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Economic dispatch using chaotic bat algorithm

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ABSTRACT

This paper presents the application of a new metaheuristic optimization algorithm, the chaotic bat algorithm for solving the economic dispatch problem involving a number of equality and inequality constraints such as power balance, prohibited operating zones and ramp rate limits. Transmission losses and multiple fuel options are also considered for some problems. The chaotic bat algorithm, a variant of the basic bat algorithm, is obtained by incorporating chaotic sequences to enhance its performance. Five different example problems comprising 6, 13, 20, 40 and 160 generating units are solved to demonstrate the effectiveness of the algorithm. The algorithm requires little tuning by the user, and the results obtained show that it either outperforms or compares favorably with several existing techniques reported in literature.

1. Introduction

ED (Economic Dispatch) is an important optimization problem in electric power systems which aims at allocating optimum generation values to minimize the cost while simultaneously satisfying several equality and inequality constraints. A naive approach would be to consider the cost curve as continuous and quadratic in nature. In practice, thermal power plants have a non-smooth and non-convex cost curve due to valve point loading effects [1] and discontinuities due to POZs (prohibited operating zones).

Given the importance of the ED problem, several attempts have been made to solve it using a variety of methods, classical and non-classical. Classical, gradient based methods like linear programming, Base-Point and Participation Factors, λ-iteration method [1], gradient method [2], branch and bound [3], quadratic programming [4] were initially proposed. The presence of POZs and multiple fuel options make the ED problem discontinuous [5]; this is a complication that gradient based methods find difficult to deal with, since gradient based methods are for smooth and continuous objective functions. Several modifications to the gradient based methods have been presented recently, to address these complications, like the dimensional steepest decline method [5] and Big-M method [6]. These involve additional computation to account for these complications.

Non-classical, metaheuristic methods owe their popularity to their ability to deal easily with these difficulties in the ED problem. The metaheuristic methods include the GA (Genetic Algorithm) [8,9], EP (Evolutionary Programming) [10], PSO (Particle Swarm Optimization) and its variants [11–15], neural networks [16,17], DE (Differential Evolution) [18] and so on.

The metaheuristic techniques in turn can be loosely classified into several categories like evolutionary algorithms, SI (swarm intelligence) and immune algorithms. SI techniques are inspired by the flocking behavior of agents like birds, bees and bats. In these techniques, the agents belonging to the swarm or population interact locally with each other and their environment in an organized and decentralized fashion to reach the required optimum solution.

Several SI algorithms have been developed and applied to solve the ED problem like the PSO [11], FA (Firefly Algorithm) [19], Bat Algorithm [20], DS (Differential Search) [21], and ABC (Artificial Bee Colony Algorithm) [22].

A recent advancement in optimization using metaheuristic algorithms has been to enhance existing algorithms by using chaotic sequences to tune the parameters that control the performance of the algorithm to improve diversity and avoid premature convergence [23], [24]. Chaos is characterized by ergodicity, stochastic

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Table 1
Table of abbreviations.

Optimization technique/algorithm	Abbreviation
Artificial Bee Colony	ABC
Backtracking Search Algorithm	BSA
Bat Algorithm	BA
Biogeography-Based Optimization	BBO
Chaotic Bat Algorithm	CBA
Chemical Reaction Optimization	CRO
Classical Evolutionary Programming	CEP
Conventional Genetic Algorithm with Multiplier Updating	CGA_MU
Cross-Entropy Method and Sequential Quadratic Programming	CE-SQP
Differential Evolution	DE
Differential Evolution with Biogeography-Based Optimization	DE/BBO
Differential search	DS
Dimensional Steepest Decline Method	DSD
Distributed Auction-Based Algorithm	AA(Dist.)
Evolutionary Programming	EP
Evolutionary Programming with Sequential Quadratic Programming	EP-SQP
Firefly Algorithm	FA
Genetic Algorithm – Ant Colony Optimization (special class)	GA-API
Genetic Algorithm – Binary	GA Binary
Group Search Optimizer	GSO
Hopfield Modeling	HM
Hybrid Chemical Reaction Optimization with Differential Evolution	HCRO-DE
Hybrid Differential Evolution	HDE
Improved Fast Evolutionary Programming	IFEP
Improved Genetic Algorithm with Multiplier Updating	IGA_MU
Modified Artificial Bee Colony Algorithm	MABC
Multiple Tabu Search	MTS
New Particle Swarm Optimization with Local Random Search	NPSO-LRS
Oppositional Real Coded Chemical Reaction Optimization	ORCCRO
Particle Swarm Optimization	PSO
Particle Swarm Optimization with the Sequential Quadratic Programming	PSO-SQP
Passive Congregation-based Particle Swarm Optimization	PC-PSO
Quantum Particle Swarm Optimization	QPSO
Random Drift Particle Swarm Optimization	RDPSO
Real Coded Chemical Reaction Optimization	RCCRO
Self-Tuning Hybrid Differential Evolution	STHDE
Self-Organizing Hierarchical Particle Swarm Optimization	SOH-PSO
Simple Particle Swarm Optimization	SPSO
Simulated Annealing	SA
Society-Civilization Algorithm	SCA
Species-based Quantum Particle Swarm Optimization	SQPSO
Tabu Search	TS
θ-Particle Swarm Optimization	θ –PSO

properties, and regularity [25]. Chaotic maps or sequences have been used instead of random numbers in different stages of an algorithm to improve the performance [26]. Several chaos based algorithms have been presented to solve various engineering optimization problems including the ED problem [27,28].

