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Payment mechanisms for electricity markets with uncertain supply

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ABSTRACT

This paper provides a framework for deriving payment mechanisms for intermittent, flexible and inflexible electricity generators who are dispatched according to the optimal solution of a stochastic program that minimizes the expected cost of generation plus deviation. The first stage corresponds to a pre-commitment decision, and the second stage corresponds to real-time generation that adapts to different realizations of a random variable. By taking the Lagrangian and decoupling in different ways we study two payment mechanisms with different properties.

1. Introduction

In response to pressure to reduce CO_2 emissions and increases in the penetration of renewables, electricity pool markets are procuring increasing amounts of electricity from intermittent sources such as wind and solar. This pressure can cause difficulties for the independent system operator (ISO) when dispatching generators. These difficulties arise because many thermal generators are inflexible and need to be informed of their generation obligations before the output of renewables is known.

In markets with large amounts of inflexible generation, generators can be provided with a pre-commitment setpoint before the generation output of renewables is known. The setpoint is determined by assuming that renewables generate their expected output capacity. The ISO then dispatches generators at the commencement of the trade period, with the objective of minimizing the cost of generation plus deviation from the setpoint, in order to manage fluctuations between the expected and the realized output capacity of intermittent renewables. We refer to this dispatch mechanism as the traditional mechanism.

As noted by Chao et al. in [3] this approach can be suboptimal in maximizing consumer welfare. The expected costs of corrective actions arising from surpluses or shortfalls are not priced into the market clearing problem, because the system operator does not consider uncertainty. Dispatches derived via the traditional mechanism therefore may have a small fuel cost and a large expected cost of correction.

With small quantities of intermittent renewable penetration the price of failing to consider uncertainty is small, as flexible generators can be re-dispatched at a low cost to meet shortfalls. However, as intermittent renewables increase their penetration, the cost of corrective actions becomes significant. Limits on flexible plants mean that expensive and inflexible plants need to be re-dispatched to meet large shortfalls in wind or solar generation. Therefore, when the penetration of intermittent renewables is high, a new dispatch mechanism is required.

The first alternative to the traditional dispatch mechanism was proposed by Bouffard and Galiana in [2], which involves clearing a co-optimized pool and reserve market using a two-stage stochastic program. The first stage is a capacity market which is cleared before random events pertaining to the availability of plants and transmission lines take on a realization. The second stage is a market clearing problem where only units committed in the first stage may be dispatched.

Subsequently, several authors including [9,7,4,5,10] have proposed explicitly modelling uncertainty in the generation output of intermittent renewables as a random variable. Existing ancillary market services are supplemented with a real-time market, which is cleared after the generation capacity of renewables is realized. This can be achieved by casting the market clearing problem within a two-stage stochastic programming framework, where the first stage corresponds to a pre-commitment setpoint, and the second stage corresponds to a real-time dispatch under a particular realization of the generation output of renewables. We refer to this dispatch mechanism as the *Stochastic Dispatch Mechanism*, or SDM.

In SDM, the pre-commitment setpoint can be interpreted as the output level at which generators prepare to generate before wind generation is realized. The real-time dispatch quantity is the generation quantity which the generator produces throughout the trade period, after the generation of renewables is known. A deviation to increase or decrease a generators real-time dispatch

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output from its day-ahead setpoint causes the generator to incur a cost, in the form of e.g. wear-and-tear on generation equipment.

To replicate the dispatch found by SDM and maximize expected total social welfare, it is necessary to pay market participants in a manner which causes their optimal behaviour to align with the optimal dispatch found by a system operator when solving SDM. In the literature (see [9,7,4,10]) this is achieved by compensating and charging market participants with a single-settlement payment mechanism, where the amount charged to each participant under each possible realization of renewable generation is made known apriori, and the actual charge to each participant occurs after renewable generation is realized.

In single-settlement payment mechanisms, two desirable attributes are revenue adequacy and cost recovery. A payment mechanism is revenue adequate if and only if the system operator collects at least as much from consumers as it pays generators. A payment mechanism exhibits cost recovery if and only if generators are paid at least as much as their fuel costs.

