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Earning potential in multilevel marketing enterprises

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1. Introduction

ABSTRACT

Government regulators and other concerned citizens warily view multilevel marketing enterprises (MLM) because of their close operational resemblance to exploitative pyramid schemes. We analyze two types of MLM network architectures — the unilevel and binary, in terms of growth behavior and earning potential among members. We show that network growth decelerates after reaching a size threshold, contrary to claims of unrestricted growth by MLM recruiters. We have also found that the earning potential in binary MLM's obey the Pareto "80–20" rule, implying an earning opportunity that is strongly biased against the most recent members. On the other hand, unilevel MLM's do not exhibit the Pareto earning distribution and earning potential is independent of member position in the network. Our analytical results agree well with field data taken from real-world MLM's in the Philippines. Our analysis is generally valid and can be applied to other MLM architectures.

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Multilevel marketing enterprises (MLM) attract millions of members and generate billions of dollars in annual sales and revenues [1]. Their potential for future growth is strong since an increasing number of people are getting affordable access to reliable mobile communication and the Internet [2].

Invariably, a MLM member sells a product and to earn, he or she must recruit new members who will operate under his/her supervision. One becomes a new member by buying the product from a member/recruiter and/or by paying a one-time MLM membership fee. The successful recruiter earns a commission from the product sale and/or membership payment. The recent member/buyer in turn, earns by enlisting his own (new) members and a chain of recruitments is created that expands the MLM network in terms of membership and connections.

Sustained membership growth that keeps the commission stream going is the key to making money in a MLM. Of course, an MLM member can also earn from sales to non-members — an unattractive option that requires a high-quality product that could compete in the "open" market. Such a product is normally the outcome of costly and painstaking product research and development.

Note that a MLM member/buyer does not have to actually pay for the product for as long as he is able to 'move' it downstream to his new recruit(s). MLM membership is enticing because it endows one with the opportunity to make money endlessly without an initial capital except the one-time membership fee, if there is one. In emergent economies (e.g. Philippines), MLM's thrive particularly well since people are generally poor and dream of getting rich quickly and easily.

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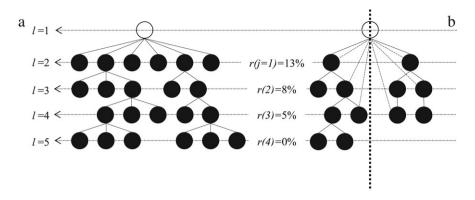


Fig. 1. Basic MLM structures: (a) unilevel and (b) binary with the topmost node designated as first level (l = 1) buyer/recruiter (unfilled circle). Left and right side of a binary MLM is separated by thick dotted symmetry line. Agents recruited directly by the topmost agent are linked to him via a dashed line. Shown are the rebate rates r(j) used in a unilevel (Forever Living) and binary (Legacy) MLM's in the Philippines. Table 1 presents the incomes of the topmost node in the networks shown above.

MLM's rely on the formation of cooperative social networks among members who play the multiple roles of product consumers, sellers and distributors all at the same time [1]. Membership growth is often fueled by the recruitment of acquaintances, friends and colleagues who are easier to engage (by members) than strangers [3]. Direct selling accounted for 69.8% of MLM income in 2004 (Direct Selling Association/USA, 2004). An MLM product can achieve brand recognition through face-to-face endorsements and testimonials by members, which is much less costly to finance than traditional advertising via print, television and radio [4].

Inevitably, MLM's elicit the attention of government regulators since they operate in ways that closely resemble with those practiced in illegal pyramid schemes [2]. Together with civil society groups, regulators act to promote public welfare and to uphold an innate sense of moral fairness [5]. One consumer advocacy group, Pyramid Scheme Alert, was organized in 2000 to expose and halt the spread of pyramid schemes (see http://www.pyramidschemealert.org). The U.S. Federal Trade Commission utilizes a "70% rule" for establishing the legal status of an MLM. The rule requires that participants first sell 70% of their previously purchased inventory (preferably to non-members) before procuring new orders [6]. Normally, a MLM is not allowed to operate if it has no product to sell and members earn only from the membership fees of new recruits.

It is worthwhile to gain a better understanding of the growth dynamics of MLM's since they affect the lives of many people in society. Here, we analyze two types of commonly employed MLM configurations – the unilevel and binary structures (see Fig. 1). A row is designated as level *l*, with l = 1 assigned to the first member (founder) of the MLM network. New recruits are placed below the level of a recruiting agent. If *Y* is recruited by *X* and is part of *X*'s portfolio, then *Y* is referred to as a *downline* of *X*. In Fig. 1(a), the founding (l = 1) member has a portfolio of 22 downlines, while in Fig. 1(b) the l = 1 member has 12 downlines.

