Demand Response Scheduling by Stochastic SCUC

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***Abstract*—Considerable developments in the real-time telemetry of demand-side systems allow independent system operators (ISOs) to use reserves provided by demand response (DR) in ancillary service markets. Currently, many ISOs have designed programs to utilize the reserve provided by DR in electricity markets. This paper presents a stochastic model to schedule reserves provided by DR in the wholesale electricity markets. Demand-side reserve is supplied by demand response providers (DRPs), which have the responsibility of aggregating and managing customer responses. A mixed-integer representation of reserve provided by DRPs and its associated cost function are used in the proposed stochastic model. The proposed stochastic model is formulated as a two-stage stochastic mixed-integer programming (SMIP) problem. The first-stage involves network-constrained unit commitment in the base case and the second-stage investigates security assurance in system scenarios. The proposed model would schedule reserves provided by DRPs and determine commitment states of generating units and their scheduled energy and spinning reserves in the scheduling horizon. The proposed approach is applied to two test systems to illustrate the benefits of implementing demand-side reserve in electricity markets.**

***Index Terms*—Demand response, mixed-integer programming, security cost, stochastic security-constrained unit commitment, uncertainty.**

I. NOMENCLATURE

i	Index of generating units.
l	Index of transmission line.
t	Index of time.
b	Index of bus.
d	Index of DRPs.
NG	Number of generating units.
NT	Number of scheduling hours.
NB	Number of buses.
ND	Number of DRPs.
NS	Number of scenarios.
NG_b	Number of generating units connected to bus b.
L_b	Number of transmission lines connected to bus b.
NN_i	Number of segments of piecewise linear cost function of generating unit i.
NQ_d	Number of discrete points in offer package of DRP d.
SUC_{it}	Startup cost of unit i at time t.
MC_i	Minimum production cost of unit i.
I_{it}	Commitment state of unit i at time t.
P_{it}	Real power generation of unit i at time t.
P_{it}^n	Real power generation of unit i in segment n at time t.
$P_i^{\min}$	Lower limit of real generation of unit i.
$P_i^{\max}$	Upper limit of real generation of unit i.
K_i	Startup cost of unit i.
SR_{it}^U	Scheduled up-spinning reserve of unit i at time t.
SR_{it}^D	Scheduled down-spinning reserve of unit i at time t.
RU_i	Ramp-up limit of unit i (MW/min).
RD_i	Ramp-down limit of unit i (MW/min).
T_i^{on}	Minimum up time of unit i.
T_i^{off}	Minimum down time of unit i.
X_{it}^{on}	On time of unit i at time t.
X_{it}^{off}	Off time of unit i at time t.
F_{lt}	Real power flow of line l at time t.
$F_l^{\max}$	Maximum capacity of line l.
X_l	Reactance of line l.
δ_{ls}	Voltage angle of sending-end bus of line l.
δ_{lr}	Voltage angle of receiving-end bus of line l.
P_{bt}^D	Load demand of bus b at time t.
DRR_{dt}	Scheduled reserve of DRP d at time t.
U_{dt}^k	Binary variable associated with point k of DRP d at time t; 1 if the point is scheduled and 0 otherwise.
cc_{dt}^k	Capacity cost of point k of DRP d at time t.
ec_{dt}^k	Energy cost of point k of DRP d at time t.
CCDRP_{dt}	Capacity cost of reserve provided by DRP d at time t.
ECDRP_{dt}	Energy cost of reserve provided by DRP d at time t.

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μ_{it}^{n}	Slope of segment n of the piecewise linear cost function of unit i at time t.
μ_{it}^{UC}	Offered capacity cost of unit i for providing up-spinning reserve at time t.
μ_{it}^{DC}	Offered capacity cost of unit i for providing down-spinning reserve at time t.
μ_{it}^{UE}	Offered energy cost of unit i for providing up-spinning reserve at time t.
μ_{it}^{DE}	Offered energy cost of unit i for providing down-spinning reserve at time t.
$\mathrm{sr}_{it,s}^{U}$	Deployed up-spinning reserve of unit i at time t in scenario s.
$\mathrm{sr}_{it,s}^{D}$	Deployed down-spinning reserve of unit i at time t in scenario s.
$f_{lt,s}$	Real power flow of line l at time t in scenario s.
$\delta_{ls,s}$	Voltage angle of sending-end bus of line l in scenario s.
$\delta_{lr,s}$	Voltage angle of receiving-end bus of line l in scenario s.
$\mathrm{drr}_{dt,s}$	Deployed reserve of DRP d at time t in scenario s.
$u_{dt,s}^{k}$	Binary variable associated with point k of DRP d at time t; 1 if the point is deployed in scenario s and 0 otherwise.
$\mathrm{LC}_{bt,s}$	Involuntary load curtailment in bus b at time t in scenario s.
VOLL_{bt}	Value of lost load in bus b at time t.
p_s	Probability of scenario s.
T	System lead time (h).
τ	Spinning reserve market lead time (min).

II. Introduction

DEMAND RESPONSE (DR) is a tariff or program established to motivate changes in electric consumption by end-use customers in response to changes in the price of electricity over time. DR offers incentives designed to induce lower electricity use at times of high market prices or when grid reliability is jeopardized [1]. Dramatic increases in demand for electric power have made the use of DR more attractive to both customers and system operators.

As the above definition implicitly emphasizes, DR programs can be divided into two major programs: time-based DR programs, and incentive-based DR programs. Both type of DRs are currently under operation in many ISOs around the world. The time-based DR programs are established to overcome flat or averaged electricity pricing flaws. Many types of these programs are designed in different independent system operators (ISOs), from which time-of-use tariffs, critical-peak pricing, and real-time pricing are the three well-known programs. The incentive-based DR programs offer payments for customers to reduce their electricity usage during periods of system need or stress. The incentive-based DR programs substantially have market-based structures, and can be offered in both retail and wholesale markets. Different types of incentive-based programs span over long-term to mid-term, short-term, and even real-time offered programs, each of which has its own goal of operation.

