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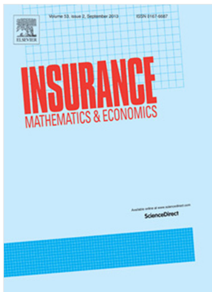

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Life Insurance Settlement and the Monopolistic Insurance Market

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Abstract: We analyze the effects of life insurance settlement on insurance contract design, the insurer's profit and welfare. Policyholders face not only mortality risks but also heterogeneous liquidity risks which lead the policyholders to surrender or settle the policies. It is assumed that the insurer cannot discriminate policyholders based on liquidity risks, and that no cost is incurred in surrender and settlement. We characterize the conditions for the endogenous existence of a settlement market, and find that the settlement market, if it exists, raises insurance premium. The effects of settlement on profit and welfare depend on the market structure. In the monopolistic insurance market, the settlement market lowers the insurer's profit, and consumer welfare increases whenever demand increases and possibly increases even when demand decreases. This finding is in contrast with most of the existing studies reporting that settlement never has a positive effect on welfare. In the competitive insurance market, welfare always decreases.

Keywords: life insurance, settlement, surrender, welfare
JEL codes: D420, G220

Life Insurance Settlement and the Monopolistic Insurance Market

1. Introduction

Life settlement is a transaction in which a policyholder sells her insurance policy to a third-party investor, known as a settlement provider. The life settlement market is a secondary market for life insurance. As a result of settlement transaction, the policyholder receives the settlement price while the investor becomes the beneficiary of the life insurance. Life settlements have been used by policyholders for securing cash. Without the settlement market, the policyholder may have to surrender the policy to receive cash payment, i.e., surrender value. In effect, the life settlement market provides the policyholder with an alternative to surrender. Given that the surrender value is generally lower than the actuarial value of the insurance contract, a policyholder may opt to settle the policy rather than surrender it.

Life settlement may also benefit financial investors since settlements can provide investors with investment opportunities that are not correlated with the existing portfolios. The inclusion of life settlements in portfolios lowers portfolio risks. According to Gatzert (2010), settlement transactions are allowed in Germany, the U.K., and the U.S. For example, terminally ill Acquired Immune Deficiency Syndrome (AIDS) patients are allowed to sell their policies in the settlement market in the U.S. According to the European Life Settlement Association (2013), 55 billion dollars of face value has been purchased in U.S. from 2001 to 2012. During the same period, 35 billion dollars of face value has been purchased by investors in Europe such as Luxembourg, Switzerland, Netherlands and Germany. Conning Research & Consulting (2008) reported that the size of the U.S. settlement market was 12 billion dollars in 2007. Conning Research & Consulting (2014) also forecast an average annual gross market potential for life settlements of 180 billion dollars from 2014 to 2023.

It is potentially possible that the existence of secondary markets enhances efficiency, for example, by correcting imbalance between demand and supply or stimulating new products in the primary markets (Bréchet and Picard, 2010; Chambers, Garriga and Carlos, 2009). However, these positive findings do not seem to apply to the insurance market. In insurance literature, settlement market is generally claimed to have a negative effect on efficiency as shown below.

Doherty and Singer (2003) argue that the settlement market may enhance consumer welfare since it can reduce the monopsony power of the insurer. They observe that without the settlement market, the policyholder would surrender the policy to the insurer, with the insurer playing a role similar to that of a monopsonistic firm. The introduction of a settlement market effectively increases competition among buyers, which lowers the monopsony rent of the insurer. However, it is notable that they do not analyze fully the interaction between the insurer and the settlement market in an equilibrium model. Therefore, it is not clear if their conclusion is applicable in the long run.

Hendel and Lizzeri (2003, HL hereafter) and Daily, Hendel and Lizzeri (2008, DHL hereafter) consider a dynamic contract in a competitive insurance market. The insurance contract is of one-sided commitment, which means that only the insurer's commitment is binding. They assume that the policyholder's income is growing and that the insurer can observe the policyholder's health risk (symmetric learning). DHL assume that the cash surrender value (CSV) is zero and insurance contract is "front-loaded" in which policyholders pay front-loaded premiums.[1] Front

[1] According to HL, front-loading is understood as the prepayment of some of the future premium.

loading allows the policyholder to avoid reclassification risk in the following period, where reclassification risk refers to the changes in insurance premium following mortality risk changes. When a policyholder surrenders her policy, the insurer may earn a surrender profit from the prepaid front-loaded premium. DHL argue that a settlement market lowers consumer welfare because it exposes the policyholder to the reclassification risk.

Fang and Kung (2010, FK hereafter) extend the discrete model of DHL into a continuous model. Unlike DHL, FK allow a positive CSV with a value dependent on the policyholder's health condition. The FK's findings are in line with those of DHL. FK show that the optimal CSV is equal to zero and that the settlement deteriorates consumer welfare because consumers lose the opportunity to hedge the reclassification risk.

Using a simulation based on actuarial assumptions, Gatzert, Hoermann and Schmeiser (2009, GHS hereafter) argue that the introduction of a settlement market may worsen the insurer's profit. Unlike the aforementioned studies, they consider the case in which mortality is heterogeneous and the surrender rate is affected by health status. Given that policyholders with bad health choose to settle their contracts while those with good health choose to surrender, high-risk policyholders remain in the insurer's pool. This result lowers the surrender profit and the overall profit of the insurer. The profit reduction increases premiums.

Zhu and Bauer (2013, ZB hereafter) are focused on the empirical finding that the realized return of settlement investment is much lower than the expected return. Similar to GHS, they consider the heterogeneity in mortality (health) risk. They employ a one-period expected utility model to calculate the fair settlement price in a competitive settlement market. They point out that adverse selection over health risk can be a reason for the discrepancy and propose a pricing formula for settlement.

On the other hand, let us note that welfare improvement is possible when consumers are irrational. For example, the behavioral approaches of Gottlieb and Smetters (2014) and Fang and Wu (2017) argue that settlement may improve consumer welfare when consumers exhibit overconfidence about the surrender possibility. Settlement provides an opportunity to correct the overconfidence in their approaches. Whereas some welfare implications are common, our analysis is based on the standard approach in which consumers are rational. As our concern is with rational agents, comparison will be confined within this standard approach.

Adding to this line of research, this paper aims to investigate the effects of the settlement market on the design of insurance contracts and consumer welfare. While we analyze both the monopolistic insurance market and the competitive market, our main focus will be on the monopolistic case. Studying the monopolistic market will provide an important intuition regarding the strategic reaction of the insurer to settlement, which has been ignored in the literature focusing on the competitive market. Considering the monopolistic market can also partially capture the reality that a few numbers of insurers exercise market powers, due to, for example, reputation and financial stability. Comparing the monopolistic case with the competitive case will allow us to understand better the effects of settlement on the insurance market.

We set up a two-period model to compare between the equilibrium outcomes with settlement and without settlement. Policyholders are assumed to face heterogeneous liquidity risks. Facing liquidity needs, policyholders should decide whether to surrender or settle their policies. This assumption reflects the fact that settlement market is often used by policyholders who need cash for medical treatment or urgent care. Policyholders are homogeneous except for liquidity risks. Our assumption allows us to focus on the pure interaction between liquidity risks and settlement.

We also assume that the settlement market, when it exists, is competitive as in existing studies.[2]

We consider the case in which the insurer is not allowed to offer a menu of contracts that discriminate among policyholders based on the liquidity needs as in DHL and ZB.[3] In fact, our approach can be considered a hybrid of the existing ones in the following sense. As of now, two distinguished approaches seem to exist in the analysis of settlement. In one ("economics") approach utilized in DHL and FK, death benefits are endogenously determined on the basis of mortality risks. Settlement affects insurance market through its effects on the interaction between the change of mortality risk and the surrender incentives. In comparison, death benefits are fixed in the other ("actuarial") approach utilized in GHS and ZB. Then the effect of settlement on the insurance market is analyzed directly from its effects on pricing and profits. On the other hand, surrender value is treated as given or a fixed function of death benefit in both approaches except for FK. Each approach has its own strength and weakness. While the former approach is built upon the rationality of economic agents, it seems to somewhat deviate from the reality in which life insurance contracts do not fully discriminate across policyholders' risks. This deviation may imply that its findings do not fully describe or apply to the reality. On the other hand, the latter approach seems to better reflects the contract design in practice. However, it tends to disregard the strategic aspects between economic agents. These observations lead us to combine the two approaches. We consider the case in which death benefit is fixed for a mortality risk[4] and contracts are non-discriminatory across liquidity risks, reflecting reality, whereas the strategic interactions between insurers and settlement are allowed, reflecting the rational reactions of economic agents. This approach will allow us to trace the strategic effects of settlement in a more realistic setting, while keeping the model simple to analyze.

The findings of the analysis for the monopolistic insurance market can be summarized as follows. First, we provide the conditions for the endogenous existence of a settlement market. Second, we find that cash surrender value may well be positive, unlike in DHL and FK. Third, we also find that the introduction of a settlement market lowers the insurer's profit, but increases insurance premium. The insurer's profit is reduced due to the competitive pressure from the settlement market. The insurer, however, increases premium in order to recover the loss. Fourth, consumer welfare may increase or decrease, which depends on the distribution of liquidity risks and the utility shapes of policyholders.

In addition, we provide the analyses for the competitive insurance market and the monopolistic settlement market for comparison. Settlement increases insurance premium and lowers consumer welfare when the insurance market is competitive, while it does not affect the main results when the settlement market is monopolistic.

The novelty of this study can be noted as follows. First, the existence of a settlement market is endogenously determined. Existing studies virtually presume that the existence of a settlement market is endogenously given, when it is allowed. However, a settlement market does not have to exist even if it is allowed, when the insurer(s) strategically sets a higher surrender value than the

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[2] However, we also provide the analysis for the case of the monopolistic settlement market for comparison.

[3] In reality, menus with a limited number of contracts are offered. Nevertheless, the assumption is acceptable as long as the number of contracts is too small to fully discriminate over liquidity risks, which is the case because liquidity risk is continuous.

[4] This assumption implies that the death benefit is the same for all policyholders as they face a homogeneous mortality risk.

settlement price. The existing studies tend to ignore the strategic interaction between insurers and the settlement market.[5] Note that the strategic reaction can be better understood in the monopoly setting. For this reason, we are more focused on the monopoly case, although we consider both monopolistic and competitive markets.

Second, the cash surrender value is endogenously determined and may well be positive. In contrast, DHL and FK assume or claim that cash surrender value is zero. In GHS and ZB, surrender value is assumed to be a given function of death benefit. Our finding suggests that the positive surrender value in the real world may result from the strategic decision of the insurer.

Third, our finding points to the possibility of welfare improvement under the monopolistic insurance market. Welfare can be improved if the contract change attracts a sufficient number of potential policyholders. This finding stands in sharp contrast with standard economics studies claiming that settlement market never improves welfare. It is also important to note that welfare improvement in this paper depends not only on monopoly but also on the non-discriminatory feature of insurance contracts.[6]

Fourth, this paper focuses on the liquidity risks of policyholders as a rationale for surrender/settlement. In contrast, the standard studies (DHL and FK) consider a loss of bequest motive a rationale for surrender. Liquidity risk seems to make an important rationale for settlement, as it better reflects the history of settlement.[7]

The remainder of the paper proceeds as follows. Section II describes the model for the monopolistic insurer. Section III examines the range of surrender value. Section IV discusses the existence of the settlement market. Section V investigates the equilibrium insurance contract when settlement is not allowed. Section VI studies the equilibrium insurance contract when settlement is allowed. Section VII studies the effects of the settlement market on the insurance contract and on welfare. Section VIII provides an illustrative example, and Section IX compares our results with those of the competitive insurance market and the monopolistic settlement market. Section X concludes.

