

## Optimal reconfiguration and capacitor allocation in radial distribution systems for energy losses minimization

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### ABSTRACT

This paper presents an algorithm for reconfiguration associated with capacitor allocation to minimize energy losses on radial electrical networks considering different load levels. The proposed model is solved using a mixed integer non-linear programming approach, in which a continuous function is used to handle the discrete variables. The primal–dual interior point technique is applied to solve the optimization problem at each step. The Lagrange multipliers are used to evaluate a new proposed sensitivity index for distribution system reconfiguration. The association of reconfiguration with capacitor allocation is achieved through the combination of two sequential solution based approaches. The performance of the algorithm is analyzed in three systems available in the literature.

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### 1. Introduction

Reconfiguration and capacitor allocation procedures in radial electrical distribution systems (EDS) are attractive alternatives for technical losses reduction [1,2]. The reconfiguration problem consists in determining a radial and connected network topology, through the definition of the maneuverable switches states (open or closed). The switching devices include: (i) sectionalizing or normally closed (NC) switches; (ii) tie or normally open (NO) switches. The capacitor allocation problem involves the placement and determination of the type, size and number of capacitors when they are required to be installed in the system [3]. The placement consists in the state determination (on/off) of the capacitor bank switches for reduction of reactive power flow on distribution lines [4]. This leads to a mixed integer non-linear programming formulation which has a combinatorial nature, requiring a large number of simulations for practical distribution systems solutions. Further complexities to the problem are added by the radial and connectivity constraints of the distribution network.

Reconfiguration approaches are well discussed in [5–10,11] whereas capacitor allocation is addressed in [3,12–14]. Singh

et al. [11] propose a heuristic method for reconfiguration that performs a sequential switch opening based on the branch power flow. Segura et al. [14] present a constructive heuristic algorithm to solve the optimal capacitor placement in EDS using a specialized interior point method. In this approach, the discrete variables are relaxed as a strategy to identify the most attractive bus to add capacitors.

Refs. [15–23] consider both the capacitor allocation and the reconfiguration problems.

An algorithm that performs the capacitor allocation after the reconfiguration, in order to reduce losses, is proposed in [15]. This algorithm uses power flow analysis and represents the hourly-seasonal load variation. The algorithm considers the power loss reduction for each load level independently, and evaluates the energy loss reduction through some proposed loss factors. The reconfiguration is performed using an analytical approach based on linear programming with voltage drop approximation technique. Unfortunately, the handling of discrete variables is not well explained. Besides, the decoupled analysis among different load levels may cause the algorithm to miss some good quality solutions.

In [16], a technique that performs the reconfiguration after the capacitor allocation for energy loss reduction, considering daily load curves is presented. This work applies the concept of system state characterization to decrease the computational effort

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required. The *Branch Exchange* technique [6] is used for reconfiguration.

Preponis et al. [17] describe a method in which the *Branch Exchange* technique is used for the reconfiguration, and the capacitor allocation problem is solved through dynamic programming [24]. The energy loss reduction is estimated by the summation of power loss reductions for all different network loading conditions over the study period. These loading conditions are represented by typical daily load curves. However, the daily load curves are not considered in the problem formulation, and so better quality solutions may not be achieved.

Jiang and Baldick [18] propose an algorithm for reconfiguration and capacitor control in order to reduce active power losses in distribution systems. The reconfiguration is performed via *Simulated Annealing* (SA), and a discrete optimization algorithm, based on load flow analysis, is used to find the capacitor control.

Rong et al. [19] present an algorithm for active power loss reduction. The capacitor allocation is solved via genetic algorithm, and the system reconfiguration is solved through a heuristic search. This search method determines the branches candidates to be opened, based on the nodal voltages and power flow analysis.

In [20], the capacitor allocation is solved by using a genetic algorithm combined with power flow analysis. For the reconfiguration, a simplified *Branch Exchange* method is used, based on the calculation of the operating point deviation due to a given branch switching.

In [21], a method based on genetic algorithm for both the reconfiguration and the capacitor allocation problems, aiming at the active power loss reduction, is proposed. Venkatesh and Ranjan [22] present a methodology for the reconfiguration and the capacitor allocation, using an evolutionary algorithm so as to generate candidate solutions for both problems. The methodology maximizes the system loadability. Another approach for reconfiguration with capacitor allocation is proposed in [23], by using ant colony search algorithm, in order to reduce active power losses.

From the literature review it is seen that the application of Lagrange multipliers as sensitivity index for reconfiguration of distribution systems has not been explored in previous works. Besides, the daily load curve is not considered in [18–20,23] meaning that the system might not be in the optimal configuration for all load levels.

The present work describes a methodology for both optimal reconfiguration and capacitor allocation, considering the daily load curve represented by a given number of load levels. The proposed algorithm for reconfiguration considers an initial meshed topology and performs a sequential opening of switches, until an optimal radial configuration is obtained. The major contribution of this paper is the proposition of a new sensitivity index obtained from the Lagrange multipliers for the switching variables. The opening of each switch is based on this proposed index. Both the maneuverable switches, which are modeled by a continuous function, and the different load levels are incorporated into a single optimal power flow calculation (OPF) [3], not yet used for reconfiguration in the literature. The primal–dual interior point method is used for the OPF solutions [25,26]. Comparisons with existing methods and test systems in the literature are discussed, to show the advantages of the proposed technique.

## 2. OPF problem

The modeling of the switches in the optimal power flow (OPF) is performed through the use of the sigmoid function [27]. This function as given by (1), allows the (on/off) integer variable to be represented by a continuous function which is differentiable and can be modeled in the OPF problem.

$$CH_{km}(x_{km}) = \frac{e^{\alpha \cdot x_{km}} - 1}{e^{\alpha \cdot x_{km}} + 1} \quad x_{km} \geq 0 \quad (1)$$

where  $x_{km}$  represents the optimization variable associated with the maneuverable switch coupled to branch  $k-m$  and  $CH_{km}(x_{km})$ , represents the position value of the maneuverable switch of branch  $k-m$ .