This paper presents the application of one such chaos based metaheuristic algorithm, the CBA (Chaotic Bat Algorithm), a SI algorithm to the ED problem. Five test systems of varying complexities have been solved by CBA to demonstrate its performance. The CBA is easy to implement and shows promising results.

Table 2
Optimal generations and cost obtained by the CBA for Test System 1 (6 generators with loss, POZ and ramp rate limits).

Unit	P_j^{min}	P_j^{max}	POZ	Generation
1	100	500	[210, 240]; [350, 380]	447.4187
2	50	200	[90, 110]; [140, 160]	172.8255
3	80	300	[150, 170]; [210, 240]	264.0759
4	50	150	[80, 90]; [110, 120]	139.2469
5	50	200	[90, 110]; [140, 150]	165.6526
6	50	120	[75, 85]; [100, 105]	86.7652
Cost ($/hr)			15,450.2381	
Transmission loss (MW)			12.9848	

The rest of the paper is organized as follows. Section 2 describes the ED problem, Section 3 introduces the CBA algorithm, and Section 4 explains its application to the ED problem. Section 5 presents the results of the CBA applied to five test systems, and Section 6 concludes the paper.

2. ED problem formulation

The objective of the ED problem is to minimize the fuel cost of thermal power plants for a given load demand subject to various constraints.

2.1. Objective function

The objective is to minimize the quadratic fuel cost function of the thermal units, given by

$$\min_{P \in R^{N_g}} \quad F = \sum_{j=1}^{N_g} F_j(P_j) = \sum_{j=1}^{N_g} \left(a_j + b_j P_j + c_j P_j^2 \right) \tag{1}$$

where N_g is the total number of generating units, $F_j(P_j)$ is the fuel cost of the jth generating unit in \$/hr, P_j is the power generated by the jth generating unit in MW, and a_j, b_j and c_j are cost coefficients of jth generator.

Table 3
Comparison of fuel costs and statistical results for 50 trial runs for Test System 1.

S. no.	Algorithm	Best fuel cost($/hr)	Mean fuel cost($/hr)	Max. fuel cost($/hr)	Standard deviation	Average computation/run time (seconds)
1	**CBA**	**15,450.2381**	**15,454.76**	**15,518.6588**	**2.965**	**0.704**
2	DE [46]	15,449.5826	15,449.62	15,449.6508	NA	3.634 s
3	GAAPI [32]	15,449.78	15,449.81	15,449.85	NA	NA
4	MABC [41]	15,449.8995	15,449.8995	15,449.8995	6.04 × 10-8	0.62 s
5	NPSO-LRS [34]	15,450	15,454	15,452	NA	NA
6	MTS [39]	15,450.06	15,451.17	15,450.06	0.9287	1.29
7	GA Binary [32]	15,451.66	15,469.21	15,519.87	NA	NA
8	TS [39]	15,454.89	15,472.56	15,454.89	13.7195	20.55
9	SA [39]	15,461.1	15,488.98	15,461.1	28.3678	50.36

NA — not applicable/available.

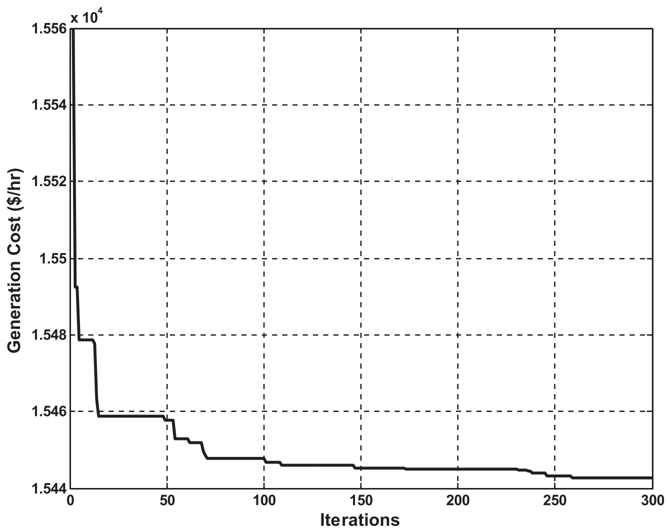

Fig. 1. Convergence characteristic of the CBA for the Test System 1 (6-generators).

The objective function when the valve-point loading effect is taken into account becomes:

$$\min_{P \in R^{N_g}} \quad F = \sum_{j=1}^{N_g} F_j(P_j)$$
$$= \sum_{j=1}^{N_g} \left(a_j + b_j P_j + c_j P_j^2\right) + \left| e_j \sin\left(f_j\left(P_j^{\min} - P_j\right)\right)\right| \quad (2)$$

where e_j and f_j are constants of the valve-point effect of generators.

Table 4
Optimal generations and cost obtained by the CBA for Test System 2 (13 generators with valve point loading).

Unit	P_j^{min}	P_j^{max}	Generation
1	0	680	628.3185
2	0	360	149.5997
3	0	360	222.7491
4	60	180	109.8666
5	60	180	109.8666
6	60	180	109.8666
7	60	180	109.8666
8	60	180	60.0000
9	60	180	109.8663
10	40	120	40.0000
11	40	120	40.0000
12	55	120	55.0000
13	55	120	55.0000
Cost ($/hr)		17,963.8339	

If there are multiple fuel options, the fuel cost of the jth generator is given by

$$F_j(P_j) = \begin{cases} a_{j1}P_j^2 + b_{j1}P_j + c_{j1}P_j, & \text{fuel } 1, \quad P_j^{\min} \le P_j \le P_{j1} \\ a_{j2}P_j^2 + b_{j2}P_j + c_{j2}P_j, & \text{fuel } 2, \quad P_{j1} \le P_j \le P_{j2} \\ \vdots & \\ a_{jk}P_j^2 + b_{jk}P_j + c_{jk}P_j, & \text{fuel } k, \quad P_{jk-1} \le P_j \le P_j^{\max} \end{cases} \quad (3)$$

For a generator with k fuel options there are k discrete regions.