2. Monopolistic insurance market

2.1. Model description

We consider a monopolistic insurance market in a two-period model. Time is denoted by $t = 0$, $t = 1$, and $t = 2$. The discount factor is denoted by ρ. A potential policyholder purchases life insurance in $t = 0$, and the death event occurs with probability p_1 in $t = 2$. Insurance premium is denoted by Q, and the death benefit of insurance is given as D. The premium is

[5] A notable exception is Doherty and Singer (2003) which conjecture ADB (Accelerated Death Benefits) as a competitive reaction to the emergence of life settlement. However, it is not clear whether settlement exists as an equilibrium outcome because the strategic interactions are not fully considered in their approach. In contrast, settlement is determined endogenously as an equilibrium outcome in our model.

[6] The possibility that prohibiting discrimination can enhance welfare is also noted in literature. For example, Polborn, et al. (2006) argue that prohibiting insurers from discriminating policyholders regarding genetic information can enhance welfare.

[7] Let us note that Gottlieb and Smetters (2014) also consider liquidity risk in the behavioral approach.

decomposed into the pure premium and the loading premium denoted by R. We assume that policyholders are homogeneous except for their liquidity risks.[8] Liquidity risk is measured by the probability that the policyholder urgently needs cash, which is denoted by q, distributed on [0, 1]. It is assumed that the event of needing cash occurs immediately prior to the (possible) death event at $t = 1$. When the policyholder needs cash, the policyholder has to surrender the policy to the insurer and receive surrender value S if there is no settlement market. We assume that $S \geq 0$, i.e., the negative surrender value is not feasible. Note that the policyholder can choose between surrender and settlement when a settlement market exists.

We assume that the settlement market, if it exists, is perfect and competitive. For technical simplicity, the settlement price is assumed to be equal to the actuarially fair price of death, , which implies a zero risk premium. This assumption is satisfied if investors are risk neutral. Even if investors are risk averse, the assumption can be satisfied when the CAPM holds, because mortality can be considered uncorrelated with the capital market return.[9]

The population of potential policyholders is distributed over the liquidity risk. The population density function and the cumulative density function of q are denoted by $f(q)$ and $F(q)$,

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8 The purpose of the assumption is to simplify analysis, which allows us to focus on the pure effect of settlement. Once we understand the effect of settlement given homogeneous consumers except for liquidity risks, the results can be extended to more heterogeneous cases. For example, when consumers exhibit different mortality risks, our approach can be applied to each mortality risk group. However, caution is needed when consumers are heterogeneous in unobservable characteristics.

9 As pointed by a referee, the zero risk premium assumption is seldom observed in reality. We appreciate the referee for raising this point. Insurance literature reports that the risk premiums for CAT risk and longevity risk are large (Mitchell et al., 1999; Froot, 2001; Bauer, Phillips, and Zanjani, 2013). It is possible to incorporate the positive risk premium (RP) into the analysis by replacing settlement price $\rho p_1 D$ by $\rho p_1 D - RP$. Applying this replacement in section 2, we note that the modification does not alter the results of subsections 2.2 and 2.4, in which settlement is not allowed. The policyholder's surrender behavior is still based on the relative values of S^I and $\rho p_1 D$. When settlement is allowed in other subsections, the modification may lead to some technical changes as given below. These changes, however, do not qualitatively alter the main implications of the analysis. Some examples of technical changes include the following cases among others, related with Propositions 2 and 5 and consumer welfare (CW).

$$R_s * f(q_s*) = \rho[u(W_1 - y + \rho p_1 D - RP) - \rho p_1 v(D) - u(W_1 - y)] \frac{[1 - F(q_s*)]}{u'(W_0 - Q_s*)} \quad (32)$$

$$\rho q_s *[u(W_1 - y + \rho p_1 D - RP) - \rho p_1 v(D) - u(W_1 - y)] = u(W_0) - \rho^2 p_1 v(D) - u(W_0 - Q_s*) \quad (33)$$

$$Q_s* = \rho^2 p_1 D + R_s * \quad (34)$$

$$CW_s = \rho[\int_{q_s*}^1 qf(q)dq - q_s *\{1 - F(q_s*)\}][u(W_1 - y + \rho p_1 D - RP) - \rho p_1 v(D) - u(W_1 - y)] \quad (35)$$

$$\frac{[\int_{q_s*}^1 qf(q)dq - q_s *\{1 - F(q_s*)\}]}{[\int_{q*}^1 qf(q)dq - q\ *\{1 - F(q\ *)\}]} > \frac{[u(W_1 - y + S*) - \rho p_1 v(D) - u(W_1 - y)]}{[u(W_1 - y + \rho p_1 D - RP) - \rho p_1 v(D) - u(W_1 - y)]} \quad (37)$$

respectively. Policyholders will be identified with their liquidity risks throughout this paper.[10] The monopolistic insurer is not supposed to offer contracts which discriminate over liquidity needs because, for example, liquidity needs are not insurance risks.[11] The time line of the model is depicted in Figure 1.

Figure 1 around here

Let us denote the endowment income of policyholders as W_0 at t = 0. At t = 1, the policyholder is exposed to a liquidity risk: the loss of y with probability q, where y indicates an income loss after a liquidity shock. As a result, the policyholder's income becomes $W_1 - y$ with probability q or W_1 with probability $1-q$. The income at t = 2 is denoted as W_2.

The policyholder's utility is composed of two parts following DHL and FK. If the policyholder is alive and her consumption is W, the utility is denoted as $u(W)$. The policyholder's own utility becomes zero if she dies. However, the policyholder is considerate of her dependent. If she dies and her dependent inherits W, the utility becomes $v(W)$, where $v(.)$ denotes the utility incorporated for the dependent, reflecting a bequest motive. Utility functions are strictly concave and twice differentiable: $u'(W) > 0, u''(W) < 0$ and $v'(W) > 0, v''(W) < 0$. For simplicity, we assume that $v(0) = 0$.

Suppose that there is no settlement market. Let us first consider the benchmark case in which the insurer can discriminate among policyholders based on the liquidity needs.

The policyholder's expected utility with no insurance can be written as[12]

$$Eu_0 = u(W_0) + \rho q_i u(W_1 - y) + \rho(1-q_i)u(W_1) + \rho^2(1-p_1)u(W_2) \quad (1)$$

To pinpoint the pure effects on the insurance market, we suppose that the policyholder has no access to a capital market.[13] When the policyholder with insurance faces liquidity needs, she has to choose between surrendering and keeping insurance. If she chooses to surrender her policy, her utility becomes $u(W_1 - y + S_i)$. On the other hand, if she chooses to keep the policy, her utility becomes $u(W_1 - y) + \rho p_1 v(D)$. The surrender decision will be made based on the relative

[10] It is also assumed that separate insurance for liquidity risks is not available, because, for example, liquidity needs are not verifiable thus uninsurable.

[11] The assumption of non-discriminatory contracts can be considered a simplified approach for the case in which the number of discriminatory contracts is small compared with that of liquidity risk types where the menu of discriminatory contracts are offered. We also conjecture that the major implications of our analysis are not affected even if the Rothschild-Stiglitz discrimination is allowed. That is because the demand change by low liquidity risks is a main factor for our results, which does not seem to be qualitatively altered under the Rothschild-Stiglitz discrimination model. The difference will be the magnitude of demand change: the demand change is from zero to one in our model, while it is from low partial to higher in the Rothschild-Stiglitz discrimination model.

[12] The term "policyholder" will be used even if she does not purchase insurance.

[13] Even if a capital market exists, policyholders may not fully save for the future cash needs. As long as it is the case, policyholders still face the same decision problem as presented in the paper. Our approach simplifies analysis without sacrificing major implications.

sizes of the utilities. Throughout the paper, we assume that $u(W_1+S)<u(W_1)+\rho p_1 v(D)$ to rule out the case in which policyholders choose to surrender even if they face no liquidity needs. Note that this condition will hold when W_1 is sufficiently large, compared with S and D.

Thus, the expected utility of a policyholder with liquidity risk q_i with insurance paying premium Q_i is determined as follows:

$$Eu_1 = u(W_0-Q_i)+\rho q_i \max\{u(W_1-y+S_i), u(W_1-y)+\rho p_1 v(D)\}+\rho(1-q_i)u(W_1) \\ +\rho^2(1-p_1)u(W_2)+\rho^2(1-q_i)p_1 v(D) \qquad (2)$$

The difference between the two expected utilities with and without insurance is called the net benefit of policyholder q_i from insurance, denoted by $NB(q_i)$. $NB(q_i)$ can be expressed as follows:

$$NB(q_i) = Eu_1 - Eu_0 \\ = u(W_0-Q_i)+\rho q_i \max\{u(W_1-y+S_i), u(W_1-y)+\rho p_1 v(D)\}+\rho^2(1-q_i)p_1 v(D) \\ -u(W_0)-\rho q_i u(W_1-y) \qquad (3)$$

As full discrimination is possible, the monopolistic insurer will offer a contract to each policyholder to make her net benefit zero. That is, $NB(q_i)=0$ for all q_i.

Now, let us turn to the main case in which discrimination is not allowed. The monopolistic insurer has to offer the same premium and surrender value to all policyholders. Given the insurance contract, policyholders with different liquidity risks may have different preferences. Consequently, only those who enjoy non-negative net benefits will purchase insurance. Let us refer to the marginal policyholder (liquidity risk) with a zero net benefit as a *target policyholder* (*liquidity risk*), denoted by q^T. Let us denote the contract by (Q,S,q^T).

Technically, the target policyholder's net benefit is as follows:

$$NB(q^T) = u(W_0-Q)+\rho q^T \max\{u(W_1-y+S), u(W_1-y)+\rho p_1 v(D)\}+\rho^2(1-q^T)p_1 v(D) \\ -[u(W_0)+\rho q^T u(W_1-y)]=0 \qquad (4)$$

Then, the net benefit of a policyholder with liquidity risk q_i is expressed as:

$$NB(q_i) = u(W_0-Q)+\rho q_i \max\{u(W_1-y+S), u(W_1-y)+\rho p_1 v(D)\}+\rho^2(1-q_i)p_1 v(D) \\ -[u(W_0)+\rho q_i u(W_1-y)] \qquad (5)$$

The policyholder will purchase insurance if and only if her net benefit is non-negative. Now, let us define the *indifference surrender value* S^I as the surrender value that makes the policyholder indifferent between surrendering and keeping insurance:

$$u(W_1 - y + S^I) = \rho p_1 v(D) + u(W_1 - y)$$

The policyholder will keep insurance if $S < S^I$ and surrender if $S > S^I$. The following lemma describes the insurance demand given surrender value in each case. With $S = S^I$, policyholders are indifferent between surrendering and keeping insurance. However, the insurer's profit is significantly different depending on the policyholder's decision. With this concern, we will postpone the analysis of this case until the next section.

Lemma 1. Suppose that the surrender value is given by S.

(1) If $S < S^I$, then all potential policyholders purchase insurance.
(2) If $S > S^I$, then the potential policyholders with higher liquidity risks than the target liquidity risk purchase insurance.

