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Distributed control algorithm for optimal reactive power control in power grids

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ABSTRACT

Reactive power generation has been commonly used for power loss minimization and voltage profile improvement in power systems. However, the opportunity cost of reactive power generation should be considered since it affects the frequency control capability of the generator to some degree. This paper proposed a distributed nonlinear control based algorithm to achieve the optimal reactive power generation for multiple generators in a power grid. The reactive power control setting update for each generator only requires local measurement and information exchange with its neighboring buses. It is demonstrated that the proposed algorithm can reduce the non-convex objective function monotonically till convergence and achieve comparable solutions to the centralized technique: particle swarm optimization with faster convergence speed. The proposed algorithm has been tested on the IEEE 9-bus, 39-bus and 162-bus systems to validate its effectiveness and scalability.

Introduction

Reactive power generation has been widely used to improve the voltage of the buses as well as minimize the power loss in the conventional power systems. Abundant control and optimization techniques have been developed for the optimal reactive power control such as linear programming [1], gradient method [2], interior point method [3,4], and sequential quadratic programming algorithm [5]. Shortcomings of these solutions include sensitivity to initial conditions and mathematical restriction on objective functions, such as convexity. Recently, numerous computational intelligent based methods have been proposed to overcome the shortcomings of the traditional algorithms such as, Gravitational Search Algorithm [6], Differential Evolution Algorithm [7], Enhanced Genetic Algorithm [8], Artificial Bee Colony [9], and Particle Swarm Optimization (PSO) [10]. However, all these algorithms require sophisticated communication network for global information collection and are usually implemented offline in a centralized way.

Distributed control and optimization techniques can improve the respond speed effectively by relieving the communication and computational burden [11,12], and they are proved to be suitable for online applications that require frequent control setting update [13]. In [14], the authors propose a decentralized nonlinear auto-adaptive controller for reducing system losses only by the optimal management of the reactive power. Whereas, Di Fazio et al. [11] propose a decentralized approach with off-line coordination to improve the voltage profile of smart feeders, and furthermore Maknouninejad et al. [12] demonstrate that minimizing the voltage deviation naturally contribute to the reduction of active power loss. In contrast, Zhang et al. [13] propose a subgradient based distributed algorithm to minimize the approximated convex objective function of both power loss and voltage deviation directly, but still the cost of reactive power generation is not considered. If cost of the reactive power is not taken into account, unnecessary generation of reactive power may compromise their primary objective, i.e. real power generation for frequency control. Hence, the objective of this paper is to deal with the optimal reactive power control problem considering a multi-objective function which includes power loss, voltage deviation and cost of the reactive power generation simultaneously.

In this paper, a distributed nonlinear control based algorithm has been proposed to minimize the formulated multi-objective function. The proposed strategy is implemented based on a multi-agent system (MAS)-based framework, where each generator in the power network is assigned with an agent. Each agent first calculates the available reactive power generation capacity based on

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the rated apparent power and the predicted active power generation requirement; then updates the reactive power generation based on the designed nonlinear control law, using information exchange among neighboring agents only. The proposed distributed algorithm enables the sharing of computational and communication burden among multiple agents and its convergence is guaranteed through rigorous stability analysis. It has been tested with 9-bus, 39-bus and 162-bus systems, which proves its effectiveness and scalability. The major contributions of the proposed distributed nonlinear control based algorithm are summarized as follows:

(1) The active power loss, voltage deviation and the opportunity cost of the reactive power generation are included in the objective function, which is formulated as a non-convex optimization problem.
(2) Non-linear control technique, using the formulated objective function as a Lyapunov candidate, has been employed to minimize the non-convex objection function.
(3) Distributed algorithm using only local information exchange among neighboring agents is implemented to reduce the communication and computational burden.
(4) It is demonstrated that the approximated relation for power loss by ignoring the voltage angle difference between neighboring buses can still achieve comparable results at normal level of generation/load condition.

The rest of the paper is organized as follows. Section 'Problem formulation' describes the problem formulation of the multi-objective function for optimal reactive power control. Proposed non-linear control based algorithm design is presented in section 'Proposed algorithm design'. Section 'Simulation studies' discusses and analyses the simulation results, and section 'Conclusions' provides the conclusions.

Problem formulation

Optimal reactive power control of power system can lead to minimal active power loss and improved voltage profile. Notice that minimization of the cost of reactive power generation is another important factor which cannot be ignored, simultaneously. Therefore, the objective function is formulized as the combination of three sub-functions

$$f = W_1 P_{loss} + W_2 D_V + W_3 C_Q \tag{1}$$

where W_1, W_2 and W_3 are the weight coefficients, which describe the preference of the reactive power suppliers. P_{loss}, D_V and C_Q are the power loss, voltage deviation and cost of reactive power generation, respectively.

The objective function given in (1) is consisted of three terms. The first term is related to the active power loss, which can be derived from power flow equation [15,16]

$$P_{Gi} - P_{Li} - V_i \sum_{j=1}^{n} V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) = 0 \tag{2}$$

where P_{Gi}, P_{Li} are the generation and the load at bus i, respectively, and remaining is the power flow from bus i. Y_{ij} and θ_{ij} are the magnitude and angle of element of the Y bus matrix between bus i and j, V_i, V_j and δ_i, δ_j are bus voltages and angles of i and j, respectively.

The total power loss P_{loss} in the power system can be obtained by calculating the difference between the total generation and total load

$$P_{loss} = \sum_{i=1}^{n} P_{G,i} - \sum_{i=1}^{n} P_{L,i}. \tag{3}$$

The total power loss P_{loss} can be obtained by taking the summation of Eq. (2) for i = 1, . . . , n.

$$P_{loss} = \sum_{i=1}^{n} P_{Gi} - \sum_{i=1}^{n} P_{Li} = \sum_{i=1}^{n}\sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i). \tag{4}$$

Using $\delta_{ji} = \delta_j - \delta_i$,

$$P_{loss} = \sum_{i=1}^{n}\sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_{ji}) \tag{5}$$

The second term of the objective function, which is the deviation between bus voltage magnitude and its reference

$$D_V = \sum_{i=1}^{n} (V_i - V_i^*)^2 \tag{6}$$

where V_i^* is the reference voltage for bus i.