The sigmoid function is chosen based on the experience of the authors in handling discrete variables in electrical power system optimization problems [3,28,29].

The OPF has been adopted for the EDS optimal reconfiguration because it allows the identification of the best switch to be opened through the new sensitivity index. The OPF problem can be formulated as:

$$\text{Min } FOB = \sum_{u=1}^{NT} \left[ \sum_{k=1}^{NB} \left[ \sum_{m \in \Omega k} [CH_{km}(x_{km}) \cdot (ce_u \cdot T_u \cdot L_{km,u})] \right] \right] \quad (2)$$

Subject to:

$$Pg_{k,u} - Pl_{k,u} + \sum_{m \in \Omega k} CH_{km}(x_{km}) \cdot P_{km,u} = 0 \quad (\lambda p_{k,u}) \quad (2.1)$$

$$Qg_{k,u} - Ql_{k,u} + \sum_{m \in \Omega k} CH_{km}(x_{km}) \cdot Q_{km,u} = 0 \quad (2.2)$$

$$L_{km,u} = g_{km} \cdot [V_{k,u}^2 + V_{m,u}^2 - 2 \cdot V_{k,u} \cdot V_{m,u} \cdot \cos(\theta_{km,u})] \quad (2.3)$$

$$0 \leq x_{km} \leq 20 \quad (2.4)$$

$$\bar{Z}^{\min} \leq \bar{Z}_u \leq \bar{Z}^{\max} \quad (2.5)$$

where  $FOB$  is the objective function,  $u$  the given load level,  $NT$  the total number of load levels,  $NB$  the total number of busbars,  $\Omega k$  the set of busbars directly connected to busbar  $k$ ,  $ce_u$  the energy price (US\$/kWh) for load level  $u$ ,  $T_u$  the time interval the EDS operates with load level  $u$ ,  $L_{km,u}$  the active power loss of branch  $k-m$  on load level  $u$ ,  $Pg_{k,u}$  the active power generation at busbar  $k$  on load level  $u$ ,  $Pl_{k,u}$  the active power load at busbar  $k$  on load level  $u$ ,  $P_{km,u}$  the active power flow through branch  $k-m$  on load level  $u$ ,  $\lambda p_{k,u}$  the Lagrange multipliers associated with the real power constraint (2.1) on load level  $u$ ,  $Qg_{k,u}$  the reactive power generation at busbar  $k$  on load level  $u$ ,  $Ql_{k,u}$  the reactive power load at busbar  $k$  on load level  $u$ ,  $Q_{km,u}$  the reactive power flow through branch  $k-m$  on load level  $u$ ,  $g_{km}$  the conductance of branch  $k-m$ ,  $V_{k,u}$  the voltage magnitude at busbar  $k$  on load level  $u$ ,  $\theta_{km,u}$  the phase angle between busbars  $k$  and  $m$  on load level  $u$ ,  $\bar{Z}_u$  the vector containing the other OPF variables that have low and upper limits on load level  $u$ , and  $\bar{Z}^{\min}$ ,  $\bar{Z}^{\max}$  is the low and upper limits, respectively, of the variables  $\bar{Z}_u$ .

Eq. (2) defines the OPF objective function, and is associated with the minimum cost of the total energy losses in the system for all load levels being considered [3]. If the branch  $k-m$  has no maneuverable switch,  $CH_{km}(x_{km})$  is fixed at the value 1.

Eqs. (2.1) and (2.2) correspond to the constraints of real and reactive power balance, respectively. The active power loss calculation of branch  $k-m$  is given by (2.3). In addition, expression (2.4) defines the limits of the variables associated with the switch of branch  $k-m$  ( $x_{km}$ ).

The other optimization variables have their limits established in (2.5). In this set of constraints, nodal voltage magnitude and active power generation limits are included.

The proposed OPF allows the use of as many load levels as required. The sparse Hessian matrix has an augmented structure, including the sub-matrices associated with each load level. The load levels are coupled by the switch position variables ( $CH_{km}$ ). Further details of the load fluctuation modeling can be provided in [3].

The OPF problem in (2) is solved using the primal–dual interior point method [14,25,26]. Once this solution is obtained, the maneuverable switch position values ( $CH_{km}$ ) lie in the continuous interval [0, 1]. However, in practice, the switch states are discrete,

i.e., on ( $CH_{km} = 1$ ) or off ( $CH_{km} = 0$ ). Therefore, a strategy for the switches states definition is required. This strategy is based on the new sensitivity index calculation, presented hereafter.

### 2.1. Sensitivity index

The sensitivity index proposed for the switch state determination corresponds to a measure of the impact that the opening of a given branch causes in the OPF objective function. This index is given by:

$$SS_{km} = \frac{\sum_{u=1}^{NT} SS_{km,u}}{NT} = \frac{\sum_{u=1}^{NT} CH_{km}(x_{km}) \cdot (\lambda p_{k,u} - \lambda p_{m,u}) \cdot \frac{N_{km,u}}{(V_{k,u} - V_{m,u})}}{NT} \quad (3)$$

where  $SS_{km}$  is the sensitivity index for the switch  $k$ – $m$  state definition,  $SS_{km,u}$  the sensitivity index for the switch  $k$ – $m$  on load level  $u$ , and  $N_{km,u}$  is the MVA-load flow through branch  $k$ – $m$ , on load level  $u$ .

