Collection and Distribution of Returned-Remanufactured Products in a Vehicle Routing Problem with Pickup and Delivery Considering Sustainable and Green Criteria

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Abstract

As increasing transportation costs mounts pressure on the businesses, there has been an increasing interest on vehicle routing problem (VRP) as a viable and effective solution. Both industry and academia are continuously looking for new approaches to save transport cost and time while increasing profit margins. This endeavor will eventually reduce costs of delivering goods and services for customers, and therefore enhancing the competitiveness of firms involved. Particularly, transportation cost savings could have potential impacts on marketing activities of remanufactured and recycled products in reverse logistics chains. Therefore, developing practical solutions for VRP of original and remanufactured products is one the emerging topics in the current transportation research. In this article, we propose a multi-objective non-linear programming model for the green vehicle routing problem (GVRP), including original and remanufactured products distribution (both delivery and pickup) of end of life (EOL) products. Through the appropriate fuzzy approach the model is linearized, validated, and solved. The results show considerable level of improved performance under the model configurations and proposed solution approach. The obtained results clearly indicate that the proposed mathematical model is capable of reducing the fuel cost, distribution center set-up cost and supplying vehicles, as well as minimizing air pollution. Finally, using a real case study the reliability and viability of the proposed model is verified.

Keywords: Vehicle routing problem; Green logistics; Greenhouse gas; Fuel consumption; Multi-objective; Reverse logistics.

1. Introduction

Since the price of remanufactured products is typically lower than the original products, the cost effectiveness of their distribution becomes a critical focus (Ferrer &

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Swaminathan 2010). Therefore, cost minimization and route management is a vital step in making the marketing and availability of remanufactured products feasible and profitable. In this paper, a vehicle routing problem (VRP) is proposed with the focus of distributing remanufactured and original products simultaneously in shared channels, while considering the pollution levels emitted by the system. This strategy is beneficial to undertake the distribution of remanufactured products at lower costs and achieving a great level of sustainability. The proposed VRP approach considers that the destination points of the demands for original and remanufactured products as the collection points of the returned products. Using this strategy, the collection cost of End of Life (EOL) items and remanufacturing process becomes more profitable.

While increasing transportations costs is becoming a serious concern for many business organizations, complexity of decision making parameters contribute to the difficulty of VRP research. Some of these factors include traffic condition, government regulation, punctuality and sustainability aspects. Thus, VRP is categorized in Np-Hard problems (non-deterministic polynomial-time) (Lenstra and Kan 1981) that requires a significant amount of time and mathematical knowledge for solving. However, when adding the revers and green logistics constraints, VRP issues becomes even more complex.

As Today’s world is marching toward better usage of limited resources, secondhand products and supply chain residuals are becoming valuable resources for remanufacturing processes. Effective remanufacturing has several benefits for the society and consumption cycle, such as reducing the raw materials cost, lowered workforce and disposal benefits. As natural resources become scarce, many companies will eventually move toward multiple levels of recycling and remanufacturing. This move is somehow a necessity for sustainability of future business environment and demands advanced research to identify solutions for optimum pickup and delivery of original, secondhand and waste in the supply chains.

In addition to the financial benefits of effective VRP, sustainable issues are crucial criteria in willingness to pay for remanufactured products (Michaud and Llerena 2011). For example, one of the main social responsibilities of a distribution system is to minimize the fuel consumption (Kara et al. 2007). Lowering fuel consumption can be achieved through lower kilometers travelled, and therefore the level of CO2 emission could be significantly reduced. In a reverse logistics context, remanufacturing issues refer to a situation where original products are distributed, and then secondhand products are collected from customers to repair and redistribute in a different market with (typically) lower price. Also, green issue relates to logistics activities with less harm to the environment. Thus, the necessity of Green Vehicle Routing Problem (GVRP) is quite evident. The lack of such approaches is evident in review studies such as Brækers et al. 2016 and Montoya-Torres et al. 2015.
Knowing the above considerations, a multiple objective mixed-integer mathematical programming model is developed to incorporate two objective functions: minimizing the total cost of distribution and minimizing the total energy consumed in the distribution system by the vehicles. The level of CO2 emissions produced in the system is considered as a constraint in this model. The unique contribution of this model can be realized in the simultaneous distribution of both remanufactured and original products in shared channels. Even though integrated distribution approach plays an essential role in making collecting and redistributing of EOLs profitable, to the best of our knowledge this approach is rarely considered in the available literature. Besides identifying opportunities to optimize distribution of firsthand products, we explore distribution of secondhand products. To put it differently, the focus of this paper is on distribution and delivery of first and secondhand products along with picking up used products.

The remaining of this paper is organized as follows. Section 2 provides a brief review of the most relevant literature to this work. The problem is formulated and presented Section 3, including the mathematical model and linearization process. The numerical results, sensitivity analysis, and a real case study are presented in Sections 4, 5, and 6 respectively. Finally, Section 7 concludes the main findings of this research and provides implications for future research.

2. Literature review

VRP research has a bourgeoning literature which is being briefly and purposely discussed in this article. The truck dispatching problem was first introduced by Dantzig and Ramser (1959). The authors proposed a model in which a fleet of homogenous trucks deliver the gas required by a number of stations from a single operating center. This model aimed to minimize the distances traveled between the stations by the trucks. Afterwards, Clarke and Wright (1964) extended this topic into a linear optimization problem which was normally dealt with transportation and logistics activities involving a set of customers, depots, and a fleet of trucks with different capacities. The work of Clarke and Wright (1964) is probably of the first that is similar to the contemporary VRP models. Since then, researches in this field have been seeking for methods and models to optimize the routing problem and offer a better solution. Malandraki and Daskin (1992) introduced routing problem models, which were time-dependent and considered different speeds. This was an important step to include time-based attributes into VRP models.