In order to better implementation of DR programs, new market participants designated as demand response providers (DRP) are introduced in wholesale electricity markets. A DRP participates in electricity markets as a medium between ISO and retail customers, and has the responsibility of aggregating and managing customer responses to ISO offered programs. The ISO-sponsored DR programs have requirements such as minimum curtailment level. Many of retail customers do not satisfy these requirements. The DRP enrolls customers to participate in different DR programs, and offers the aggregated responses in the ISO's program. In this way, all customers, even small ones, have an opportunity to participate in DR programs. In the FERC order 719, it is emphasized that ISOs can permit DRPs to bid DR on behalf of retail customers directly into the ISO's organized markets [2]. The DRP is also responsible to provide customers with telemetry systems needed for monitoring and control of their electricity consumption. It should also be noted that customers who satisfy these requirements can participate solely in DR programs.

The FERC order 719 requires ISOs to accept bids from DR resources in their markets for ancillary services, on a basis comparable to other resources [2]. Considerable developments in demand-side real-time telemetry systems allow ISOs to use demand-side provided reserves in ancillary service markets. To this end, many ISOs developed certain programs designated as ancillary service demand response (ASDR) programs. The NYISO has developed the ICAP/SCR program and utilize it during reserve shortage events [3]. The PJM interconnection implemented the day-ahead scheduling reserve market (DASR) and is intended to provide incentives for demand resources to provide day-ahead scheduling reserves [4]. The ERCOT designed the load acting as a resource (LaaR) program, which allows customers who meet certain performance requirements to provide operating reserve [5]. The ISO New England started the real-time DR program in 2005, which requires customers to commit mandatory energy reductions on a predefined notice from the ISO [6].

Considerable efforts have been devoted to solve the security-constrained unit commitment (SCUC) problem in the four past decades [7]–[12]. The state-of-the-art method for the solution of the SCUC problem is presented using the Benders decomposition [13]. The method decomposes the SCUC problem into the UC master problem and two subproblems for checking network constraint at the base case and contingencies. The method of [13] has further been developed in [14] to consider system ac load flow constraints in the SCUC problem.

The SCUC problem can be considered as a large-scale mathematical programming problem which is subjected to system components unavailability and load forecast errors. Stochastic programming (SP) is introduced in [15] to deal with uncertainties in mathematical programming problems. Reference [16] might be the first that formulated the unit commitment problem as a stochastic programming model without considering network security constraints. In [17], the market-clearing problem with security is formulated as a stochastic programming problem with uncertainty affecting only the objective function. The long-term stochastic SCUC model is developed in [18], which simulates the impact of uncertainty and allocation of fuel resources and emission allowance when solving the long term SCUC problem.

This paper presents a short-term stochastic SCUC model that simultaneously schedules generating units' energy and spinning reserve and also reserve provided by demand response resources. The proposed stochastic SCUC model is formulated as a two-stage stochastic mixed-integer programming (SMIP) model. The first-stage involves network-constrained unit commitment in the base case and the second-stage checks security assurance in system scenarios. The second-stage recourse function in the proposed two-stage model is cost of providing security in system scenarios. This is the cost of deploying available resources to return the system to the load-supply balance state. The Monte Carlo simulation method is used to simulate random outages of generating units and transmission lines. The scenario reduction method is also adapted to reduce the number of scenarios and the computational burden of the model.

In the proposed stochastic model, ISO runs the ASDR program to provide operating reserve from DRPs at load buses. Naturally, the reserve provided by DRPs is different from that of generating units. It should therefore be appropriately modeled to reflect its actual condition. A model for reserve provided by DRPs and its associated cost function is presented in this paper, and its mixed-integer representation is developed to be used in the proposed stochastic SCUC model.

The rest of this paper is organized as follows. In Section III, the proposed DR program and market structure are introduced. The proposed stochastic SCUC problem is defined and elaborated in this section. Section IV presents the proposed mixed-integer representation of DRP reserves and the associated cost functions. The formulation of proposed two-stage stochastic SCUC is presented in Section V. Section VI presents the solution method of stochastic programming. In Section VII, case studies are presented and discussed. Conclusions are given in Section VIII.

III. Problem Definition

A. Demand Response Program

The focus of this paper is to schedule operating reserves provided by DR. It is assumed that ISO runs the ASDR program for providing operating reserves. DRPs submit offers to participate in this program. The reserve provided by DRPs are analogous to up-spinning reserve provided by generating units. The enrolled customers would reduce their demand in the predefined lead time to provide the service. In this paper, it is assumed that customers will not provide down-spinning reserve services.

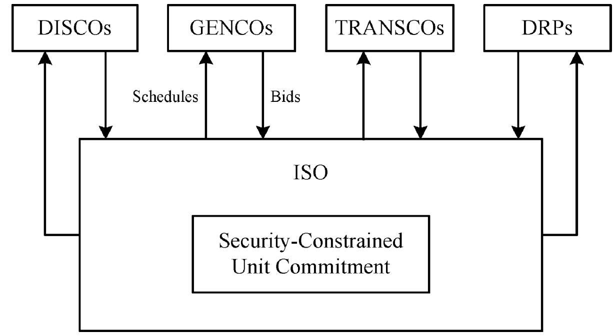

Fig. 1. Correspondence between ISO and main market participants.

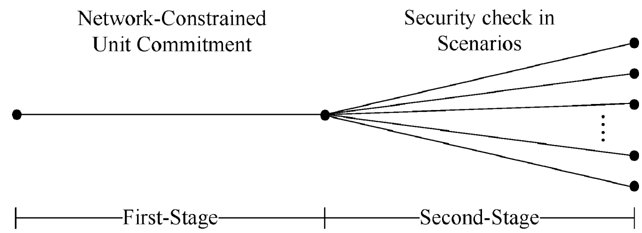

Fig. 2. Sequence of decisions in the SCUC problem.

B. Day-Ahead Market Structure

Fig. 1 shows that ISO receives bid-quantity offers from generating companies (GENCOs) to provide energy, up- and down-spinning reserve services, as well as DRPs' offer to provide reserves. ISO will also receive hourly load demands from DISCOs. It clears energy and spinning reserve markets and schedules DRP reserves simultaneously by applying SCUC.