2.2. Optimization constraints

The equality and inequality constraints for the ED problem are the real power balance criterion, and real power generation limits as given by the following two equations:

$$\sum_{j=1}^{N_g} P_j = P_D + P_L \quad (4)$$

$$P_j^{\min} \le P_j \le P_j^{\max} \quad (5)$$

where P_j is the generation of the jth generating unit in MW. P_D is the total power demand in MW,
$P_j^{\min}$ and $P_j^{\ \max}$ are the minimum and maximum power generation limits of the jth generator, and
P_L represents the line losses in MW which is calculated using B-coefficients, given by

$$P_L = \sum_{j=1}^{N_g} \sum_{i=1}^{N_g} P_j B_{ji} P_i + \sum_{j=1}^{N_g} B_{0j} P_j + B_{00} \quad (6)$$

where P_i and P_j are the real power injection at ith and jth buses, respectively, and
B_{ij} is the loss coefficients which can be assumed to be constant under normal operating conditions.

2.3. Practical operating constraints of generators

2.3.1. POZ (Prohibited operating zones)

The prohibited zones are due to steam valve operation or vibrations in the shaft bearings. The feasible operating zones of unit j can be described as follows:

Table 5
Comparison of fuel costs and statistical results for 50 trial runs for Test System 2.

S. no.	Algorithm	Best fuel cost($/hr)	Mean fuel cost($/hr)	Max. fuel cost($/hr)	Standard deviation	Average computation/run time (seconds)
1	**CBA**	**17,963.83**	**17,965.4889**	**17,995.2256**	**6.8473**	**0.97**
2	HCRO-DE [43]	17,960.38	17,960.59	17,961.04	0.069	4.91
3	MABC [41]	17,963.83	17,963.8293	17963.83	2.26×10^{-4}	38.2 s
4	CE-SQP [33]	17,963.85	17,965.97	NA	NA	NA
5	PSO-SQP [36]	17,969.93	18,029.99	NA	NA	33.97
6	CGA_MU [5]	17,975.34	–	NA	NA	NA
7	EP-SQP [36]	17,991.03	18,106.93	NA	NA	121.93
8	EP [36]	17,994.07	18,127.06	NA	NA	157.43
9	IFEP [49]	17,994.07	18,127.06	18267.42	NA	NA
10	PSO [36]	18,030.72	18205.78	NA	NA	77.37
11	CEP [49]	18,048.21	18190.32	18404.04	NA	NA

NA – not applicable/available.

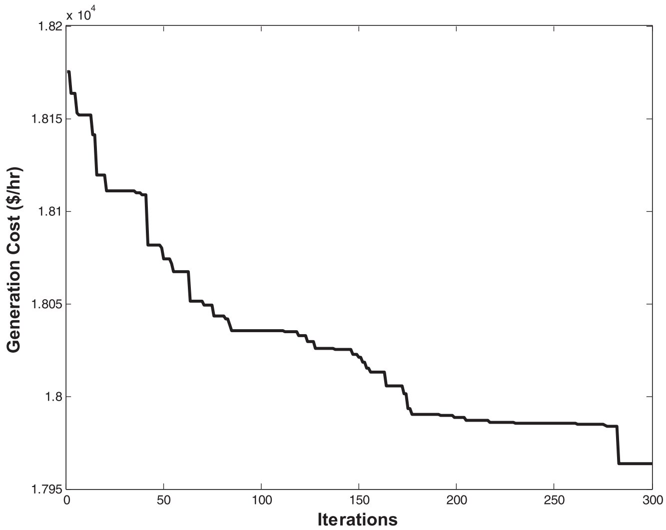

Fig. 2. Convergence characteristic of the CBA for the Test System 2 (13-generators).

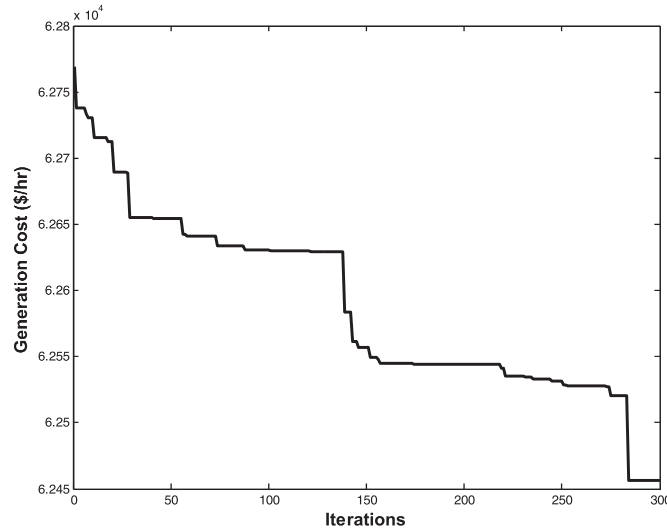

Fig. 3. Convergence characteristic of the CBA for the Test System 3 (20-generators).