Proof. See the Appendix.//

If $S < S^I$, then no policyholder will surrender. In this case, the net benefits of all policyholders become zero. Therefore, all potential policyholders purchase insurance. If $S > S^I$, a policyholder facing liquidity needs will choose surrendering over keeping insurance. This also implies that the net benefit increases in liquidity risk. Thus, policyholders with liquidity risks higher than the target will purchase insurance.

Because the demand function is significantly affected by the relative sizes between the surrender value and the indifference surrender value, our analysis will consider those cases separately. Note that the insurer will eventually choose the optimal surrender value by comparing among the profits under $S < S^I$, $S > S^I$, and $S = S^I$.

The insurer designs insurance contracts to maximize the (expected) profit, denoted by π. The insurer's profit is the difference between the premium earned and the expected payout. Finally, let us define consumer welfare (CW) as the sum of the net benefit of all consumers.[14][15]

Our next task is to identify the relevant range in which the optimal surrender value is located.

2.2. The optimal range of surrender value when settlement is not allowed

As a first step, let us narrow down the relevant range in which the optimal surrender value exists. As shown in subsection 2.2.1. (iii), it turns out that the relative sizes between S^I and $\rho p_1 D$ are important.

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[14] We adopt the so-called utilitarian approach which defines consumer welfare as the sum of consumer utilities. The approach is often used in economics studies, including Harsanyi (1955), Sen (1970), Sheshinski (1972), and Apps and Rees (1988). However, note that it is debatable as utilities are not addable in principle. Therefore, our welfare effect should be interpreted with caution and understood as a possibility.

[15] It is notable that we assume that the welfare of investors is zero given the competitive settlement market.

2.2.1. The case of $S^I \geq \rho p_1 D$

(i) $S < S^I$: In this case, no policyholder would surrender her policy. This implies that the surrender value is not relevant as long as it is less than S^I. Now, the premium can be expressed as follows:

$$Q = \rho^2 p_1 D + R \qquad (6)$$

The net benefit of policyholder i becomes

$$NB(q_i) = u(W_0 - Q) + \rho^2 p_1 v(D) - u(W_0) = 0 \qquad (7)$$

This result implies that all potential policyholders purchase insurance, confirming lemma 1 (1). While this case corresponds to no specific target risk, let us treat this case as if the target risk is zero ($q^T = 0$) and the policyholders with liquidity risks higher than the target will purchase insurance. This treatment will be consistent with the case of $S > S^I$ considered later.

Given the observation above, the insurer's profit can be written as $\pi = Q - \rho^2 p_1 D = R$. Now, the insurer's problem conditional on $S < S^I$ can be stated as follows.

$$\underset{R}{Max} \quad \pi = \pi(R \mid S < S^I) = R \qquad (8a)$$

$$s.t. \quad u(W_0 - \rho^2 p_1 D - R) + \rho^2 p_1 v(D) - u(W_0) = 0 \qquad (8b)$$

Given W_0, p_1, and D, the target loading premium will be determined at the point at which the net benefit of all policyholders is zero. The following lemma summarizes the above observation.

Lemma 2. Suppose that settlement is not allowed. Given $S < S^I$, the following results hold at the optimum.

(1) The profit and the policyholder's utility are invariant to the surrender value.
(2) The profit is equal to the target loading premium, which is determined as follows:
$$\pi = R = W_0 - \rho^2 p_1 D - u^{-1}(u(W_0) - \rho^2 p_1 v(D)) \qquad (9)$$
(3) The insurance premium is:
$$Q = W_0 - u^{-1}(u(W_0) - \rho^2 p_1 v(D)) \qquad (10)$$

Proof. Expressions (9) and (10) are obtained by transforming expressions (8b) and (7) respectively, using the inverse function of utility. //

In this case, CW is zero since the net benefit of all policyholders is zero.

(ii) $S > S^I$: In this case, lemma 1 indicates that demand is determined so that the policyholders with liquidity risks higher than the target will purchase insurance. The premium can be written as

$$Q = \rho q^T S + \rho^2 (1-q^T) p_1 D + R \quad (11)$$[16]

By definition of the target risk, we have

$$NB(q^T) = u(W_0 - Q) + \rho q^T u(W_1 - y + S) + \rho^2 (1-q^T) p_1 v(D) - u(W_0) - \rho q^T u(W_1 - y) = 0 \quad (12)$$

The insurer's profit becomes

$$\pi = \pi(Q, S, q^T, R \mid S > S^I) = Q[1 - F(q^T)] - \rho S \int_{q^T}^1 q f(q) dq - \rho^2 p_1 D \int_{q^T}^1 (1-q) f(q) dq \quad (13)$$

By plugging (11) into (13), we have the following problem:

$$\underset{Q,S,q^T,R}{Max} \quad \pi(Q, S, q^T, R \mid S > S^I)$$

$$= [\rho S - \rho^2 p_1 D][q^T \{1 - F(q^T)\} - \int_{q^T}^1 q f(q) dq] + R\{1 - F(q^T)\} \quad (14a)$$

$$s.t. \quad u(W_0 - Q) + \rho q^T u(W_1 - y + S) + \rho^2 (1-q^T) p_1 v(D) - [u(W_0) + \rho q^T u(W_1 - y)] = 0 \quad (14b)$$

$$Q = \rho q^T S + \rho^2 (1-q^T) p_1 D + R \quad (14c)$$

(iii) $S = S^I$: In this case, policyholders are indifferent between surrendering and keeping insurance. However, the insurer's profit is significantly affected by the decision of the policyholders as shown below. As a tie-break rule, we will assume that the policyholders behave in accordance with the insurer's interest. This assumption is justified by the observation that the insurer can slightly change the surrender value if the policyholders behave against its interest (see below for details).

If policyholders are assumed to keep insurance, case (i) is applied and the profit is R as in (10). On the other hand, if policyholders are assumed to surrender, case (ii) is applied with zero target risk and the profit, from (14a), becomes

$$\pi = [\rho S - \rho^2 p_1 D][-\int_0^1 q f(q) dq] + R \quad (15)$$

This expression is less than R if and only if the first term is negative. This occurs when $S^I \geq \rho p_1 D$, which is the case. In this case, the policyholders are assumed to keep insurance. For the justification of this assumption, observe that if the policyholders are going to surrender, the insurer can induce them to keep insurance by slightly lowering the surrender value to $S^I - \varepsilon$

[16] R is interpreted as the margin for target policyholders. In general, the margin may differ for different policyholders due to different liquidity risks.

for arbitrarily small $\varepsilon > 0$. In this case, $S = S^I$ is considered to be a limit case when ε goes to zero. By a similar reasoning, if $S^I < \rho p_1 D$, the policyholders are assumed to surrender when $S = S^I$, which will be used in section 2.2.2.

Let us denote Q^*, S^*, q^*, and R^* for the (globally) optimal values of Q, S, q^T, and R. The above observation indicates that the location of S* is affected by the relative sizes of profits from (i), (ii), and (iii), evaluated at the optimum. The resulting profit is determined by

$$\pi = \max \left\{ \begin{matrix} W_0 - \rho^2 p_1 D - u^{-1}(u(W_0) - \rho^2 p_1 v(D)), \\ [\rho S^* - \rho^2 p_1 D][q^* \{1 - F(q^*)\} - \int_{q^*}^1 qf(q)dq] + R\{1 - F(q^*)\} \end{matrix} \right\} \quad (16)$$

As a result, we have the following:

$S^* \le S^I$ with $\pi = W_0 - \rho^2 p_1 D - u^{-1}(u(W_0) - \rho^2 p_1 v(D))$ or,

$S^* > S^I$ with $\pi = [\rho S^* - \rho^2 p_1 D][q^* \{1 - F(q^*)\} - \int_{q^*}^1 qf(q)dq] + R\{1 - F(q^*)\}$

2.2.2. The case of $S^I < \rho p_1 D$

When $S < S^I$, all potential policyholders purchase insurance by lemma 1, and the profit becomes R. When S is equal to or slightly greater than S^I, policyholders will surrender by lemma 1, and the relevant profit is close to $[\rho S - \rho^2 p_1 D][-\int_0^1 qf(q)dq] + R$, given by (15). This profit is greater than R when $S < \rho p_1 D$. This implies that the maximum profit among $S \ge S^I$ is greater than that with $S < S^I$. As a result, we can rule out the case of $S < S^I$, and the relevant profit is expressed as

$[\rho S^* - \rho^2 p_1 D][q^T \{1 - F(q^T)\} - \int_{q^T}^1 qf(q)dq] + R\{1 - F(q^T)\}$, which is given by (14a).

The next lemma summarizes the discussion above.

Lemma 3. Suppose that settlement is not allowed. The relevant range in which the optimal surrender value S^* exists is given as follows.

(1) If $S^I \ge \rho p_1 D$, then both $S^* \le S^I$ and $S^* > S^I$ are potentially possible.

(a) When $S^* \le S^I$, the insurer sells insurance to all policyholders and policyholders keep insurance when they face liquidity needs. The profit is determined as $\pi = W_0 - \rho^2 p_1 D - u^{-1}(u(W_0) - \rho^2 p_1 v(D))$.

(b) When $S^* > S^I$, the insurer sells insurance to a portion of policyholders and policyholders surrender when they face liquidity needs. The profit is determined as $\pi = [\rho S^* - \rho^2 p_1 D][q^* \{1 - F(q^*)\} - \int_{q^*}^1 qf(q)dq] + R\{1 - F(q^*)\}$.

(2) If $S^I < \rho p_1 D$, then $S^* \ge S^I$. In this case, the insurer sells insurance to a portion of policyholders, and policyholders surrender when they face liquidity needs. The profit is

determined as $\pi = [\rho S^* - \rho^2 p_1 D][q^*\{1 - F(q^*)\} - \int_{q^*}^{1} qf(q)dq] + R\{1 - F(q^*)\}$.

Proof. See the text above. //

The findings of this section are also summarized in the second and third rows of Table 1. While lemma 3 shows the possible location of the optimal surrender value S^*, it does not show other values, such as Q^*, q^*, and R^*. A detailed analysis will be provided in section 2.4. Let us first show that the settlement market does not exist even if settlement is allowed, when $S^I \geq \rho p_1 D$.

Table 1 around here.

2.3. Non-existence of a settlement market under $S^I \geq \rho p_1 D$

When $S^I > \rho p_1 D$, the settlement market cannot exist because policyholders will strictly prefer keeping insurance or surrender to settlement by definition of S^I. When $S^I = \rho p_1 D$, policyholders are indifferent between keeping insurance and settlement, which also implies that the insurer's profit is not altered. As a result, settlement has no effect on the insurance market. For expository convenience, we treat this case as if the policyholders choose to keep insurance so that the settlement does not exist. This observation is stated in the following lemma.

Lemma 4. Suppose that $S^I \geq \rho p_1 D$. The settlement market does not exist, even if settlement is allowed.

Proof. See the text above. //

The result of lemma 4 is also summarized in the fourth row of Table 1. Lemma 4 implies that settlement may have an impact on the insurance market only if $S^I < \rho p_1 D$. In what follows, we will compare between the equilibrium outcomes when settlement is not allowed and when it is allowed, in the case of $S^I < \rho p_1 D$.