There is no limit to the total number of new recruits that can be managed by an MLM member. However, the number of possible immediate downlines under him depends on the MLM architecture. It may be uncapped and capped for a unilevel and binary network, respectively. In a unilevel MLM, a member can position all of his recruits as immediate downlines in one and the same level. In a binary MLM on the other hand, a member is allowed only two immediate downlines. However, he can strategically position all subsequent recruits as downlines of his own downlines or as downlines of downlines of its downlines, and so on. He is strongly motivated to maintain lateral symmetry for the binary MLM network (see Fig. 1(b)) since it entitles him to a *pairing bonus*.

A pairing bonus is introduced to encourage a binary MLM member to always recruit two new members (instead of just one) and fulfill his quota of new members. In the ideal scenario of unlimited growth, the preservation of symmetry in a binary MLM network is highly desirable because it leads to geometric growth that brings in huge profits especially for the 'old' members (with low *l* values). The rapid growth also stifles the development of other MLM enterprises that compete for new members from one and the same population.

Accurately predicting the earning prospect of members in an MLM network requires knowledge of its growth mechanism [7]. Typically, MLM recruiters persuade their potential recruits by presenting diagrams that illustrate "explosive" growth via "exponential" recruitment [6]. We investigate the validity of the *unlimited growth* claim and evaluate the earning potential of the various MLM members. In the next section, we describe our agent-based simulation procedure and the analytic approximations that are employed to study MLM dynamics. We then compare our theoretical predictions with field data that are obtained from real-world MLM's.

2. Dynamical model of multilevel marketing

2.1. Agent-based simulations

Underlying the operation of an MLM enterprise is a social network of "friendships" or "acquaintances" between

individuals (agents). We generate the social network using the Watts–Strogatz small-world model [8–10]. A regular network with $k_{WS} = 14$ (friendship) links for each agent is initialized. The links are rewired with probability $\alpha = 0.01$. Eventually, the regular network evolves into a small-world network representing a social network that is then utilized to simulate the growth of an MLM network.

At time t = 1, an agent is chosen randomly out of a total of N agents, to represent the founder of the MLM network. Then at t = 2, the founder randomly recruits an agent from a pool of "friends". In each succeeding time-step, only one member in the existing MLM network is permitted to recruit from its own pool of "friends". Only one new agent is added to the network per unit time Δt .

We assume that recent members are more aggressive recruiters than the older ones. In a binary MLM, the probability that an agent who joined at an earlier time t_i , can recruit a new member at a later time t reduces exponentially as: exp $(-\beta (t - t_i))$, where β is the decay constant. However, older members who are already supervising a larger number of recruits, may have better chances of directly attracting new recruits due to "preferential attachment" [11,12]. Prospective members tend to prefer attaching themselves to recruiters with established reputation [13]. Our agent-based model captures the two opposing features in the recruitment probability.

2.2. Unilevel MLM network

Let f(l) be the number of members in level l and N the total number of members. Both f(l) and N are continuous variables and the rate in which f(l) increases with N is directly proportional to f(l-1)/N, the density of active members at l-1. The motivation to recruit newer agents is represented by the branching rate [14] that increases as l^{γ} with $\gamma \ge 0$. Consequently, f(l) satisfies the dynamical equation:

$$\frac{\mathrm{d}f(l)}{\mathrm{d}N} = \frac{l^{\lambda}f(l-1)}{N}.\tag{1}$$

Initially, the network consists only of its founding member and the normalized solution to Eq. (1) is:

$$f(l) = \frac{N}{\sum_{j} (j!)^{\gamma - 1} (\ln N)^{j}} \frac{(\ln N)^{l}}{(l!)^{1 - \gamma}}.$$
(2)

The network cross-section has a maximum value at l_{max} . For $l \le l_{\text{max}}$, f(l) increases as a power of $\ln N$ while for $l > l_{\text{max}}$, f(l) decays with l!.

2.3. Binary MLM network

The binary MLM is harder to analyze due to the constraint that is imposed on the maximum number of possible downlines and the role that is played by preferential attachment. By mathematical induction, the number of downlines at level *l* is 2f(l-1) - f(l). In a network of size *N*, the total number of possible downlines is N + 1. The ratio [2f(l-1) - f(l)] / (N + 1) thus represents the probability that a new downline is placed at level *l*.