Table 6
Optimal generations and cost obtained by the CBA for Test System 3 (20 generators with loss).

Unit	P_j^{min}	P_j^{max}	Generation	Unit	P_j^{min}	P_j^{max}	Generation
1	150	600	512.7176	11	100	300	150.2262
2	50	200	169.0294	12	150	500	292.7687
3	50	200	126.8788	13	40	160	119.1206
4	50	200	102.8739	14	20	130	30.8427
5	50	160	113.6932	15	25	185	115.8196
6	20	100	73.5788	16	20	80	36.2513
7	25	125	115.3014	17	30	85	66.8602
8	50	150	116.4061	18	30	120	87.9713
9	50	200	100.4357	19	40	120	100.8124
10	30	150	106.0647	20	30	100	54.3106
Cost ($/hr)						62,456.6328	
Transmission loss (MW)						91.9632	

$$P_j \in \begin{cases} P_j^{\min} \leq P_j \leq P_{j,1}^{l} \\ P_{j,\,k-1}^{u} \leq P_j \leq P_{j,k}^{l}, \quad k = 2, 3, \ldots n_j, \quad j = 1, 2, \ldots n \\ P_{j,\,n_j}^{u} \leq P_j \leq P_j^{\max} \end{cases} \tag{7}$$

where n_j is the number of prohibited zones of jth generator. P_{jk}^{l}, P_{jk}^{u} are lower and upper power outputs of the kth prohibited zone of the jth generator, respectively.

2.3.2. Ramp rate limits

The physical limitations of starting up and shutting down of generators impose ramp rate limits, which are modeled as follows. The increase in generation is limited by

$$P_j - P_j^0 \leq UR_j \tag{8}$$

Table 7
Comparison of fuel costs and statistical results for 50 trial runs for Test System 3.

S. no.	Algorithm	Best fuel cost ($/hr)	Mean fuel cost ($/hr)	Max fuel cost ($/hr)	Standard deviation	Average computation/run time (seconds)
1	**CBA**	**62456.6328**	**62456.6348**	**62,501.6714**	**0.3879**	**1.16**
2	GSA [40]	62456.6332	62456.6336	62456.6353	NA	30.45
3	HM [16]	62456.6341	–	NA	NA	6.355
4	λ-Iteration [16]	62456.6391	–	NA	NA	33.757
5	BSA [40]	62456.6925	62457.1517	62458.1272	NA	14.477
6	BBO [40]	62456.7793	62456.7928	62456.7928	NA	NA

NA – not applicable/available.

Table 8
Optimal generations and cost obtained by the CBA for Test System 4 (40 generators with valve point loading effects).

Unit	P_j^{min}	P_j^{max}	Generation	Unit	P_j^{min}	P_j^{max}	POZ	Generation
1	36	114	110.8000	21	254	550	–	523.2794
2	36	114	110.8000	22	254	550	–	523.2794
3	60	120	97.3999	23	254	550	–	523.2795
4	80	190	179.7331	24	254	550	–	523.2794
5	47	97	87.7999	25	254	550	–	523.2794
6	68	140	140.0000	26	254	550	–	523.2794
7	11	300	259.5997	27	10	150	–	10.0000
8	13	300	284.5997	28	10	150	–	10.0000
9	13	300	284.5997	29	10	150	–	10.0000
10	13	300	130.0000	30	47	97	–	87.7999
11	94	375	94.0000	31	60	190	–	190.0000
12	94	375	94.0000	32	60	190	–	190.0000
13	12	500	214.7598	33	60	190	–	190.0000
14	12	500	394.2793	34	90	200	–	164.7998
15	12	500	394.2794	35	90	200	–	194.3971
16	12	500	394.2794	36	90	200	–	200.0000
17	22	500	489.2795	37	25	110	–	110.0000
18	22	500	489.2794	38	25	110	–	110.0000
19	24	550	511.2794	39	25	110	–	109.9999
20	242	550	511.2793	40	242	550	–	511.2793
Cost ($/hr) 121,412.5468								

Similarly, the decrease is limited by

$$P_j^0 - P_j \le DR_j \tag{9}$$

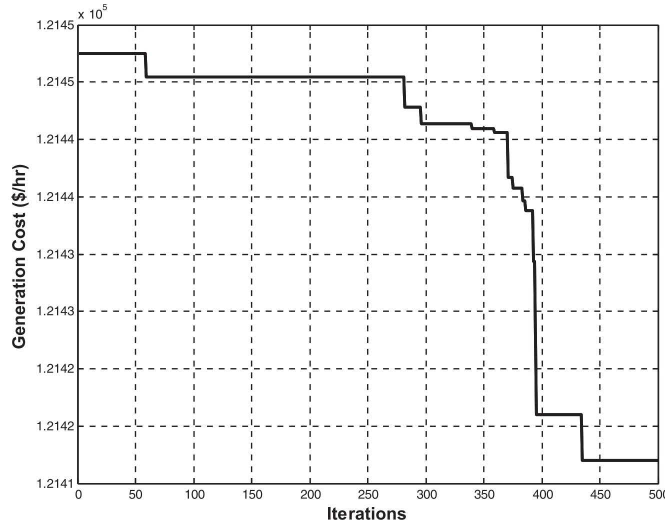

Fig. 4. Convergence characteristic of the CBA for the Test System 4 (40-generators).

where P_j^0 is the previous output power, UR_j and DR_j are the up-ramp limit and the down-ramp limit, respectively, of the jth generator.