2.4. Equilibrium under $S^I < \rho p_1 D$ when settlement is not allowed

From lemma 3, we know that $S^* \geq S^I$ given that $S^I < \rho p_1 D$. Thus, the relevant insurer's problem is (14). The Lagrangian to problem (14) is written as follows:

$$
\begin{aligned}
L = {} & [\rho S - \rho^2 p_1 D][q^T\{1 - F(q^T)\} - \int_{q^T}^{1} qf(q)dq] + R\{1 - F(q^T)\} \\
& + \lambda[u(W_0 - Q) + \rho q^T u(W_1 - y + S) + \rho^2 (1 - q^T) p_1 v(D) - u(W_0) - \rho q^T u(W_1 - y)] \\
& + \mu[Q - \rho q^T S - \rho^2 (1 - q^T) p_1 D - R]
\end{aligned}
$$

The first order conditions (FOC) are as follows:

$$L_Q = -\lambda u'(W - Q) + \mu = 0 \qquad (17)$$

$$L_S = \rho[q^T \left\{1 - F(q^T)\right\} - \int_{q^T}^{1} qf(q)dq] + \lambda\rho q^T u'(W_1 - y + S) - \mu\rho q^T = 0 \quad (18)$$

$$L_{q^T} = [\rho S - \rho^2 p_1 D]\left\{1 - F(q^T)\right\} - Rf(q^T)$$
$$+\lambda\rho[u(W_1 - y + S) - \rho p_1 v(D) - u(W_1 - y)] - \mu[\rho S - \rho^2 p_1 D] = 0 \qquad (19)$$

$$L_R = \left\{1 - F(q^T)\right\} - \mu = 0 \qquad (20)$$

$$L_\lambda = u(W_0 - Q) + \rho q^T u(W_1 - y + S) + \rho^2 (1 - q^T) p_1 v(D) - u(W_0) - \rho q^T u(W_1 - y) = 0 \qquad (21)$$

$$L_\mu = Q - \rho q^T S - \rho^2 (1 - q^T) p_1 D - R = 0 \qquad (22)$$

From the FOC, we have the next proposition, which characterizes the solution.

Proposition 1. Suppose that $S^I < \rho p_1 D$ and settlement is not allowed. At the optimum, Q^*, S^*, q^*, and R^* satisfy the following conditions.

$$\int_{q^*}^{1} qf(q)dq = q^* u'(W_1 - y + S^*) \frac{\left\{1 - F(q^*)\right\}}{u'(W_0 - Q^*)} \qquad (23)$$

$$R^* f(q^*) = \rho[u(W_1 - y + S^*) - \rho p_1 v(D) - u(W_1 - y)] \frac{\left\{1 - F(q^*)\right\}}{u'(W_0 - Q^*)} \qquad (24)$$

$$\rho q^*[u(W_1 - y + S^*) - \rho p_1 v(D) - u(W_1 - y)] = u(W_0) - u(W_0 - Q^*) - \rho^2 p_1 v(D) \qquad (25)$$

$$Q^* = \rho q^* S^* + \rho^2 (1 - q^*) p_1 D + R^* \qquad (26)$$

From expressions (17) and (20), we have $\lambda = \frac{1 - F(q^*)}{u'(W_0 - Q^*)}$. Note that λ is the shadow price measuring the value to the insurer of the one-unit increase of the net benefit of the policyholder, expressed in terms of the target policyholder. The expression states that the shadow price is positive, thus, the increase in the net benefit is still in the interest of the insurer at the optimum. The numerator of the right hand side (RHS) of the expression $(1 - F(q^*))$ is the marginal revenue when the insurer increases the premium by one dollar, which is also the shadow price of the premium increase (μ). The denominator measures the marginal decrease of utility ($u'(W_0 - Q^*)$) following the one-dollar increase in premium. Thus, the RHS measures the tradeoff between the marginal revenue and the marginal utility, more specifically, the unit revenue increase per marginal utility decrease following a premium increase. The expression indicates that the value loss through the utility decrease from the premium increase ($\lambda u'(W_0 - Q^*)$) is equal to the value increase through the revenue increase $(1 - F(q^*))$ at the optimum.

Equation (23) characterizes the optimal surrender value. The RHS measures the profit increase following the increase of S through the increase of the policyholder's utility. The increase of S, however, also incurs the additional expected surrender payment cost, which is measured by the left hand side (LHS). At the optimum, the marginal cost is equal to the marginal revenue.

Equation (24) characterizes the optimal target risk, which has an interpretation similar to that of (23). If the insurer increases the target risk, the demand decreases thus, the insurer's profit decreases as much as the lost margin for the target population, which is measured by the LHS. The RHS measures the profit increase through the increase of the policyholder's utility. Again, the marginal cost is equal to the marginal revenue. Equations (25) and (26) are the constraints.

Given Q^*, S^*, q^*, and R^*, CW can be expressed as

$$CW = u(W_0 - Q^*)\{1 - F(q^*)\} + \rho u(W_1 - y + S^*)\int_{q^*}^{1} qf(q)dq + \rho^2 p_1 v(D)\int_{q^*}^{1}(1-q)f(q)dq$$
$$-u(W_0)\{1 - F(q^*)\} + \rho u(W_1 - y)\int_{q^*}^{1} qf(q)dq \quad (27)$$

Using (23), we can rewrite this expression as

$$CW = \rho\{1 - F(q^*)\}[\frac{\int_{q^*}^{1} qf(q)dq}{\{1 - F(q^*)\}} - q^*][u(W_1 - y + S^*) - \rho p_1 v(D) - u(W_1 - y)] \quad (28)$$

Consumer welfare is positive due to the positive net benefit from the surrender. The term $\frac{\int_{q^*}^{1} qf(q)dq}{\{1 - F(q^*)\}}$ in (28) is the average liquidity risk of policyholders who purchase insurance. However, the insurer's contract offer is based on the target risk, q^*, which implies that the insurer acts as if all policyholders have the target risk. This difference leads to the insurer's failure to extract full rents from policyholders, which in turn leads to the positive consumer welfare.

2.5. Equilibrium under $S^I < \rho p_1 D$ when settlement is allowed

2.5.1. Surrender value

Now, let us consider the case in which settlement is allowed. For notational clarity, we add subscript *s* to indicate the existence of the settlement market. Note that S^* can be less or greater than $\rho p_1 D$, the settlement price. When $S^* > \rho p_1 D$, the settlement market cannot exist because the surrender value is higher than the settlement price, so no policyholder will choose settlement. In this case, the result is not affected by the settlement possibility because the insurer has no incentive to change the contract. In contrast, when $S^* \le \rho p_1 D$, policyholders will prefer settling policies to surrendering if the insurer does not adjust the surrender value. This implies that the settlement market may exist. However, the existence is not clear yet because the insurer may want to change its contract terms to deter the formation of the settlement market. What is relevant is if S_s* (not $S*$) is greater than $\rho p_1 D$. The next lemma, however, shows that $S_s* \le \rho p_1 D$ whenever $S* \le \rho p_1 D$, implying that the settlement market exists when $S* \le \rho p_1 D$.

Lemma 5. Suppose that $S^I < \rho p_1 D$ and settlement is allowed. The settlement market exists ($S_s* \le \rho p_1 D$) if and only if $S* \le \rho p_1 D$ or the following condition holds:

$$\int_{q*}^{1} qf(q)dq \ge q*u'(W_1 - y + \rho p_1 D)\frac{\{1-F(q*)\}}{u'(W_0 - Q*)} \qquad (29)$$
$$where \quad Q* = \rho^2 p_1 D + R*$$

Proof. See the Appendix.//

Given that settlement is allowed, condition (29) implies that the revenue increase from the marginal increase of S (RHS) is less than the additional surrender payment (LHS) evaluated at $S = \rho p_1 D$. Thus, the optimal surrender value should be less than or equal to the settlement price ($S* \le \rho p_1 D$). To see why $S_s* \le \rho p_1 D$ is equivalent to $S* \le \rho p_1 D$, suppose that $S* \le \rho p_1 D$ and settlement is not allowed. As $S*$ is the optimal value, the profit should decrease as S increases beyond $S*$. Specifically, the profit is higher with $S = \rho p_1 D$ than with $S > \rho p_1 D$. Now, note that the existence of the settlement market corresponds to the case in which the insurer sets the surrender value at $\rho p_1 D$, as the profit and the net benefits of policyholders are the same. The insurer needs to set the surrender value above $\rho p_1 D$ if it wants to deter the formation of the settlement market. As the profit is higher under the former case, the insurer is willing to allow the existence of the settlement market. Therefore, $S_s* \le \rho p_1 D$ as long as $S* \le \rho p_1 D$. This result is also summarized in the fourth row of Table 1.

As far as profit and consumer utility are concerned, the existence of the settlement market is equivalent to the case in which the surrender value is $\rho p_1 D$. Thus, we will technically treat it as if the surrender value is $\rho p_1 D$ in what follows. Now, let us further narrow down our focus to the case in which the settlement market exists or (29) holds.

2.5.2. Target liquidity risk, target loading premium, and welfare

The net benefit of the target risk, q_s^T is zero by definition.

$$NB(q_s^T) = u(W_0 - Q_s) + \rho q_s^T u(W_1 - y + \rho p_1 D) + \rho^2 (1 - q_s^T) p_1 v(D) - u(W_0) - \rho q_s^T u(W_1 - y) = 0 \quad (30)$$

Then, the profit of the insurer is $\pi_s = Q_s[1 - F(q_s^T)] - \rho^2 p_1 D[1 - F(q_s^T)] = R_s[1 - F(q_s^T)]$. The insurer's profit maximization problem becomes

$$\underset{Q_s, R_s, q_s^T}{Max} \quad \pi_s(Q_s, R_s, q_s^T) = R_s[1 - F(q_s^T)] \qquad (31a)$$

$$s.t. \quad u(W_0 - Q_s) + \rho q_s^T u(W_1 - y + \rho p_1 D) + \rho^2 (1 - q_s^T) p_1 v(D) - u(W_0) - \rho q_s^T u(W_1 - y) = 0 \quad (31b)$$

$$Q_s = \rho^2 p_1 D + R_s \qquad (31c)$$

Let us denote the optimal solutions as Q_s*, R_s*, and q_s* with $S_s* = \rho p_1 D$. By solving the above problem, we obtain the following result.

Proposition 2. Suppose that $S^I < \rho p_1 D$ and (29) holds. At the optimum, Q_s*, q_s*, and R_s* satisfy the following conditions as settlement is allowed.

$$R_s * f(q_s*) = \rho[u(W_1 - y + \rho p_1 D) - \rho p_1 v(D) - u(W_1 - y)] \frac{[1 - F(q_s*)]}{u'(W_0 - Q_s*)} \qquad (32)$$

$$\rho q_s *[u(W_1 - y + \rho p_1 D) - \rho p_1 v(D) - u(W_1 - y)] = u(W_0) - \rho^2 p_1 v(D) - u(W_0 - Q_s*) \qquad (33)$$

$$Q_s* = \rho^2 p_1 D + R_s * \qquad (34)$$

Proof. See the Appendix. //

Note that (32) is the same as (24) except for the additional constraint that the surrender value is equal to $\rho p_1 D$. The interpretation of (32) is similar to (24). The insurer determines q_s* where the marginal revenue equals the marginal cost. (33) also indicates that the net benefit of the target consumer is zero under the restriction that $S_s* = \rho p_1 D$.

Consumer welfare (CW_s) can be written as follows:

$$CW_s = \rho[\int_{q_s*}^{1} qf(q)dq - q_s*\{1 - F(q_s*)\}][u(W_1 - y + \rho p_1 D) - \rho p_1 v(D) - u(W_1 - y)] \qquad (35)$$

As in the case without settlement, consumer welfare is nonnegative.

2.6. The effects of settlement under $S^I < \rho p_1 D$

Let us analyze the effects of settlement on the insurance contract. By comparing the cases with and without the settlement market, we obtain the following results.