Bektas (2006) reviewed the multiple traveling salesman problem (mTSP), which is the general form of TSP problem. In this problem, several salesmen can sell products in a distribution chain and its development resulted in the modern VRP. However, until recently, a major focus of VRP problems were on reducing transportation costs.

Later on, Soler et al. (2009) studied time-dependent VRP with high accuracy and published several articles on this topic. With regards to the routing problems and
considering time windows i.e., the time-dependency of the model, Hashimoto et al. (2010), Kok et al. (2012), Kritzinger et al. (2012) have been pivotal in advancing this area.

Sbihi and Eglese (2007), Pokharel and Mutha (2009), and Govindan et al. (2015) studies were focused on the hard and soft time windows, the effect of traffic information and avoiding transport congestion in VRP. In this approach, time and speed are required to calculate fuel consumption and greenhouse gas emissions. From these developments, two main focus were evident; first, reverse logistics in picking up goods which passed their expiration date, and second, simultaneous distribution and pickup of secondhand products. The latter is relatively a new area with potential research opportunities.

With regards to internalizing pollution costs into the VRP general problem, Bektaş and Laporte (2011) introduced the concept of pollution-routing problem (PRP). Their study identified important tradeoffs between several factors such as vehicle load, speed and cost. The authors also suggest that solving PRP is largely more complex, but there is potential savings in total cost. Xiao et al. (2012) conducted a research on minimizing cost of diesel vehicles and increasing the efficiency of the GVRP problem. Their model aimed to control the sources of pollution, optimize routing, and to have a positive effect in reducing the total expenses of green routing. Furthermore, Cruz et al. (2012) proposed a heuristic algorithm to solve VRP involving pickup and delivery, which resulted in an acceptable accuracy level. As mentioned earlier, large savings exist in pickup and delivery of goods in shared channels, which has not been researched comprehensively.

In recent years, GVRP has been a focal point of attention for many researchers, mainly because its potential benefit for the environment and financial benefits. GVRP incorporates certain methods to minimize the carbon dioxide emitted from vehicles. For example, a previous study by Jabir et al (2015), measures of carbon dioxide emission were included. In this approach, canonical capacitated vehicle routing problem and multi-objective optimization model was proposed to tackle the conflicting objectives of the emission reduction while maintaining the economic benefits. Furthermore, Kazemian and Aref (2015) proposed a green view on capacitated time-dependent VRP with time windows. Their model was developed in a realistic distribution configuration as it incorporated different speed limits for different times of a day.

Later on, Madankumar and Rajendran (2016) proposed a case of VRP that addresses routing issues in a semiconductor supply chain. The study used two mixed integers linear programming (MILP) models for solving the GVRP with pickup and delivery. Furthermore, Xiao and Konak (2016) studied the GVRP with time scheduling. In their study, authors sought to minimize the greenhouse gases emitted from vehicles in a logistics system which considered the products delivery and pickup. They proposed a MILP model with consideration of heterogeneous vehicles, variable times, and time constraints of the vehicle. Authors suggested a hybrid algorithm to solve the problem.
Also, the results suggest that under traffic congestion, distance/time-based schedules do not always result in lowered emission.

Cherkesly et al. (2016) introduced models and algorithms for the pickup and delivery VRP with time windows and multiple stacks. The numerical results of the proposed model revealed the advantage of using the multi-stack approach in VRP pricing problem. The effect on carbon emissions of consolidation of shipments on trucks in a VRP setting was studied by Turkensteen and Hasle (2017). The study revealed that emission savings are larger when small vehicles are set for delivery and pickup locations that are relatively in distance of each other. However, if a vehicle calls many supply and demand points before returning to the depot, the carbon emission savings are not significant or even emission surges for consolidation routes.

A recent study by Ting et al. (2017) addresses a unique variant of pickup and delivery problem (PDP) as multi-vehicle selective pickup and delivery problem (MVSPDP). This study aims to minimize the VRP cost for a fleet of vehicles collecting and supplying goods, subject to the constraints on vehicle capacity and distance. The authors claim that three metaheuristic algorithms are capable to effectively solve the MVSPDP. Also, results suggest that tabu search (TS) outperforms genetic algorithm (GA) and scatter search (SS) in solution accuracy and convergence speed. Alvarez and Munari (2017) developed a VRP model with time windows and multiple deliverymen. They successfully developed a branch-price-and-cut (BPC) exact algorithm in order to solve the model. The results of their study suggest that hybrid approach outperforms the BPC algorithm employed as standalone method in terms of solution accuracy and running time.

Finally, Toro et al. (2017) proposed a mathematical model for capacitated location-routing problem (CLRP) considering environmental impacts of the distribution system. Based on a bi-objective MILP, authors suggest that employment of more vehicles potentially results to greater fuel savings in the long term, and thus lowering emission levels. Also, higher number of vehicle involved in a distribution of shorter routes results in less pollution when prioritizing high demand customers. From a distribution policy perspective, the results of Toro et al. (2017) study is beneficial in decision making as it presents several strategic planning alternatives.

To sum up this section, over the last decade research in VRP and PDP has extensively developed to embrace the sustainability issues and costs associated with the environmental implications of transportation and logistics activities (Lin et al., 2014). Likewise, in this article we address the issue of pollution in a remanufacturing routing problem by extending the classical VRP function to a non-linear mathematical model with two objectives of minimizing greenhouse gases and total distribution costs. While there is an increasing number of research on GVRP, gap still exists between the practicality of current GVRP models and the complexity of real distribution problems.
3. Problem presentation

In the proposed problem of this research, we consider a case of redistribution for products that are repairable and or reusable. In other words, the distributor delivers the firsthand products and collects the secondhand products from the market to repair them. Subsequently, the distributor again delivers the firsthand products and repaired secondhand ones (with lower price) to the market. While minimizing the distribution costs in this problem, we also aim to minimize emission produced by vehicle involved in the distribution system. The assumptions of the model are depicted in below:

- The model considers a multi-depot and multi-product situation where modes of vehicles, depot and vehicle capacities are different.
- The vehicles must return to the depot where they start distributing.
- The demand for the original and remanufactured products are determined and related together with positive correlation.
- The flow of returned products is deterministic and is associated with the flow of original products with positive correlation.
- The demand of each retailer must be met by just one visit of a vehicle and there is no strategy of multiple sourcing.