The cost of reactive power generation, contributed by generator, is given by [17]

$$C_{QG} = \sum_{i \in N_G} a_{Q_i} Q_{G_i}^2 + b_{Q_i} Q_{G_i} + c_{Q_i} \tag{7}$$

where N_G is the index set of generators, Q_{Gi} is the reactive power generation from generator i. a_{Qi}, b_{Qi}, c_{Qi} are the reactive power cost coefficients of generator i, which are determined from active power cost coefficients a_{Pi}, b_{Pi}, c_{Pi}, respectively, by the modified triangle method [9,18,19]

$$C_Q = \sum_{i \in N_G} a_{p_i} \sin^2 \sigma_i Q_i^2 + b_{p_i} \sin \sigma_i Q_i + c_{p_i} \tag{8}$$

where σ_i is the angle difference between voltage and current.

Proposed algorithm design

The optimal reactive power control of multiple generators in the power grid has been formulated as a nonlinear multi-objective function, which is minimized by controlling the reactive power generation of generators in this section.

Distributed nonlinear control based algorithm

Since the multi-objective function represented by Eq. (1) is definitely positive in nature, it is a viable Lyapunov candidate for the control of the targeted nonlinear systems. According to the nonlinear control theory, the condition for monotonically decreasing objective function is given as

$$\frac{\partial f}{\partial t} = \sum_{i \in N_G} \frac{\partial f}{\partial Q_{Gi}} \cdot \frac{\partial Q_{Gi}}{\partial t} \leqslant 0 \tag{9}$$

To ensure the absolute negativity of the derivative term of the objective function w.r.t. time, a control law is designed as

$$\frac{\partial Q_{Gi}}{\partial t} = -\frac{\partial f}{\partial Q_{Gi}}. \tag{10}$$

Substitute Eq. (10) into Eq. (9) yield

$$\frac{\partial f}{\partial t} = -\sum_{i \in N_G} \left(\frac{\partial f}{\partial Q_{Gi}} \right)^2 \leqslant 0. \tag{11}$$

The control law in Eq. (10) can be easily realized using the following approximation [20]

$$\frac{\partial f}{\partial Q_{Gi}} \approx \frac{f(Q_{Gi}[k]) - f(Q_{Gi}[k-1])}{Q_{Gi}[k] - Q_{Gi}[k-1]}. \tag{12}$$

However, as pointed out in [13], this kind of approach is less accurate and sensitive to the selection of time interval between control updates. To improve the control accuracy, it is desirable to discover the partial derivative term of the objective function w.r.t. Q_{Gi} based on the present states of the system.

The gradient of the objective function w.r.t. control variable Q_{Gi} is determined as follows

$$\frac{\partial f}{\partial Q_{Gi}} = W_1 \frac{\partial P_{loss}}{\partial Q_{Gi}} + W_2 \frac{\partial D_V}{\partial Q_{Gi}} + W_3 \frac{\partial C_{QG}}{\partial Q_{Gi}}. \tag{13}$$

Eq. (13) can be further expanded as [12,13]:

$$\frac{\partial f}{\partial Q_{Gi}} = W_1 \frac{\partial P_{loss}}{\partial V_i} \frac{\partial V_i}{\partial Q_{Gi}} + W_2 \frac{\partial D_V}{\partial V_i} \frac{\partial V_i}{\partial Q_{Gi}} + W_3 \frac{\partial C_Q}{\partial Q_{Gi}}. \tag{14}$$

As shown in Eq. (14), the gradient of power loss w.r.t. Q_{Gi} can be calculated as the product of two terms, where the first term is determined as follows

$$\frac{\partial P_{loss}}{\partial V_i} = \frac{\partial}{\partial V_i} \left[\sum_{i=1}^{n} \sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_{ji}) \right] \tag{15}$$

Eq. (15) can be further simplified as

$$\frac{\partial P_{loss}}{\partial V_i} = 2V_i G_{ii} + 2 \sum_{j=1, j \neq i}^{n} V_j Y_{ij} \cos(\theta_{ij} + \delta_{ji}) = 2 \sum_{j=1}^{n} V_j Y_{ij} \cos(\theta_{ij} + \delta_{ji}). \tag{16}$$

The second term of the power loss gradient can be derived from the reactive power flow [12,21,22]

$$Q_{Gi} - Q_{Di} = \sum_{j=1}^{n} V_i V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \tag{17}$$

where Q_{Gi}, Q_{Di} are the reactive power generation and load at bus i, and B_{ij} is the imaginary part of the Y_{ik}.

Eq. (17) can be expanded as

$$Q_{Gi} - Q_{Di} = \sum_{j=1, j \neq i}^{n} V_i V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) - V_i^2 B_{ii}. \tag{18}$$

In this paper, the local reactive power load is considered as constant. Therefore, the second term of the power loss gradient is calculated as

$$\frac{\partial Q_{Gi}}{\partial V_i} = \sum_{j=1, j \neq i}^{n} V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) - 2V_i B_{ii}. \tag{19}$$

The R.H.S. of Eq. (19) can be easily rewritten as

$$\frac{\partial Q_{Gi}}{\partial V_i} = \frac{V_i \sum_{j=1, j \neq i}^{n} V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) - V_i^2 B_{ii}}{V_i} - \frac{V_i^2 B_{ii}}{V_i}. \tag{20}$$

The nominator of the first term on the R.H.S of Eq. (20) can be replaced by $Q_{Gi} - Q_{Di}$ according to Eq. (17)

$$\frac{\partial Q_{Gi}}{\partial V_i} = \frac{Q_{Gi} - Q_{Di}}{V_i} - V_i B_{ii}. \tag{21}$$