The first paper to study this aspect of pricing was by Wong and Fuller [9] who introduced a two-stage dispatch mechanism and suggested several payment mechanisms to compensate generators that were revenue adequate and recovered costs on average. A similar approach was taken by Pritchard et al. [7] who focused on wind intermittency. The dispatch mechanism of [7] was improved by Zakeri et al. in [10], who observed that welfare could be enhanced by removing the first-stage constraint on supply meeting demand, and suggested an alternative payment mechanism, which is revenue adequate in every scenario and provides generator cost recovery on average.

In this paper, we revisit the dispatch mechanism of [10] in a classical Lagrangian framework for modelling perfectly competitive partial equilibrium. We define a price of information, the amount which a generator needs to be paid to enforce nonanticipativity on its first-stage generation. The price of information leads to a discriminatory variation on the pricing mechanism of [10], giving cost recovery in every scenario, expected revenue adequacy and expected revenue equivalence for all agents compared to [10]'s mechanism, assuming that agents behave in the risk-neutral price-taking manner assumed in [7] and [10]. This discriminatory variation explicitly identifies the uplift payment required to make all generators whole, and ensures that the expected uplift payment to each generator is zero.

2. The stochastic dispatch mechanism

Following [7] and [10], we consider a SDM based on the DC-Load Flow model of a constrained electricity transmission network. We follow [1] in letting uncertainty be represented by the scenario $\omega \in \Omega$, which occurs with probability $P(\omega)$. We assume that a realization of ω prescribes all uncertainty due to intermittent generation in an electricity pool market, and that the sample space Ω is a finite set containing all possible future realizations of ω. Although the real distribution of wind is not finite, Ω can be viewed as an approximation, obtained for example by sampling from the true distribution if this is available. The dispatch models obtained are then Sample Average Approximations for which one can derive asymptotic convergence results as the sample size increases (see [8] for a general theory, and [10] for asymptotic results in the context of SDM). We note that any computational dispatch model must work with a finite Ω, and since our focus in this paper is on different payment mechanisms for these models, we restrict attention to this case.

Our notation closely follows that of [7] and [10] in which deterministic variables are denoted by lower case Roman symbols, random variables by upper case Roman symbols, and prices by lower case Greek symbols.

The sets and indices in the Stochastic Program (SP) solved to obtain the day-ahead setpoint in SDM are defined as follows:

- i is the index of a generator. We assume perfect competition, so the ownership of generation does not affect the solution. This means that each agent i can be thought of as operating a single unit i.
- $\mathcal{N}$ is the set of all nodes in the transmission network.
- $j(i)$ is the node $j \in \mathcal{N}$ where generator i is located.
- $a_{in} = 1$ when agent i is located at node n and 0 otherwise.
- $\mathcal{T}(n)$ is the set of all offers at node n.
- $\mathcal{F}$ is a set constraining the flows in the network to meet thermal limits and the DC-Load Flow constraints imposed by Kirchhoff's Laws. We assume that $0 \in \mathcal{F}$.

The decision variables in SP are defined as follows:

- x_i is the day-ahead setpoint level which generator i is advised to prepare to produce before the generation capacity of intermittent renewables is known.
- $X_i(\omega)$ is the real-time dispatch produced by generator i in scenario ω.
- $U_i(\omega)$ and $V_i(\omega)$ are the amounts which generator i deviates up/down by in scenario ω. That is, $U_i(\omega) = \max(X_i(\omega) - x_i, 0)$, and $V_i(\omega) = \max(x_i - X_i(\omega), 0)$.
- $F(\omega) \in \mathcal{F}$ is the vector of branch flows in the network in scenario ω.
- $\tau_n(F(\omega))$ is the net amount of energy flowing from the grid into node n in scenario ω. We assume that τ_n is a concave function of F with $\tau_n(0) = 0,\ \forall n \in \mathcal{N}$.

The shadow prices for the constraints in SP are defined as follows:

- $\lambda_n(\omega)$ is the marginal cost of generating one additional unit of electricity at node n in scenario ω under SP. As the constraint on supply meeting demand is an inequality, we require that $\lambda_n(\omega) \geq 0,\ \forall n \in \mathcal{N}$.
- $\rho_i(\omega)$ is the marginal cost of agent i generating one additional unit of electricity in scenario ω under SP.
- $\bar{\lambda}_n = \mathbb{E}_\omega[\lambda_n(\omega)]$ is the expected value of the shadow price $\lambda_n(\omega)$ in SP.