The mathematical presentation of the problem is presented in the following section.

3.1 Mathematical model

**Indices**

<table>
<thead>
<tr>
<th>$i$</th>
<th>Product</th>
<th>$(1 \leq i \leq I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>Distribution center</td>
<td>$(1 \leq d \leq D)$</td>
</tr>
<tr>
<td>$c, c'$</td>
<td>Demand points</td>
<td>$(1 \leq c \leq C)$</td>
</tr>
<tr>
<td>$v$</td>
<td>Vehicle</td>
<td>$(1 \leq v \leq V)$</td>
</tr>
</tbody>
</table>

**Parameters**

- $\text{cap}_{dist-s}$: The capacity of distribution center $d$ to supply product $i$
- $\text{cap}_{dist-r}$: The capacity of distribution center $d$ to receive returned products $i$
- $\text{cap}_{v}^{veh}$: The capacity of vehicle $v$
- $\text{cost}_{v}^{veh}$: The cost of preparing vehicle $v$
- $\text{cost}_{d}^{dist}$: The cost of setting up distribution center $d$
- $\text{del}_{ic}$: Delivered demands of product $i$ to demand point $c$
The collected product i (returned) from demand point c
\( f_v \)  
Rate of fuel consumption per unit distance by vehicle v
\( \text{cost}_{\text{fuel}} \)  
The cost of each unit of fuel
\( \xi \)  
The rate of greenhouse gas emissions per each unit of fuel
\( \text{bigm} \sim \infty \)  
A very large number

**Variables**

\[ x_{dist}^d \]
- If distribution center d sets up
- Binary
- Otherwise

\[ x_{veh}^v \]
- If the vehicle v is provided
- Binary
- Otherwise

\[ x_{veh}^v \]
- If vehicle v travels from demand point to demand point
- Binary
- Otherwise

\[ \beta_{vd} \]
- If vehicle v is allocated to distribution center d
- Binary
- Otherwise

\[ c_{del}^i \]
- The number of products i in vehicle v deliverable to demand points before the servicing to the demand point c
- Integer

\[ \alpha_{veh}^{pic} \]
- The number of products i in vehicle v collected from demand points after servicing demand point c
- Integer

**Objective function**

\[
\min z^{\text{pollution}} = \xi \times \left( \sum_{v,c > 1, c > 1} f_v \times \text{dis}_{xc}^v \times x_{veh}^v + \sum_{v,c > 1, d} f_v \times (x_{veh}^v + x_{del}^v) \times \text{dis}_{dc} \times \beta_{vd} \right) 
\]

(1)

The first objective is to minimize air pollution which is in fact the GVRP objective of the problem. It minimizes the greenhouse gas emitted by reducing fuel consumption.

\[
\min z^{\text{cost}} = \text{cost}_{\text{fuel}} \times \left( \sum_{v,c > 1, c > 1} f_v \times \text{dis}_{xc}^v \times x_{veh}^v + \sum_{v,c > 1, d} f_v \times (x_{veh}^v + x_{del}^v) \times \text{dis}_{dc} \times \beta_{vd} \right) + \sum_d \text{cost}_{\text{dist}} \times x_{dist}^d + \sum_v \text{cost}_{\text{veh}} \times x_{veh}^v
\]

(2)

The second objective is to minimize the cost of fuel, cost of setting up distribution centers and cost of preparing the vehicles.

**Subject to:**
\[
\sum_{v,c} \alpha^{del}_{vc} \times \beta_{vd} \leq \text{cap}^{dist-s}_{vd} \quad \forall i, d \tag{3}
\]
\[
\sum_{v,c} \alpha^{pic}_{vc} \times \beta_{vd} \leq \text{cap}^{dist-r}_{vd} \quad \forall i, d \tag{4}
\]
\[
\sum_{c} \alpha^{del}_{vc} \leq \text{bigm} \times \sum_{d} \beta_{vd} \quad \forall i, v \tag{5}
\]
\[
\sum_{c} \alpha^{pic}_{vc} \leq \text{bigm} \times \sum_{d} \beta_{vd} \quad \forall i, v \tag{6}
\]
\[
\sum_{d} \beta_{vd} \leq 1 \quad \forall v \tag{7}
\]
\[
\sum_{v} \beta_{vd} \leq \text{bigm} \times x^{dist}_{d} \quad \forall d \tag{8}
\]
\[
\sum_{c} x^{cc}_{v} \leq 1 \quad \forall v, c \tag{9}
\]
\[
\sum_{c} x^{cc}_{vc} = \sum_{c} x^{cc}_{vc} \quad \forall v, c \tag{10}
\]
\[
\alpha^{del}_{vc} \leq \text{bigm} \times \sum_{c} x^{cc}_{vc} \quad \forall i, v, c \tag{11}
\]
\[
\alpha^{pic}_{vc} \leq \text{bigm} \times \sum_{c} x^{cc}_{vc} \quad \forall i, v, c \tag{12}
\]
\[
\alpha^{del}_{vc} - \alpha^{del}_{vc} + \text{cap}^{veh}_{v} \times x^{cc}_{vc} + (\text{cap}^{veh}_{v} - \text{del}_{ic} - \text{del}_{ic}) \times x^{cc}_{vc} \leq \text{cap}^{veh}_{v} - \text{del}_{ic} \quad \forall i, v, \hat{c}, c \tag{13}
\]
\[
\alpha^{pic}_{vc} - \alpha^{pic}_{vc} + \text{cap}^{veh}_{v} \times x^{cc}_{vc} + (\text{cap}^{veh}_{v} - \text{pic}_{ic} - \text{pic}_{ic}) \times x^{cc}_{vc} \leq \text{cap}^{veh}_{v} - \text{pic}_{ic} \quad \forall i, v, \hat{c}, c \tag{14}
\]
\[
\sum_{i} \alpha^{del}_{vc} + \sum_{i} \alpha^{pic}_{vc} - \sum_{i} \text{del}_{ic} \leq \text{cap}^{veh}_{v} \quad \forall v, c \tag{15}
\]
\[
\alpha^{del}_{vc} \geq \text{del}_{ic} + \sum_{c} \text{del}_{ic} \times x^{cc}_{vc} \quad \forall i, v, \hat{c} \tag{16}
\]
\[
\alpha^{pic}_{vc} \geq \text{pic}_{ic} + \sum_{c} \text{pic}_{ic} \times x^{cc}_{vc} \quad \forall i, v, c \tag{17}
\]