Proposition 3. Suppose that $S^I < \rho p_1 D$ and (29) holds. The insurer's profit is lower when settlement is allowed than when settlement is not allowed.

Proof. The existence of the settlement market effectively imposes additional constraints ($S_s* = \rho p_1 D$) on the insurer. The insurer should solve the profit maximization problem with an additional constraint. Therefore, the profit decreases when settlement is allowed. That is:

$$\pi_s = \pi_s(Q_s*, S_s* = \rho p_1 D, q_s*, R_s* \big| S^I < \rho p_1 D) \le \pi = \pi(Q*, S*, q* R* \big| S^I < \rho p_1 D) \quad (36)$$

Equality holds only when $S*$ equals $\rho p_1 D$.//

The lower profit can be interpreted as a result of competition between the insurer and the settlement providers, reflecting the reduced monopsony power of the insurer. Now, the comparative statics shows the following results.

Proposition 4. Suppose that $S^I < \rho p_1 D$ and (29) holds. When settlement is allowed, the following holds, compared with the case in which settlement is not allowed.
(1) The premium is higher.
(2) The target liquidity risk may be higher or lower.

Proof. See the Appendix.//

Settlement will lead to increases in the death benefit payment and thus a reduction in profit. The insurer may react to this change by increasing the premium and/or increasing the number of sales. If the decrease in utility due to the increase in premium is less in size than the increase in the utility of target risk, then the target liquidity risk increases. Note that the lowered target risk implies a demand increase. On the other hand, if the decrease in utility following the increase in premium is greater than the increase in the utility of target risk, then target risk decreases. The sign of demand change depends on the utility shape, population distribution, and the wealth level of the policyholders.

By comparing welfare with and without the settlement market, we obtain the next proposition.

Proposition 5. Suppose that $S^I < \rho p_1 D$ and (29) holds. Settlement allowance leads to the following change, compared with the case in which settlement is not allowed. Consumer welfare increases if condition (37) holds. Condition (37) always holds when demand increases, and it may hold even when demand decreases.

$$\frac{[\int_{q_s*}^{1} qf(q)dq - q_s*\{1-F(q_s*)\}]}{[\int_{q*}^{1} qf(q)dq - q\ *\{1-F(q\ *)\}]} > \frac{[u(W_1 - y + S*) - \rho p_1 v(D) - u(W_1 - y)]}{[u(W_1 - y + \rho p_1 D) - \rho p_1 v(D) - u(W_1 - y)]} \quad (37)$$

Proof. See the Appendix.//

Recall that demand increases when the net benefit of the target policyholder increases by proposition 4. The demand increase has two positive effects on consumer welfare. First, consumer welfare increases as much as the net benefits of the new policyholders. Second, the net benefit of the existing policyholders also increases as demand increases (see the appendix for technical details). Consequently, consumer welfare increases.

If demand decreases, the effects on policyholders are not in one direction. First, policyholders who leave the market experience the loss of utilities. Similarly, policyholders who are close to the target risk also experience the loss of utilities. However, some policyholders who are at a distance from the target risk may obtain a gain in utilities.[17] When the sum of the utility gains is greater than the sum of utility losses, consumer welfare may increase.

The results from proposition 3 through proposition 5 are summarized in the fifth row of Table 1.

3. Competitive insurance market

3.1. Equilibrium when settlement is not allowed

The above discussion is based on the monopolistic insurance market. Now we consider a competitive insurance market. We denote the premium and surrender value as Q_c and S_c, respectively, and $S_c \geq 0$. We also denote the optimal premium and surrender value as Q_c* and S_c*, respectively. We suppose that the mean value of q is δ. All other assumptions are identical to those in the monopolistic insurance market.

In a competitive equilibrium without settlement market, the premium and surrender value are determined to maximize consumer welfare. In addition, insurers sell insurance to all consumers and the expected profit of insurers should be zero. The problem can be written as

$$\underset{Q_c, S_c}{Max} \quad CW = u(W_0 - Q_c) + \rho\delta \max\{u(W_1 - y + S_c), \rho p_1 v(D) + u(W_1 - y)\} + \rho^2 p_1 (1-\delta) v(D)$$
$$-\{u(W_0) + \rho\delta u(W_1 - y)\} \qquad (38a)$$
$$s.t. \quad Q_c = \rho S_c \delta + \rho^2 p_1 D(1-\delta) \qquad (38b)$$

As in the monopoly case, we examine equilibria under the cases in which $S^I \geq \rho p_1 D$ and $S^I < \rho p_1 D$.

3.1.1. The case of $S^I \geq \rho p_1 D$

We first suppose that $S^I \geq \rho p_1 D$. Then, the range of surrender value can be following cases: $S_c < S^I$, $S_c > S^I$, and $S_c = S^I$.

(i) $S_c < S^I$: In this case, no policyholder will opt to surrender. Then, consumer welfare and the premium become $u(W_0 - Q_c) - u(W_0) + \rho^2 p_1 v(D)$ and $Q_c = \rho^2 p_1 D$, respectively.

(ii) $S_c > S^I$: The problem (38) is written as follows:

[17] Technically, the slope of the demand function in q is steeper when settlement is allowed than when it is not. Thus, two demand curves meet each other at most once in [q_s*, 1].

$$\underset{Q_c,S_c}{Max}\ CW = u(W_0 - Q_c) + \rho\delta u(W_1 - y + S_c) + \rho^2 p_1(1-\delta)v(D) - \{u(W_0) + \rho\delta u(W_1 - y)\} \quad (39a)$$

$$s.t.\quad Q_c = \rho S_c\delta + \rho^2 p_1 D(1-\delta) \quad (39b)$$

Then, the Lagrangian and the first order conditions are as follows:

$$\begin{aligned} L &= u(W_0 - Q_c) + \rho\delta u(W_1 - y + S_c) + \rho^2 p_1(1-\delta)v(D) - \{u(W_0) + \rho\delta u(W_1 - y)\} \\ &\quad + \lambda_c[Q_c - \rho S_c\delta - \rho^2 p_1 D(1-\delta)] \qquad (40) \\ L_{Q_c} &= -u'(W_0 - Q_c) + \lambda_c = 0 \qquad (41) \\ L_{S_c} &= \rho\delta u'(W_1 - y + S_c) - \lambda_c\rho\delta = 0 \qquad (42) \\ L_{\lambda_c} &= Q_c - \rho S_c\delta - \rho^2 p_1 D(1-\delta) = 0 \qquad (43) \end{aligned}$$

The optimal surrender value is determined at the point in which the marginal utility of surrender value ($\rho\delta u'(W_1 - y + S_c)$) is equal to the marginal cost of surrender value ($\rho\delta u'(W_0 - Q_c)$).

(iii) $S_c = S^I$: As opposed to the monopoly case, the insurer's profit is the same as zero when policyholders select either surrender or keeping insurance, while consumer welfare is influenced by the policyholder's choice. Thus, we use the policyholder's interest as a tie-break rule likewise monopoly case.

If policyholders are assumed to keep insurance, case (i) is applied and consumer welfare is $u(W_0 - Q_c) - u(W_0) + \rho^2 p_1 v(D)$. On the other hand, if policyholders are assumed to surrender, case (ii) is applied and consumer welfare is the same as the welfare in (39a). The expression (39a) is less than $u(W_0 - Q_c) - u(W_0) + \rho^2 p_1 v(D)$ because the premium is higher than that under the case of keeping insurance when $S^I \geq \rho p_1 D$. Thus, we regard that policyholders keep insurance in case of $S_c = S^I$. That is, for arbitrarily small $\varepsilon > 0$, insurers set the surrender value as $S^I - \varepsilon$. All these results are summarized as follows.

$$CW = \max \left\{ \begin{array}{l} u(W_0 - \rho^2 p_1 D) - u(W_0) + \rho^2 p_1 v(D) \\ u(W_0 - \rho S_c{*}\delta + \rho^2 p_1 D(1-\delta)) + \rho\delta u(W_1 - y + S_c{*}) + \rho^2 p_1(1-\delta)v(D) \\ -\{u(W_0) + \rho\delta u(W_1 - y)\} \end{array} \right\} \quad (44)$$

3.1.2. The case of $S^I < \rho p_1 D$

Suppose that $S_c < S^I$. Policyholders are assumed to keep insurance and consumer welfare is $u(W_0 - Q_c) - u(W_0) + \rho^2 p_1 v(D)$ for similar reasons to those discussed in (iii) in Section 9.1.1.1. On the other hand, if S_c is equal to or slightly greater than S^I, consumer welfare is greater than $u(W_0 - \rho^2 p_1 D) - u(W_0) + \rho^2 p_1 v(D)$. This is because the premium is less than $\rho^2 p_1 D$ and the utility $\rho\delta[u(W_1 - y + S_c) - \rho p_1 v(D) - u(W_1 - y)]$ is positive. As a result, we can rule out the case of $S_c < S^I$.

We obtain the following proposition from the above discussion.

Proposition 6. Suppose that the insurance market is competitive and that settlement is not allowed. At the optimum, we have the following results.

(1) Insurers sell insurance to all policyholders.

(2) If $S^I \geq \rho p_1 D$, then both $S_c^* \leq S^I$ and $S_c^* > S^I$ are potentially possible.

(a) When $S_c^* \leq S^I$, policyholders keep insurance when they face liquidity needs. Consumer welfare is determined as $u(W_0 - \rho^2 p_1 D) - u(W_0) + \rho^2 p_1 v(D)$.

(b) When $S_c^* > S^I$, policyholders surrender when they face liquidity needs. Consumer welfare is determined as

$$u(W_0 - \rho S_c{}^* \delta + \rho^2 p_1 D(1-\delta)) + \rho\delta u(W_1 - y + S_c{}^*) + \rho^2 p_1 (1-\delta) v(D)$$
$$-\{u(W_0) + \rho\delta u(W_1 - y)\}$$

(3) If $S^I \geq \rho p_1 D$, then $S_c^* \geq S^I$. In this case, policyholders surrender when they face liquidity needs. Consumer welfare is determined as

$$u(W_0 - \rho S_c{}^* \delta + \rho^2 p_1 D(1-\delta)) + \rho\delta u(W_1 - y + S_c{}^*) + \rho^2 p_1 (1-\delta) v(D)$$
$$-\{u(W_0) + \rho\delta u(W_1 - y)\}$$

Proof. See the text above. //

3.2. Equilibrium when settlement is allowed

We also add subscript *s* to indicate the existence of a settlement market. Suppose that $S^I > \rho p_1 D$. Then a settlement market cannot exist since policyholders will prefer keeping insurance or surrender to settlement with the same logic of monopoly case. When $S^I = \rho p_1 D$, settlement does not affect the insurance market, thus, we treat this case as if the settlement does not exist as in the monopoly case. Now we limit the following discussion on the case that $S^I < \rho p_1 D$.

Suppose that $S^I < \rho p_1 D$ and that settlement is allowed. The settlement market cannot exist when $S_c^* > \rho p_1 D$. From (41) and (42), we obtain the following condition for $S_c^* \leq \rho p_1 D$:

$$\rho(1+\rho) p_1 D \geq W_0 - W_1 + y \qquad (45)$$

Now, we suppose that (45) holds. If settlement is allowed, the premium Q_{cs} increases by $\rho^2 p_1 D$ and surrender value S_{cs} is equal to $\rho p_1 D$ for similar reason with the monopolistic insurance market. Consequently, consumer welfare decreases because policyholders cannot smooth the consumption between t = 0 and t = 1. These results are summarized in proposition 7.