The problem data in SP are defined as follows:

- c_i is the marginal cost incurred by generator i.
- $D_n(\omega)$ is the consumer demand at node n in scenario ω.
- $r_{u,i}$ and $r_{v,i}$ are the marginal costs incurred by generator i for deviating up or down in its generation. We require that $r_{u,i} > c_i > r_{v,i}$ to avoid providing generator i with arbitrage opportunities when it is choosing whether to ramp up or down.
- $G_i(\omega)$ is the maximum output capacity of generator i in scenario ω.

The day-ahead setpoint is found by solving the following stochastic program for x, as defined by Zakeri et al. in [10]:

$$\begin{aligned}
\text{SP: } \min\ & c^\top x + \sum_{\omega} P(\omega)(r_u^\top U(\omega) + r_v^\top V(\omega)) \\
\text{s.t. } & \sum_{i \in \mathcal{T}(n)} X_i(\omega) + \tau_n(F(\omega)) \geq D_n(\omega),\ \forall n, \forall \omega \in \Omega,\ [P(\omega)\lambda_n(\omega)], \\
& x + U(\omega) - V(\omega) = X(\omega),\ \ \forall \omega \in \Omega,\ [P(\omega)\rho(\omega)], \\
& F(\omega) \in \mathcal{F}, \quad \forall \omega \in \Omega, \\
& 0 \leq X(\omega) \leq G(\omega),\ \ U(\omega), V(\omega), x \geq 0, \quad \forall \omega \in \Omega.
\end{aligned}$$

Observe that we have matched each constraint with a corresponding shadow price in square brackets. Since $\lambda_n(\omega)$ is the marginal cost of meeting additional demand at node n in scenario ω, and $\rho_i(\omega)$ is generator i's marginal cost of generation in scenario ω, each of these must be weighted by probability $P(\omega)$.

After the optimal day-ahead setpoint x^* is found, the intermittent generation scenario $\omega = \hat{\omega}$ is realized, and the ISO follows the dispatch defined by $\left(X^*(\hat{\omega}), U^*(\hat{\omega}), V^*(\hat{\omega}), F^*(\hat{\omega})\right)$. In a competitive market setting, agents respond to price signals, so the remuneration from these prices for each agent's dispatch provides incentives to respond as the dispatch dictates. To model this, we decouple SP using a Lagrangian approach.

3. Payment mechanisms

Assume that SP satisfies a constraint qualification such as a Slater condition (see [1]). Then we can solve SP by minimizing a Lagrangian. We choose first to use multipliers only on energy balance constraints to yield:

$$\begin{aligned}\mathscr{L} &= \sum_i c_i x_i \\ &\quad + \sum_\omega P(\omega) \sum_i \left(r_{u,i} U_i(\omega) + r_{v,i} V_i(\omega)\right) \\ &\quad + \sum_\omega P(\omega) \sum_n \Big(D_n(\omega) - \sum_i a_{in} X_i(\omega) - \tau_n(F(\omega))\Big)\lambda_n(\omega),\end{aligned}$$

which is to be minimized, subject to the following constraints:

$$\begin{aligned}&x_i + U_i(\omega) - V_i(\omega) = X_i(\omega),\ \forall i,\ \forall \omega \in \Omega, \\ &0 \le X_i(\omega) \le G_i(\omega),\ U_i(\omega), V_i(\omega), x_i \ge 0, \\ &F(\omega) \in \mathcal{F}.\end{aligned}$$

All Lagrange multipliers should be nonnegative. Observe that $\sum_n a_{in}\lambda_n(\omega) = \lambda_{j(i)}(\omega)$. Rearranging $\mathscr{L}$ and expressing as a maximization we obtain:

$$\begin{aligned}\max &- \sum_i c_i x_i - \sum_n \sum_\omega P(\omega)\lambda_n(\omega) D_n(\omega) \\ &+ \sum_i \sum_\omega P(\omega)\lambda_{j(i)}(\omega) X_i(\omega) \\ &+ \sum_n \sum_\omega P(\omega)\lambda_n(\omega)\tau_n(F(\omega)) \\ &- \sum_i \sum_\omega P(\omega)\left(r_{u,i}U_i(\omega) + r_{v,i}V_i(\omega)\right) \\ \text{s.t. } & x + U(\omega) - V(\omega) = X(\omega),\ \forall \omega \in \Omega, \\ & 0 \le X_i(\omega) \le G_i(\omega),\ U_i(\omega), V_i(\omega), x_i \ge 0, \\ & F(\omega) \in \mathcal{F}.\end{aligned}$$