Calculate the reciprocal of Eq. (21) yield

$$\frac{\partial V_i}{\partial Q_{Gi}} = \frac{V_i}{Q_{Gi} - Q_{Di} - V_i^2 B_{ii}}. \tag{22}$$

The gradient of power loss w.r.t. Q_{Gi} can now be calculated by multiplying Eqs. (16) and (22)

$$\begin{aligned} \frac{\partial P_{loss}}{\partial Q_{Gi}} &= 2 \sum_{j=1}^{n} V_j Y_{ij} \cos(\theta_{ij} + \delta_{ji}) \frac{V_i}{Q_{Gi} - Q_{Di} - V_i^2 B_{ii}} \\ &= 2 \frac{V_i \sum_{j=1}^{n} V_j Y_{ij} \cos(\theta_{ij} + \delta_{ji})}{Q_{Gi} - Q_{Di} - V_i^2 B_{ii}}. \end{aligned} \tag{23}$$

Usually, the voltage angle difference between the neighboring buses is very small, hence, $\cos(\delta_{ji}) \approx \cos 0 = 1$. By adopting this approximation, Eq. (23) becomes

$$\frac{\partial P_{loss}}{\partial Q_{Gi}} = 2 \frac{V_i \sum_{j=1}^{n} V_j G_{ij}}{Q_{Gi} - Q_{Di} - V_i^2 B_{ii}}. \tag{24}$$

According to Eq. (14), the gradient of voltage deviation w.r.t. Q_{Gi} is calculated as the product of two terms, where the first term is determined as follows

$$\frac{\partial D_V}{\partial V_i} = 2(V_i - V_i^*). \tag{25}$$

The second term for the gradient of voltage deviation is the same as Eq. (22). Thus, the gradient of voltage deviation can be determined as

$$\frac{\partial D_V}{\partial Q_{Gi}} = 2(V_i - V_i^*) \left(\frac{V_i}{Q_{Gi} - Q_{Di} - V_i^2 B_{ii}} \right) = \frac{2V_i (V_i - V_i^*)}{Q_{Gi} - Q_{Di} - V_i^2 B_{ii}}. \tag{26}$$

The third term, which is the derivative of the reactive power generation cost w.r.t. Q_{Gi} is calculated as

$$\begin{aligned} \frac{\partial C_{QG}}{\partial Q_{Gi}} &= 2a_{Pi} \cdot \left(Q_{Gi}^2 \cdot \sin \sigma_i \cdot \frac{\partial \sin \sigma_i}{\partial Q_{Gi}} + \sin^2 \sigma_i \cdot Q_{Gi}\right) \\ &\quad + b_{Pi} \left(Q_{Gi} \cdot \frac{\partial \sin \sigma_i}{\partial Q_{Gi}} + \sin \sigma_i\right) \end{aligned} \tag{27}$$

Next, the term $\partial \sin \sigma_i / \partial Q_{G_i}$ can be derived from power triangle relationship

$$\sin \sigma_i = \frac{Q_{G_i}}{\sqrt{P_{G_i}^2 + Q_{G_i}^2}}. \tag{28}$$

P_{Gi} from generator i is considered as constant when the system is connected to the main grid and the generators are controlled in power regulation mode. Thus, the derivative term is given by

$$\frac{\partial \sin \sigma_i}{\partial Q_{G_i}} = \frac{P_{G_i}^2}{(P_{G_i}^2 + Q_{G_i}^2)^{\frac{3}{2}}}. \tag{29}$$

Substitute Eq. (29) into Eq. (27) yield

$$\begin{aligned} \frac{\partial C_{QG}}{\partial Q_{Gi}} &= 2a_{Pi} Q_{Gi} \left(\frac{Q_{Gi} \sin \sigma_i P_{Gi}^2}{(P_{Gi}^2 + Q_{Gi}^2)^{\frac{3}{2}}} + \sin^2 \sigma_i \right) \\ &\quad + b_{Pi} \left(\frac{Q_{Gi} P_{Gi}^2}{(P_{Gi}^2 + Q_{Gi}^2)^{\frac{3}{2}}} + \sin \sigma_i \right). \end{aligned} \tag{30}$$

Now Eq. (13), which is the gradient of the objective function w.r.t. Q_{Gi}, can be calculated using Eqs. (24), (26), and (30):

$$\begin{aligned} \frac{\partial f}{\partial Q_{Gi}} &= \frac{2V_i \left(W_1 \sum_{j=1}^{n} V_j G_{ij} + W_2 (V_i - V_i^*) \right)}{Q_{Gi} - Q_{Di} - V_i^2 B_{ii}} + 2W_3 a_{Pi} Q_{Gi} \\ &\quad \cdot \left(\frac{Q_{Gi} \cdot \sin \sigma_i P_{Gi}^2}{(P_{Gi}^2 + Q_{Gi}^2)^{\frac{3}{2}}} + \sin^2 \sigma_i \right) \\ &\quad + W_3 b_{Pi} \left(\frac{Q_{G_i} P_{Gi}^2}{(P_{Gi}^2 + Q_{Gi}^2)^{\frac{3}{2}}} + \sin \sigma_i \right) \end{aligned} \tag{31}$$

It is worthy to note that only admittance of the transmission line connecting two buses and local information, such as bus voltage, voltage reference, present active and reactive power generation, and generation cost coefficients are required to calculate the gradient. No global parameter of the system is required.

Eq. (31) can be used to update the control variable of the reactive power generation and attain the optimal solution. The derivative of Q_{Gi} w.r.t. time can be approximated by

$$\frac{\partial Q_{Gi}}{\partial t} \approx \frac{Q_{Gi}[k+1] - Q_{Gi}[k]}{\Delta t} \tag{32}$$

Eq. (32) can be rewritten as

$$Q_{Gi}[k+1] = Q_{Gi}[k] + \frac{\partial Q_{Gi}}{\partial t}\Delta t \tag{33}$$

where Δt is the time interval for control setting update.

