26}$	0.70	1.08	1.07	1.15
96	1.14	$\underline{0.99}$	$\underline{0.56}$	$\underline{0.40}$	2.65	3.29	$\underline{2.44}$	$\underline{2.17}$	0.36	0.42	$\underline{0.31}$	$\underline{0.26}$	0.74	1.24	1.17	1.21
98	1.24	1.39	$\underline{0.72}$	$\underline{0.46}$	2.85	4.57	3.13	$\underline{2.55}$	0.39	0.55	$\underline{0.38}$	$\underline{0.29}$	0.79	1.65	1.44	1.40
100	1.35	3.49	1.40	$\underline{0.80}$	3.09	11.2	6.09	4.55	0.41	1.15	0.66	0.46	0.84	3.52	2.68	2.41
	L=2%															
90	0.75	$\underline{0.44}$	$\underline{0.38}$	$\underline{0.42}$	1.90	$\underline{1.66}$	$\underline{1.63}$	$\underline{1.73}$	0.38	$\underline{0.24}$	$\underline{0.24}$	$\underline{0.26}$	0.59	0.74	0.84	0.81
92	0.77	$\underline{0.44}$	$\underline{0.35}$	$\underline{0.35}$	1.95	$\underline{1.67}$	$\underline{1.57}$	$\underline{1.65}$	0.38	$\underline{0.24}$	$\underline{0.22}$	$\underline{0.23}$	0.60	0.75	0.85	0.83
94	0.80	$\underline{0.44}$	$\underline{0.33}$	$\underline{0.29}$	2.01	$\underline{1.69}$	$\underline{1.53}$	$\underline{1.56}$	0.39	$\underline{0.24}$	$\underline{0.21}$	$\underline{0.21}$	0.61	0.77	0.83	0.83
96	0.86	$\underline{0.42}$	$\underline{0.31}$	$\underline{0.26}$	2.09	$\underline{1.69}$	$\underline{1.50}$	$\underline{1.47}$	0.30	$\underline{0.24}$	$\underline{0.21}$	$\underline{0.19}$	0.63	0.78	0.83	0.80
98	0.90	$\underline{0.39}$	$\underline{0.29}$	$\underline{0.23}$	2.18	$\underline{1.67}$	$\underline{1.49}$	$\underline{1.40}$	0.31	$\underline{0.23}$	$\underline{0.20}$	$\underline{0.18}$	0.66	0.82	0.84	0.79
100	0.93	$\underline{0.38}$	$\underline{0.27}$	$\underline{0.22}$	2.27	$\underline{1.62}$	$\underline{1.45}$	$\underline{1.37}$	0.32	$\underline{0.25}$	$\underline{0.20}$	$\underline{0.18}$	0.68	0.88	0.90	0.93
	L=3%															
90	0.60	$\underline{0.30}$	$\underline{0.29}$	$\underline{0.34}$	1.59	$\underline{1.20}$	$\underline{1.30}$	$\underline{1.43}$	0.24	$\underline{0.19}$	$\underline{0.23}$	$\underline{0.23}$	0.53	0.60	0.71	0.79
92	0.60	$\underline{0.28}$	$\underline{0.26}$	$\underline{0.28}$	1.60	$\underline{1.14}$	$\underline{1.20}$	$\underline{1.32}$	0.24	$\underline{0.18}$	$\underline{0.20}$	$\underline{0.20}$	0.53	0.58	0.69	0.79
94	0.61	$\underline{0.25}$	$\underline{0.23}$	$\underline{0.23}$	1.61	$\underline{1.05}$	$\underline{1.1}$	$\underline{1.19}$	0.25	$\underline{0.17}$	$\underline{0.17}$	$\underline{0.18}$	0.53	0.56	0.65	0.76
96	0.63	$\underline{0.21}$	$\underline{0.20}$	$\underline{0.18}$	1.64	$\underline{0.91}$	$\underline{0.98}$	$\underline{1.04}$	0.25	$\underline{0.16}$	$\underline{0.16}$	$\underline{0.15}$	0.54	0.53	0.61	0.70
98	0.63	$\underline{0.17}$	$\underline{0.16}$	$\underline{0.16}$	1.64	$\underline{0.71}$	$\underline{0.81}$	$\underline{0.86}$	0.25	$\underline{0.14}$	$\underline{0.14}$	$\underline{0.14}$	0.54	0.46	0.54	0.62
100	0.61	$\underline{0.13}$	$\underline{0.13}$	$\underline{0.13}$	1.63	$\underline{0.48}$	$\underline{0.58}$	$\underline{0.65}$	0.25	$\underline{0.12}$	$\underline{0.12}$	$\underline{0.12}$	0.54	0.35	0.44	0.51
	L=4%															
90	0.48	$\underline{0.21}$	$\underline{0.24}$	$\underline{0.28}$	1.32	$\underline{0.88}$	$\underline{1.05}$	$\underline{1.19}$	0.21	$\underline{0.16}$	$\underline{0.17}$	$\underline{0.20}$	0.47	0.49	0.61	0.69
92	0.47	$\underline{0.19}$	$\underline{0.20}$	$\underline{0.23}$	1.30	$\underline{0.78}$	$\underline{0.94}$	$\underline{1.08}$	0.21	$\underline{0.15}$	$\underline{0.16}$	$\underline{0.18}$	0.47	0.47	0.58	0.68
94	0.46	$\underline{0.16}$	$\underline{0.17}$	$\underline{0.18}$	1.28	$\underline{0.67}$	$\underline{0.81}$	$\underline{0.94}$	0.21	$\underline{0.13}$	$\underline{0.14}$	$\underline{0.15}$	0.46	$\underline{0.45}$	0.53	0.63
96	0.45	$\underline{0.14}$	$\underline{0.15}$	$\underline{0.15}$	1.26	$\underline{0.53}$	$\underline{0.67}$	$\underline{0.77}$	0.21	$\underline{0.12}$	$\underline{0.12}$	$\underline{0.13}$	0.45	$\underline{0.36}$	0.46	0.56
98	0.43	$\underline{0.12}$	$\underline{0.13}$	$\underline{0.13}$	1.21	$\underline{0.36}$	$\underline{0.50}$	$\underline{0.58}$	0.20	$\underline{0.11}$	$\underline{0.12}$	$\underline{0.12}$	0.44	$\underline{0.28}$	$\underline{0.38}$	0.46
100	0.39	$\underline{0.10}$	$\underline{0.11}$	$\underline{0.11}$	1.13	$\underline{0.20}$	$\underline{0.30}$	$\underline{0.38}$	0.19	$\underline{0.10}$	$\underline{0.11}$	$\underline{0.11}$	0.43	$\underline{0.18}$	$\underline{0.26}$	$\underline{0.32}$

The underlined numbers are the ones where OBPI dominates CPPI.

investors' preferences in the gain domain were more relevant in describing the choice between OBPI and CPPI. OBPI was the preferred strategy for expected utility and prospect utility investors who are risk averse in the gain domain and, CPPI provided a higher risk-adjusted performance for investors who are risk seeking in gains and risk averse in losses; However, no absolute dominance between OBPI and CPPI is observed for risk seeking investors. Increasing the insured percentage α was always in favor of the CPPI which was shown to be preferred for all types of investors when $\alpha = 1$. However, the gap between the risk-adjusted performance of OBPI and CPPI diminished when the reference point increased and when the market was more volatile.

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