Observe that this problem can be decoupled by generation agent. Given prices $\lambda_{j(i)}(\omega)$ each generation agent i can determine its own dispatch by solving the following stochastic program.

$$\begin{aligned}\text{SP1:}\ \max &\sum_\omega P(\omega)\Big(-c_i x_i + \lambda_{j(i)}(\omega) X_i(\omega) - r_{u,i} U_i(\omega) - r_{v,i} V_i(\omega)\Big) \\ \text{s.t. } & x_i + U_i(\omega) - V_i(\omega) = X_i(\omega),\ \forall \omega \in \Omega, \\ & 0 \le X_i(\omega) \le G_i(\omega),\ U_i(\omega), V_i(\omega), x_i \ge 0.\end{aligned}$$

The objective function of SP1 also specifies the remuneration for each agent. Here if scenario $\hat{\omega}$ is observed then purchasers in node n pay $\lambda_n(\hat{\omega}) D_n(\hat{\omega})$ and generator i is paid $\lambda_{j(i)}(\hat{\omega}) X_i(\hat{\omega})$. This is the payment scheme studied by Zakeri et al. [10]. As demonstrated by [10] the amount collected from purchasers is always at least enough to cover the payments being made to generators. This result is called revenue adequacy.

Definition 3.1. A payment mechanism is *revenue adequate* if and only if in every scenario $\omega \in \Omega$, clearing the market does not leave the system operator in a financial deficit. As shown by Philpott and Pritchard in [6], revenue adequacy is equivalent to the following statement:

$$\sum_n \lambda_n(\omega)\tau_n(F(\omega)) \ge 0,\ \forall \omega \in \Omega.$$

Proposition 1. *If $(x^*, X^*(\omega), U^*(\omega), V^*(\omega), F^*(\omega))$ solves SP, then paying $\lambda_{j(i)}(\omega) X_i(\omega)$ to generator i and charging $\lambda_n(\omega) D_n(\omega)$ to demand agent n results in revenue adequacy in every scenario.*

Proof. See [10]. □

It is not hard to see that in some circumstances a generator might not be compensated for the short-run costs of its first-stage dispatch. For example, if $x_i^* > 0$ and $X_i^*(\hat{\omega}) = 0$ in the realized scenario then no revenue will be earned to cover the cost $c_i x_i^* + r_{v,i} V_i^*(\hat{\omega})$. If this is the case, then there is no incentive for generator i to participate in the market, unless the payments can be made whole in some manner.

Definition 3.2. A payment mechanism exhibits *cost recovery* if and only if in every scenario $\omega \in \Omega$, all generators recover their short-run (fuel and deviation) costs. That is,

$$R_i(\omega) - c_i x_i(\omega) - r_{u,i} U_i(\omega) - r_{v,i} V_i(\omega) \ge 0, \quad \forall i, \quad \forall \omega \in \Omega,$$

where $R_i(\omega)$ is generator i's revenue in scenario ω.

The payment mechanism of SP1 does not exhibit cost recovery. We say that a market clearing mechanism exhibits *expected cost recovery* if all generators recover their generation and ramping costs in expectation. This was shown to be the case for SP1 by [10].

Proposition 2. *Paying $\lambda_{j(i)}(\omega) X_i(\omega)$ to generation agent i in scenario ω results in expected cost recovery.*

Proof. See [10]. □

It is not surprising that the payment mechanism of SP1 might result in some generators not recovering their costs. In SP1 each agent is presented with scenario prices and solves a stochastic program to maximize their expected profit. The constraints

$$x_i + U_i(\omega) - V_i(\omega) = X_i(\omega), \forall \omega \in \Omega,$$

can be rewritten

$$\begin{aligned}&\bar{x}_i(\omega) + U_i(\omega) - V_i(\omega) = X_i(\omega), \forall \omega \in \Omega, \\ &\bar{x}_i(\omega) = x_i, \forall \omega \in \Omega,\end{aligned}$$

where the second set of constraints are called *nonanticipativity* constraints.