Finally, the control variable is updated according to the designed nonlinear control law as

$$Q_{Gi}[k+1] = Q_{Gi}[k] - \frac{\partial f}{\partial Q_{Gi}}\Delta t. \tag{34}$$

Implementation of the proposed algorithm

In this paper, an agent is defined as a function module that combines both physical controller and computational elements. According to the adopted MAS framework, each bus has been assigned a bus agent (BA) and each generator has been assigned a generator agent (GA), which can communicate with its neighbors and update its local information. Two buses are considered to be neighbors only if they are physically connected to each other by the transmission line. This topology can easily exchange the information of reactive power ratings, voltage magnitudes and calculate angle differences between two neighboring agents. Hence, the implementation cost of the proposed distributed algorithm can be reduced.

Each BA is responsible for obtaining the local measurement of voltage magnitude and exchanging the information with its neighboring BAs. Each GA is responsible for calculating the derivative term according to Eq. (31) and updating the reactive power generation setting according to Eq. (34). The online implementation of the proposed distributed algorithm is described as the flow chart shown in Fig. 1.

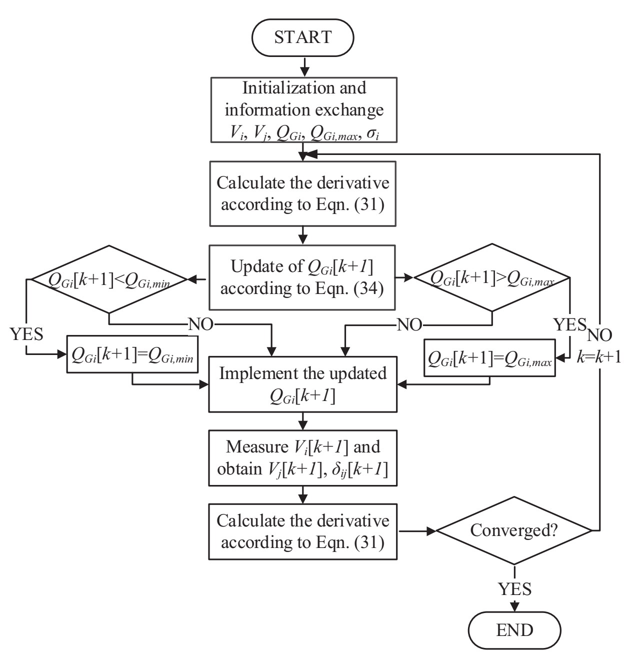

Fig. 1. Flow chart of the proposed distributed nonlinear control based algorithm.

Simulation studies

In this section, the proposed distributed nonlinear control based algorithm is first tested on the IEEE 9-bus system to demonstrate its effectiveness. To validate the possibility of application to large scale power systems, simulation results with IEEE 39-bus and 162-bus are also presented.

Case study 1: 9-bus system

The proposed distributed algorithm is applied to the modified IEEE 9-bus power system [23], in which bus 1 is a slack bus, 2 and 3 are voltage controlled buses and 4–9 are load buses. Generators are attached at 5, 6, 7, 8 and 9 buses for optimal reactive power control to minimize the objective function represented by Eq. (1). The maximum capacity of the reactive power for five generator s, namely, 5th, 6th, 7th, 8th, and 9th, are 0.80, 0.50, 0.50, 0.50, and 0.50 p.u, respectively, whereas the lower limit is kept as -0.5 p.u. Three sub-objective functions: power loss, voltage deviation and reactive power cost are weighted as 1, 10, and 0.1, respectively. However, the weights can be set to other values based on the preference of the generation suppliers. The system network data is shown in Table 1, and the cost coefficients for power generations are summarized in Table 2.

In the first scenario, the derivative of power loss w.r.t. the optimization variable is calculated according to Eq. (23) including cos (δ_{ji}). In the second scenario, the derivative of power loss is simplified by assuming cos(δ_{ji}) = 1, according to Eq. (24). In the third scenario, the objective is reduced to consider power loss only.

Fig. 2 shows the objective function converges to the value of 0.52268 in about 10 iterations. Reactive power generation update for each generator is shown in Fig. 3. Figs. 4 and 5 show the simulation results of the second scenario: the angle differences between two neighboring buses are ignored.

Comparing of Figs. 2 and 4 shows that with cos(δ_{ji}) = 1, it takes only few more iterations to converge. The objective function converges at 13th iteration as compared to 10th with nonzero δ_{ji}. And the optimal value is just a little bit higher, which is 0.52288 in comparison with 0.52268 for the nonzero δ_{ji} scenario. The reason behind this slight difference is that for nonzero δ_{ji} scenario, complete information is provided for optimization. Similarly, reactive power generation updates for two scenarios: without and with approximation can be compared in Figs. 3 and 5.

Table 1
9-Bus System Network Data.

From bus	To bus	R_{ij} (p.u)	X_{ij} (p.u)	B_{ij} (p.u)
4	1	0	0.115	0
7	2	0	0.125	0
9	3	0	0.117	0
7	8	0.025	0.144	0.149
9	8	0.036	0.202	0.209
7	5	0.096	0.322	0.306
9	6	0.117	0.340	0.358
5	4	0.030	0.170	0.176
6	4	0.051	0.184	0.158

Table 2
Cost coefficients for five generators.

Gen. no.	a_{Pi} (p.u)	b_{Pi} (p.u)
5	0.282	0.225
6	0.122	0.420
7	0.175	0.325
8	0.241	0.256
9	0.350	0.189

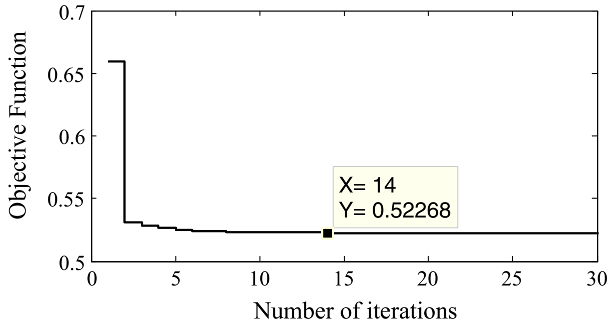

Fig. 2. Convergence of the objective function without approximation.