The above mathematical model has two objective functions, some constraints and a linearization process. Constraint (3): The capacities of the depots and vehicles, i.e., the goods capacity of the distribution center \(d\) for supplying \(i\) product equals to or more than the amount of product \(i\) in vehicle \(v\) before its delivery. Constraint (4): The demand of goods \(i\) in depot \(d\) is equal or greater than the capacity of the vehicle or the amount of goods after collecting secondhand products. Constraint (5): If the vehicle \(v\) is assigned to distribution center \(d\), the vehicle is loaded; otherwise, the vehicle is empty and does not move. Constraint (6): If the vehicle is assigned to distribution center \(d\), the vehicle \(v\) could pickup the secondhand goods after delivering goods. Constraint (7): Only one vehicle can be allocated to each depot. Constraint (8): If a distribution center \(d\) is active, no vehicle is assigned to it. Constraint (9): At most, one vehicle could travel between two demand points. Constraint (10): The vehicle travelling between the two demand nodes could not be in reverse direction. Constraint (11): If there is no demand for a good at demand point \(c\), the vehicle should not travel to the point for delivery. Constraint (12): If
vehicle \( v \) is not travelling to demand center \( c \), it will not collect any goods. Constraint (13): There is a certain deliverable capacity. Constraint (14): There is a capacity constraint with regard to pickup goods. Constraint (15): Refers to the vehicle capacity constraint and assures that all products in the vehicle should be less than the vehicle capacity. Constraint (16): Refers to the shortage constraint, i.e., the deliverable goods to the customer should not be in shortage. Constraint (17): This constraint refers to pickup goods, i.e. the total collected goods from demand nodes is at least equal to the collected goods by the vehicle.

3.2 Linearizing Process

The modelling of the problem as non-linear (NLP) was finalized in constraint (17). However, the mathematical model must be converted to a linear form. Using integer and binary numbers, the objective functions and constraints are transformed from non-linear to a linear form. In relation to subtour constraint elimination, we are looking for a Hamiltonian cycle with the minimum weight in a fully-weighted graph. For example, Miller-Tuker-Zelmin (MTZ) constraint which is also called K-VPR subtour eliminate. By analogy to Karaoglan et al. (2012) that the elimination of subtour was made in constraints (13) and (14), we effected this in constraints (10) and (11). However, the capacity constraint was observed and it was incorporated in constraints (13) and (14).

The linearization process used in this problem was based on using integer variables. Accordingly, the following auxiliary variables are defined:

\[
\begin{align*}
    x_{\beta_{vdc}} & \begin{cases} 
    1 & \text{Get value of 1 if vehicle } v \text{ which is assigned to center } d \text{ moves from demand center } c \text{ to demand center } c, \text{ otherwise get 0.} \\
    0 & \end{cases} \\
    \alpha_{p_{vdc}}^{del} & \text{Integer} \quad \text{The number of product } i \text{ assigned to vehicle } v \text{ from distribution center } d \text{ to demand point } c. \\
    \alpha_{p_{vdc}}^{pic} & \text{Integer} \quad \text{The number of collected products } i \text{ from demand point } c \text{ by vehicle } v \text{ from distribution center } d.
\end{align*}
\]

First objective function non-linear expression

\[
\begin{align*}
    \text{Min } z^{\text{pollution}} = & \xi \times \left( \sum_{v,c,d} f_v \times \text{dis}^{\text{ext}}_{v,dc} \times x_{vdc} + \sum_{v,c>d} f_v \times (x_{vdc} + x_{vcd}) \times \text{dis}_d \times \beta_{vd} \right)
\end{align*}
\]

Linear equivalent

\[
\begin{align*}
    \text{Min } z^{\text{pollution}} = & \xi \times \left( \sum_{v,c,d} f_v \times \text{dis}^{\text{ext}}_{v,dc} \times x_{vdc} + \sum_{v,c>d} f_v \times (x_{vdc} + x_{vcd}) \times \text{dis}_d \right) \\
x_{\beta_{vdc}} \leq & \beta_{vd} + (1-x_{vdc}) \times \text{bigm}
\end{align*}
\]
\[ \begin{align*}
\alpha_{vdc}^d \leq & \ x_{vdc} \cdot (1 - \alpha_{vd}) \times \text{bigm} \\
\alpha_{vdc}^d \geq & \ 1 + (x_{vdc} + \alpha_{vd} - 2) \times \text{bigm} \\
\alpha_{vdc}^d \leq & \ (x_{vdc} + \alpha_{vd}) \times \text{bigm}
\end{align*} \]  