The SCUC objective is to determine a unit commitment schedule at minimum production cost without compromising the system security constraints [13], [19], i.e., the solution will satisfy network flow and load bus constraints in the base case and contingencies. A contingency is a function of random outages of generating units and transmission lines. The random outages of generating units and transmission lines and also hourly load forecast uncertainty are modeled in the proposed approach. A two-stage SMIP model [15] is proposed in Fig. 2 for short-term SCUC. The SMIP decisions are divided into the first and second-stage decisions.

The first-stage decisions are those which have to be made before the realization of system scenarios. The decisions consist of commitment states of generating units and their scheduled energy and spinning reserve in each scheduling hour. The decision on the scheduled DRP reserves is also made in the first-stage. The system security constraints are checked after the realization of system scenarios and in the second-stage decisions. The decisions are associated with the deployment of spinning and DRP reserves, and the amount of involuntary load shedding in each scenario. The social cost of SCUC consists of the base case cost and the expected cost of providing security.

The proposed SMIP model considers the following goals:

- commit generating units and clear the energy market;
- schedule spinning reserve of each generating unit (simultaneous clearing of spinning reserve market);
- schedule DRP reserve;
- consider random outages of generating units and transmission lines;

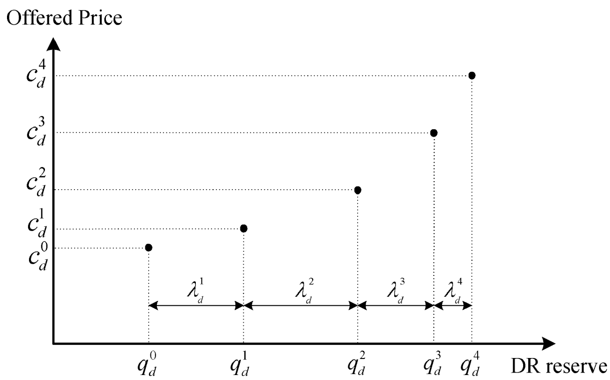

Fig. 3. DRP's bid-quantity offer package.

- deviations of power produced in scenarios as compared to the base case is measured and monetized by reserve variables;
- consider involuntary load curtailments as possible corrective actions.

IV. Demand Response Model

DRPs will aggregate discrete retail customer responses and submit a bid-quantity offer to the ISO, as shown in Fig. 3.

The discrete DRP reserve quantities are labeled as q_d^k with the associated cost of c_d^k. Here, q_d^0 should be greater than the minimum curtailment level of the ASDR program specified by ISO. A mixed-integer representation of the DRP bid-quantity is shown in (1)–(3)

$$\mathrm{DRR}_d = q_d^0 u_d^0 + \sum_{k=1}^{\mathrm{NQ_d}} \lambda_d^k u_d^k \tag{1}$$

$$\mathrm{CDRP}_d = c_d^0 q_d^0 u_d^0 + \sum_{k=1}^{\mathrm{NQ_d}} c_d^k \lambda_d^k u_d^k \tag{2}$$

$$\lambda_d^k = q_d^k - q_d^{k-1}. \tag{3}$$

Here, it is assumed that the demand decreases as prices increase and c_d^k is constrained to increase monotonically [8]. A DRP submits two set of offers to the ASDR program; the capacity cost and the energy cost of reserve. It should be noted that the energy cost of reserve is paid only if the reserve is deployed by the ISO in actual operation.

V. Problem Formulation

The formulation of the problem includes the objective function, and the first-stage and second-stage constraints.

A. Objective Function

The objective function is formulated as a standard two-stage SP problem [15]. The total cost is given in (4), in which the first line is cost of energy production including startup cost; the second line is cost of scheduling up- and down-spinning reserve; the third line is cost of scheduling DRP reserves, and the fourth line is the expected cost of providing security in scenarios

$$\begin{aligned}\sum_{t=1}^{\mathrm{NT}} \Bigg\{ \sum_{i=1}^{\mathrm{NG}} \Bigg(&\mathrm{SUC}_{it} + \mathrm{MC}_i I_{it} + \sum_{n=1}^{\mathrm{NN}} \mu_{it}^n P_{it}^n \Bigg) \\ &+ \sum_{i=1}^{\mathrm{NG}} \left(\mu_{it}^{\mathrm{UC}} \mathrm{SR}_{it}^U + \mu_{it}^{\mathrm{DC}} \mathrm{SR}_{it}^D \right) \\ &+ \sum_{d=1}^{\mathrm{ND}} \left(\mathrm{CDRP}_{dt} \right) \\ &+ \sum_{s=1}^{\mathrm{NS}} \left(p_s \mathrm{SC}_s \right) \Bigg\}. \end{aligned} \tag{4}$$

SC_s is the second-stage recourse function of the two-stage stochastic model. It is the security cost associated with scenario s as expressed below

$$\begin{aligned}\mathrm{SC}_s = &\sum_{i=1}^{\mathrm{NG}} \left(\mu_{it}^{\mathrm{UE}} \mathrm{sr}_{it,s}^U + \mu_{it}^{\mathrm{DE}} \mathrm{sr}_{it,s}^D \right) \\ &+ \sum_{d=1}^{\mathrm{ND}} \left(\mathrm{ECDRP}_{dt,s} \right) \\ &+ \sum_{b=1}^{\mathrm{NB}} \left(\mathrm{VOLL}_{bt} \mathrm{LC}_{bt,s} \right) \end{aligned} \tag{5}$$

where the first line of (5) represents cost of deploying up- and down-spinning reserve in scenario s, the second line is cost of deploying the DRP reserve in scenario s, and the third line is cost of involuntary load curtailment in scenario s. In other words, the cost of system security is the cost of deploying resources for providing security in system scenarios. In this paper, spinning reserve, DRP reserve, and involuntary load curtailment are considered as resources which can be used to maintain system security in case of system component outages.

There are two sets of variables in (4) and (5) for reserve services provided by DRPs and generating units. The first set is associated with the capacity cost offered by GENCOs and DRPs, while the other set is associated with the energy cost offered by GENCOs and DRPs. These two sets of variables are subjected to the first-stage and second-stage constraints which are presented below.