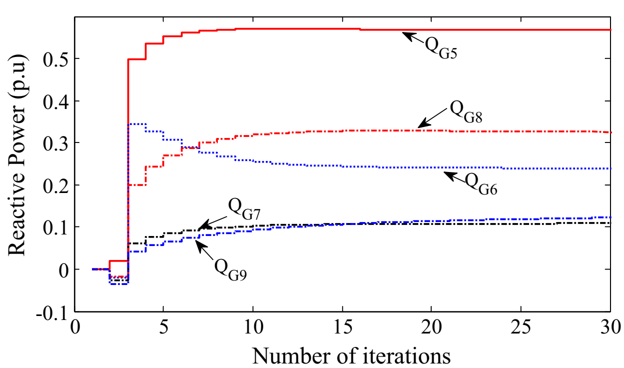

Fig. 3. Reactive power generation update without approximation.

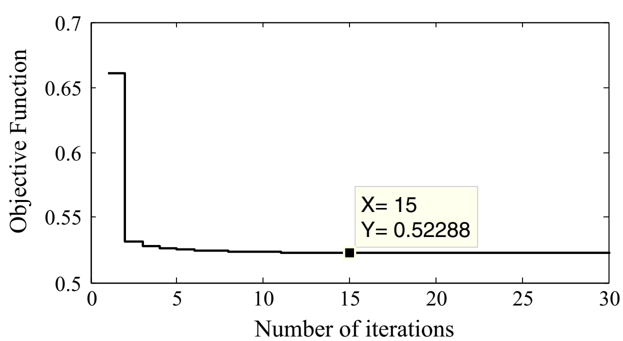

Fig. 4. Convergence of the objective function with approximation of $\cos(\delta_{ji}) = 1$.

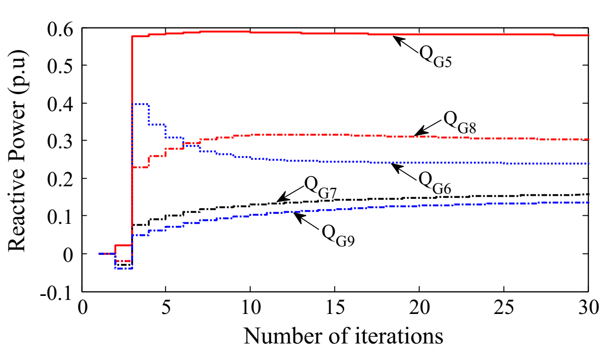

Fig. 5. Reactive power generation update with approximation of $\cos(\delta_{ji}) = 1$.

To validate the effectiveness of the proposed distributed algorithm, it is compared with centralized optimization technique of PSO as shown in Figs. 6 and 7. It demonstrates that proposed distributed algorithm converges to a lower value of 0.52268 with only 10 iterations compared to 0.52629 obtained by PSO.

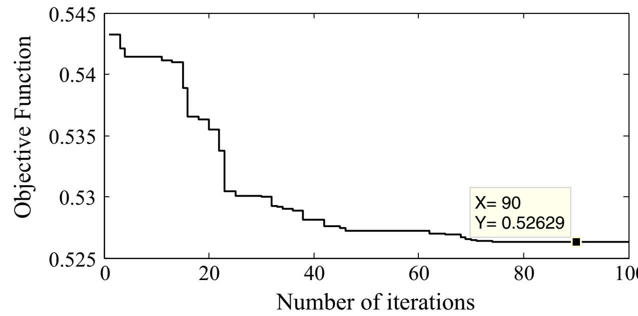

Fig. 6. Convergence of the objective function using PSO.

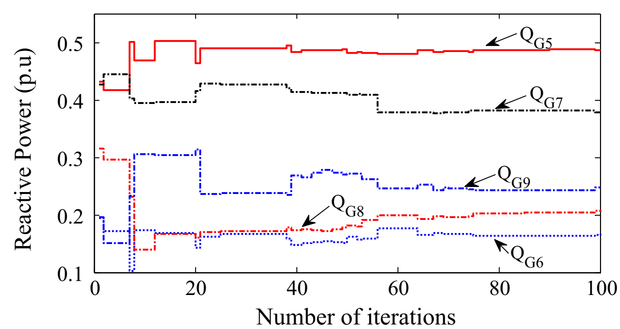

Fig. 7. Reactive power generation update using PSO.

High level of generation/load and different weight coefficients influence analysis

To analyze the effect of making approximation of $\cos(\delta_{ji}) = 1$ on overloaded systems, load on buses is increased step by step until the power system becomes heavily loaded. For each increment of load, the objective function for the two cases, with and without approximation has been presented in Fig. 8. It becomes clear that the approximated value of function deviates largely from the real value as the system becomes overloaded. Thus, it can be conferred that it may not advisable to use this approximation in case of heavily loaded power system.

However, when the power system becomes overloaded, the top priority of system operators is the security of the system and more constraints are added which may narrow down the feasible region of the optimization solution. That is why during the overloaded

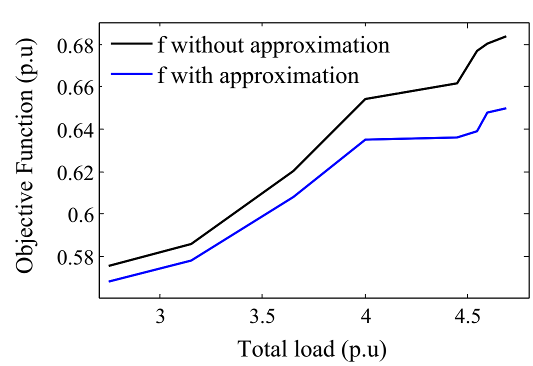

Fig. 8. Effect of varying the total load on the objective function with and without approximation of $\cos(\delta_{ji}) = 1$.

condition, power system optimization may not be very important operation to perform.