Combining (8) and (9) with (5) results in the change of the effective operating or generation limits to

$$\underline{P_j} \le P_j \le \overline{P_j} \tag{10}$$

where

$$\underline{P_j} = \max\left(P_j^{min}, P_j^0 - DR_j\right) \tag{11}$$

$$\overline{P_j} = \min\left(P_j^{max}, P_j^0 + UR_j\right) \tag{12}$$

Table 9
Comparison of fuel costs and statistical results for 50 trial runs for Test System 4.

S. no	Algorithm	Best fuel cost ($/hr)	Mean fuel cost ($/hr)	Max fuel cost ($/hr)	Standard deviation	Average computation/run time (seconds)
1	**CBA**	**121,412.5468**	**121,418.9826**	**121,436.15**	**1.611**	**1.55**
2	DSD [5]	121,412.5355	–	NA	NA	NA
3	HCRO-DE [43]	121,412.55	121,413.11	1,21,415.68	91	7.64
4	SQPSO [13]	121,412.57	121,455.7	121,709.5582	49.8076	47.24
5	MABC [41]	121,412.5918	121,431.5763	121,493.1885	18.16	1.92 min
6	DE [46]	121,412.68	121,439.89	121479.63	NA	31.5037 s
7	CE-SQP [33]	12,1412.88	121,423.65	NA	NA	NA
8	BSA [40]	121,415.614	121474.882	121524.9577	NA	13.12
9	CRO [43]	121,416.69	121,418.03	1,21,422.92	0.88	8.15
10	θ-PSO [13]	121,420.9027	121509.8423	121,852.4249	92.3956	103.9665
11	BBO [35]	121,426.953	121,508.0325	121,688.6634	NA	NA
12	QPSO [13]	121,448.21	122,225.07	121,994.0267	114.08	48.25
13	CSO [38]	121,461.6707	121,936.1926	NA	32	NA
14	SOH-PSO [11]	121,501.14	121,853.57	122,446.3	NA	NA
15	NPSO-LRS [34]	121,664.4308	122,209.3185	122,981.5913	NA	3.93
16	AA (Dist.) [37]	121,788.7	121,788.7	NA	NA	NA
17	PSO-SQP [33]	122,094.67	122,245.25	NA	NA	NA
18	EP-SQP [36]	122,323.97	122,379.63	NA	NA	997.73
19	SCA [38]	122,713.6828	125,235.1288	130,918.3914	NA	130.23/per 1000 it

NA – not applicable/available.

Table 10
Optimal generations and cost obtained by the CBA for Test System 5 (160 generators with valve point loading effects and multiple fuel option).

Unit	Generation	Unit	Generation	Unit	Generation	Unit	Generation	Unit	Generation
1	218.9236	33	274.6928	65	277.8544	97	292.7296	129	433.8804
2	208.1220	34	241.1441	66	242.0309	98	243.5103	130	276.8822
3	280.9570	35	287.9821	67	289.3348	99	430.4213	131	221.2948
4	235.0007	36	242.1802	68	237.5221	100	277.4891	132	214.8380
5	277.1804	37	293.8753	69	433.0113	101	221.7764	133	279.3951
6	238.9805	38	241.6323	70	259.9733	102	211.5289	134	238.2402
7	285.8679	39	426.8197	71	213.5490	103	277.5534	135	272.0972
8	238.2201	40	281.5554	72	202.7771	104	240.6612	136	242.3765
9	436.8050	41	216.6582	73	276.9687	105	275.5609	137	285.4913
10	272.1712	42	213.5491	74	237.1790	106	243.3427	138	241.7948
11	223.1044	43	280.4044	75	273.5552	107	298.6092	139	422.2599
12	210.9978	44	237.9394	76	238.9476	108	242.0799	140	269.0701
13	286.6814	45	274.4310	77	284.6644	109	430.7492	141	213.3330
14	235.9218	46	234.2998	78	234.7298	110	268.1856	142	210.6310
15	282.8000	47	289.8687	79	430.2759	111	216.8815	143	316.6300
16	239.2793	48	238.9028	80	286.8461	112	207.2007	144	240.4678
17	286.3819	49	431.8573	81	214.7914	113	283.5781	145	271.9684
18	242.4313	50	271.5666	82	210.3170	114	240.1776	146	244.7934
19	431.9165	51	219.0597	83	287.7874	115	274.9631	147	284.5584
20	262.5443	52	215.0891	84	238.7221	116	244.0469	148	241.2125
21	215.6173	53	281.7286	85	273.1856	117	282.9826	149	430.5254
22	211.4109	54	236.6008	86	243.2461	118	238.8939	150	276.9483
23	286.9811	55	286.7678	87	290.6811	119	424.3245	151	217.3145
24	228.5692	56	245.3649	88	244.1070	120	278.0752	152	214.5596
25	269.9395	57	297.8972	89	424.9956	121	215.5769	153	306.8994
26	241.5283	58	239.5060	90	275.4708	122	210.0023	154	238.3386
27	288.1008	59	432.9399	91	219.6006	123	291.3170	155	271.2063
28	240.1601	60	274.0136	92	206.9705	124	238.1354	156	241.7981
29	431.2663	61	217.0250	93	277.8040	125	276.2371	157	297.4385
30	266.3111	62	210.2195	94	236.9838	126	242.0846	158	247.5073
31	216.8685	63	267.9713	95	267.8490	127	293.4493	159	386.9917
32	213.9825	64	238.2421	96	243.2522	128	244.3282	160	271.0135
Cost ($/hr) – 10,002.8596									