B. First-Stage Constraints

The first-stage constraints are associated with the base case, including the following:

DC power flow equation in steady state

$$\sum_{i=1}^{\mathrm{NG_b}} P_{it} - P_{bt}^D = \sum_{l=1}^{L_b} F_{lt} \qquad \forall b, \forall t \tag{6}$$

$$F_{lt} = \frac{1}{X_l} \left(\delta_{ls} - \delta_{lr} \right) \qquad \forall l, \forall t. \tag{7}$$

Transmission flow limits in the base case

$$-F_{lt}^{\max} \le F_{lt} \le F_{lt}^{\max} \qquad \forall l, \forall t. \tag{8}$$

Generating units startup cost constraint

$$\mathrm{SUC}_{it} \geq K_i \left(I_{it} - I_{i(t-1)}\right) \qquad \forall i, \forall t. \tag{9}$$

Real power generation constraints

$$P_{it} = P_i^{\min} I_{it} + \sum_{n=1}^{\mathrm{NN_i}} P_{it}^n \qquad \forall i, \forall t \tag{10}$$

$$0 \leq P_{it}^n \leq P_i^{n,\max} \qquad \forall n, \forall i, \forall t \tag{11}$$

$$P_{it} \leq P_i^{\max} I_{it} - \mathrm{SR}_{it}^U \qquad \forall i, \forall t \tag{12}$$

$$P_{it} \geq P_i^{\min} I_{it} + \mathrm{SR}_{it}^D \qquad \forall i, \forall t. \tag{13}$$

Up- and down-spinning reserve limits

$$0 \leq \mathrm{SR}_{it}^U \leq \mathrm{RU}_i \times \tau \qquad \forall i, \forall t \tag{14}$$

$$0 \leq \mathrm{SR}_{it}^D \leq \mathrm{RD}_i \times \tau \qquad \forall i, \forall t. \tag{15}$$

Minimum up and down time constraints

$$\begin{gathered}\left[X_{i(t-1)}^{\mathrm{on}} - T_i^{\mathrm{on}}\right]\left(I_{i(t-1)} - I_{it}\right) \geq 0 \\ \left[X_{i(t-1)}^{\mathrm{off}} - T_i^{\mathrm{off}}\right]\left(I_{it} - I_{i(t-1)}\right) \geq 0 \\ \forall i, \forall t.\end{gathered} \tag{16}$$

Ramping up and down constraints

$$\begin{aligned}P_{it} - P_{i(t-1)} \leq & \left[1 - I_{it}\left(1 - I_{i(t-1)}\right)\right] \mathrm{RU}_i \\ & + I_{it}\left(1 - I_{i(t-1)}\right) P_i^{\min} \\ P_{i(t-1)} - P_{it} \leq & \left[1 - I_{i(t-1)}\left(1 - I_{it}\right)\right] \mathrm{RD}_i \\ & + I_{i(t-1)}\left(1 - I_{it}\right) P_i^{\min} \\ & \forall i, \forall t.\end{aligned} \tag{17}$$

DRP reserve constraints

$$\mathrm{DRR}_{dt} = q_{dt}^0 U_{dt}^0 + \sum_{k=1}^{\mathrm{NQ_d}} \lambda_{dt}^k U_{dt}^k \tag{18}$$

$$\mathrm{CCDRP}_{dt} = \mathrm{cc}_{dt}^0 q_{dt}^0 U_{dt}^0 + \sum_{k=1}^{\mathrm{NQ_d}} \mathrm{cc}_{dt}^k \lambda_{dt}^k U_{dt}^k. \tag{19}$$

As stated in (12) and (13), it is assumed that generating units offer maximum amount of their achievable capacity as spinning reserve. The only constraint on spinning reserve provided by generating units is their ramping capability, which is stated in (14) and (15). This will result in optimum determination of energy and spinning reserve provided by generating units according to energy and reserve requirements of the system.

C. Second-Stage Constraints

The second-stage constraints which are considered in system scenarios are as follows:

DC power flow equation in scenarios

$$\begin{aligned}\sum_{i=1}^{\mathrm{NG_b}} \xi_i^G(s) P_{it} + \sum_{i=1}^{\mathrm{NG_b}} \mathrm{sr}_{it,s}^U - \sum_{i=1}^{\mathrm{NG_b}} \mathrm{sr}_{it,s}^D & \\ + \sum_{d=1}^{\mathrm{ND_b}} \mathrm{drr}_{dt,s} - P_{bt}^D + \mathrm{LC}_{bt,s} & \\ = \sum_{l=1}^{L_b} f_{lt,s} \qquad \forall b, \forall t, \forall s &\end{aligned} \tag{20}$$

$$f_{lt,s} = \xi_l^L(s)\left[\frac{1}{X_l}\left(\delta_{ls,s} - \delta_{lr,s}\right)\right] \qquad \forall l, \forall t, \forall s. \tag{21}$$

Transmission flow limit in scenarios

$$-F_{lt}^{\max} \leq f_{lt,s} \leq F_{lt}^{\max} \qquad \forall l, \forall t, \forall s. \tag{22}$$

Deployed up- and down-spinning reserve limit

$$0 \leq \mathrm{sr}_{it,s}^U \leq \xi_i^G(s)\, \mathrm{SR}_{it}^U \qquad \forall i, \forall t, \forall s \tag{23}$$

$$0 \leq \mathrm{sr}_{it,s}^D \leq \xi_i^G(s)\, \mathrm{SR}_{it}^D \qquad \forall i, \forall t, \forall s. \tag{24}$$

Deployed DRP reserve constraints

$$0 \leq \mathrm{drr}_{dt,s} \leq \mathrm{DRR}_{dt} \tag{25}$$

$$\mathrm{drr}_{dt,s} = q_{dt}^0 u_{dt,s}^0 + \sum_{k=1}^{\mathrm{NQ_d}} \lambda_{dt}^k u_{dt,s}^k \tag{26}$$

$$\mathrm{ECDRP}_{dt,s} = \mathrm{ec}_{dt}^0 q_{dt}^0 u_{dt,s}^0 + \sum_{k=1}^{\mathrm{NQ_d}} \mathrm{ec}_{dt}^k \lambda_{dt}^k u_{dt,s}^k. \tag{27}$$

Involuntary load curtailment limit

$$0 \leq \mathrm{LC}_{bt,s} \leq \mathrm{LC}_{bt}^{\max} \qquad \forall b, \forall t, \forall s. \tag{28}$$

Here, in order to consider random outage of generating units and transmission lines, ξ is divided into $\xi(s) = \left\{\xi^G(s), \xi^L(s)\right\}$ respectively. Considering a two-state Markov model [20] for each component, the elements of these two vectors are binary random variables in which 1 represents the healthy state of a component and 0 otherwise.