Cost recovery for each generator would be possible if there were only one scenario, or if the generator's nonanticipativity constraints were relaxed, so that the first-stage decision x_i can vary with scenario (becoming $\bar{x}_i(\omega)$). This enables a generator to use perfect foresight in choosing $\bar{x}_i(\omega)$ and gain from this. Enforcing a nonanticipativity constraint incurs a cost from the loss of information, and so to ensure a nonnegative profit, each generator should be compensated with an ex-post information payment in scenarios where it makes less than its expected profit and charged an ex-post information rent in scenarios where it makes a profit greater than its expected profit.

The values of these payments and charges can be made explicit by applying a Lagrangian relaxation of nonanticipativity constraints. Equivalently, one can introduce Lagrange multipliers for the constraints linking x_i and $X_i(\omega)$ in SP. This gives the following Lagrangian.

$$\begin{aligned}\hat{\mathscr{L}} &= \sum_i c_i x_i \\ &\quad + \sum_\omega P(\omega) \sum_i \big(r_{u,i} U_i(\omega) + r_{v,i} V_i(\omega)\big) \\ &\quad + \sum_\omega P(\omega) \sum_n \Big(D_n(\omega) - \sum_i a_{in} X_i(\omega) - \tau_n(F(\omega))\Big)\lambda_n(\omega) \\ &\quad + \sum_\omega P(\omega) \sum_i \rho_i(\omega)\big(V_i(\omega) - U_i(\omega) + X_i(\omega) - x_i\big),\end{aligned}$$

which is to be minimized, subject to the following constraints:

$$0 \le X_i(\omega) \le G_i(\omega),\ U_i(\omega), V_i(\omega), x_i \ge 0,$$
$$F(\omega) \in \mathcal{F}.$$

Rearranging $\hat{\mathscr{L}}$ and expressing as a maximization yields:

$$\begin{aligned}\max &\sum_i (-c_i + \sum_\omega P(\omega)\rho_i(\omega)) x_i \\ &+ \sum_i \sum_\omega P(\omega)\left(\lambda_{j(i)}(\omega) - \rho_i(\omega)\right) X_i(\omega) \\ &+ \sum_n \sum_\omega P(\omega)\lambda_n(\omega)\tau_n(F(\omega)) \\ &- \sum_n \sum_\omega P(\omega)\lambda_n(\omega) D_n(\omega) \\ &+ \sum_i \sum_\omega P(\omega)\big((\rho_i(\omega) - r_{u,i})U_i(\omega) + (-\rho_i(\omega) - r_{v,i})V_i(\omega)\big) \\ \text{s.t. } & 0 \le X_i(\omega) \le G_i(\omega),\ U_i(\omega), V_i(\omega), x_i \ge 0, \\ & F(\omega) \in \mathcal{F}.\end{aligned}$$

This can be decoupled by generation agent. Generation agent i solves the following problem:

$$\begin{aligned}\text{SP2: } \max &\sum_\omega P(\omega)\Big((\rho_i(\omega) - c_i)x_i + (\lambda_{j(i)}(\omega) - \rho_i(\omega))X_i(\omega) \\ &+ (\rho_i(\omega) - r_{u,i})U_i(\omega) + (-\rho_i(\omega) - r_{v,i})V_i(\omega)\Big) \\ \text{s.t. } & 0 \le X_i(\omega) \le G_i(\omega),\ U_i(\omega), V_i(\omega), x_i \ge 0.\end{aligned}$$

The objective function coefficient for the first-stage decision can be rewritten as $\bar{\rho}_i - c_i$, where:

$$\bar{\rho}_i = \sum_\omega P(\omega)\rho_i(\omega).$$

Observe that SP2 then decouples into the optimization of an action x_i, and optimal actions for each scenario. When $\bar{\rho}_i > c_i$, x_i^* will be infinite, so we require $\bar{\rho}_i \le c_i$, and $(c_i - \bar{\rho}_i)x_i^* = 0$. This means x_i^* will be non zero only when generator i is paid exactly its marginal cost c_i. Similarly for finiteness we require $\rho_i(\omega) \le r_{u,i}$ and $(-\rho_i(\omega) \le r_{v,i})$, yielding $(\rho_i(\omega) - r_{u,i})U_i^*(\omega) = 0$ and $(-\rho_i(\omega) - r_{v,i})V_i^*(\omega) = 0$. We collect these results in the following lemmas.