The proposed index ( $SS_{km}$ ) in (3) is proportional to the switch position value  $CH_{km}(x_{km})$ . The lower the  $CH_{km}(x_{km})$  value, the larger is the tendency of opening branch  $k$ – $m$  for the losses minimization established by the objective function. As shown in (3),  $SS_{km}$  is given by the average of the indexes values on each period ( $SS_{km,u}$ ).

The difference ( $\lambda p_{k,u} - \lambda p_{m,u}$ ) corresponds to the impact on the objective function in (2) due to an active power flow variation through branch  $k$ – $m$ . In other words, the lower ( $\lambda p_{k,u} - \lambda p_{m,u}$ ) is, the lower is the loss increase in the EDS due to the branch  $k$ – $m$  opening. It should be stressed that the index  $SS_{km}$  needs the evaluation of Lagrange multipliers on all considered load levels  $u$ . These multipliers are obtained from the OPF (2) calculation.

The decision index  $SS_{km}$  is also a function of the MVA-power flow  $N_{km,u}$ , since the opening of a given switch results in active and reactive power flow redistribution for the whole system. The opening of a branch with a low  $N_{km,u}$  value causes few alterations in the network, regarding the power flow redistribution, and results in minor deviations on the system operating point.

Finally, the  $SS_{km}$  index is inversely proportional to the voltage drop in branch  $k$ – $m$  ( $V_{k,u} - V_{m,u}$ ). The branch which has large voltage drop value is a potential candidate to be opened because its power conduction must be avoided [30]. Thus the preferred switch to be opened is the one which has the lowest  $SS_{km}$  index, since its opening results in the lowest additional losses. The switches must be opened sequentially using the algorithm which is presented hereafter.

### 2.2. Reconfiguration algorithm

Fig. 1 shows a flowchart of the distribution system reconfiguration algorithm (DSRA).

In the DSRA the network is initially considered in a meshed configuration by closing all the tie switches (NO). From this configuration, the switches are opened in sequence until a radial configuration is obtained.

In Step-1 the maneuverable switches list (MSL) [9] of the system is determined.

In Step-2 an OPF simulation (2) is performed in order to evaluate the sensitivity index (3) for the switches in the MSL list. It is important to notice the variables  $x$  for the switches that are not in the MSL are fixed according to their states in the following values:  $x = 0$  for the open switches, since in this case  $CH(x) = 0$  according to (1);  $x = 20$  for the closed switches, since in this case  $CH(x) = 1$ . Thus, after the OPF simulation, the values of the variables  $x$  associated with the MSL switches are in the continuous interval  $[0, 1]$ .

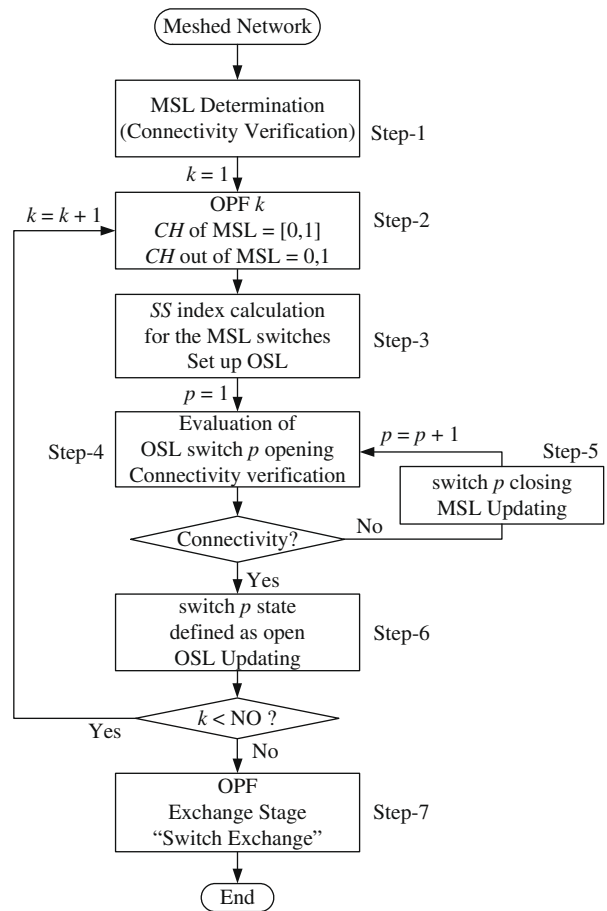


Fig. 1. Flowchart of the proposed algorithm (DSRA).

From the results obtained in Step-2 the sensitivity indexes are ranked in ascending order to produce an optimal switch list (OSL) which is determined in Step-3.

Step-4 is simply applied to check if opening a switch causes a network islanding. If the answer is yes, then this switch is removed from the MSL list and its status is set as closed as shown in Step-5, else this switch is defined as open and also is removed from the MSL list, as shown in Step-6. Step-7 of the algorithm constitutes in making exchanges among each of the open switches and the corresponding adjacent neighbors [6,8]. An exchange operation is defined as the action to close an open switch and to open a closed neighbor switch, so that radiality and connectivity are always ensured. The switch that is set as open is the one which produces the minimum total losses, considering all load periods. This procedure is repeated for all the open switches which are determined from the previous steps.

### 2.3. Proposed methods for the EDS planning

In order to evaluate potential advantages of the association of the reconfiguration algorithm with a capacitor allocation technique where both are based on Lagrange multipliers [3], a joint optimization approach involving the two optimization problems is proposed. This joint approach considers the two methods shown in Fig. 2 (R&C and C&R approaches), and consists on the alternate solutions of the reconfiguration algorithm (proposed DSRA) and the capacitor allocation algorithm (HCA) [3].