Since only the first two recruits of a member can become his own downlines, a new recruit may be placed at levels that do not necessarily succeed the level of the recruiter. This is possible if a downline exists that bridges the recruiter's level with other deeper levels. Conversely, and for as long as a bridging connection exists, a new recruit can be placed at *l* not only by members at level (l - 1), but also by those in (l - 2), (l - 3), etc. An effective fraction, λ/N , of existing members is responsible for a new recruit situated at *l*. Consequently, *f*(*l*) satisfies the dynamical equation:

$$\frac{df(l)}{dN} = \frac{\lambda}{N} \frac{2f(l-1) - f(l)}{N+1}.$$
(3)

Due to the initial conditions f(0) = 1 and $f(l \ge 1) = 0$, a possible solution for Eq. (3) is:

$$f(l) = 2^{l} \left[1 - \left(\frac{N+1}{2N}\right)^{\lambda} \sum_{j=0}^{l-1} \frac{\lambda^{j} \ln\left(\frac{N+1}{2N}\right)^{j}}{j!} \right].$$
 (4)

For $\lambda \ge 1$, Eq. (4) increases as 2^l for small *l* values, consistent with the pairing bonus incentive in binary MLM networks. As *l* increases further, the second term in the square brackets becomes significantly close to 1, causing a drastic decrease in f(l). The growth of binary and unilevel MLM's is limited, contrary to claims of unrestricted growth by MLM recruiters.

2.4. MLM business model

Let P(l) be the total income of all members at level l, with a magnitude that depends on the compensation scheme that is being offered by a particular MLM [15]. Generally, total income is due to at least one of the following types of payment: (i) Rebate, (ii) Pairing, and (iii) Referral incentive bonus.

A rebate is awarded to a member and its value represents a percentage of the sales made by its downlines below some depth of its portfolio. The total profit that is earned from rebates by all members at *l* is:

$$p_{\text{rebates}}(l) = \sum_{j=1}^{D} r(j) f(l+j) C_{\text{sales}},$$
(5)

where C_{sales} is the product price, r(j) is the rebate rate, and D is the maximum level below l up to which a member is entitled a share of earnings from product sales. The rebate rate can be expressed as:

$$r(j) = a_{Dj} j^{D-1} + a_{D-1} j^{D-2} + \dots + a_0.$$
(6)

The matrix coefficients $\{a_n\}$ are determined from the details of an MLM compensation plan using standard matrix algebra. For example, Eq. (6) implies that if r(j = 1) = 13%, r(2) = 8%, r(3) = 5% and r(j > 3) = 0, then D = 3, $a_0 = 20$, $a_1 = -8$, $a_2 = 1$.

The *pairing bonus* is a possible incentive only in binary networks. It is awarded to a member each time that he is able to fill his two immediate downlines with a pair of recruits. The pairing bonus of a member is computed by counting the number of left-right pairs below it. As shown in Fig. 1(b), the topmost node has five pairs, and each of these pairs has a corresponding cash incentive (C_{bonus}). The pairing bonus mechanism favors the preservation of binary network symmetry and high overall network earning that is maximized when the total agents separated by the symmetry line are equal. Unilevel networks do not exhibit clear symmetry lines and are unable to offer a pairing bonus. The total profit gained from pairing bonuses by all members at *l* is:

$$p_{\text{bonus}}(l) = \frac{1}{2} \sum_{j} f(l+j) C_{\text{bonus}}.$$
(7)

A member earns a *referral incentive* every time that he recruits a new agent. If C_{referral} is the incentive prize, then the total referrals profit earned by all the members in level *l* is:

$$p_{\text{referral}}(l) = \eta(l)C_{\text{referral}},\tag{8}$$

where $\eta(l)$ is the number of recruits made by all the members in level *l*. Note that total number of referrals in binary networks are not directly deducible from the binary tree representation as illustrated in Fig. 1.

In a unilevel network, $\eta(l) = f(l+1)$, since the recruits simply correspond to the downlines. For a binary network, $n(l) \approx \sum_i (\sqrt{N/i} - 1)$, where index *i* ranges from $i = \sum_{x < l} f(x)$ to $i = f(l) + \sum_{x < l} f(x)$. The values are obtained from continuum theory as applied to the Barabasi–Albert preferential attachment model [12], where the probability $\Pi(k_i)$ that a member *i* with an existing number of referrals k_i recruits an additional agent within Δt is:

$$\Pi(k_i) = \frac{k_i + 1}{\sum_{j=1}^{N} k_j + 1}.$$
(9)