Table 3 is presented to compare the values of reactive power generations (Q_{Gi}), voltage magnitudes (V_i), power loss (P_L), reactive power generation cost (C_{QG}) and objective function (f) between the proposed distributed algorithm (DA) and Centralized Algorithm (CA) for different weight coefficients. Q_{Gi}, V_i and P_L are given in per unit, where 16.5 KV and 100 MVA is selected as the base voltage and power, respectively. \$1000 is selected as base value for C_{QG}. Table 4 presents two more cases with different weight coefficients each time. Comparing Tables 3 and 4, it becomes obvious that reactive power cost as well as the objective function increases from 0.3846 and 0.6019 to 0.7693 and 0.9625 as the reactive power generation cost weight is increased from 0.5 to 1. Another important observation is that by giving more weight to reactive power cost, the generation becomes expensive and it affects the power loss, which increases from 0.1474 to 0.1906 as cost weight is increased from 0.1 to 0.5 in Table 3.

Figs. 9 and 10 show that, when the objective function consists power loss only, the power loss is reduced to 0.1358 compared to 0.1474 for the previous case, however, the reactive power generations are much larger.

Case study 2: 39-bus system

The proposed algorithm is then tested on the modified IEEE 39-bus system, where 8 generators for reactive power control are connected at bus 4, 7, 12, 16, 20, 21, 23, and 27, reference voltages are taken as given in IEEE 39-bus data. The weight for power loss, voltage deviation and reactive power cost are selected as 1, 2, and 0.2, respectively. Reactive power generation cost coefficients are provided in Table 5.

Figs. 11 and 12 show that the objective function converges at 11th iteration as compared to 90th iteration for PSO as shown in Figs. 13 and 14. The comparison of Figs. 11 and 13 demonstrates the proposed distributed method converges to a smaller value of 15.2356 than 15.24 obtained by PSO. Figs. 15 and 16 validate that, by approximating $\delta_{ji} = 0$, the proposed distributed algorithm can still achieve comparable results as that of the one considering δ_{ji} at normal loading level.

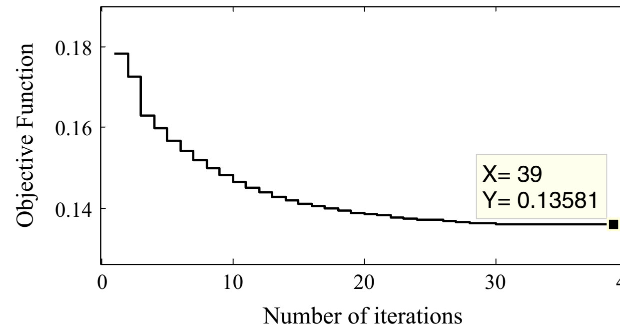

Fig. 9. Convergence of the objective function for the case with only power loss.

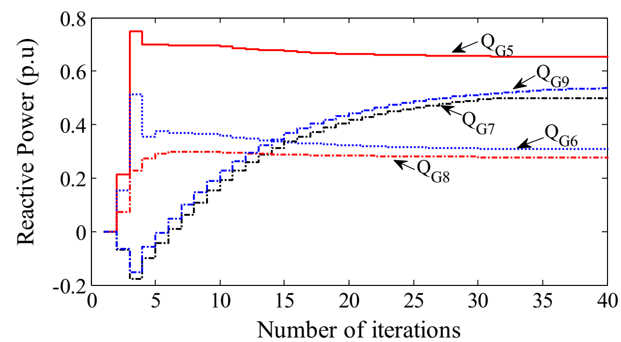

Fig. 10. Reactive power generation update for the case with only power loss.

Table 3
Summary for reactive power control results for 9-bus system.

	$W1 = 1, W2 = 10, W3 = 0.1$		$W1 = 1, W2 = 1, W3 = 0.5$	
	DA (p.u)	CA (p.u)	DA(p.u)	CA (p.u)
Q_{G5}	0.5686	0.4863	0.6377	0.6163
Q_{G6}	0.2409	0.1623	0.3631	0.3013
Q_{G7}	0.1080	0.3591	0.3155	0.3584
Q_{G8}	0.3281	0.2174	0.2587	0.2471
Q_{G9}	0.1147	0.2532	0.0463	0.0501
V_5	0.9980	0.9899	0.9705	0.9788
V_6	1.0014	0.9951	0.9787	0.9821
V_7	0.9947	1.0065	0.9828	0.9835
V_8	0.9980	1.0006	0.9778	0.9846
V_9	0.9980	1.0025	0.9804	0.9786
P_L	0.1474	0.1482	0.1906	0.1906
C_{QG}	0.3717	0.3720	0.3846	0.3846
f	0.5227	0.5262	0.6019	0.6023

Table 4
Result of higher weight on reactive power cost for reactive power control of 9-bus system.

	$W1 = 1, W2 = 1, W3 = 1$		$W1 = 1.5, W2 = 1, W3 = 1$	
	DA (p.u)	CA (p.u)	DA(p.u)	CA (p.u)
Q_{G5}	0.6377	0.5972	0.6028	0.6131
Q_{G6}	0.3632	0.3781	0.3512	0.3672
Q_{G7}	0.3156	0.3170	0.2715	0.2689
Q_{G8}	0.2588	0.2304	0.2462	0.2435
Q_{G9}	0.0464	0.0592	0.0010	0.0020
V_5	0.9706	0.9738	0.9572	0.9651
V_6	0.9787	0.9711	0.9686	0.9690
V_7	0.9828	0.9795	0.9735	0.9749
V_8	0.9778	0.9807	0.9674	0.9576
V_9	0.9805	0.9814	0.9721	0.9821
P_L	0.1906	0.1895	0.2910	0.2906
C_{QG}	0.7692	0.7705	0.7683	0.7660
f	0.9625	0.9631	1.0651	1.0704

Table 5
Cost coefficients for eight generators.