In the R&C procedure, the distribution system reconfiguration is performed firstly via DSRA, from the meshed configuration. After

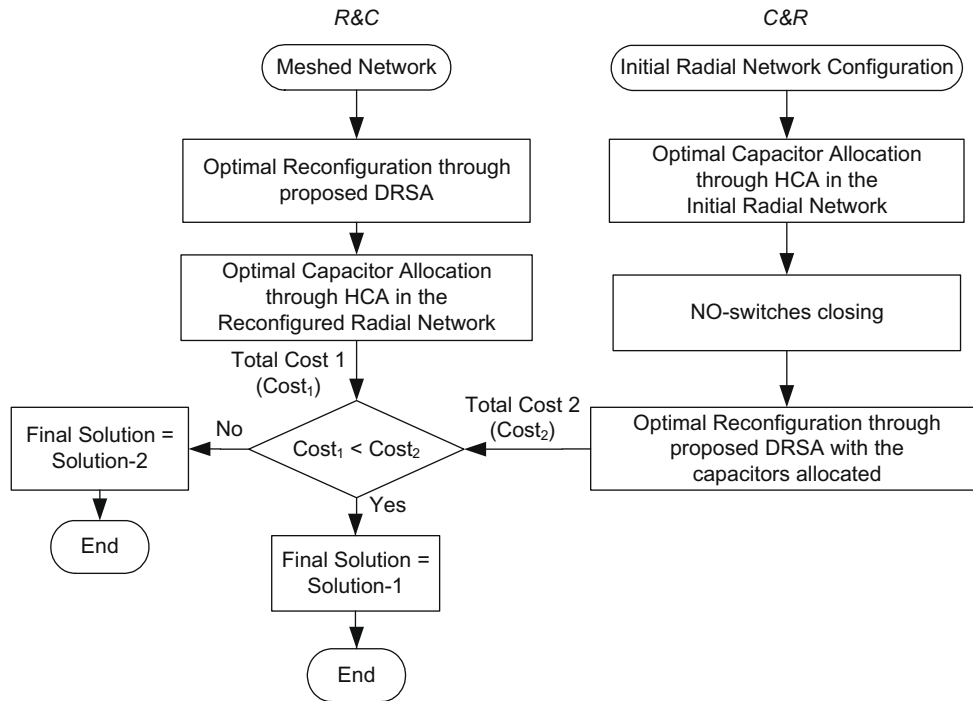


Fig. 2. Proposed methods for the EDS planning.

the reconfiguration, the capacitor allocation problem is solved via HCA.

In the C&R procedure, the capacitor allocation in the original radial topology of the system is solved first via HCA. After this problem solution, the tie switches are closed and the DRSA algorithm is applied for reconfiguration from the meshed topology.

These procedures produce two solutions, where Cost<sub>1</sub> and Cost<sub>2</sub> represent the total costs on all load levels associated with the losses and the investment in capacitors obtained, respectively, through the R&C and C&R approaches. These solutions are compared and the optimal solution associated with the least total cost,

is chosen as final solution for the combined planning problem involving reconfiguration and capacitor allocation.

### 3. Results

In this section, the results obtained with the proposed methodology are presented. Three systems from the literature are used: the 16-bus system [6], the 33-bus system [7] and the 83-bus system [31]. In all cases, the substations voltages are considered as 1.0 p.u. The simulations were performed using an AMD Athlon, 2 GHz, 1 GHz of RAM computer; and the MATLAB<sup>®</sup> software.

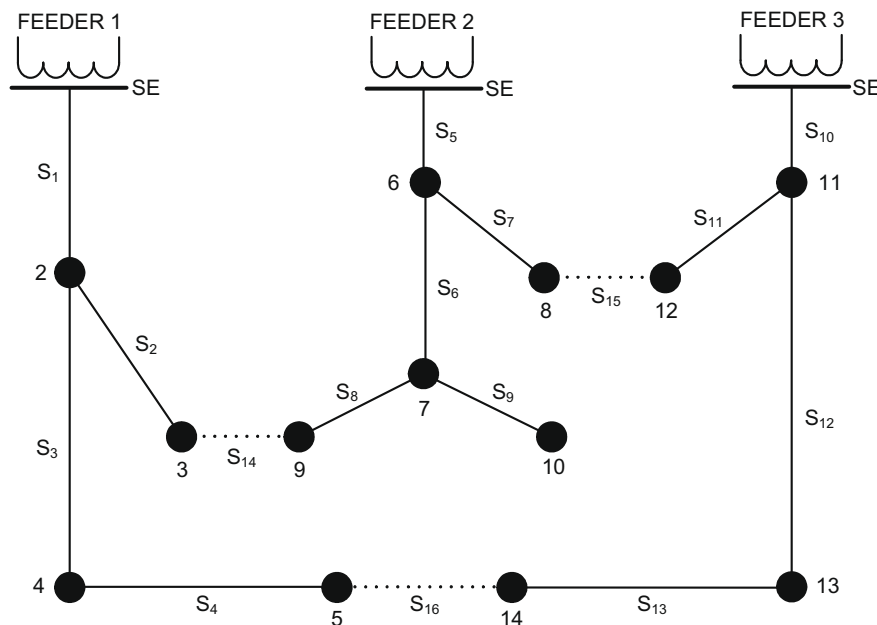


Fig. 3. 16-Bus system.

3.1. 16-Bus system

The 16-bus, 23-kV system [6] is composed of three feeders having 16 branches. Fig. 3 shows the diagram of this system. In this initial topology, the branches represented by solid lines have the normally closed switches (NC) and the dotted lines represent the normally open switches (NO). In this configuration, the total active power losses are 511.44 kW.

The DSRA solution for reconfiguration will be given step by step, but for comparison purposes a medium load level (1.0 p.u.) and power losses minimization will be considered. The limits for nodal voltages are 0.90 and 1.01 p.u.