Proposition 7. Suppose that the insurance market is competitive and that settlement is allowed.
(1) If $S^I \geq \rho p_1 D$, the settlement market does not exist.
(2) If $S^I < \rho p_1 D$, the settlement market exists when (45) holds. The existence of the settlement market leads to the premium increase and the consumer welfare decrease.

Proof. See the text above.//

The introduction of settlement never improves consumer welfare under the competitive insurance market, which is in sharp contrast with the case under the monopolistic insurance market.[18] The possible improvement of consumer welfare under the monopolistic insurance market comes from the fact that the insurer cannot discriminate over the liquidity risks so that the monopolistic insurer fails to fully extract the consumer surplus. This implies that the insurer sells insurance only to a portion of potential policyholders, which leaves room for welfare improvement as settlement is allowed.

4. The monopolistic settlement market

Suppose that both insurance and settlement market are monopoly. We limit our analysis to the case in which $S^I < \rho p_1 D$ and (29) holds. Otherwise the settlement market does not exist as in the competitive case. Let us suppose that the settlement investor offers *V*, which is in between the surrender value and an actuarial fair value of the insurance ($\rho p_1 D$) to attract policyholders from surrender when settlement is allowed. However, from the expression (31a) to (31c), we observe that as long as the insurer offers $V+\varepsilon$, $\varepsilon > 0$, as a surrender value, the profit is higher than that under $S \leq V$. That is, the insurer may offer a greater surrender value than the settlement price, and consequently the investor has to offer the settlement price greater than or equal to the surrender value. As a result, both the optimal surrender value and the settlement price equal to $\rho p_1 D$ in equilibrium. This implies that the insurer tries to reduce the rent of monopolistic investor extracting from consumers. Then, the result is identical with that under the monopolistic insurer and the competitive settlement market.

Proposition 8. Suppose that both insurance and settlement market are monopoly, $S^I < \rho p_1 D$ and (29) holds. Suppose further that settlement is allowed. Then the equilibrium is identical with that when the insurance market is monopoly and the settlement market is competitive.

Proof. See the text above.//

5. Numerical examples

In this section, we show the results of above sections by using a numerical example. We

[18] Obviously, it does not mean that consumer welfare is lower under the competitive market than under the monopolistic market.

assume that the population of policyholders over liquidity risk has a uniform distribution on [0, 1]. We also assume that the utility functions are $u(W) = 20 - 35\exp(-aW)$ and $v(W) = 15 - 15\exp(-aW)$, respectively, where a denotes risk aversion.[19] Let us set the initial values as $W_0 = W_1 = 8, D = 10, a = 0.2, y = 6$, $\rho = 0.970874$, and $p_1 = 0.2$ and search for the optimal contract.[20] In this example, the indifference surrender value (S^I) is 0.56778, which is less than the settlement price ($\rho p_1 D$), 1.94175. The optimal surrender value (S^*) is 1.71345. From this example, we can confirm the result of lemma 2 (1).

We identify that a settlement market may or may not exist (lemma 5) by changing parameter values of a, y, and p_1. As people are less risk averse, the optimal surrender value tends to be increased. This result may seem to be paradoxical because policyholders can smooth consumption between t = 0 and t = 1 from a higher surrender value. Recall that the surrender value is related to the target risk. As people become more risk averse, more people buy insurance. Thus, the insurer lowers the target risk, and the lowered target prefers a low surrender value because the lowered target implies a lowered liquidity risk. As a result, if the risk aversion is less than or equal to 0.12, a settlement market cannot exist.

The optimal surrender value decreases as p_1 increases for a similar reason as in the case of risk aversion. As the death probability increases, policyholders buy more insurance. Thus, the target risk decreases and the target prefers a low surrender value. If p_1 is less than or equal to 0.18, the optimal surrender value exceeds the settlement price, consequently, a settlement market cannot exist.

In addition, as y increases, the optimal surrender value tends to be increased to cope with the income loss. Thus, the optimal surrender value can also exceed the settlement price, and consequently a settlement market cannot exist. Furthermore, as people become wealthier, the insurer can sell insurance to more policyholders.

Second, we also find that the premium increases and demand can increase when a settlement market is allowed (proposition 4). As the wealth increases, demand can increase. We also show that consumer welfare can be enhanced when settlement is allowed (proposition 5).

These results are illustrated in Tables 4, 5, 6, and 7.

Table 4 around here

Table 5 around here

Table 6 around here

Table 7 around here

In this numerical example, consumer welfare is always improved. It is also possible to show that welfare can decrease in a different example. Let us set the initial value as $W_0 = W_1 = 10$, $D = 12$, $a = 0.3$, $y = 8$, $\rho = 0.970874$, and $p_1 = 0.2$. The utility functions are

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[19] In this study, death is treated as an exogenous event.

[20] We used MATLAB R2014a.

$u(W) = 20 - 100\exp(-aW)$ and $v(W) = 10 - 10\exp(-aW)$.[21] The liquidity risk still follows the uniform distribution: $q \sim U[0,1]$. In this example, consumer welfare may or may not be improved when demand decreases. The result is reported in Table 8.

Table 8 around here

In addition, by using the example that the initial value as $W_0 = W_1 = 17$, $D = 15.3$, $a = 0.3$, $y = 15$, $\rho = 0.970874$,and $p_1 = 0.2$, and the utility functions of $u(W) = 10 - 100\exp(-aW)$ and $v(W) = 9 - 9\exp(-aW)$, we observe that the demand increases.[22] The result is represented in Table 9.

Table 9 around here

6. Conclusion

We analyze the effects of settlement on the insurance contract design, the insurer's profit, and welfare. We consider two-period models in which the insurance market is monopolistic or competitive. Policyholders face heterogeneous liquidity risks. The liquidity risk is introduced to address the case in which policyholders urgently need cash, leading them to surrender or settle policies. It is assumed that the insurer cannot discriminate among policyholders based on their liquidity risks. It is further assumed that no costs are incurred in policy surrender or settlement. We find that a settlement market does not have to exist even if it is allowed, and derive the conditions for the endogenous existence of the settlement market. Cash surrender value may well be positive. In the monopolistic insurance market, the introduction of the settlement market lowers the insurer's profit, raises insurance premium but may increase or decrease insurance demand and consumer welfare. Consumer welfare increases whenever demand increases and possibly increases even when demand decreases. Our findings are in contrast with the existing studies in which settlement tends to lower welfare. On the other hand, we find that welfare decreases in the competitive insurance market. It is noted that the analysis is focused on the case in which the policyholder has no access to the capital market. The interaction between capital market and insurance market will be an interesting future research topic.

[21] In this case, $u(10) = 15.02129 > v(10) = 9.50213$.
[22] In this case, $u(17)=9.39033 > v(17)=8.94513$.

Figure 1. Time line of model

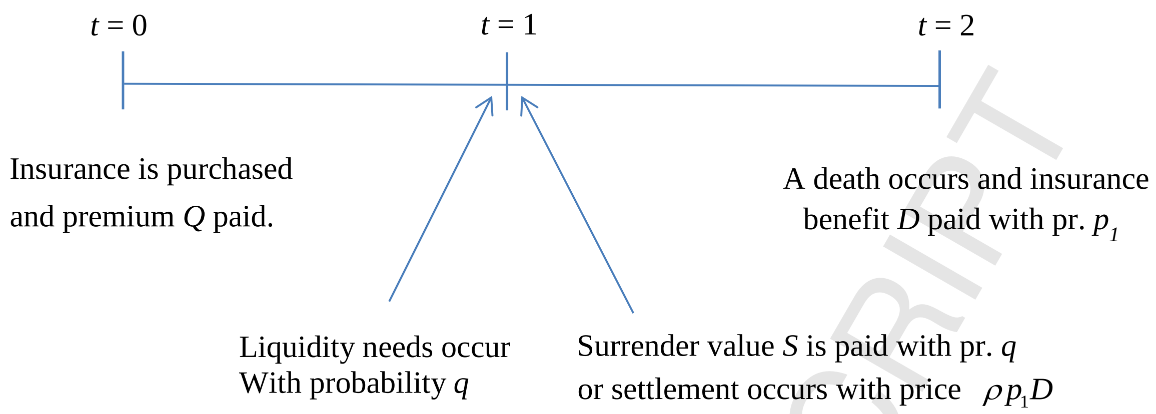

Table 1. The surrender value and the effects of settlement

	$S^I < \rho p_1 D$	$S^I \geq \rho p_1 D$
The optimal surrender value when settlement is not allowed	$S^* \geq S^I$	$S^* > S^I$ or $S^* \leq S^I$ §
Policyholder's behavior when settlement is not allowed	Surrender	Surrender, if $S^* > S^I$ Keeping insurance, if $S^* \leq S^I$
The existence of settlement market when settlement is allowed	The settlement market exists when $S^* \leq \rho p_1 D$.	The settlement market does not exist.
The effect of the settlement market	Premium ↑ Profit of insurer ↓ Demand ↑↓ §§ Consumer welfare ↑↓ §§§	Not relevant

§ $S^* > S^I : W_0 - \rho^2 p_1 D - u^{-1}(u(W_0) - \rho^2 p_1 v(D)) > [\rho S^* - \rho^2 p_1 D][q^*\{1-F(q^*)\} - \int_{q^*}^{1} qf(q)dq] + R\{1-F(q^*)\}$

$S^* \leq S^I : W_0 - \rho^2 p_1 D - u^{-1}(u(W_0) - \rho^2 p_1 v(D)) \leq [\rho S^* - \rho^2 p_1 D][q^*\{1-F(q^*)\} - \int_{q^*}^{1} qf(q)dq] + R\{1-F(q^*)\}$

§§ If the decrease in utility due to the increase in premium is less in size than the increase in the utility of target risk, then the demand decreases. Otherwise, the demand increases.

§§§ Consumer welfare increases when the following condition holds:

$$\frac{[\int_{q_s^*}^{1} qf(q)dq - q_s^*\{1-F(q_s^*)\}]}{[\int_{q^*}^{1} qf(q)dq - q^*\{1-F(q^*)\}]} > \frac{[u(W_1 - y + S^*) - \rho p_1 v(D) - u(W_1 - y)]}{[u(W_1 - y + \rho p_1 D) - \rho p_1 v(D) - u(W_1 - y)]}$$

Table 2. Summary of used variables and parameters

Variables and Parameters	Summary
W_0	Income at t = 0
W_1	Income at t = 1
y	Income loss
q	Probability of liquidity shock(liquidity risk)
q^T	Target liquidity risk when settlement is not allowed
q_s^T	Target liquidity risk when settlement is allowed
Q	Insurance premium when settlement is not allowed
Q_s	Insurance premium when settlement is allowed
Q_c	Insurance premium when settlement is not allowed in competitive insurance market
Q_{cs}	Insurance premium when settlement is allowed in competitive insurance market
D	Death benefit
S	Surrender value when settlement is not allowed
S_s	Surrender value when settlement is allowed
S_c	Surrender value when settlement is not allowed in competitive insurance market
S_{cs}	Surrender value when settlement is allowed in competitive insurance market
S^I	Indifference surrender value
R	Loading premium when settlement is not allowed
R_s	Loading premium when settlement is allowed
p_1	Death probability
ρ	Discount factor
π	Profit of insurer when settlement is not allowed
π_s	Profit of insurer when settlement is allowed
CW	Consumer welfare when settlement is not allowed
CW_s	Consumer welfare when settlement is allowed
a	Risk aversion

Table 3. Summary of Base Scenarios for Numerical examples.