If Δt is sufficiently small so that only one agent is recruited per unit time, then the sum in the denominator simplifies to 2t - 1. If k_i is taken as a continuous variable [12] then Eq. (9) satisfies:

$$\frac{\partial k_i}{\partial t} = \frac{k_i + 1}{2t - 1} \tag{10}$$

which yields the following solution for the number of recruits by agent *i* at the limit that: $N = t \gg 1$:

$$k_i = \sqrt{N/i} - 1,\tag{11}$$

where i = 1 represents the first member, i = 2 the second member, and so on until the *N*th member (see Fig. 2). The total number of recruits $\eta(l)$ made by all members in level *l* may be estimated as:

$$\eta(l) \approx \sum_{\substack{i=\sum_{x
(12)$$

Combining Eqs. (5), (7) and (8) yields the general expression for P(l) and the average profit that is gained by each member in level *l* is P(l)/f(l).

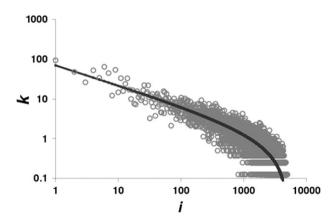


Fig. 2. Number of referrals k (open circles) versus agent number i. Solid curve: $\sqrt{N/i} - 1$. Founder of network is represented by i = 1.

Table 1

Actual earnings of two MLM members with downlines that are similar to the network structures illustrated in Fig. 1

Incentive type	Forever living products (unilevel)	Auto load advantage (unilevel)	Legacy Philippines, Inc. (binary)	First quadrant (binary)
Rebates (Eq. (5)) Pairing bonus (Eq. (7))	845 units 0 unit	160 units 0 unit	312 units 500 units	0 units 500 units
Referral bonus (Eq. (8))	600 units	600 units	600 units	600 units
Total	1445 units	760 units	1412 units	1100 units

Parameters used: $C_{sales} = 1000$ units, $C_{bonus} = 100$ units, $C_{referral} = 100$ units.

We consider the following MLM's that are doing business in the Philippines: (1) *Forever Living Products* (FLP; items being sold: health and beauty), (2) *Auto Load Advantage* (ALA; prepaid credits for mobile communication), (3) *Legacy Philippines Incorporated* (Legacy; shoes, bags, apparels), and (4) *First Quadrant* (FQ; clothing, shoes, beauty products, vitamin supplements). Both FLP and ALA are unilevel MLM's, while Legacy and FQ are binary MLM's.

For FLP the rebate rates are: r(j = 1) = 13%, r(2) = 8%, and r(3) = 5% ($a_0 = 20$, $a_1 = -8$, $a_2 = 1$). For ALA the rates are: r(j = 1) = 1%, r(2) = 1%, and r(3) = 1% ($a_0 = 1$, $a_1 = a_2 = 0$). For the binary plans, both Legacy and FQ reward hardworking agents with referral and pairing bonuses. However, only Legacy is offering rebates which is practiced by the unilevel companies. For Legacy, the rebate rates are: r(j = 1) = 13%, r(2) = 8%, and r(3) = 5% ($a_0 = 10$, $a_1 = -6$, $a_2 = 1$). For FQ, the rates are: r(l) = 0 ($a_0 = a_1 = a_2 = 0$). All the above-mentioned plans provide referral incentives which are offered by the other two MLM's. The information used here was taken from recruitment materials and advertisements given to prospective recruits. Table 1 provides an example of the earnings of the topmost nodes (members) in the networks illustrated in Fig. 1.

3. Results and discussion

3.1. Network cross-section

The network cross-section refers to the distribution of members among levels. Fig. 3(a) illustrates the cross-section of unilevel MLM networks for different network sizes *N*. The graph plots data from agent-based simulations that agree with the analytic curves generated via Eq. (2). Contrary to claims promoted by MLM's [6], the growth of a unilevel MLM tapers off when the network levels have reach a certain number. The slowdown happens even when nobody renounces membership in the network. The decrease results primarily from the finiteness of the population of possible recruits within the social network of each member-recruiter.

Since the recruitment success rate does not improved markedly for a member at level l = 30 (see Fig. 3(a)), an agent can earn more through rebates and referral incentives, by joining another smaller rival MLM. Agents at the top of the unilevel network with low *l* numbers, earn considerably more than those below them.

Fig. 3(b) plots the cross-section of a binary MLM and compares it with field data from a real-world binary MLM (Legacy). The data plot exhibits a relative peak shift towards l > 20, that might be caused by the resignation of a significant number of agents at $l \sim 15$, due to *allelomimesis* [16,17] or sympathy [18].