Lemma 3. *For every generation agent i, the optimal dispatch policy $(x_i^*, X_i^*(\omega), U_i^*(\omega), V_i^*(\omega))$ satisfies:*

$$(\bar{\rho}_i - c_i)x_i^* + (\rho_i(\omega) - r_{u,i})U_i^*(\omega) + (-\rho_i(\omega) - r_{v,i})V_i^*(\omega) = 0.$$

Lemma 4. *For every generation agent i with $X_i^*(\omega) > 0$, it follows that $\rho_i(\omega) \le \lambda_{j(i)}(\omega)$.*

Proof. We take the contrapositive. Suppose $\rho_i(\omega) > \lambda_{j(i)}(\omega)$. Then since $X_i(\omega) \ge 0$ it follows that the optimal choice of $X_i(\omega)$ is $X_i^*(\omega) = 0$. □

We use Lemmas 3 and 4 to yield the following result:

Proposition 5. *For every i, if agent i has made the optimal choice of x_i^* then paying:*

$$\bar{\rho}_i x_i^* + (\lambda_{j(i)}(\omega) - \rho_i(\omega))X_i^*(\omega) + \rho_i(\omega)U_i^*(\omega) - \rho_i(\omega)V_i^*(\omega),$$

results in cost recovery in every scenario.

Proof. Assume that generation agent i has seen the realization of ω and is making its second-stage decision. Then agent i's profit in scenario ω is:

$$\begin{aligned}\phi_i^*(\omega) = \max_{X_i(\omega), U_i(\omega), V_i(\omega)} & (\bar{\rho}_i - c_i)x_i^* + (\lambda_{j(i)} - \rho_i(\omega))X_i(\omega) \\ & + (\rho_i(\omega) - r_{u,i})U_i(\omega) + (-\rho_i(\omega) - r_{v,i})V_i(\omega) \\ \text{s.t. } & 0 \le X_i(\omega) \le G_i(\omega),\ U_i(\omega), V_i(\omega) \ge 0.\end{aligned}$$

Now, by Lemma 3, we know that:

$$(\bar{\rho}_i - c_i)x_i^* + (\rho_i(\omega) - r_{u,i})U_i^*(\omega) + (-\rho_i(\omega) - r_{v,i})V_i^*(\omega) = 0.$$

Therefore, $\phi_i^*(\omega) = (\lambda_{j(i)} - \rho_i(\omega))X_i^*(\omega)$. Furthermore, we know from Lemma 4 that $(\lambda_{j(i)} - \rho_i(\omega))X_i^*(\omega) \ge 0$. Therefore, $\phi_i^*(\omega) \ge 0,\ \forall \omega \in \Omega$. □

Proposition 5 gives cost recovery for the payment mechanism from SP2 because $\bar{\rho}_i = c_i$ whenever $x_i^* > 0$. This discriminatory mechanism means each generator that is dispatched in the first stage has their exact costs for this dispatch paid by the ISO. This might require the ISO to suffer a negative rent if a particular scenario is realized, but as shown in Corollary 7 the ISO rental will be positive in expectation. It follows that the ISO will not incur a deficit in the long run if we assume that the random outcomes in each dispatch instance are i.i.d.

The payment mechanism in Proposition 5 can be rewritten as follows:

Proposition 6. *For every i, if generation agent i makes the optimal choice of x_i^*, $X_i^*(\omega)$, $U_i^*(\omega)$ and $V_i^*(\omega)$ then paying:*

$$\bar{\rho}_i x_i^* + (\lambda_{j(i)}(\omega) - \rho_i(\omega))X_i^*(\omega) + \rho_i(\omega)U_i^*(\omega) - \rho_i(\omega)V_i^*(\omega),$$

gives the same profit as paying agent i:

$$(\bar{\rho}_i - \rho_i(\omega))x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega).$$