(20)  

(21)  

(22)  

\[
\begin{align*}
\text{Min} \ z^{\text{cost}} = & \ \text{cost}^{\text{fuel}} \times ( \sum_{v,c} f_v \times \text{dis}_{ve} \times x_{vdc} + \sum_{v,c} f_v \times (x_{vdc} + x_{vcd}) \times \text{dis}_{dc} \times \alpha_{vd} ) + \\
\sum_d \text{cost}^{\text{dist}}_d \times x_d + \sum_v \text{cost}_v^{\text{veh}} \times x_v
\end{align*}
\]

Second objective function non-linear expression

\[
\begin{align*}
\text{Min} \ z^{\text{cost}} = & \ \text{cost}^{\text{fuel}} \times ( \sum_{v,c} f_v \times \text{dis}_{ve} \times x_{vdc} + \sum_{v,c} f_v \times (x_{vdc} \cdot x_{vcd}) \times \text{dis}_{dc} \times \alpha_{vd} ) + \\
\sum_d \text{cost}^{\text{dist}}_d \times x_d + \sum_v \text{cost}_v^{\text{veh}} \times x_v
\end{align*}
\]

Linear equivalent (23)

\[
\begin{align*}
\sum_{v,c} \alpha_{vdc}^d \times \alpha_{vd} \leq & \ \text{cap}^{\text{dist}}_{id} \quad \forall i,d \\
\sum_{v,c} \alpha_{vdc}^d \leq & \ \text{cap}^{\text{dist}}_{id} \quad \forall i,d \\
\alpha_{vdc}^d \geq & \ \alpha_{vdc}^d - (1 - \alpha_{vd}) \times \text{bigm} \\
\alpha_{vdc}^d \leq & \ \alpha_{vdc}^d \\
\alpha_{vdc}^d \leq & \ \text{bigm} \times \alpha_{vd}
\end{align*}
\]

Non-linear expression (24)  

Linear equivalent (25)  

(26)  

(27)  

\[
\begin{align*}
\sum_{v,c} \alpha_{vdc}^d \times \alpha_{vd} \leq & \ \text{cap}^{\text{dist}}_{id} \quad \forall i,d \\
\sum_{v,c} \alpha_{vdc}^d \leq & \ \text{cap}^{\text{dist}}_{id} \quad \forall i,d \\
\alpha_{vdc}^d \geq & \ \alpha_{vdc}^d - (1 - \alpha_{vd}) \times \text{bigm} \\
\alpha_{vdc}^d \leq & \ \alpha_{vdc}^d \\
\alpha_{vdc}^d \leq & \ \text{bigm} \times \alpha_{vd}
\end{align*}
\]

Non-linear expression (28)  

Linear equivalent (29)  

(30)  

(31)  

The first objective function (18) (green function) is linearized through changing variables with support of constraints 19 to 22. Constraint 19 is relaxed, if vehicle \(v\) does not pass through demand center \(c\) to demand center \(c\). Also, constraint 20 is relaxed if vehicle \(v\) is not assigned to distribution center \(d\). Otherwise, vehicle \(v\) can move from \(c\) to \(c\) or not.
In addition, if two binary variables $\beta_{vd}$ and $\chi_{vcc}$ of constraint 21 get value of 1, vehicle $v$ of distribution center $d$ will surely travel from $c$ to $c$. Similarly, in constraint 22, if two binary variables $\beta_{vd}$ and $\chi_{vcc}$ get value 0, vehicle $v$ of center $d$ will not travel from $c$ to $c$.

The second objective function (23), which is the cost objective, is linearized through changing variables with support of constraints 24 to 27. Constraint 24 guarantees that capacity of distribution center $d$ of product $i$ must be higher than the products in the assigned vehicles. Constraints 25, 26, and 27 are relaxed if vehicle $v$ is not assigned to the distribution center $d$. Otherwise, the number of deliverable product $i$ in vehicle $v$ of distribution center $d$ has to be higher than the number of deliverable product $i$ in vehicle $v$ to supply center $c$. The constraints 28 to 31 are the linear equivalents of constraints 3 and 4 (the capacity constraints).

4. Numerical Results

This section presents the numerical results of the model and testing. We first demonstrate the solution of the model with a software application, and then the validation is presented as follows. In order to validate the research model, the code was developed by GAMS IDE/Cplex. The code created the exact solution of the model for random example problems in different scales (from small to medium). Accordingly, an algorithmic was designed and formulated in MATLAB. Table 1 presents the parameter ranges in the algorithm for the random generation of problems.

Table 1: Parameter ranges in the algorithm for the random generation of instances

<table>
<thead>
<tr>
<th>Number</th>
<th>Parameter</th>
<th>Distribution function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indices</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$I$</td>
<td>The number of products is taken from the user</td>
</tr>
<tr>
<td>2</td>
<td>$D$</td>
<td>The number of potential distribution centers is taken from the user</td>
</tr>
<tr>
<td>3</td>
<td>$C$</td>
<td>The number of customers is taken from the user</td>
</tr>
<tr>
<td>4</td>
<td>$V$</td>
<td>The number of vehicles is taken from the user</td>
</tr>
<tr>
<td>Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$del_{ic}$</td>
<td>Follows the uniform discrete distribution function between $a$ and $b$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discrete uniform $[a,b]$</td>
</tr>
<tr>
<td>2</td>
<td>$pic_{ic}$</td>
<td>Follows the uniform distribution function between $a$ and $b$ by the product uniform distribution between 0.15 and 0.25.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Round (discrete uniform $[a,b] \times$ uniform $[0.15,0.25]$)</td>
</tr>
<tr>
<td>3</td>
<td>$cap_{id}^{dist-s}$</td>
<td>This value is calculated based on the coefficient of $del_{ic}$ $\sum_{c} del_{ic} \times rn_{id} \ \forall i,d; \quad 1 \leq \sum_{d} rn_{id} \leq 1.5 \quad \forall i$</td>
</tr>
<tr>
<td>4</td>
<td>$cap_{id}^{dist-r}$</td>
<td>This value is calculated based on the coefficient of $pic_{ic}$</td>
</tr>
</tbody>
</table>
This value is calculated based on the coefficient of $del_{ic}$

$$\sum_{i} pi_{ic} \times rn_{id} \forall i, d; \quad 1 \leq \sum_{d} rn_{id} \leq 1.5 \quad \forall i$$

$$\sum_{i} \frac{del_{ic}}{v} \times rn_{v} \forall v; \quad 1 \leq rn_{v} \leq 1.5 \quad \forall v$$