No.	a_{Pi} (p.u)	b_{Pi} (p.u)	No.	a_{Pi} (p.u)	b_{Pi} (p.u)
4	0.27	0.24	20	0.22	0.30
7	0.16	0.34	21	0.13	0.42
12	0.19	0.29	23	0.21	0.28
16	0.33	0.15	27	0.30	0.18

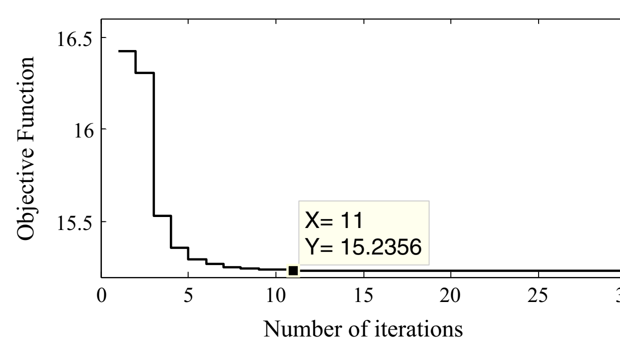

Fig. 11. Convergence of the objective function without approximation for 39-bus system.

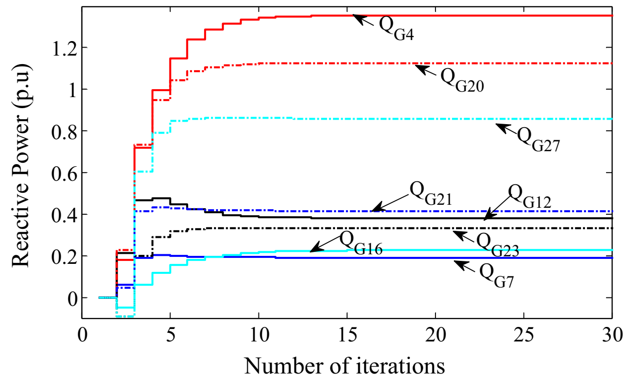

Fig. 12. Reactive power update without approximation for 39-bus system.

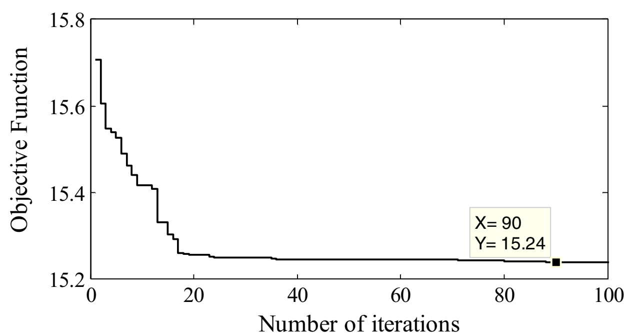

Fig. 13. Convergence of the objective function using PSO for 39-bus system.

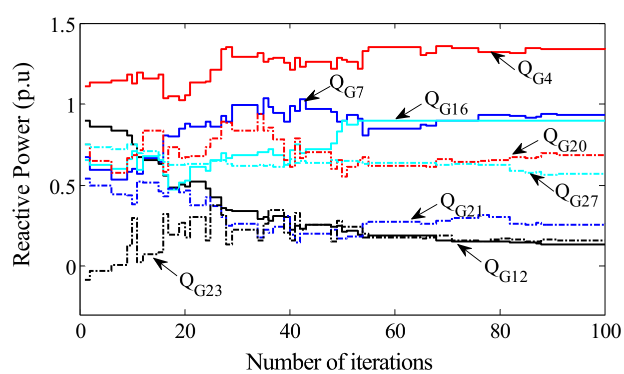

Fig. 14. Reactive power update using PSO for 39-bus system.

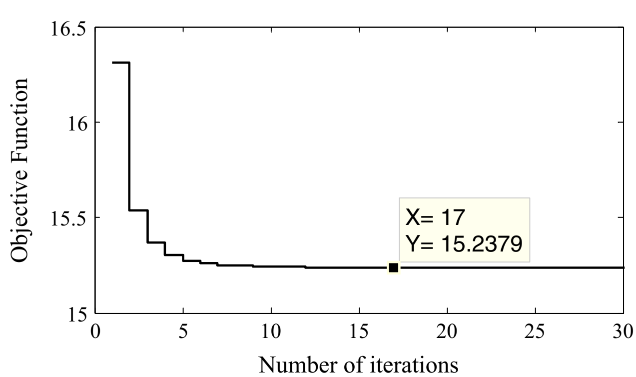

Fig. 15. Convergence of the objective function with $\cos(\delta_{ji}) = 1$ for 39-bus system.

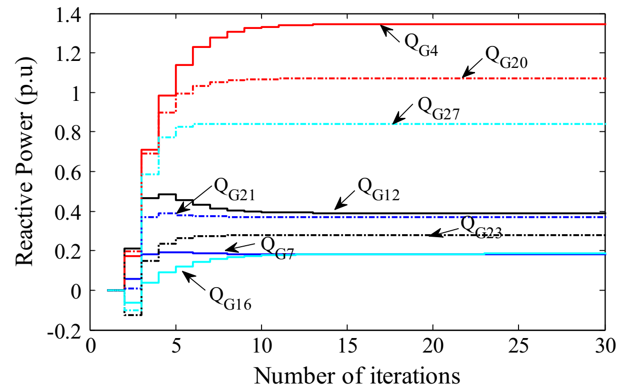

Fig. 16. Reactive power generation update with approximation of $\cos(\delta_{ij}) = 1$ for 39-bus system.