In the proposed two-stage stochastic model, decision on generating units' commitment states is only made in the first-stage. Besides, the real power generation of the committed units at the base case should satisfy the DC power flow constraint expressed in (6). The power generation variable P_{it} does not change in any scenario s. Instead, $\mathrm{sr}_{it,s}^U$, $\mathrm{sr}_{it,s}^D$, $\mathrm{drr}_{dt,s}$, and $\mathrm{LC}_{bt,s}$, are determined such that the DC power flow (20) is satisfied in each scenario s. The most economic portfolio of the above alternatives is selected by the model to alleviate the adverse impacts of random outages of generating units and transmission lines.

The relationship between the first and the second-stage reserve variables is specified in (23)–(25). In addition, (23) and (24) indicate that only healthy generating units in scenario s would provide spinning reserves.

VI. Solution Method

The first step in solving a SP problem is to model the uncertainties associated with the system [15]. The basic two-state Markov model shown in Fig. 4 is used to represent generating unit and transmission line status [20].

TABLE I
FAILURE RATES AND MEAN DOWN TIMES OF THE 6-BUS SYSTEM

Component	G1	G2	G3	L1	L2	L3	L4	L5	L6	L7
Failure Rate (f/yr)	0.001	0.00083	0.0022	0.24	0.02	0.51	0.33	0.02	0.48	0.02
Mean Down Time (h)	-	-	-	16	768	16	16	768	16	768

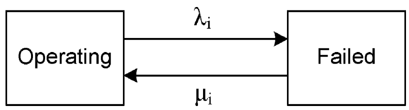

Fig. 4. Two-state model of generating unit/transmission line i.

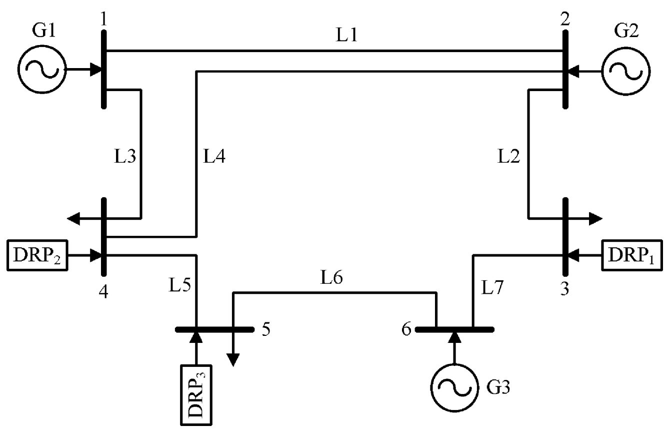

Fig. 5. One-line diagram of the six-bus system.

The time-dependent probabilities of the operating and failed states are calculated as follows:

$$P^i_{\text{failed}}(T) = \frac{\lambda_i}{\lambda_i + \mu_i} - \frac{\lambda_i}{\lambda_i + \mu_i} e^{-(\lambda_i + \mu_i)T} \tag{29}$$

$$P^i_{\text{operating}}(T) = 1 - P^i_{\text{failed}}(T). \tag{30}$$

In the case of generating units, the system lead time T is relatively short such that the failed unit may not be repaired or replaced within this short period [20]. Under this assumption, (29) and (30) can be approximated by

$$P^i_{\text{failed}}(T) = 1 - e^{-\lambda_i T} \cong \lambda_i T = ORR_i \tag{31}$$

$$P^i_{\text{operating}}(T) = 1 - ORR_i. \tag{32}$$

This, however, is not the case for transmission lines [20]. The next step in the solution of the proposed SP model is to generate system scenarios. The Monte Carlo simulation approach is used in this paper to simulate the failed and operating state of generating units and transmission lines.

The dimensionality of a SP problem depends considerably on the number of scenarios. The scenario generation algorithms normally generate many scenarios such that computational burden associated with the resulting SP problem is cumbersome or even there could be no feasible solution for it. However, scenario reduction methods can appropriately be adapted to reduce the number of generated scenarios such that a tradeoff is made between the computational burden and accuracy of the results [21]. In this paper, the probability metrics based scenario reduction methods [22] are used to reduce the number of generated scenarios.

TABLE II
DRP OFFERS

k	0	1	2
q^k_{dt} (MW)	33% of total Response	66% of total response	100% of total response
cc^k_{dt} ($/MW)	22	23	24
ec^k_{dt} ($/MW)	$20 \times cc^0_{dt}$	$20 \times cc^1_{dt}$	$20 \times cc^2_{dt}$

The reduction method determines a subset of the initial generated scenario set and assigns new probabilities to the selected scenarios. The probabilities associated with all deleted scenarios are then set to zero. The new probability of a selected scenario is equal to the sum of its former probability and the probabilities associated with all deleted scenarios which are closest to it based on the specified distance [22]. The new set of probabilities associated with the selected scenarios is such that it covers most of the probability space of the problem.

VII. NUMERICAL EXAMPLES

The proposed method for the scheduling of DR reserve is demonstrated on a six-bus system and on the IEEE-RTS.

A. Six-Bus System

The six-bus system shown in Fig. 5 is used to demonstrate the features of the proposed model. Failure rate and mean down time of generating units and transmission lines are presented in Table I. Additional data associated with system are extracted from [18]. The spinning reserve market lead time is assumed to be 10 minutes. The ramping rates of the three units are considered to be 5.5 MW, 5.0 MW, and 2.0 MW, respectively. The cost curves of generating units given as a quadratic function in [18] are approximated by three linear segments between the minimum and maximum generating units capability. It is assumed that generating units offer energy and capacity cost of up- and down-spinning reserves at the rates of 100% and 40% of their highest incremental cost of producing energy, respectively. The minimum up and down time constraints are not considered in this study.