Proof. Agent i's actions satisfy:

$$x_i^* + U_i^*(\omega) - V_i^*(\omega) = X_i^*(\omega),\quad \forall \omega \in \Omega.$$

It follows that:

$$\begin{aligned}&\bar{\rho}_i x_i^* + (\lambda_{j(i)}(\omega) - \rho_i(\omega))X_i^*(\omega) + \rho_i(\omega)U_i^*(\omega) - \rho_i(\omega)V_i^*(\omega) \\ &= \bar{\rho}_i x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega) + \rho_i(\omega)(U_i^*(\omega) - X_i^*(\omega) - V_i^*(\omega)) \\ &= \bar{\rho}_i x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega) - \rho_i(\omega)x_i^* \\ &= (\bar{\rho}_i - \rho_i(\omega))x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega). \quad \square\end{aligned}$$

Proposition 6 shows that each generation agent is paid exactly the same as in the first payment scheme except they are compensated by $(\bar{\rho}_i - \rho_i(\omega))x_i^*$ for the first-stage dispatch. The expected value of this compensation will be zero, as this compensation will be negative in some scenarios, and so in expectation generators will receive the same amount under both payment schemes.

A market clearing mechanism is said to exhibit *revenue adequacy in expectation* if the rental collected by the ISO is nonnegative in expectation. The first payment scheme satisfies this because it is revenue adequate in every scenario, and the discussion above demonstrates that the second payment scheme additionally pays a price of information which has an expected value of zero, meaning that the second scheme is revenue adequate in expectation. We formalize this as corollaries to Proposition 6.

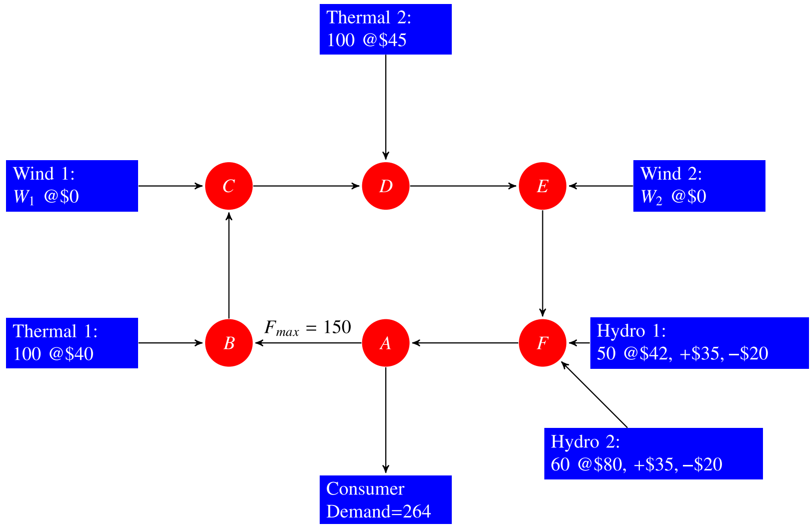

Fig. 1. The six node example from Pritchard et al. [7].

Table 1
First and second stage dispatches.

Agent	First stage dispatch	Mean 2nd stage dispatch	Std deviation 2nd stage dispatch
Thermal 1	74	74	0
Wind 1	n/a	60	20
Thermal 2	40	40	0
Wind 2	n/a	60	20
Hydro 1	40	30.5	17.2
Hydro 2	0	4	9.4
Sum agents	n/a	268.5	7.9

Table 2
Dispatch summary statistics.

Agent	% Not dispatched	Min 2nd stage dispatch	Max 2nd stage dispatch
Thermal 1	0	74	74
Wind 1	0	30	90
Thermal 2	0	40	40
Wind 2	0	30	90
Hydro 1	4	0	50
Hydro 2	80	0	40
Sum agents	0	264	294

Corollary 7. *If $(x^*, X^*(\omega), U^*(\omega), V^*(\omega), F^*(\omega))$ solves SP, then paying each generation agent $(\bar{\rho}_i - \rho_i(\omega))x_i^* + \lambda_{j(i)}(\omega)X_i^*(\omega)$ results in revenue adequacy in expectation.*