	Scenario 1	Scenario 2	Scenario 3
Utility function	$u(W)=20-35\exp(-aW)$ $v(W)=15-15\exp(-aW)$	$u(W)=20-100\exp(-aW)$ $v(W)=10-10\exp(-aW)$	$u(W)=10-100\exp(-aW)$ $v(W)=9-9\exp(-aW)$
Parameters	Initial value	Initial value	Initial value
W_0	8	10	17
W_1	8	10	17
D	10	12	15.3
a	0.2	0.3	0.3
y	6	8	15
ρ	0.970874	0.970874	0.970874
p_1	0.2	0.2	0.2

Numerical Example 1 (Scenario 1).
W_0=W_1 = 8, D = 10, a = 0.2, y = 6, $\rho = 0.970874$, $p_1 = 0.2$, $u(W) = 20 - 35\exp(-aW)$, $v(W) = 15 - 15\exp(-aW)$, q

Table 4. Change in demand, premium, insurer's profit and consumer welfare when risk aversion changes

	a	**0.1**	**0.12**	**0.15**	**0.17**	**0.2**	**0.22**	**0.25**	**0.**
	S^I	0.66407	0.64002	0.60875	0.59081	0.56778	0.55477	0.53843	0.
$S \geq S^I$ **and settlement is not allowed**	S^*	1.99431	1.94611	1.86675	1.80886	1.71345	1.64279	1.52134	1.
	q_*	0.73567	0.69478	0.63744	0.60111	0.54819	0.51319	0.45927	0.
	Q^*	2.35346	2.39918	2.46472	2.50566	2.56121	2.59278	2.62702	2.
	π_*	0.11207	0.15578	0.23173	0.28870	0.38295	0.45136	0.56217	0.
	CW	0.11328	0.16726	0.25969	0.32551	0.42497	0.48858	0.57378	0.
Settlement is allowed	$\rho p_1 D$	1.94175	1.94175	1.94175	1.94175	1.94175	1.94175	1.94175	1.
	q_s^*	0.73345	0.69458	0.64149	0.60884	0.56299	0.53406	0.49248	0.
	Q_s^*	2.30546	2.39525	2.53103	2.62149	2.75575	2.84362	2.97193	3.
	π_s^*	0.11202	0.15578	0.23154	0.28801	0.38044	0.44657	0.55154	0.
	CW_s	0.11093	0.16697	0.26760	0.34340	0.46649	0.55295	0.68650	0.
Change in q, Q, π and welfare	Δq	**Settlement market does not exist.**	**Settlement market does not exist.**	0.00405	0.00774	0.01480	0.02086	0.03321	0.
	ΔQ			0.06631	0.11583	0.19454	0.25084	0.34491	0.
	$\Delta\pi$			-0.00019	-0.00069	-0.00251	-0.00479	-0.01063	-0.
	ΔCW			0.00791	0.01789	0.04152	0.06437	0.11272	0.

Notes: Table 4 presents the change in welfare and demand as risk aversion changes. All these cases correspond t
indifference surrender value (0.56778) is less than the settlement price (1.94175). We can confirm that the profit,
π_2 in all cases. If $a \leq 0.12$, then the settlement market cannot exist because the settlement price is higher than t
optimal surrender value decreases as risk aversion increases. As risk aversion increases, target risk decreases wit
market. The premium increases when settlement is allowed. Consumer welfare can be improved even if the dem

Table 5. Change in demand, premium, insurer's profit and consumer welfare when income loss changes

	y	**4.8**	**5**	**5.5**	**5.7**	**6**	**6.5**	**6.6**
	S^I	0.73359	0.70270	0.63141	0.60509	0.56778	0.51088	0.50
$S \geq S^I$ **and settlement is not allowed**	S^*	1.06479	1.13723	1.49276	1.58682	1.71345	1.90168	1.93
	q_*	0.48254	0.49881	0.54181	0.54589	0.54819	0.54649	0.54
	Q^*	1.73118	1.82746	2.24397	2.37424	2.56121	2.86291	2.92
	π_*	0.24689	0.26444	0.31837	0.34303	0.38295	0.45704	0.47
	CW	0.13277	0.16940	0.30175	0.34936	0.42497	0.56755	0.59
Settlement is allowed	$\rho p_1 D$	1.94175	1.94175	1.94175	1.94175	1.94175	1.94175	1.94
	q_s^*	0.60845	0.59944	0.57969	0.57270	0.56299	0.54854	0.54
	Q_s^*	2.44902	2.49697	2.62235	2.67473	2.75575	2.89738	2.92
	π_s^*	0.22077	0.24505	0.30984	0.33737	0.38044	0.45696	0.47
	CW_s	0.25459	0.28533	0.36992	0.40704	0.46649	0.57642	0.60
Change in q, Q, π and welfare	Δq	0.12591	0.10063	0.03788	0.02680	0.01480	0.00205	0.00
	ΔQ	0.71783	0.66951	0.37837	0.30049	0.19454	0.03447	0.00
	$\Delta \pi$	-0.02613	-0.01938	-0.00853	-0.00566	-0.00251	-0.00008	0.00
	ΔCW	0.12182	0.11593	0.06817	0.05768	0.04152	0.00887	0.00

Notes: Table 5 describes the change in welfare and demand as the size of the income loss changes. If the income to 6.7, then the optimal surrender value is greater than the settlement price, so the settlement market cannot exist the premium increases as income loss increases. As the income loss increases, the optimal surrender value increa

Table 6. Change in demand, premium, insurer's profit and consumer welfare when death probability changes

	p_1	0.18	0.19	0.2	0.22	0.25	0.27	0.3
	S^I	0.50801	0.53780	0.56778	0.62827	0.72040	0.78278	0.8778
$S \geq S^I$ **and settlement is not allowed**	S^*	1.75520	1.73465	1.71345	1.66876	1.59444	1.53796	1.4348
	q_*	0.55487	0.55160	0.54819	0.54092	0.52852	0.51878	0.5001
	Q^*	2.55849	2.55998	2.56121	2.56268	2.56140	2.55685	2.5389
	π_*	0.38105	0.38200	0.38295	0.38489	0.38794	0.39008	0.3935
	CW	0.45004	0.43776	0.42497	0.39769	0.35156	0.31614	0.2516
Settlement is allowed	$\rho p_1 D$	1.74757	1.84466	1.94175	2.13592	2.42718	2.62136	2.9126
	q_s^*	0.55440	0.55859	0.56299	0.57241	0.58808	0.59959	0.6185
	Q_s^*	2.55181	2.65498	2.75575	2.95062	3.22810	3.40442	3.6576
	π_s^*	0.38105	0.38140	0.38044	0.37496	0.35903	0.34411	0.3165
	CW_s	0.44857	0.45833	0.46649	0.47805	0.48386	0.48040	0.4649
Change in *q*, *Q*, π and welfare	Δq	**Settlement market does not exist.**	0.00700	0.01480	0.03148	0.05957	0.08081	0.1183
	ΔQ		0.09500	0.19454	0.38795	0.66671	0.84757	1.1186
	$\Delta \pi$		-0.00060	-0.00251	-0.00993	-0.02890	-0.04596	-0.0769
	ΔCW		0.02058	0.04152	0.08035	0.13230	0.16426	0.2132

Notes: Table 6 reports the change in welfare and demand when death probability changes. If the death probabilit then the optimal surrender value exceeds the settlement price. Thus, the settlement market cannot exist. With the market, the premium increases. As the death probability increases, target risk decreases with and without settlem increases. If the death probability is greater than or equal to 0.4, then $S < S^I$, so no policyholder will not choose

Table 7. Change in demand, premium, insurer's profit and consumer welfare when wealth changes

	W	**6.5**	**7**	**7.5**	**8**	**8.5**	**8.8**
	S^I	0.41432	0.45995	0.51088	0.56778	0.63141	0.67321
$S \geq S^I$ **and settlement is not allowed**	S^*	1.90098	1.85640	1.79729	1.71345	1.57576	1.40587
	q_*	0.61171	0.59571	0.57539	0.54819	0.50714	0.46171
	Q^*	2.72080	2.68273	2.63241	2.56121	2.44404	2.29774
	π_*	0.33684	0.34916	0.36418	0.38295	0.40741	0.42675
	CW	0.54874	0.50536	0.46478	0.42497	0.37971	0.33501
Settlement is allowed	$\rho p_1 D$	1.94175	1.94175	1.94175	1.94175	1.94175	1.94175
	q_s^*	0.61317	0.59934	0.58286	0.56299	0.53871	0.52144
	Q_s^*	2.75575	2.75575	2.75575	2.75575	2.75575	2.75575
	π_s^*	0.33676	0.34880	0.36314	0.38044	0.40158	0.41662
	CW_s	0.55739	0.52238	0.49211	0.46649	0.44555	0.43532
Change in *q*, *Q*, *π* and welfare	Δq	0.00146	0.00363	0.00747	0.01480	0.03158	0.05973
	ΔQ	0.03495	0.07302	0.12334	0.19454	0.31171	0.45801
	$\Delta\pi$	-0.00008	-0.00037	-0.00104	-0.00251	-0.00583	-0.01013
	ΔCW	0.00865	0.01703	0.02733	0.04152	0.06584	0.10031

Notes: Table 7 presents the change in welfare and demand when wealth changes. As people are wealthier, the op
decreases. When settlement is allowed, the premium increases and consumer welfare can increase. In case that th
equal to 9.5, demand increases.

Numerical Example 2 (Scenario 2).

$W_0 = W_1$ = 10, D = 12, a = 0.3, y = 7, $\rho = 0.970874$, $p_1 = 0.2$, $u(W) = 20 - 100\exp(-aW)$, $v(W) = 10 - 10\exp(-aW)$, $q \sim U[0,1]$

Table 8. Change in demand, premium, insurer's profit and consumer welfare when income loss changes.

Table 6	y	**3.5**	**4**	**5**	**7**	**8**
	S^I	0.74017	0.62668	0.45261	0.24076	0.17667
$S \geq S^I$ **and settlement is not allowed**	S^*	1.12090	1.26436	1.53085	1.98409	2.17310
	q_*	0.73408	0.70625	0.65401	0.56233	0.52222
	Q^*	1.82421	2.10585	2.68685	3.92027	4.57129
	π_*	0.11420	0.16456	0.30125	0.73219	1.02954
	CW	0.04222	0.09990	0.31285	1.43238	2.59882
Settlement is allowed	$\rho p_1 D$	3.49515	3.49515	3.49515	3.49515	3.49515
	q_s^*	0.98619	0.89272	0.75756	0.59364	0.53842
	Q_s^*	3.41400	3.58155	3.95443	4.84779	5.36335
	π_s^*	0.00029	0.02019	0.13603	0.59103	0.90933
	CW_s	0.00059	0.04416	0.33270	1.88986	3.39399
Change in q**,** Q**,** π **and welfare**	Δq	0.25211	0.18647	0.10355	0.03130	0.01619
	ΔQ	1.58979	1.47571	1.26758	0.92752	0.79206
	$\Delta \pi$	-0.11392	-0.14437	-0.16522	-0.14115	-0.12022
	ΔCW	-0.04162	-0.05574	0.01985	0.45748	0.79516

Notes: Table 8 shows that consumer welfare decreases in case of decrease in demand with settlement.

Numerical Example 3 (Scenario 3).