There are three DRPs in load buses with a format shown in Fig. 3. The DRPs data are presented in Table II, which consist of three discrete points, i.e., 33%, 66%, and 100% of the total response of customers. The vector of random variables contains three random variables for generating units and seven for transmission lines. A total of 53 scenarios are generated. The backward reduction method is used to reduce the number of the scenarios to ten, with ten random variables in each scenario. The relative distance between the generated and reduced scenarios is set to be 10%.

The analyses are conducted for a 5-h scheduling horizon and the load model shown in Table III. Three different case studies

TABLE III
SYSTEM LOAD IN SCHEDULING HOURS

Hour	1	2	3	4	5
Load (MW)	150	225	250	200	125

TABLE IV
SCHEDULING RESULTS – CASE 1

	Unit	Hour				
		1	2	3	4	5
P_{it} (MW)	G1	100	130	154	145	100
	G2	0	10	10	0	0
	G3	50	85	86	55	25
SR_{it}^{U} (MW)	G1	50	55	40.12	55	25
	G2	0	30	45.63	0	0
	G3	0	0	8.37	20	0
SR_{it}^{D} (MW)	G1	0	30	54	45	0
	G2	0	0	0	0	0
	G3	0	0	0	0	0
Expected Load Curtailment Cost ($/h)		733.37	733.37	802.48	1042.32	733.37

TABLE V
SCHEDULING RESULTS – CASE 2

	Unit	Hour				
		1	2	3	4	5
P_{it} (MW)	G1	100	155	155	133	100
	G2	0	0	0	0	0
	G3	50	70	95	67	25
SR_{it}^{U} (MW)	G1	50	55	55	55	25
	G2	0	0	0	0	0
	G3	0	20	5	20	0
SR_{it}^{D} (MW)	G1	0	55	55	33	0
	G2	0	0	0	0	0
	G3	0	0	0	0	0
DR_{dt} (MW)	DRP1	0	3	5	1.3	0
	DRP2	0	6	10	5.3	0
	DRP3	0	6	10	5.3	0
Expected Load Curtailment Cost ($/h)		733.37	980.53	1276.26	745.72	733.37

are conducted to illustrate the impacts of utilizing DR reserve. In Case 1, only generating units can provide reserves. In Case 2, in addition to the generating units, DRPs can enroll 10% of their consumers to participate, while in Case 3, 20% of the consumers agree to participate. The remaining load in each bus is set as the maximum involuntary load curtailment in that bus at a cost of 7000 $/MWh. The proposed model was solved using the mixed-integer programming solver CPLEX 11.2.0 [23] on a *DELL vostro 1500* computer with a 2.2 GHz dual-core processor and 2 GB of RAM. The computation times for all the three cases are less than 1 s, while the upper bound on the duality gap is set to zero.

The results are presented in Tables IV–VI. Table IV presents the optimal results associated with Case 1, in which units G1 and G3 are committed at all hours, while the expensive unit G2 is committed only at peak hours 2 and 3 and is loaded at its minimum capacity. It can also be seen from Table IV that unit G2 provides considerable amounts of up-spinning reserve at hours 2 and 3. As a matter of fact, unit G2 is committed at these hours to provide up-spinning reserve because the capacities associated

TABLE VI
SCHEDULING RESULTS – CASE 3

	Unit	Hour				
		1	2	3	4	5
P_{it} (MW)	G1	100	146	155	131.7	100
	G2	0	0	0	0	0
	G3	50	79	95	68.3	25
SR_{it}^{U} (MW)	G1	50	55	55	55	25
	G2	0	0	0	0	0
	G3	0	20	5	18.3	0
SR_{it}^{D} (MW)	G1	0	46	55	31.7	0
	G2	0	0	0	0	0
	G3	0	0	0	0	0
DR_{dt} (MW)	DRP1	0	6	6.7	2.7	0
	DRP2	0	12	13.3	5.3	0
	DRP3	0	6	20	5.3	0
Expected Load Curtailment Cost ($/h)		733.37	758.08	856.95	733.37	733.37

with the cheaper units G1 and G3 are not sufficient to supply both energy and up-spinning reserve requirements.

The last row of Table IV presents the expected hourly cost of load curtailment. In this study, involuntary load curtailment is required at off-peak hours 1 and 5. Based on the proposed model, it is more economic to curtail loads instead of scheduling reserve in those scenarios with low likelihood of occurrences.

Tables V and VI summarize the optimal results for Cases 2 and 3, respectively. The expensive unit G2 which was committed in Case 1, is not committed in Cases 2 and 3 due to utilizing DRP reserves. The generating unit schedules in energy and reserve markets in Cases 2 and 3 are different from those of Case 1. As expected, the DRP reserve is only utilized at peak hours 2 to 4. In Case 2, the entire DRP reserve is scheduled at hour 3; while a part of this reserve is used at hours 2 and 4. In Case 3, when 20% of the system load is offered by DRPs as reserve, more reserve is scheduled. However, the entire DRPs reserve in not scheduled at any given hours.

The expected cost of load curtailment at hour 3 in Cases 2 and 3 is investigated here. Although DRPs provide 25 MW reserve at hour 3 in Case 2, the expected cost of load curtailment at this hour is higher than that of Case 1. The reason for this is that the credibility of scenarios is not high enough to commit the expensive unit G2. Instead, the load is curtailed in rarely occurred scenarios. The subject is different in Case 3. In this case, DRPs offer 50 MW of reserve at hour 3, from which 40 MW is scheduled. However, the expected cost of load curtailment at this hour is still greater than that of Case 1. In other words, when a sufficient DRP reserve is available in Case 3, it may be worthwhile to curtail more loads involuntarily instead of scheduling more DRP reserves.