Step-1. Set up the list of maneuverable switches:

$$MSL = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}, S_{15}, S_{16}\}$$

Step-2. First OPF execution to obtain the switch positions, Lagrange multipliers, voltages and MVA-power flows.

Step-3. Sensitivity index calculation, first execution:

This step computes the sensitivity indexes *SS* from (3), for all the MSL switches. These indexes are presented in Table 1.

Next, the optimal switch list (OSL) is assembled with the switches arranged in ascending order of respective indexes. Table 2 presents OSL-1, which is defined in the first execution of this Step-3.

It can be observed that switch *S*<sub>16</sub> is associated with the lowest index *SS*. Consequently, this is the first switch evaluated for opening.

Step-4. First execution:

It is seen that the opening switch *S*<sub>16</sub> does not cause network islanding. Then it will be removed from the MSL list and set as open. Since this switch is in series with *S*<sub>3</sub>, *S*<sub>4</sub>, *S*<sub>12</sub> and *S*<sub>13</sub>, as shown in Fig. 3, these switches also are removed from MSL, which is updated to produce MSL-1:

$$MSL-1 = \{S_1, S_2, S_5, S_6, S_7, S_8, S_{10}, S_{11}, S_{14}, S_{15}\}$$

At the end of this first execution (*k* = 1), only one switch was opened (*S*<sub>16</sub>). The number of open switches is thus, lower than the number of tie switches (NO = 3). Therefore, the counter of open switches *k* is increased and the algorithm is executed again from Step-2, considering the new MSL-1 list.

**Table 1**  
SS index, OPF-1, 16-bus system.

Switch	SS (×10 <sup>6</sup> )	Switch	SS (×10 <sup>6</sup> )	Switch	SS (×10 <sup>6</sup> )
<i>S</i> <sub>1</sub>	9.5054	<i>S</i> <sub>6</sub>	8.3398	<i>S</i> <sub>12</sub>	3.4212
<i>S</i> <sub>2</sub>	6.2298	<i>S</i> <sub>7</sub>	0.4911	<i>S</i> <sub>13</sub>	3.6427
<i>S</i> <sub>3</sub>	2.5979	<i>S</i> <sub>8</sub>	2.3238	<i>S</i> <sub>14</sub>	3.4496
<i>S</i> <sub>4</sub>	0.8357	<i>S</i> <sub>10</sub>	6.6628	<i>S</i> <sub>15</sub>	1.2646
<i>S</i> <sub>5</sub>	9.8568	<i>S</i> <sub>11</sub>	2.6531	<b><i>S</i><sub>16</sub></b>	0.2944

**Table 2**  
OSL-1, 16-bus system.

Switches	<b><i>S</i><sub>16</sub>-<i>S</i><sub>7</sub>-<i>S</i><sub>4</sub>-<i>S</i><sub>15</sub>-<i>S</i><sub>8</sub>-<i>S</i><sub>3</sub>-<i>S</i><sub>11</sub>-<i>S</i><sub>12</sub>-<i>S</i><sub>14</sub>-<i>S</i><sub>13</sub>-<i>S</i><sub>2</sub>-<i>S</i><sub>10</sub>-<i>S</i><sub>6</sub>-<i>S</i><sub>1</sub>-<i>S</i><sub>5</sub></b>
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**Table 3**  
OSL-2, 16-bus system.

Switches	<b><i>S</i><sub>7</sub>-<i>S</i><sub>15</sub>-<i>S</i><sub>8</sub>-<i>S</i><sub>11</sub>-<i>S</i><sub>14</sub>-<i>S</i><sub>2</sub>-<i>S</i><sub>10</sub>-<i>S</i><sub>6</sub>-<i>S</i><sub>1</sub>-<i>S</i><sub>5</sub></b>
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**Table 4**  
OSL-3, 16-bus system.

Switches	<b><i>S</i><sub>8</sub>-<i>S</i><sub>14</sub>-<i>S</i><sub>10</sub>-<i>S</i><sub>2</sub>-<i>S</i><sub>6</sub>-<i>S</i><sub>1</sub>-<i>S</i><sub>5</sub></b>
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**Table 5**  
Branch exchanges, 16-bus system.

Open switches	Branch exchanges
<i>S</i> <sub>7</sub>	Closing <i>S</i> <sub>7</sub> and opening <i>S</i> <sub>5</sub> Closing <i>S</i> <sub>7</sub> and opening <i>S</i> <sub>15</sub>
<i>S</i> <sub>8</sub>	Closing <i>S</i> <sub>8</sub> and opening <i>S</i> <sub>6</sub> Closing <i>S</i> <sub>8</sub> and opening <i>S</i> <sub>14</sub>
<i>S</i> <sub>16</sub>	Closing <i>S</i> <sub>16</sub> and opening <i>S</i> <sub>4</sub> Closing <i>S</i> <sub>16</sub> and opening <i>S</i> <sub>13</sub>

**Table 6**  
DSRA solution, 16-bus system.

Topology	Loss (kW)	Reduction (%)	Open switches
Initial	511.44	–	<i>S</i> <sub>14</sub> - <i>S</i> <sub>15</sub> - <i>S</i> <sub>16</sub>
Proposed DSRA	<b>466.13</b>	8.86	<i>S</i> <sub>16</sub> - <i>S</i> <sub>7</sub> - <i>S</i> <sub>8</sub>

After the second OPF simulation, switch *S*<sub>7</sub> presented the lowest index *SS*, as shown in Table 3.