$W_0 = W_1$ = 16.5, D = 15.3, a = 0.3, y = 15, $\rho = 0.970874$, $p_1 = 0.2$, $u(W) = 10 - 100\exp(-aW)$, $v(W) = 9 - 9\exp(-aW)$, $q \sim U[0,1]$

Table 9. Change in demand, premium, insurer's profit and consumer welfare when wealth changes.

Table 7	W	**16**	**16.5**	**17**
	S^I	0.07876	0.09168	0.10676
$S \geq S^I$ **and settlement is not allowed**	S^*	2.87267	2.84601	2.81360
	q_*	0.29276	0.28795	0.28220
	Q^*	9.48721	9.47104	9.45119
	π_*	4.71340	4.74567	4.78393
	CW	9.96973	8.58571	7.39211
Settlement is allowed	$\rho p_1 D$	2.97087	2.97087	2.97087
	q_s^*	0.29258	0.28783	0.28223
	Q_s^*	9.54569	9.54569	9.54569
	π_s^*	4.71238	4.74403	4.78133
	CW_s	10.19555	8.83432	7.66356
Change in q, Q, π and welfare	Δq	-0.00018	-0.00012	0.00002
	ΔQ	0.05848	0.07465	0.09450
	$\Delta \pi$	-0.00102	-0.00165	-0.00260
	ΔCW	0.22582	0.24861	0.27145

Notes: Table 9 shows that demand increases. In this example, $u(16)$ = 9.17703 > $v(16)$ = 8.92593.

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Appendix

1. Proof of Lemma 1

If the net benefit is greater than 0, potential policyholders with liquidity risk q_i will buy insurance contracts. Thus, we have:

$$\begin{aligned} & 0 = NB(q^T) \le NB(q_i) \\ & \Rightarrow \rho q^T \max\{u(W_1 - y + S), u(W_1 - y) + \rho p_1 v(D)\} - \rho^2 q^T p_1 v(D) - \rho q^T u(W_1 - y) \\ & \quad \le \rho q_i \max\{u(W_1 - y + S), u(W_1 - y) + \rho p_1 v(D)\} - \rho^2 q_i p_1 v(D) - \rho q_i u(W_1 - y) \end{aligned} \quad \text{(A.1)}$$

From this relation, we obtain the following:

$$(q_i - q^T)\rho[\max\{u(W_1 - y + S), u(W_1 - y) + \rho p_1 v(D)\} - \rho p_1 v(D) - u(W - y)] \ge 0 \quad \text{(A.2)}$$

In (A.2), if $u(W_1 - y + S) > u(W_1 - y) + \rho p_1 v(D)$, then $q_i \ge q^T$. On the other hand, if $u(W_1 - y + S) = u(W_1 - y) + \rho p_1 v(D)$, the net benefit of all policyholders is the same as zero. //

2. Proof of Lemma 5

From (22), $S^* \le \rho p_1 D$ if (29) is satisfied. On the other hand, let us suppose that the premium Q' is as follows, given the surrender value S', $S' \ge \rho p_1 D$, when the settlement market exists:

$$Q' = \rho q' S' + \rho^2 (1 - q') p_1 D + R' \quad \text{(A.4)}$$

Thus, the profit maximization problem can be stated as follows:

$$\begin{aligned} & \underset{Q',S'R',q'}{Max} \quad \pi(Q', S'R', q') \\ & = Q'F(q') - \rho S' \int_{q'}^{1} q f(q) dq - \rho^2 p_1 D \int_{q'}^{1} (1 - q) f(q) dq \\ & = [\rho S' - \rho^2 p_1 D][q'(1 - F(q')) - \int_{q'}^{1} q f(q) dq] + R'(1 - F(q')) \\ & s.t. \quad u(W - Q') + \rho q' u(W - y + S') + \rho^2 (1 - q') p_1 v(D) - u(W) - \rho q' u(W - y) = 0 \\ & \qquad Q' = \rho q' S' + \rho^2 (1 - q') p_1 D + R' \qquad \text{(A.5)} \end{aligned}$$

The Lagrangian becomes

$$\begin{aligned} & \mathcal{L} = [\rho S' - \rho^2 p_1 D][q'(1 - F(q')) - \int_{q'}^{1} q f(q) dq] + R'(1 - F(q')) \\ & + \gamma[u(W - Q') + \rho q' u(W - y + S') + \rho^2 (1 - q') p_1 v(D) - u(W) - \rho q' u(W - y)] + \eta[S' - \rho p_1 D] \\ & where \quad \eta \ge 0 \qquad \text{(A.6)} \end{aligned}$$

For S', the first-order condition is

$$\mathcal{L}_{S'} = \rho[q'(1-F(q')) - \int_{q'}^{1} qf(q)dq] + \gamma\rho q' u'(w-y+S') + \eta = 0 \qquad \text{(A.7)}$$

We show that $\mathcal{L}_S = \mathcal{L}_S(Q_2*, S_2*, q_2*, R_2*) = 0$ in (17). If we assume that the optimal S_2*, q_2*, and R_2* are unique, for $S', \mathcal{L}_S(Q', S', q', R') \le 0$. Thus, for S', q', and R', $\mathcal{L}_{S'}$ should be negative and $\mathcal{L}_S(Q', S', q', R') + \eta = 0$. As a result, η should be positive to satisfy (A.7), and we obtain $\rho p_1 D = S'$ by complementary slackness condition. //

3. Proof of Proposition 2

The Lagrangian is written as

$$\begin{aligned}\mathcal{L} &= R_s(1-F(q_s^T)) \\ &\quad + \lambda_s[u(W-Q_s) + \rho q_s^T u(W-y+\rho p_1 D) + \rho^2(1-q_s^T)p_1 v(D) - u(W) - \rho q_s^T u(W-y)] \\ &\quad + \mu_s[Q_s - \rho^2 p_1 D - R_s] \qquad \text{(A.8)}\end{aligned}$$

The first-order conditions are obtained as follows:

$$\mathcal{L}_{Q_s} = -\lambda_s u'(W-Q_s) + \mu_s = 0 \qquad \text{(A.9)}$$

$$\mathcal{L}_{q_s} = -R_s f(q_s^T) + \lambda_s \rho[u(W-y+\rho p_1 D) - \rho p_1 v(D) - u(W-y)] = 0 \qquad \text{(A.10)}$$

$$\mathcal{L}_{R_s} = (1-F(q_s)) - \mu_s = 0 \qquad \text{(A.11)}$$

$$\mathcal{L}_{\lambda_s} = u(W-Q_s) + \rho q_s^T u(W-y+\rho p_1 D) + \rho^2(1-q_s^T)v(D) - u(W) - \rho q_s^T u(W-y) = 0 \qquad \text{(A.12)}$$

$$\mathcal{L}_{\mu_s} = Q_s - \rho^2 p_1 D - R_s = 0 \qquad \text{(A.13)}$$

From (A.9) to (A.13), we obtain proposition 2. //

4. Proof of Proposition 4

By total differentiation of expressions (23) and (25), we obtain the following relation:

$$\begin{aligned} u''(W-Q^*)R^* f(q^*)\frac{dQ^*}{dS^*} &= u'(W-Q^*)f(q^*)\frac{dR^*}{dS^*} - \rho u'(W_1-y+S^*)(1-F(q^*)) \\ &\quad + \left\{u'(W-Q^*)R^* f'(q^*) + \rho[u(W_1-y+S^*) - \rho p_1 v(D) - u(W_1-y)]f(q^*)\right\}\frac{dq^*}{dS^*} \qquad \text{(A.14)}\end{aligned}$$

$$\frac{dQ^*}{dS^*} = \rho q^* \frac{u'(W_1-y+S^*)}{u'(W-Q^*)} + \frac{\rho[u(W_1-y+S^*) - \rho p_1 v(D) - u(W_1-y)]}{u'(W-Q^*)}\frac{dq^*}{dS^*} \qquad \text{(A.15)}$$

As the surrender value increases by the settlement price, the premium, target risk, and target loading premium change. There can be four possible cases: (i) $dR^* > 0$, $dq^* > 0$; (ii) $dR^* > 0$, $dq^* < 0$; (iii) $dR^* < 0$, $dq^* > 0$; and (iv) $dR^* < 0$, $dq^* < 0$. However, because of total differentiation, both (A.14) and (A.15) indicate that not all these cases can be excluded.

Next, the premium increases when $dq^* > 0$ regardless of the sign of dR^*. In addition, we know the premium also increases when $dR^* > 0$ even though $dq^* < 0$ by comparing (25) and (34). Finally, the premium increases when $dR^* < 0$ and $dq^* < 0$ because

$$u'(W - Q^*)\left\{R^* f'(q^*) + \frac{\rho[u(W_1 - y + S^*) - \rho p_1 v(D) - u(W_1 - y)]}{u'(W - Q^*)} f(q^*)\right\} > 0$$ in (A.14).

As a result, the premium always increases and demand can increase or decrease. //

5. Proof of Proposition 5

First, we obtain (A.16) by (27) and (35).

$$\begin{aligned}\Delta CW &= CW_s - CW \\ &= \rho\Big[\int_{q_s^*}^{1} qf(q)dq - q_s^*\{1 - F(q_s^*)\}\Big][u(W_1 - y + \rho p_1 D) - \rho p_1 v(D) - u(W_1 - y)] \\ &\quad -\rho\Big[\int_{q_s^*}^{1} qf(q)dq - q_s^*\{1 - F(q_s^*)\}\Big][u(W_1 - y + S^*) - \rho p_1 v(D) - u(W_1 - y)] \qquad \text{(A.16)}\end{aligned}$$

Since $\dfrac{d\rho[\int_{q^*}^{1} qf(q)dq - q^*\{1 - F(q^*)\}]}{dq^*} = -1 + F(q^*) \le 0$, consumer welfare increases when the target risk is lower. On the other hand, suppose the target increases. Then, by (A.2) in the Appendix, the expressions $\rho(q_i - q^*)[u(W_1 - y + S^*) - u(W_1 - y) - \rho p_1 v(D)]$ and $\rho(q_i - q_s^*)[u(W_1 - y + \rho p_1 D) - u(W_1 - y) - \rho p_1 v(D)]$ meet where

$$q_i^* = \frac{q_s^*[u(W_1 - y + \rho p_1 D) - u(W_1 - y) - \rho p_1 v(D)] - q^*[u(W_1 - y + S^*) - u(W_1 - y) - \rho p_1 v(D)]}{[u(W_1 - y + \rho p_1 D) - u(W_1 - y + S^*)]} \qquad \text{(A.17)}$$

If $q_i^* = 1$, consumer welfare always decreases. However, if $q_i^* < 1$, consumer welfare can increase when the following condition holds:

$$\begin{aligned}&\int_{q_s^*}^{1} \begin{matrix}\rho(q_i - q_s^*)[u(W_1 - y + \rho p_1 D) - u(W_1 - y) - \rho p_1 v(D)] \\ -\rho(q_i - q^*)[u(W_1 - y + S^*) - u(W_1 - y) - \rho p_1 v(D)]dq_i\end{matrix} \\ &> \int_{q^*}^{q_s^*} \begin{matrix}\rho(q_i - q^*)[u(W_1 - y + S^*) - u(W_1 - y) - \rho p_1 v(D)] \\ -\rho(q_i - q_s^*)[u(W_1 - y + \rho p_1 D) - u(W_1 - y) - \rho p_1 v(D)]dq_i\end{matrix} \qquad \text{(A. 18)}\end{aligned}$$

From (A.17) and (A.18), we have condition (37).//