Analysis Method of Voltage Stability for Bulk Power System by P-V and Q-V Curves Considering Dynamic Load

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***Abstract*—A novel analysis method of the voltage stability for bulk power system is proposed by introducing the P-V and the Q-V curves based on the dynamic load. Firstly, a bulk power system is exactly rewritten into a simple nonlinear decentralized system by the power flow calculation. Next, the dynamic load is modeled in a higher order polynomial form identified from the observed data at a load-bus. Then, the system and the load P-V and Q-V curves are constructed, and finally the reactive power is shown to be more sensitive to the voltage stability than the active one by considering the curves.**

***Keywords*—Dynamic load, Electric power system, Nonlinear load model, P-V curve, Q-V curve, Voltage stability.**

I. INTRODUCTION

IN a power system model, the load is often assumed as an infinite-bus, which denotes that the load-bus voltage will not vary with time, and always be fixed, since power system loads keep being very difficult to model. Therefore, it has been said that a realistic dynamic modeling of composite loads is one of the greatest unsolved problems of electric power field [1] [2]. On the other hand, a dynamic load model has been proposed [3]-[5], and is constructed on a basis of the observed load characteristics, where the model is given in a polynomial form with a differential equation [6].

Recently, the load voltage has been discussed as an integral part of power system response, and is an important aspect of system stability [7]-[9]. Therefore, voltage instability and collapse can not be separated from the general problem of system stability, where there are several factors which relate to the voltage instability such as increase loading, reactive power constraints, on-load tap changer dynamics and load characteristics [10]. For studying the voltage stability, one approach is generally to analyze the system and the load P-V curves and the Q-V curves. From the curves, a system operation point can be obtained as an intersection of the system and the load curves. However, the existing methods [3]-[10] do not consider dynamic loads and characteristics into the P-V and the Q-V curves in polynomial form.

The aim of this paper is to propose an analysis method of voltage stability for bulk power system by introducing P-V and Q-V curves considering dynamic load [11]-[14]. Firstly, it is shown in Section II that a bulk power system can be rewritten exactly into a nonlinear decentralized system by looking over the upper system at a load-bus, and the decentralized system consists of one-load and one generator regarded as a complex power source. In Section III, from the observed data of load-bus voltage, active power and reactive power, the dynamic load is modeled in a higher order polynomial form. Then, the system and the load P-V curves and the Q-V curves are constructed. In Section IV, by matching the dynamic load to the Japanese standard one-machine system model [15]. Finally, it is concluded that the voltage stability for bulk power system can be analyzed by constructing the decentralized system and introducing the curves derived from the observed dynamic load characteristics.

II. POWER FLOW IN BULK POWER SYSTEM

A bulk power system is a large scale nonlinear control one with many generators and loads as shown in Figure 1, and the power flow at a load-bus of power system is illustrated in Figure 2, where δ denotes the phase angle of the voltage $\dot{V}$, the current $\dot{I}_L$, the active power P_L and the reactive power Q_L at the load-bus, respectively.

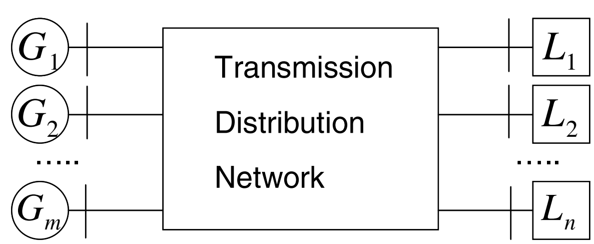

Fig. 1. Bulk power system.

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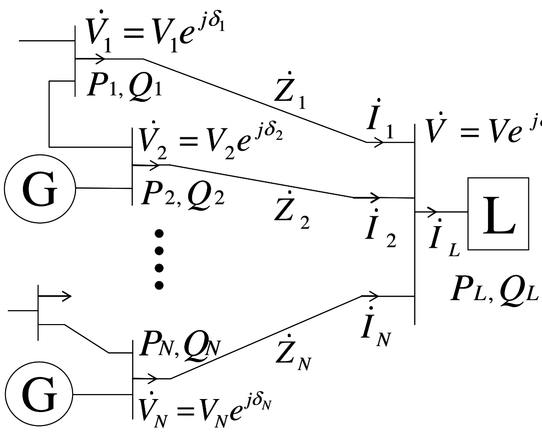


Fig. 2. Power flow at a load-bus.