Corollary 8. *A sufficient condition for revenue adequacy in scenario $\hat{\omega}$ is:*

$$\left(\rho(\hat{\omega}) - \bar{\rho}\right)^\top x^* \geq 0.$$

Corollary 9. *If all agents act as risk-neutral price takers, then the payment mechanism of charging consumer n the amount $\lambda_n(\omega)D_n(\omega)$ and paying generator i the amount $\lambda_{j(i)}X_i(\omega)$ results in the same expected profit for each generator, each consumer and the ISO as the payment mechanism of charging consumer n the amount $\lambda_n(\omega)D_n(\omega)$ and paying generator i the amount $(\bar{\rho}_i - \rho_i(\omega))x_i(\omega) + \lambda_{j(i)}X_i(\omega)$.*

4. A six node example

To illustrate the differences between the two payment mechanisms, we analyse the payoffs to agents that would occur in the six node example described by [7].

The transmission network is depicted in Fig. 1, where there are two inflexible thermal generators who cannot deviate in the second stage, two flexible hydro generators who can ramp up or down at costs of 35 and 20 per unit, and two intermittent wind generators. The generation capacity of each generator and cost per unit generation are indicated by "X@\$$Y$". The wind generators independently produce one of the following amounts with equal probability: $\{30, 50, 60, 70, 90\}$, resulting in 25 scenarios each having probability 0.04. There is a single consumer who requires 264 units of generation in each scenario and in the second stage a transmission constraint dictates that at most 150 units can be transmitted from node A to node B or vice versa. The reactances of all lines are assumed to be identical, meaning $\frac{5}{6}$ of the power generated by Thermal 1 flows via the constrained line, and $\frac{2}{3}$ of the power generated by Wind 1 flows via the constrained line. In order to prevent dual degeneracy, we impose quadratic losses on all transmission lines, with a loss coefficient of 10^{-8}. That is, $\tau_n(f) = \sum_k(f_{nk} - 10^{-8}f_{nk}^2) + \sum_k(-f_{kn} - 10^{-8}f_{kn}^2)$, where f_{nk} is a flow into node n from node k, and f_{kn} is a flow out of node n to node k.

Summary statistics regarding the first and second stage dispatches are available in Tables 1 and 2, and payoffs of each participant under the payment mechanisms considered in this paper are available in Tables 3 and 4. The Expected Profit numbers illustrate the equivalence of payoffs in expectation, and the Std Deviation values indicate how the two mechanisms allocate risk to agents and the ISO in each case.

Table 3
Statistics for the payment mechanism corresponding to SP1.

Agent	Expected profit	Std deviation	% Negative profit	Min profit	Max profit
Thermal 1	0	3423.4	68	−2960	5 550
Wind 1	2010.2	2083.2	0	0	7 630
Thermal 2	0	1703.8	68	−1800	2 800
Wind 2	2288.8	1689.2	0	0	6 900
Hydro 1	444	1724.7	64	−800	3 300
Hydro 2	0	0	0	0	0
ISO	2250	3812.5	0	0	17 325
Sum agents	6993	9328.1	24	−5560	22 000

Table 4
Statistics for the payment mechanism corresponding to SP2.

Agent	Expected profit	Std deviation	% Negative profit	Min profit	Max profit
Thermal 1	0	0	0	0	0
Wind 1	2010.2	2083.2	0	0	7 630
Thermal 2	0	0	0	0	0
Wind 2	2288.8	1689.2	0	0	6 900
Hydro 1	444	792	0	0	1 900
Hydro 2	0	0	0	0	0
ISO	2250	5736.2	64	−5560	15 505
Sum agents	6993	9328.1	24	−5560	22 000

Indeed, for a worst case risk measure, Table 3 identifies a scenario where wind generator 1 can produce up to 90 units, wind generator 2 can produce up to 90 units, agents lose $5560 and the ISO loses nothing. In contrast, the same scenario in Table 4 leads to an ISO shortfall of $5560 and all agents recovering their costs.

This scenario also shows why it is not possible in a stochastic dispatch to have both revenue adequacy and cost recovery in every scenario. Here 114 units of demand are satisfied by the inflexible thermal generators, leaving 150 units of demand to be satisfied. However, 180 units of wind generation are available, meaning that 30 units of wind generation must be shed, resulting in a real-time marginal price at each node of zero. In this scenario, either the thermal generators or the ISO must experience a shortfall.

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