Combining this with (2), the ED problem can be formulated as

$$\min_{P \in R^{N_g}} \quad F = \sum_{j=1}^{N_g} F_j(P_j) = \sum_{j=1}^{N_g} \left(a_j + b_j P_j + c_j P_j^2\right) + \left| e_j \sin\left(f_j\left(P_j^{\min} - P_j\right)\right)\right| \tag{13a}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{j=1}^{N_g} P_j = P_D + P_L \\ & \max\left(P_j^{\min}, P_j^0 - DR_j\right) \le P_j \le P_{j,1}^l \\ & P_{j,k-1}^u \le P_j \le P_{j,k}^l, k = 2, 3, \ldots n_j, j = 1, 2, \ldots N_g \\ & P_{j,n_j}^u \le P_j \le \min\left(P_j^{\max}, P_j^0 + UR_j\right) \end{aligned} \tag{13b}$$

3. Chaotic bat algorithm

3.1. The basic bat algorithm

The BA (bat algorithm) is a new swarm intelligence algorithm proposed by Xin-She Yang, inspired by the echolocation phenomenon in bats [29]. Bats found in nature are of various sizes ranging from microbats weighing 1.5–2 g to giant bats which weigh up to 1 kg [29]. Echolocation is a technique used by bats to navigate and locate prey. Bats use frequency modulated signals to sense distance, where each pulse lasts a few thousandths of a second (8–10 ms) within the frequency range 25–100 KHz. Typically bats emit 10–20 such bursts per second. However, when hunting, they can emit over 200 pulses per second in the form of short bursts.

Another characteristic of these pulses is loudness; bats emit extremely loud pulses (in the ultrasonic region) while hunting or searching for a prey (up to 110 dB). But when homing towards the prey these pulses are feebler.

Based on the behavior described above, the bat algorithm updates the velocity and position iteratively as follows.

Table 11
Comparison of fuel costs and statistical results for 50 trial runs for Test System 5.

S. no.	Algorithm	Best fuel cost ($/hr)	Mean fuel cost ($/hr)	Max fuel cost ($/hr)	Standard deviation	Average computation/run time (seconds)
1	**CBA**	**10,002.8596**	**10,006.3251**	**10,045.2265**	**9.5811**	**5.71**
2	ORCCRO [44]	10,004.2	10,004.21	10,004.45	NA	0.019/iteration
3	DE/BBO [44]	10,007.05	10,007.56	10010.26	NA	0.56/iter
4	BBO [44]	10,008.71	10,009.16	10,010.59	NA	0.62/iter
5	RCCRO [45]	10,009.5183	10,009.5222	10,009.5827	NA	50.216/iter
6	IGA_MU [42]	10,042.4742	10,042.4742	NA	NA	174.62 CPU AV
7	CGA_MU [42]	10,143.7236	10,143.7236	NA	NA	621.3 CPU AV

NA – not applicable/available.

The frequency of every bat is given by

$$f^i = f^{\min} + r_1\left(f^{\max} - f^{\min}\right) \tag{14}$$

where r_1 is a uniformly distributed random number in the range [0 1]; $f^{\min}$ and $f^{\max}$ are the minimum and maximum allowable frequencies while f^i is the frequency for ith bat.

For the present ED problem, the values of $f^{\min}$ and $f^{\max}$ are set to 0 and 100, respectively, as given in [29].

3.1.1. Exploration or diversification

The velocity V^i and position X^i are updated by

$$V^i(t+1) = V^i(t) + f^i\left(X^i(t) - X^{\text{best}}(t)\right) \tag{15}$$

$$X^i(t+1) = X^i(t) + V^i(t+1) \tag{16}$$

where t is the current iteration number, X^{best} is the location (solution), that has the best fitness in the current population. At initialization, V^i is assumed to be 0.

3.1.2. Exploitation or intensification

Local search or exploitation is by means of a random walk. Two parameters, the pulse emission rate $R^i(t)$ and loudness $A^i(t)$ are updated at each iteration, for every bat in the population. Depending on $R^i(t)$, a local search is conducted, either around the best solution or a randomly chosen solution:

$$X^{i,\text{new}}(t) = \begin{cases} X^{\text{best}}(t) + r_3 A^i(t) & \text{if} \quad r_2 > R^i(t) \\ X^r(t) + r_3 A^i(t) & \text{else} \end{cases} \tag{17}$$

where r_2 $[0,\ 1]$ and r_3 $[-1,\ 1]$ are uniformly distributed random numbers, and r $[1,\ 2,\ \ldots,\ N_b]$, $r \neq i$ is a randomly chosen integer.

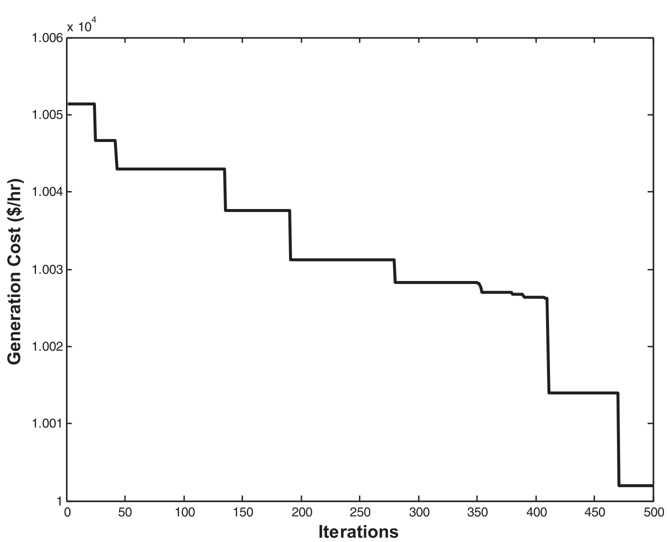

Fig. 5. Convergence characteristic of the CBA for the Test System 5 (160-generators).