Again, the opening of *S*<sub>7</sub> does not cause network islanding. Then it will be removed from the MSL-1 list and set as open.

$$MSL-2 = \{S_1; S_2; S_5; S_6; S_8; S_{10}; S_{14}\}$$

After the third OPF simulation (*k* = 3), switch *S*<sub>8</sub> presented the lowest index *SS*, as shown in Table 4.

It is seen from Fig. 3 that the opening of *S*<sub>8</sub> does not cause network islanding, thus it will be set as open.

Step-7 (*Branch Exchange*) aims to perform a given number of switch exchanges in the network topology as seen in Table 5.

In this test case the set of switches which produced minimum losses remained unchanged.

Table 6 shows the initial and final configurations as well as the corresponding total losses. It is seen that a reduction of 8.86% is achieved.

It is seen that the final solution obtained by the proposed algorithm (DSRA) is identical to that obtained using other methods [6,8,10,30,31].

3.2. 33-Bus system

The 33-bus 12.66-kV system [7], 3715 kW and 2300 kVAr, is composed of five tie switches (NO). Fig. 4 shows the system diagram, where the NO switches are *S*<sub>33</sub>, *S*<sub>34</sub>, *S*<sub>35</sub>, *S*<sub>36</sub> and *S*<sub>37</sub>. Switch *S*<sub>1</sub> is not maneuverable since its opening would leave the system disconnected from the main substation. The analysis is based on realistic daily load curves from a small demonstration zone for distribution automation innovation in China [16]. The energy price is 0.5 US\$/kWh and the time period under study is 6 months [16]. The nodal voltage limits are 0.85 p.u. and 1.01 p.u.

The proposed R&C and C&R procedures were applied under the above conditions, considering capacitor banks of 1110 kVAr to be installed, in order to compare the results with those obtained in [16]. It is important to notice that in [16] a continuous variable for the capacitance to be allocated is considered. Besides, as in [16], 24 load levels from the daily load curve are represented. In this case, the proposed method and the method described in [16] obtained identical results.

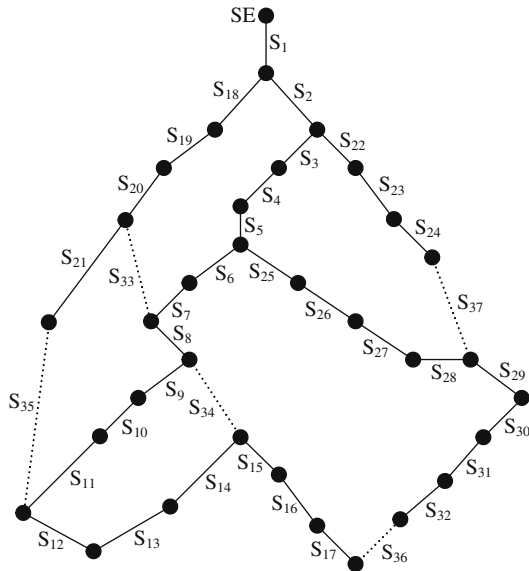


Fig. 4. 33-Bus system.

Table 7  
33-Bus system planning solutions.

Method	[16]	DSRA (R&C)	DSRA (C&R)
Open switches	S <sub>7</sub> , S <sub>9</sub> , S <sub>14</sub> , S <sub>32</sub> , S <sub>37</sub>	S <sub>7</sub> , S <sub>9</sub> , S <sub>14</sub> , S <sub>32</sub> , S <sub>37</sub>	S <sub>9</sub> , S <sub>14</sub> , S <sub>28</sub> , S <sub>33</sub> , S <sub>36</sub>
Capacitors allocation	29(1)	7(1), 23(1), 29(2)	6(1), 23(1), 29(2)
Total losses (MW h)	476.2	436.1	462.9
Losses cost (US\$)	238393.9	218069.5	231431.1
Capacitors cost (US\$)	5106.0	16200.0	16200.0
Total cost (US\$)	<b>243499.9</b>	<b>234269.5</b>	<b>247631.1</b>

Table 8  
Impact of load levels in reconfiguration through the DSRA.

Load levels	24, 12	6, 3
Open switches	S <sub>7</sub> , S <sub>9</sub> , S <sub>14</sub> , S <sub>32</sub> , S <sub>37</sub>	S <sub>7</sub> , S <sub>9</sub> , S <sub>14</sub> , S <sub>32</sub> , S <sub>28</sub>
Total losses (MW h)	641.3	644.0

Another simulation was carried out so as to evaluate the impact of varying the predefined size of the capacitors banks. In this case, banks of 450 kVar were considered. According to [16], the investment cost for this fixed capacitor size is 9 US\$/kVar. It is seen from Table 7 that the R&C strategy presented a solution associated with a larger investment in capacitors and with a smaller operation cost, i.e., smaller losses than the approach in [16]. The total cost including the investment and operation is smaller than the obtained in [16], showing the advantages of considering the predefined capacitor banks size as proposed by the R&C approach.

Some additional analyses were performed to examine the impact of the number of load levels. In these analyses, the daily curves in [16] were segmented into 12, 6 and 3 load levels for the reconfiguration process. It is seen from Table 8 that the final topology is affected. As expected, the solution obtained considering 24 and 12 load levels presents lower losses than the solution for 6 and 3 load levels.

### 3.3. 83-Bus system

The 83-bus 11.4-kV system, of the Taiwan Power Corporation (TPC) [31], consists of 11 feeders, two substations and 96 branches.