Table 2. Parameters of the small and medium scale problems

<table>
<thead>
<tr>
<th>Instances</th>
<th>Product (i)</th>
<th>Distribution Center (d)</th>
<th>Customer (c)</th>
<th>Vehicle (v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P01</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>P02</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>P03</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>P04</td>
<td>5</td>
<td>4</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>P05</td>
<td>6</td>
<td>4</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>P06</td>
<td>7</td>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>P07</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>P08</td>
<td>7</td>
<td>5</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>P09</td>
<td>8</td>
<td>5</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>P10</td>
<td>8</td>
<td>6</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Min</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Max</td>
<td>8</td>
<td>6</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

According to Table 2, the parameters of the problem is increasing from top to bottom, the value of indices in the next problem is remained constant or increased. Because the
proposed model is in bi-objectives form, using the method presented by Zimmerman (1978), we will turn it into a single-objective model. Based on this approach, the model is presented as:

\[
\begin{align*}
    \text{Max } & \lambda \\
    \text{Subject to:} & \lambda \leq \mu_{z_1}^+ (x) \\
                      & \lambda \leq \mu_{z_1}^- (x) \\
    \mu_{z_1}^+(x) &= \begin{cases} 
    1 & z_k(x) > z_k^+ \\
    0 & z_k(x) < z_k^-
    \end{cases} \\
    f_{\mu_{z_1}^+} &= \frac{z_k^+ - z_k(x)}{z_k^+ - z_k^-}, \quad \text{if } z_k(x) \leq z_k^-
    \mu_{z_1}^-(x) &= \begin{cases} 
    1 & z_l(x) > z_l^- \\
    0 & z_l(x) < z_l^+
    \end{cases} \\
    f_{\mu_{z_1}^-} &= \frac{z_l^- - z_l(x)}{z_l^- - z_l^+}, \quad \text{if } z_l(x) \geq z_l^+
\end{align*}
\]

In which \(z_k^-\) and \(z_l^-\) are the lower limits, \(z_k^+\) and \(z_l^+\) are the higher limits and \(\mu_{z_1}^+(x)\) and \(\mu_{z_1}^-(x)\) are the functions of minimizing and maximizing respectively.

In the above single-objective method, the weight of the objective functions are not considered. Thus, the following approach is developed for the membership functions with applying their weights:

\[
\begin{align*}
    \text{Max } & \lambda = \sum_i w_i \times \lambda_i \\
    \text{Subject to:} & \lambda_i \leq \mu_i (x) \\
                      & \sum_i w_i = 1
\end{align*}
\]

Now, with the implementation of the model in GAMS win32 24.1.2/ Cplex using the parameters in different scales (for weights of \(w_1=0.5\), \(w_2=0.5\)), the obtained results are presented in Table 3:

Table 3. Values of objective function and running time
According to Table 3, the software was capable to solve all ten problems in less than 1000 seconds. Now, to validate the proposed model, using one of the examples we carry out sensitivity analysis based on the objective function coefficients. Hence, we expect that with increase in the coefficient, the value does not get worse and vice versa. Table 4 presents the results of the sensitivity analysis for the problem No. 5 (P05).

Table 4. The objective function values and run-time

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Weight of first objective function</th>
<th>Weight of second objective function</th>
<th>Value of first objective function</th>
<th>Value of first objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>S01</td>
<td>(w_1 = 0.2)</td>
<td>(w_2 = 0.8)</td>
<td>1645.7</td>
<td>322678900</td>
</tr>
<tr>
<td>S02</td>
<td>(w_1 = 0.3)</td>
<td>(w_2 = 0.7)</td>
<td>1634.6</td>
<td>347163120</td>
</tr>
<tr>
<td>S03</td>
<td>(w_1 = 0.4)</td>
<td>(w_2 = 0.6)</td>
<td>1602.1</td>
<td>364153850</td>
</tr>
<tr>
<td>S04</td>
<td>(w_1 = 0.5)</td>
<td>(w_2 = 0.5)</td>
<td>1602.1</td>
<td>364153850</td>
</tr>
<tr>
<td>S05</td>
<td>(w_1 = 0.6)</td>
<td>(w_2 = 0.4)</td>
<td>1584.6</td>
<td>384090700</td>
</tr>
<tr>
<td>Scenario</td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>Objective Function 1</td>
<td>Objective Function 2</td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
<td>-------</td>
<td>----------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>S06</td>
<td>0.7</td>
<td>0.3</td>
<td>1572.8</td>
<td>392780320</td>
</tr>
<tr>
<td>S07</td>
<td>0.8</td>
<td>0.2</td>
<td>1572.8</td>
<td>392780320</td>
</tr>
</tbody>
</table>

The trend of changes in the objective functions based on varying coefficients also is depicted in Figures 1 and 2.