3.2. Earning prospect

Total profits per level P(l) is calculated for the unilevel and binary MLM networks using compensation plans from realworld MLM's. The profit that is earned by an individual member at level l is the ratio between P(l) and f(l). We employ

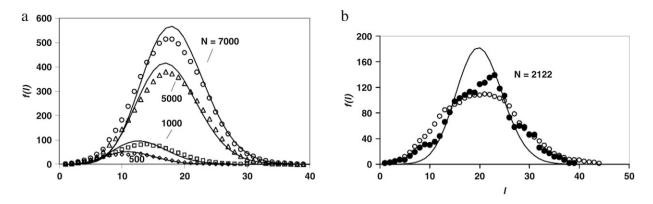


Fig. 3. Network cross-section for different network size *N*. Agent distribution f(l) per network level in (a) unilevel and (b) binary MLM. In (a) agent-based simulation results (polygons) are compared with solid curves based on Eq. (2) with $\gamma = 1/4$. Simulations use small-world network online results [9], with average path length between 5 and 7. Parameter $\alpha = 0.005$, 0.01, 0.01, 0.011 for N = 500, 1000, 5000, 7000, respectively, preserving $k_{WS} = 14$. In (b) simulation (open circles) uses $\beta = 0.0085$ while solid curve is based on Eq. (4) with $\lambda/N = 0.0068$. Also plotted in (b) are data values (filled circles) from a real-world MLM (Legacy).

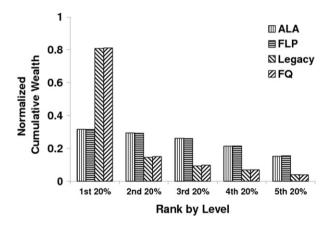


Fig. 4. Earnings versus node level number. Normalized cumulative profit of compensation plans from four real-world MLM's (Unilevel: ALA, FLP; Binary: Legacy, FQ). Horizontal axis shows five sequential indices corresponding to intervals covering 20% of total membership. Results from analytic procedure and agent-based simulation differ by less than 1%.

the Pareto "80–20" rule as a fairness criterion [19]. A MLM network is deemed unfair if at least 80% of the total MLM profit is earned by only 20% of its members. We partition the MLM membership into five sequentially-indexed intervals. Each interval consists of 20% of the total membership such that index 1 represents the first (earliest) wave of members, index 2 the second wave, and so forth. The P(l)/f(l) values for members within an interval are added up and normalized against the total profit of the entire network. Results that are based on four real-world compensation plans are plotted in Fig. 4.

The plot reveals distinct equitability patterns between unilevel and binary MLM's. The Pareto rule is obeyed by the binary MLM — the first 20% of members earn 80% of the total network profit. For the unilevel MLM, the first 20% of members only contribute a share of about 30% of the total earnings. Clearly, the unilevel MLM is more equitable than the binary MLM.

The greater equitability of unilevel MLM's is partly due to the absence of constraint in the number of downlines that can be handled by a member. In a unilevel MLM, recent members can earn comparatively with older ones by simply recruiting more downlines. On the other hand, the earning prospects of members in binary MLM's that limit the number of downlines for a member, strongly favor those who joined earlier.

3.3. Network strategies and related ideas

Our analysis can be also utilized to design a new MLM strategy. For instance, it can be used to improve the earning equitability of MLM compensation plans for a fixed total profit by: (1) Increasing the effective depth from which an agent can extract profits (*D*), and/or (2) Decreasing the variation in rebate rates [$\Delta r = r(l) - r(l+1)$] across successive downlines (optimal when $\Delta r = 0$).

MLM's are competing business schemes (binary and unilevel) that coexist in an environment with finite resources. Their case is similar to the coexistence of two languages in a community [20], with the added advantage that MLM's can be analyzed over shorter time scales than languages [21]. Unlike previous approaches that separate the role of competition [22]

or local dynamics [23] in the emergence of self-organization in complex systems, the MLM's are analyzed with both social interaction mechanisms present simultaneously. In an MLM, agents compete for market share while also engaging in the construction of the network structure. Dynamical complications are contained since MLM's have well-defined structural and interaction rules – a feature that is quite common in real-world systems [24].

4. Summarv

We have shown analytically that MLM networks cannot exhibit unlimited growth, contrary to the sales pitch of MLM recruiters. We have also quantified the earning potential of members in an MLM network and found that in binary MLM's, the earning profile obeys the Pareto "80–20" rule. In a binary MLM, the ability to earn from commissions is strongly biased against recent members due to the limited number of downlines that a member is allowed to operate. Our analytical results compare favorably with field data from real-world MLM enterprises that operate in the Philippines. Our analytical approach has general validity and can be adapted to other types of MLM architectures.

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