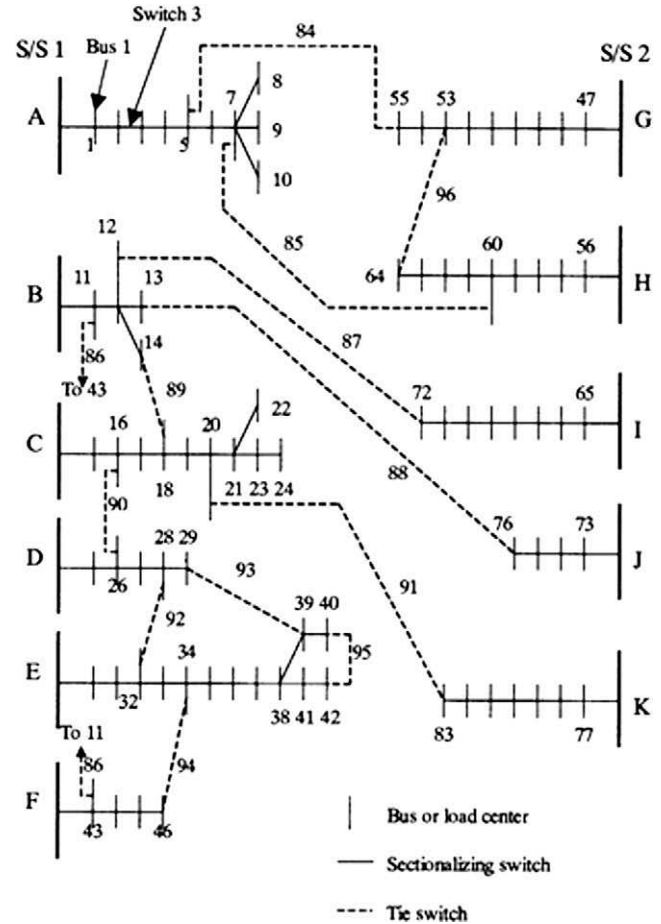


Fig. 5. 83-Bus system.

Fig. 5 shows the diagram of this system, which has a total load of 28,350 kW and 20,700 kVar. The branches represented by dotted lines correspond to the 13 NO switches as follows: S<sub>84</sub>, S<sub>85</sub>, S<sub>86</sub>, S<sub>87</sub>, S<sub>88</sub>, S<sub>89</sub>, S<sub>90</sub>, S<sub>91</sub>, S<sub>92</sub>, S<sub>93</sub>, S<sub>94</sub>, S<sub>95</sub>, S<sub>96</sub>. The total number of maneuverable switches is 89.

This analysis is based on two typical load curves which were obtained from a real Brazilian distribution system [32]. For the Group-1 of feeders (feeders A–F going out from substation S/S1), the load curve shown in the Fig. 6 is adopted, whereas the load curve of Fig. 7 is used for Group-2 (feeders G–K from S/S2).

The load curves were segmented into four load levels (N1–N4), as described in Table 9.

The time (h) is associated with a 1 year period of system operation and corresponds to the daily period of each load level according to Figs. 6 and 7. In this case, L3 corresponds to the peak load level; so its energy price is higher than those of other levels, according to the Brazilian hourly seasonal energy price scenario.

Table 10 presents the losses on this 83-bus system for: (i) the initial topology; (ii) the topology found in [10,31] considering only one load level; (iii) the topology proposed through the DSRA algorithm considering the four load levels simultaneously in the two feeders. The nodal voltage limits are 0.90 p.u. and 1.01 p.u.

It can be observed that this distribution system reconfiguration considering the four load levels (N1–N4) implies a loss reduction in the Group-1, and a loss increase in the Group-2. This happens because the reconfiguration leads to a load transfer from the Group-1 to the Group-2. However, the total loss in the two feeders is reduced.

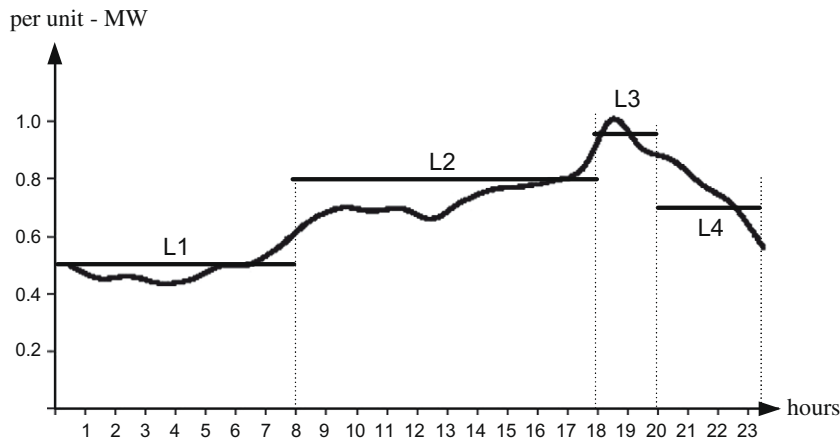


Fig. 6. Typical load curve of the Group-1.

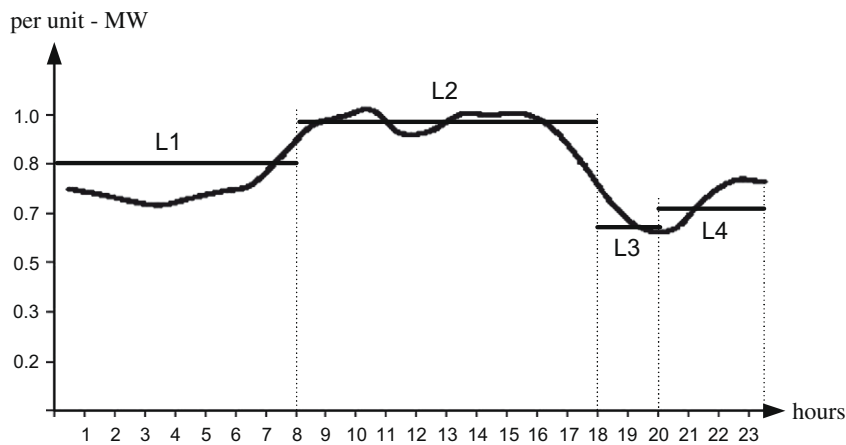


Fig. 7. Typical load curve of the Group-2.