Case study 3: 162-bus system

Scalability of the proposed algorithm is investigated by testing on the modified IEEE 162-bus system [24]. 16 generators are attached at various buses for optimal reactive power control. Weight coefficient for reactive power generation cost, power loss and voltage deviation is set to be 0.1, 1 and 1 respectively. Cost coefficients for power generations are provided in Table 6.

The optimal solution of the objective function converges at 44th iteration for the proposed distributed algorithm without approximation, whereas the one with approximation of $\cos(\delta_{ij}) = 1$ converges at 51st iteration as shown in Figs. 17 and 19, respectively. The reactive power generation updates are shown in Figs. 18 and 20. One possible disadvantage is that a few more iterations are needed. The simulation results indicate that the proposed distributed algorithm for optimal reactive power control has great potential to be applied to large power systems.

For large power system, transmission systems are equipped with power line communication (PLC) for transmission of data

Table 6
Cost coefficients for 16 generators.

No.	a_{Pi} (p.u)	b_{Pi} (p.u)	No.	a_{Pi} (p.u)	b_{Pi} (p.u)
3	0.28	0.22	84	0.31	0.20
15	0.12	0.42	94	0.13	0.40
22	0.17	0.32	100	0.22	0.36
27	0.35	0.18	124	0.20	0.32
36	0.41	0.15	125	0.29	0.21
45	0.15	0.39	126	0.37	0.16
67	0.26	0.31	147	0.19	0.29
68	0.32	0.21	148	0.23	0.34

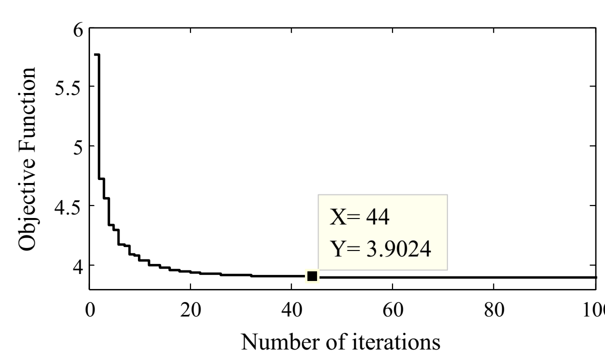

Fig. 17. Convergence of the objective function for 162-bus system without approximation.

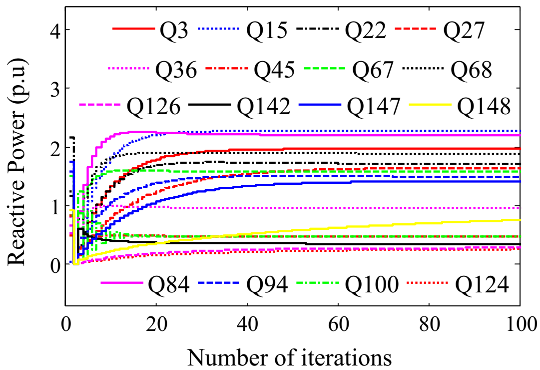

Fig. 18. Reactive power generation update for 162-bus system without approximation.

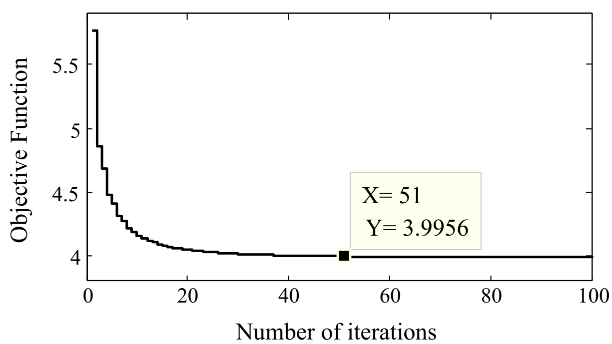

Fig. 19. Convergence of the objective function for 162-bus system with approximation.

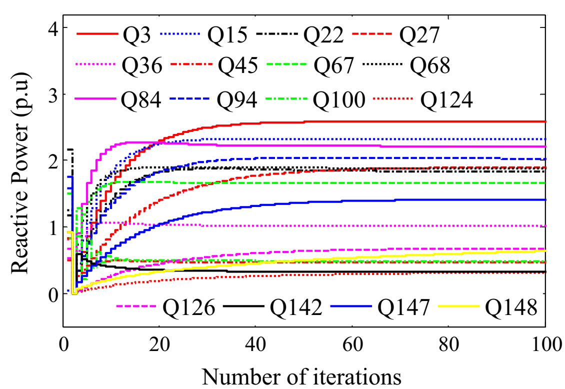

Fig. 20. Reactive power generation update for 162 bus system with approximation.

and for protection purposes. It is reasonable to assume that same PLC can be used to share the bus information among neighboring buses. As shown by the results that, the power loss and voltage deviation can be significantly reduced by optimizing the reactive power generation in a fully distributed way. Also, if compared with centralized control, it does not require a powerful centralized processor, and is computational efficient. In addition, due to the property of distributed control, it is less sensitive to single-point-failure, thus more reliable. [13,25]. Above all, distributed algorithm can be much faster than the centralized approach ideally, which is a desirable property to cope with the sudden fast variations of the power transmission system [26].

Conclusions

This paper proposed a distributed nonlinear control based algorithm for optimal reactive power control of multiple generators in a power grid. Active power loss, voltage deviation and reactive power generation cost are taken into consideration, and the optimal reactive power control of multiple generators is formulized as a non-convex problem. Only information exchange among neighboring buses is used to achieve the optimal solution, thus, the computational and communication burden are reduced compared to centralized algorithms. It has been demonstrated that by approximating $\cos(\delta_{ji}) = 1$, the calculation for online application is simplified and can still provide comparable results to that of the one without approximation at normal level of generation/load condition. The effectiveness of the proposed distributed nonlinear control based algorithm is validated by comparing to the centralized algorithm: PSO, for IEEE 9-bus, 39-bus and 162-bus systems.

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