In other words, $X^r(t)$ is a randomly chosen solution in the current iteration, and different from the ith solution.

A fitness based, tournament type of selection, in which the competitors are the old and new solutions is implemented. The fitter solution replaces the less fit one, with a probability $A^i(t)$

$$X^i(t) = X^{i,\text{new}}(t), \text{if } f\left(X^{i,\text{new}}(t)\right) < f\left(X^i(t)\right) \text{ and } r_4 < A^i(t)\ \forall i,\ i\,[1,\ 2,\ \ldots,\ N_b] \tag{18}$$

where r_4 $[0,\ 1]$ is a uniformly distributed random number.

Lastly, the loudness A^i and pulse emission rate R^i of each bat are updated as the iterations progress:

$$A^i(t+1) = \alpha A^i(t) \tag{19}$$

$$R^i(t+1) = R^i(0)[1 - \exp(-\gamma t)] \tag{20}$$

where $A^i(0)[1,\ 2]$, $R^i(0)[0,\ 1]$, both randomly generated, within their respective limits. The values $\alpha = \gamma = 0.9$, as in [29].

3.2. Chaotic bat algorithm

As mentioned in Section 1, chaotic sequences have been used instead of random numbers to improve the performance of metaheuristic methods. A variant of the basic bat algorithm with chaotic sequences has been presented in [24]. The loudness A^i can be a crucial factor that affects the tradeoff between exploration and exploitation of the algorithm. In the basic BA, this is monotonically decreased as the iterations progress as per (19), since α is less than unity. However, better results have been reported when the loudness A^i has been varied chaotically, using a chaotic map or sequence. A further choice is to choose a chaotic sequence from different maps. In [24], thirteen different chaotic sequences were tried to tune A^i to optimize three unimodal and three multimodal functions. The best results were reported for the sinusoidal map (or sequence). This is referred to as 'CBA-III' in [24], and used to solve the ED problem in this paper. The variation of the loudness A^i using the sinusoidal map is given by

$$A^i(t+1) = a\left\{A^i(t)\right\}^2 \sin\left(\pi A^i(t)\right) \tag{21}$$

where a is set to 2.3 as in [24], and $A^i(0)$ $[0,\ 1]$, and is randomly generated.

4. Implementation of CBA to ED problem

Step 0: The initialization is done as follows:

- Generation values are randomly generated within the specified limits for every generating unit in each solution or bat.
- For units with POZ, if the randomly generated value falls in the POZ, it is fixed at the nearest limit that is violated.
- If a unit has ramp-rate limits, the power output is uniformly distributed between the effective lower and upper limits.

Generate N_b number of bats or solutions, each comprising N_g number of generating units:

$$\begin{bmatrix} p_1^1 & p_2^1 & \cdots & p_{N_g}^1 \\ p_1^2 & p_2^2 & \cdots & p_{N_g}^2 \\ \vdots & & & \\ p_1^{N_b} & p_2^{N_b} & \cdots & p_{N_g}^{N_b} \end{bmatrix} = \begin{bmatrix} P^1 \\ P^2 \\ \vdots \\ P^{N_b} \end{bmatrix} \quad (22)$$

Step 1: Calculate the fitness values of all the bats using the objective or fitness function F, in (13a).
Step 2: For ith bat, define pulse frequency f^i using (14).
Step 3: Update the velocity and position (which is a vector of generation values) of each bat using (15) and (16), respectively.
Step 4: Generate a new solution by random walk using (17).
Step 5: Select the fitter of the old and new solutions, with a probability $A^i(t)$, using (18).
Step 6: Update the values of R^i and A^i using (20) and (21), respectively.
Step 7: Check if effective generation limits and POZ limits are violated. Fix the generation at the limit that is violated. This takes care of the inequality constraints. After this is done, any violation of the power balance equality constraint (4) is dealt with by using a penalty factor approach. By this approach, (13) is modified to

$$\min_{P \in R^{N_g}} L = F + \lambda \left| \sum_{j=1}^{N_g} P_j - (P_D + P_L) \right| \quad (23)$$

where λ is the penalty coefficient, and a positive real number. It is fixed at a large value.

Step 8: Repeat steps 1 to 7 until the maximum number of iterations is reached.

5. Results and discussion

To test the effectiveness of the CBA algorithm, five different test systems of varying computational difficulty levels have been solved. The results obtained are compared with the optimization techniques listed in Table 1.

To compare the performance of the CBA, 50 independent trial runs are made and the results of the best and mean fuel costs are tabulated for each test system. The value of the penalty coefficient λ is fixed at 100. The number of bats is 40 and the maximum number of iterations is 300 for the first three test systems, and 500 for the last two test systems. The programs are implemented in Matlab® on a personal computer with a 3.3 GHz processor and 4 GB RAM, running on Windows 7.

The average computation or run time varied between 0.704 s for the first 6-generator problem, to 5.71 s for the last 160 generator problem. The results obtained are compared with several techniques reported in the literature [3–49] whose abbreviations are listed in Table 1 in alphabetical order. The best fuel costs of these results are arranged in ascending order, to facilitate easy comparison, in all the subsequent tables that have the comparisons.