It is important to stress that the optimal system configuration obtained for the four load periods of time is different from the configuration obtained by [10,31], which consider only one load level. The topology determined by the DSRA for the four load levels (N1–

N4) presents lower losses than the methods proposed in [10,31], in which only one load level is considered. This aspect demonstrates the effectiveness of the proposed reconfiguration index (3) for handling the different load periods in a coupled manner. From the results obtained it is seen that the representation of load curve levels must be considered for the EDS reconfiguration.

Table 9  
Load levels, 83-bus system analysis.

Load level	L1	L2	L3	L4
Load factor (p.u.) Group-1	0.50	0.80	0.95	0.70
Load factor (p.u.) Group-2	0.80	0.95	0.60	0.70
Time (h)	2920	3650	730	1460
Energy price (US\$/kW h)	0.06	0.06	0.108	0.06

The computational time was 1.43 min. The inclusion of the load levels in the analysis does not introduce a noticeable increase of processing time.

Table 11 presents the R&C and C&R solutions for this system, considering the loading conditions of Table 9. In this case, the cost associated with the investment in capacitors is 4 US\$/kVAr, each capacitor bank being 200 kVAr, and the maximum number of banks per busbar is 3. It is seen that the results obtained from

Table 10  
83-Bus system reconfiguration solutions.

Configuration	Initial	[10,31]	Proposed (DSRA)
Open switches	S <sub>84</sub> , S <sub>85</sub> , S <sub>86</sub> , S <sub>87</sub> , S <sub>88</sub> , S <sub>89</sub> , S <sub>90</sub> , S <sub>91</sub> , S <sub>92</sub> , S <sub>93</sub> , S <sub>94</sub> , S <sub>95</sub> , S <sub>96</sub>	S <sub>7</sub> , S <sub>13</sub> , S <sub>34</sub> , S <sub>39</sub> , S <sub>42</sub> , S <sub>55</sub> , S <sub>62</sub> , S <sub>72</sub> , S <sub>83</sub> , S <sub>86</sub> , S <sub>89</sub> , S <sub>90</sub> , S <sub>92</sub>	S <sub>7</sub> , S <sub>34</sub> , S <sub>39</sub> , S <sub>42</sub> , S <sub>55</sub> , S <sub>63</sub> , S <sub>72</sub> , S <sub>82</sub> , S <sub>86</sub> , S <sub>88</sub> , S <sub>89</sub> , S <sub>90</sub> , S <sub>92</sub>
Group-1 losses (MW h)	1364.7	983.0	1078.6
Group-2 losses (MW h)	1320.2	1441.5	1333.8
Total losses (MW h)	2684.9	2424.5	2412.4
Total cost (US\$)	173640.1	155773.8	<b>155279.8</b>

**Table 11**  
83-Bus system planning solutions.

Strategy	(R&C)	(C&R)
Open switches	S <sub>7</sub> , S <sub>34</sub> , S <sub>39</sub> , S <sub>42</sub> , S <sub>55</sub> , S <sub>63</sub> , S <sub>72</sub> , S <sub>82</sub> , S <sub>86</sub> , S <sub>88</sub> , S <sub>89</sub> , S <sub>90</sub> , S <sub>92</sub>	S <sub>7</sub> , S <sub>34</sub> , S <sub>39</sub> , S <sub>42</sub> , S <sub>55</sub> , S <sub>63</sub> , S <sub>72</sub> , S <sub>83</sub> , S <sub>86</sub> , S <sub>88</sub> , S <sub>89</sub> , S <sub>90</sub> , S <sub>92</sub>
Capacitors allocation	6(1), 19(2), 71(3), 79(3)	6(3), 19(3), 28(1), 31(3), 71(3), 79(3)
Group-1 losses (MW h)	1028.3	937.4
Group-2 losses (MW h)	1211.6	1229.2
Total losses (MW h)	2239.9	2166.6
Losses cost (US\$)	144346.5	139408.3
Capacitors cost (US\$)	7200.0	12800.0
Total cost (US\$)	<b>151546.5</b>	<b>152208.3</b>

the R&C approach is better than the C&R. The C&R strategy leads to a higher cost because in this procedure the capacitors are allocated without any previous network reconfiguration, requiring more reactive power support at the very beginning of the process.

It can also be observed that the R&C and C&R solutions, involving both reconfiguration and capacitor allocation (Table 11), present smaller total cost of investment (in reactive power) and operation (losses) than the solution of the DSRA involving only the reconfiguration (Table 10), showing the advantages of combining these two alternatives. In order words, the operation time considered in this case (1 year) is sufficient to recover the money investment in the capacitor banks.

#### 4. Conclusions

This work presented a methodology for optimal distribution network reconfiguration combined with optimal capacitor allocation. The main contribution of the proposed methodology is the utilization of a new sensitivity index for reconfiguration, aiming at the minimization of the total energy loss considering the daily load curve. This index allows the handling load levels in a coupled and effective manner. The association of this reconfiguration approach with the capacitor allocation technique also based on Lagrange multipliers was effective considering the results obtained.

Other points can be highlighted:

- The proposed reconfiguration algorithm presented low computational effort due to the reduced number of OPF simulations, allowing its application in the context of large scale distribution systems considering more than one load level.
- The proposed methodology explores the potential for application of Lagrange multipliers as sensitivity indexes for determining optimal configurations.
- The consideration of more than one load level in the reconfiguration and capacitor allocation planning process was explored and the results obtained are very effective.

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