Figure 1: Changes in the first objective function in different scenarios

Figure 2: Changes in the second objective function in different scenarios
According to Table 4, Figures 1 and 2, increasing the coefficient of first objective function results in a decline of its value or it remains constant. At the same time decreasing the coefficient of second objective function increases the value or it remains constant which is consistent with our expectation in the implementation of the model. Therefore, the validity of the proposed model can be confirmed.

Since our objective functions are of minimization type, the results of Table 4 shows that when the value of the objective function increases (the higher its coefficient), it does not yield a worse solution i.e. it delivers the older solution or better. Vice versa, when the coefficient of the objective function or its value decreases, the solution does not get better. In general, the model provides solutions that are directed toward an objective function with higher value.

5. Sensitivity Analysis

In order to validate the proposed model, sensitivity analysis is performed based on the modifications of objective function coefficients. Consequently, several scenarios are defined and the behavior of the model is evaluated based on the forecasted expectations. In Table 5, the weights of the objective functions are changed and the results are presented.

Table 5: The results of the sensitivity analysis of the objective function coefficients

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$W_1$</th>
<th>$W_2$</th>
<th>Objective function 1</th>
<th>Objective function 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>S01</td>
<td>0.2</td>
<td>0.8</td>
<td>1933.4</td>
<td>359112300</td>
</tr>
<tr>
<td>S02</td>
<td>0.3</td>
<td>0.7</td>
<td>1801.1</td>
<td>360991140</td>
</tr>
<tr>
<td>S03</td>
<td>0.4</td>
<td>0.6</td>
<td>1744.5</td>
<td>362143500</td>
</tr>
<tr>
<td>S04 (P05)</td>
<td>0.5</td>
<td>0.5</td>
<td>1602.1</td>
<td>364153840</td>
</tr>
<tr>
<td>S05</td>
<td>0.6</td>
<td>0.4</td>
<td>1541.6</td>
<td>368401220</td>
</tr>
<tr>
<td>S06</td>
<td>0.7</td>
<td>0.3</td>
<td>1513.3</td>
<td>370432190</td>
</tr>
<tr>
<td>S07</td>
<td>0.8</td>
<td>0.2</td>
<td>1513.3</td>
<td>370432190</td>
</tr>
</tbody>
</table>

The schematic performance of two objective functions are also illustrated in Figures 3 and 4.
Using the information in Table 5 and Figures 3 and 4, with increasing the coefficient of the first/second objective function, the related values decrease or remain unchanged. Vice versa, decreasing the coefficient of the first/second objective functions improve the relevant values, or keep the them unchanged. This is absolutely rational and likely based on the Zimmerman (1987) methodology.

In addition, in order to complete the analysis, the Pareto optimal is presented in Figure 5 for each scenario. This can help the decision makers in real world problems to find their best choice in relation to their utility.
Figure 5: Pareto optimal (efficient frontier) based on different scenarios

Figure 5 introduces the efficient frontier of two objective functions based on each scenario. Using this approach the best combination of the objective functions in the proposed model can be found. In other words, the results of this sensitivity analysis is beneficial to evaluate the validity of the model based on various scenarios of objective function coefficients.

6. Case Study

In order to validate the proposed model under real circumstances, a real world case study was identified which satisfied the objectives and constrains of this research. The case study comprises a distribution system for one of the most well-known Iranian newspapers, Hamshahri. In this case, we are particularly looking at two districts of 11 and 12 in Tehran Metropolitan area. There are 36 newsstands in these two districts. Newspapers are distributed to newsstands between 6 to 7 AM every morning, using four vans.

The entire operations is handled through one distribution center. When the vans deliver the new newspapers, they pickup the unsold ones from the newsstands that hold them to return to the distribution center. Then, the newsstands that need old newspapers (mainly for non-reading purposes) order them through the center. In other words, the vans can deliver both new and old newspapers at the same time. The data required to test the model for this case was collected from two main sources. First, the data on distribution center set up costs, demand for newspapers (new and old), transportation costs and other relevant distribution activities cost was directly obtained from the distribution center. Second, the detailed data on emission produced by the vans was collected from the engineering department of the car manufacturer (Iran-Khodro). It is noteworthy to
mention that the van model used in this distribution system has passed the relevant standards of air pollution and received Euro 4 standard. This is an obligation to obtain registration in Tehran Metropolitan Region. The following tables (6 and 7) provide the specific figures of the case study.

Table 6: Indices of the case study

<table>
<thead>
<tr>
<th>Indices</th>
<th>Interval</th>
<th>Description</th>
<th>Number in Districts 11 and 12 of Tehran</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$1 \leq i \leq I$</td>
<td>Product</td>
<td>1</td>
</tr>
<tr>
<td>d</td>
<td>$1 \leq d \leq D$</td>
<td>Distribution center</td>
<td>1</td>
</tr>
<tr>
<td>c,c</td>
<td>$1 \leq c \leq C$</td>
<td>Newsstand</td>
<td>36</td>
</tr>
<tr>
<td>v</td>
<td>$1 \leq v \leq V$</td>
<td>Van</td>
<td>3 or 4</td>
</tr>
</tbody>
</table>