5.1. Test system 1

This is a small system comprising six generators and meeting a load demand of 1263 MW, and includes transmission loss, POZ and ramp rate limits. The system data are taken from [47,48]. Table 2 presents the optimal generation values, transmission loss and cost obtained by CBA. The optimal cost and the corresponding transmission loss obtained are 15,442.7048 \$/hr and 12.4468 MW, respectively. It may be noted that the generation values (Column 5) satisfy the generation limit constraints (Columns 2 and 3) and do not fall in the POZs (Column 4).

Table 3 shows the comparison of the statistical results of different algorithms that have been reported recently. It is seen that, the best fuel cost obtained by the CBA is comparable with those of the other methods. It is seen that the average computation or run time too of the CBA is on the lower side, in comparison.

Fig. 1 shows the graph of the convergence of solutions with iterations for the CBA algorithm for a typical run. It can be seen that smooth convergence is obtained by the CBA.

5.2. Test system 2

This is a slightly larger test system, and consists of 13 generators meeting a load demand of 1800 MW. It includes valve point loading, which has the effect of introducing several local minima [33], thereby making the cost curve multimodal. Transmission losses are not considered for this case. The system data are taken from [49]. Table 4 presents the optimal generations and the costs obtained. The optimal cost obtained by CBA is 17,963.8339 \$/hr. It may be noted that the generations satisfy the generation limit constraints.

Table 5 shows the comparison of the statistical results of the CBA and other algorithms that have been reported recently. It is seen that the best and mean fuel costs obtained by the CBA are the least of all methods with the exceptions of HCRO-DE [43] and MABC [41]. It is to be noted that the value of the e_j coefficient in the cost equation of generator 3 is assumed to be 200 in this paper (as in [49]), but this value is assumed to be 150 in [43]. This explains the lower cost obtained by HCRO-DE. It is seen that the average computation or run time of the CBA is much lower than those of the other methods.

Fig. 2 shows the graph of the convergence of solutions with iterations for the CBA algorithm for a typical run, for Test System 2. It can be seen that smooth convergence is obtained by the CBA.

5.3. Test system 3

This system consists of twenty generators supplying a demand of 2500 MW. Transmission losses are included in this system. The cost coefficients and B-coefficients data are taken from [16]. Table 6 presents the optimal generation values, cost and transmission loss obtained by the CBA. The optimal cost and the corresponding transmission loss obtained are 62,456.6328 \$/hr and 91.9632 MW, respectively. It is seen that the all the generation limit constraints are satisfied.

Table 7 shows the comparison of the statistical results of the CBA and other algorithms that have been reported recently. It is seen that, the best fuel cost obtained by the CBA is the least of all methods, and the mean fuel cost is the least of all methods with the sole exception of GSO [40]. As for the Test System 2, it is seen that the average computation time of the CBA is much lower than those of the other methods.

Fig. 3 shows the graph of the convergence of solutions with iterations for the CBA algorithm for a typical run, for Test System 3. It can be seen that smooth convergence is obtained by the CBA.

5.4. Test system 4

This system consists of forty generators supplying a demand of 10,500 MW. It includes valve point loading effects, due to which a number of local minima are introduced, thereby making the problem more complex. The system data are taken from [49].

Table 8 presents the optimal generation values and cost obtained by the CBA. The optimal cost is 121,412.5468 $/hr. It may be noted that the generations satisfy the generation limit constraints.

The results obtained are compared with those obtained by several other techniques reported recently, as shown in Table 9. The best fuel cost by the CBA is the least of all, with the exception of DSD [5]. The mean fuel cost too compares favorably with the other methods.

Fig. 4 shows the graph of the convergence of solutions with iterations for the CBA algorithm for a typical run, for Test System 4. It can be seen that smooth convergence is obtained by the CBA.

5.5. Test system 5

This system consists of sixteen replicas of a ten unit system [42], thereby making it a system with hundred and sixty generators, supplying a demand of 43,200 MW. It has multiple fuel options and includes valve point loading effects resembling a practical power system as in [42]. The cost function is piecewise non-smooth and this is a problem of a high dimensionality, thereby making this a challenging optimization problem. Table 10 presents the optimal generation values and cost obtained by the CBA. The optimal cost is 10,002.8596 $/hr.

The comparison of statistical results of the CBA and other methods are given in Table 11. The best fuel cost obtained by the CBA is the least of all the methods, and the mean fuel cost is the least with the exception of ORCCRO [44].

Fig. 5 shows the graph of the convergence of solutions with iterations for the CBA algorithm for a typical run, for Test System 5. It can be seen that smooth convergence is obtained by the CBA.

6. Conclusion

A new metaheuristic method that requires little tuning by the user, the CBA, was successfully applied to solve ED problems of various levels of complexity. The proposed CBA method incorporated the sinusoidal chaotic map to tune the loudness A^i of each bat to improve the performance of the algorithm.

The CBA method was applied to five test systems with various constraints such as power balance, POZs, valve point loading effects, and ramp rate limits. Transmission losses and multiple fuel options have also been included in some systems. The CBA either outperforms or compares favorably with the other existing techniques against which it was compared.

The results show that the CBA is capable of handling high dimensional problems with several constraints, as demonstrated by its application to the Test System 5.

Given that the CBA has shown good performance with the ED problem, it can be applied to other optimization problems in the area of power systems. Comparative study of chaotic sequences to optimization problems could also be a good topic for future work.

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