In addition, providing reserve by DR resources alleviates transmission lines congestion caused by outage of system components. Table VII shows line flows after outage of generating unit G3 in the three cases. It can be seen from this table that in Case 1 in which no DR reserve is provided, lines 2 and 3 are reached their maximum capacity. The reason for this is that, outage of unit G3 in this case is compensated by units G1 and G2. However, in Cases 2 and 3, outage of unit G3 is compensated partly by the local reserve resources provided by DRPs. This will therefore alleviate the lines congestion.

TABLE VII
LINES FLOW AFTER OUTAGE OF UNIT G3 (MW)

Line	L1	L2	L3	L4	L5	L6	L7
Case 1	94.12	**100**	**100**	49.75	49.75	-50.25	50.25
Case 2	110	73.96	**100**	36.04	50.55	-39.45	39.45
Case 3	111.16	77.64	98.84	33.52	45.69	-34.31	34.31

TABLE VIII
SIX-BUS SYSTEM COSTS

	Operation Cost ($)	Energy Cost ($)	Spinning Reserve Scheduling Cost ($)	DRP Reserve Scheduling Cost ($)	Expected Load Curtailment Cost ($)
Case 1	28,739.65	20,774.32	3,895.82	-	4,044.90
Case 2	28,546.48	19,764.90	3,014.47	1,181.83	4,469.24
Case 3	28,474.03	19,827.07	2,928.98	1,740.33	3,815.13

As shown in Table VII, in Case 2, only line 2 is reached its maximum capacity, while in Case 3, all line flows are within their allowed limits.

Table VIII presents the costs associated with the three case studies. It can be seen that the total system cost in Case 2 is reduced when DRP reserve is utilized. Also, an increase in customer response in Case 3 could result in further reduction in the total cost. The cost of energy supply is considerably reduced in Cases 2 and 3. The reason for this is that the expensive unit G2 is not committed in these cases. The energy cost in Case 3 is slightly higher than that of Case 2 as shown in Table III. The reason for this is that unit G3 is loaded more in hours 2 and 4 of Case 3 than that of Case 2 (see Tables V and VI). This can be recognized by investigating the outage scenarios. When L1 is on outage, due to the capacity limitation of L3, capacity output of unit G1 must be reduces to 100 MW. In other words, some down-spinning reserve should be provided by unit G1 in this scenario. In Case 2, outage of line L1 is compensated by utilizing 20 MW up-spinning reserve provided by unit G3, 15 MW reserve provided by DRPs, and 20 MW involuntary load curtailment. In addition, 55 MW down-spinning reserve should be provided by unit G1. In Case 3, outage of L1, is compensated by 20 MW up-spinning reserve provided by unit G3, 24 MW reserve provided by DRPs, 2 MW involuntary load curtailment, and 46 MW down-spinning reserve provided by unit G1. Therefore, load curtailment in Case 3 is 18 MW less than that of Case 2. This reduction in load curtailment is replaced by 9 MW excess in DRP reserve schedule and 9 MW excess in scheduled energy of unit G3, without facing any transmission limit violations. So, it can be seen that when the customer response is increased in Case 3, the expected cost of involuntary load curtailment decreases. This result clearly shows that the utilization of ASDR program can reduce the risk of involuntary load curtailment.

The price of DRP reserve would considerably affect the scheduled reserve. Fig. 6 shows the sum of scheduled reserves provided by the three DRPs in Case 3 as a function of price. The zero change in price corresponds to the capacity prices given in Table II. In Fig. 6, an increase in price of DRP reserve would decrease the scheduled reserve. At peak hour 3, the system is under stress and therefore some reserve is scheduled even at high prices.

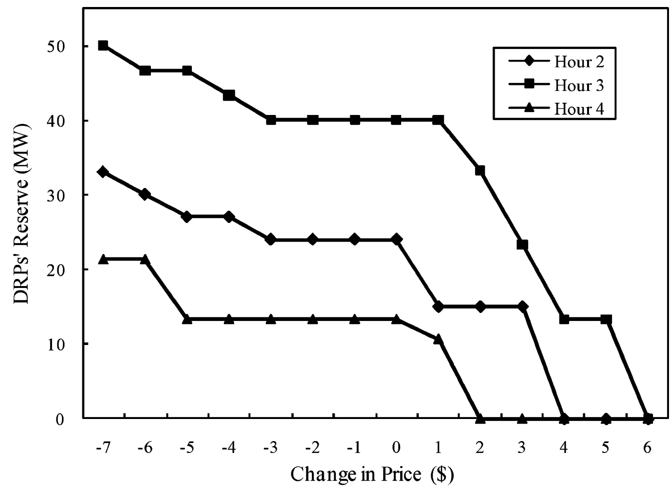

Fig. 6. Variation of scheduled DRP reserve with respect to price in Case 3.

TABLE IX
DRP OFFERS

k	0	1	2
q_{dt}^{k} (MW)	33% of total response	66% of total response	100% of total response
cc_{dt}^{k} ($\$/\mathrm{MW}$)	11	12	13
ec_{dt}^{k} ($\$/\mathrm{MW}$)	$20 \times cc_{dt}^{0}$	$20 \times cc_{dt}^{1}$	$20 \times cc_{dt}^{2}$

B. The IEEE-RTS

The proposed model is applied over a 24-h horizon to the IEEE-RTS [24] including hydro units. It is assumed that generating units submit their offers for energy at the incremental heat rates given in [24]. Also, similar to the six-bus system example, generating units offer energy and capacity cost of up- and down-spinning reserves at the rates of 100% and 40% of their highest incremental cost of energy, respectively. All other data for the system, including startup cost, upper and lower limits on power generation, ramp rates, minimum up and down times, etc., are directly extracted from [24]. The hourly load corresponds to a weekday in summer while the peak load of the day is assumed to be 2850 MW. The cost of involuntary load curtailment is assumed to be 8000 $/MWh during peak hours (hours 10 to 22), and 4000 $/MWh during off-peak hours for all buses. It is assumed that one DRP is founded in each of 17 load buses. The DRPs offer to participate in the ISO's ASDR program is presented in Table IX.