Table 7: Parameter ranges in the algorithm for the case study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>More Explanation</th>
<th>Unit</th>
<th>Number in the case study</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cap_{dist-s}^{d}$</td>
<td>The capacity of distribution center $d$ to supply product $i$</td>
<td>New newspapers, firsthand product</td>
<td>---</td>
<td>5,000</td>
</tr>
<tr>
<td>$cap_{dist-r}^{d}$</td>
<td>The capacity of distribution center $d$ to receive returned products $i$</td>
<td>Old newspapers, secondhand product</td>
<td>---</td>
<td>6,000</td>
</tr>
<tr>
<td>$cap_{veh}^{c}$</td>
<td>The capacity of distribution center $d$ to receive returned products $i$</td>
<td>The capacity of distribution center $c$ for old newspapers</td>
<td>---</td>
<td>2,000</td>
</tr>
<tr>
<td>$cost_{veh}^{c}$</td>
<td>The capacity of vehicle $v$</td>
<td>The price of buying a mono-fuel van</td>
<td>Toman</td>
<td>13,000,000</td>
</tr>
<tr>
<td>$cost_{veh}^{c} \times 3$</td>
<td>The cost of preparing vehicle $v$</td>
<td>The cost of buying 3 or 4 vans</td>
<td>Toman$^2$</td>
<td>39,000,000 52,000,000</td>
</tr>
<tr>
<td>$cost_{dist}^{d}$</td>
<td>The cost of setting up distribution center $d$</td>
<td>The cost of renting an underground area to the size of 220 m$^2$ in District 11</td>
<td>Toman</td>
<td>170,000,000</td>
</tr>
<tr>
<td>$del_{ic}$</td>
<td>Delivered demands of product $i$ to demand point $c$</td>
<td>The number was randomly taken from newsstand 7 in Hor</td>
<td>---</td>
<td>125</td>
</tr>
</tbody>
</table>

---

$^2$ 1 Iranian Tooman is equal to 0.00026 USD
Now armed with these data, we analyze two different cases and examine the validity of the model. The cases are based on the mathematical model proposed in Section 3 but in two different approaches. In other words, in the first case, we consider a situation where a pickup and delivery system distributes both first and secondhand products and collects the returned secondhand products. This is the approach proposed in this research, and the suggested model. In the second case, pickup and delivery are conducted separately, i.e. one vehicle distributes first and secondhand products and another vehicle collects the returned goods. This approach is the traditional one which can be seen now in almost all of the companies. The results are presented in Table 8.

Table 8. Comparison of costs in different scenarios

<table>
<thead>
<tr>
<th></th>
<th>Weight of first objective function</th>
<th>Weight of second objective function</th>
<th>Value of first objective function</th>
<th>Value of second objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>First case (our proposed approach)</td>
<td>$W_1 = 0.5$</td>
<td>$W_2 = 0.5$</td>
<td>107.7</td>
<td>209,000,000</td>
</tr>
<tr>
<td>Second case (traditional)</td>
<td>$W_1 = 0.5$</td>
<td>$W_2 = 0.5$</td>
<td>121.2</td>
<td>231,000,000</td>
</tr>
</tbody>
</table>
In this comparison, the weights of two objective functions are equal and the cost figures are calculated on a monthly basis (30 days). It is important to note that in the second case, the models of distribution and pickup are separate and the costs were summed up in the calculation. As demonstrated in Table 8, in both objective functions, the first and second objective functions have lower values in the first case. In other words, the distribution system could be operated with lower costs in the joint pickup and delivery scenario and this is what we propose in this paper. Finally, the results of Table 8 prove the absolute efficiency of the proposed approach (first case) in comparison with the traditional one (second case) in both costs (10.5% lower cost) and green issues (12.5% greener).

7. Conclusions

In this article, we examined a case of returned-remanufactured products in a VRP setting with pickup and delivery and considering green criteria. In this bi-objective problem, we consider both delivery and pickup of the products at the same time through shared channels. Accordingly, a non-linear GVRP model was first developed and then it was transformed into a linear programming model for further analysis and simulation. The major contribution of our model can be realized in the simultaneous distribution of both remanufactured and original products in shared channels. Most importantly, the green issue is considered here as an important part of sustainable distribution.

Since practicality and accuracy of the model was a main concern in this model, we did not use metaheuristic algorithm, mainly because it could not generate acceptable results. In order to validate our model, a real world case study was identified and incorporated into our model. The result was then analyzed and compared in different scenarios. The analysis strongly suggest that pick and delivery of first and second hand at the same time produce lower distribution costs and emission levels.

This study also provides implications for mangers and practitioners. Many business organizations across the globe are finding several profitable areas in remanufacturing and redistribution of goods. However, managing transportation costs is key to this success. Savings in transportation activities potentially enhance marketability and availability of remanufactured products in supply chains. As the price of remanufactured products are typically lower, effective route management must be considered in every distribution policy. In particular, distribution of firsthand and secondhand products through shared system provides enormous financial and environmental benefits for the manufacturer, distributor and consumers. This is evident by an increasing number of cases involved in selling and distribution of second hand cellphones and running shoes from Far East in Middle East countries. Finally, the results of the case study prove the absolute efficiency of the proposed approach (simultaneous collection and delivery) in comparison with the traditional one (separate collection and delivery) in both cost (10.5% lower cost) and environmental impacts (12.5% greener).
This study has two limitations. First, the proposed model is an absolute model. Therefore, there are limitations with the use of real data. Second, this model is relatively new and similar models are not widely available. As the result, it is difficult to evaluate the relative efficiency of this model in terms of solution quality. It is recommended to evaluate the efficiency based on other factors, such as run time and loss number of limitation. Knowing this limitation, the use of data envelopment analysis (DEA) is recommended in the future research.

This paper also identifies several gaps of research and directions for future studies which are categorized as follows:

1) Extension of the model by adding the third objective, which is maximizing the profit of the second-hand products. Furthermore, it is advised to consider the amounts of return products as a stochastic parameter in order to make the model more compatible with the real distribution systems. Besides, the model can be extended by incorporating time windows.

2) Using DEA for the product distribution and setting the inputs and outputs and achieving the efficiency of one is suggested for future research. This provides several areas of investigation and efficiency opportunities.

3) The proposed model can be examined in large-scale distribution systems where the new approach of this study and the classical ones are compared to each other more precisely.

6) Finally, applications of network analysis in this type of problems and creating time schedules is highly suggested for future research. This will ultimately minimize the run-time and efficiency of the distribution system, but also closes the gap between theoretical models and practical considerations of distribution systems.

References