The vector of random variables contains 70 random variables with 32 for generating units and 38 for transmission line availability status. A total of 3967 scenarios are generated using the Monte Carlo simulation. Using the reduction procedure the number of scenarios is reduced from 3967 with 70 random variables in each scenario to 63 scenarios in less than 7 min. The relative distance between the generated and reduced scenarios is set to be 10%.

Two case studies are conducted on this system. In Case 1, it is assumed that no DRP exists. In Case 2, it is assumed that the ISO runs the ASDR program and DRPs' offers are available. DRPs can enroll 10% of their local consumers to participate in the program. The total operating cost as well as the detailed costs

TABLE X
THE IEEE-RTS COSTS

	Operation Cost ($)	Energy Cost ($)	Spinning Reserve Scheduling Cost ($)	DRP Reserve Scheduling Cost ($)	Expected Load Curtailment Cost ($)
Case 1	593884.54	517784.82	57133.39	-	18226.39
Case 2	589869.89	510843.06	50466.54	18572.49	8388.03

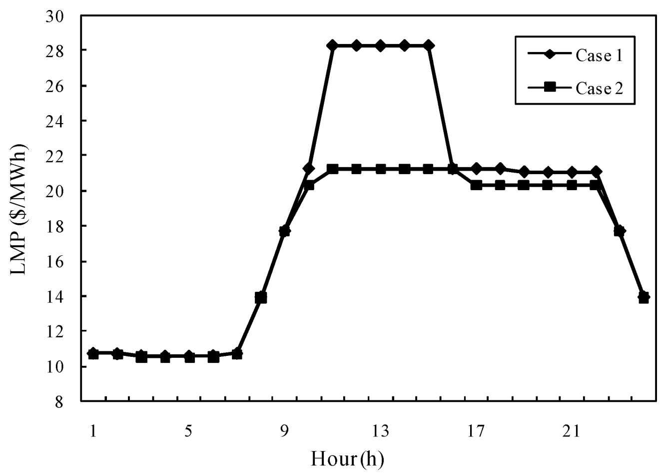

Fig. 7. LMPs of bus 1 over the 24-h horizon.

of energy, spinning reserve scheduling, DRP reserve scheduling, and expected cost of load curtailment are presented in Table X.

As shown in Table X, the cost of utilizing DRP reserve in Case 2 increases, while all other costs including the total operating cost reduce. The total operating cost reduces by $4014 for the horizon. In Case 1, where no demand-side reserve is available, the cheaper U76 units produce a small amount of energy at peak hours while the expensive U197 units produce more energy. Therefore, the required amount of up-spinning reserve should only be provided by the committed U76 units in Buses 1, 2.

The situation changes in Case 2 in which there are DRPs in load buses. In this case, DRPs located at buses 1, 2 provide some portions of required up-spinning reserve and U76 units produce more energy. So, in Table X, the energy production cost in Case 2 is smaller than that of Case 1. Also LMPs are reduced at system buses. Fig. 7 depicts the LMP at bus 1 for the two cases over the 24-h horizon. It can be seen from the figure that compare to Case 1, LMPs are reduced in Case 2, at hours with DRP reserves. The LMP reduction demonstrates one of the DR benefits. In Case 1, customers do not respond to high prices at peak hours and therefore they should pay more for their electricity consumption. In Case 2, only few customers participate in the ISO's ASDR program. In this case, all customers benefit and pay lower prices for their consumption.

Table X shows that the cost of spinning reserve scheduling and the expected load curtailment cost are lower in Case 2 compared to Case 1. These results indicate that in addition to the reduction in spinning reserve provision, the utilization of DRP reserve can significantly reduce involuntary load curtailments (about 54% in this example). The latter shows another important benefit of utilizing DR. In Case 2, a few customers voluntarily participated in the ISO's ASDR program and the risk of involuntary load curtailment reduces for all customers.

As the second-stage variables and constraints are defined for each scenario, the proposed two-stage stochastic mixed-integer programming problem becomes a large-scale mathematical problem with a large number of binary and continuous variables and constraints. This problem is partly overcome by scenario reduction.

Another issue is the number of binary variables to model the DRP reserves. In the IEEE-RTS, there are 17 DRPs with three binary variables associated with each DRP. These variables are defined for the base case and 63 scenarios over the 24-h horizon. So there are 78 336 binary variables as compared to 768 binary variables associated with the commitment state of generating units. However, the large number of binary variables does not increase the computation time dramatically. The computation times of Cases 1 and 2 in the IEEE-RTS system are 72.75 s and 96.13 s, respectively, while the upper bound on the duality gap is set to be 1% in both cases. There are three observations here. First, the proposed mixed-integer representation of DRP reserve is linear, which does not require any complex inequality constraint for linearity. Second, the DRP reserves are completely unbundled from other variables and there is no constraint which ties these variables. The third issue is that there are no conflicting constraints among binary variables of DRP reserve. The only constraint is that the reserves deployed in scenarios would be bounded by the scheduled DRP reserves. Therefore, adding a large number of binary variables associated with DRP reserves does not add any significant computational time in the proposed method.

VIII. CONCLUSIONS

In this paper, a stochastic model to schedule reserve provided by DR resources in wholesale electricity market has been presented. The demand-side reserve resources are modeled by DR providers. A model for the reserve provided by DRPs and its associated cost function are developed. The proposed stochastic model is formulated as a two-stage SMIP problem. Network-constrained unit commitment is performed in the first stage while security constraints are taken into account for each scenario in the second stage. The Monte Carlo simulation approach is used to simulate random outages of generating units and transmission lines. To overcome the dimensionality of the proposed stochastic model, a scenario reduction method is used to reduce the number of scenarios. Using the proposed model, commitment states of generating units, their energy and spinning reserve schedules, as well as scheduled reserve of DRPs are simultaneously determined.

The applicability of the proposed stochastic model is illustrated using a six-bus system and the IEEE-RTS. A number of case studies are conducted on both systems. The results presented demonstrate the benefits of customers' response to ASDR program of ISO. Finally, the computational burden of the proposed model is discussed. It has been shown that the developed model for the DRPs' reserve does not impose any significant computational problem to the proposed stochastic model.

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