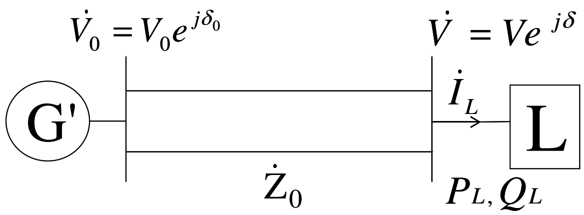


Fig. 3. An equivalent one-machine one-load decentralized system.

Using the transmission line impedance $\dot{Z}$, P_L and Q_L can be rewritten from eq. (1) to eq. (2) as follows;

$$\begin{aligned} P_L + jQ_L &= \dot{V}\bar{\dot{I}}_L \qquad (1) \\ &= \dot{V}\overline{\left(\dot{I}_1 + \dot{I}_2 + \cdots + \dot{I}_N\right)} \\ &= \dot{V}\overline{\left(\frac{\dot{V}_1 - \dot{V}}{\dot{Z}_1} + \frac{\dot{V}_2 - \dot{V}}{\dot{Z}_2} + \cdots + \frac{\dot{V}_N - \dot{V}}{\dot{Z}_N}\right)} \\ &= \dot{V}\overline{\left\{\left(\frac{1}{\dot{Z}_1}\dot{V}_1 + \frac{1}{\dot{Z}_2}\dot{V}_2 + \ldots + \frac{1}{\dot{Z}_N}\dot{V}_N\right) - \left(\frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_2} + \ldots + \frac{1}{\dot{Z}_N}\right)\dot{V}\right\}} \\ &= \dot{V}\overline{\left\{\frac{\left(\dfrac{\frac{1}{\dot{Z}_1}\dot{V}_1 + \frac{1}{\dot{Z}_2}\dot{V}_2 + \ldots + \frac{1}{\dot{Z}_N}\dot{V}_N}{\frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_2} + \ldots + \frac{1}{\dot{Z}_N}}\right) - \dot{V}}{1 \Big/ \left(\frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_2} + \ldots + \frac{1}{\dot{Z}_N}\right)}\right\}} \\ &\equiv \dot{V}\overline{\left(\frac{\dot{V}_0 - \dot{V}}{\dot{Z}_0}\right)}, \qquad (2) \end{aligned}$$

where

$$\dot{V}_0 \equiv \sum_{k=1}^{N}\left(\dot{V}_k / \dot{Z}_k\right) \Big/ \sum_{k=1}^{N}\left(1/\dot{Z}_k\right) , \qquad (3)$$

$$\dot{Z}_0 \equiv 1 \Big/ \sum_{k=1}^{N}\left(1/\dot{Z}_k\right) . \qquad (4)$$

Therefore, eq. (2) indicates exactly a one-machine one-load system that the transmission impedance is $\dot{Z}_0$, and the generator-bus voltage is $\dot{V}_0$ as shown in Figure 3. Thus, it should be noticed that the generator G' in Figure 3 can be regarded as an equivalent power source to the system excluding the load L and the transmission lines with impedance $\{\dot{Z}_1, \dot{Z}_2, \cdots \dot{Z}_N\}$ in Figure 2. Thus, at each load-bus of bulk power system, the upper system is found to have equivalently only one power source. Since the decentralized system is constructed without any approximation, an expression "aggregation" is not used here.

At the load-bus in Figure 3, the following equation holds;

$$P_L + jQ_L = \dot{V}\overline{\left(\frac{\dot{V}_0 - \dot{V}}{\dot{Z}_0}\right)} , \qquad (5)$$

and the active and the reactive powers can be determined. Introducing the relationship

$$Q_L = P_L \tan\phi , \qquad (6)$$

and eliminating the term $(\delta_1 - \delta_2)$ from P_L and Q_L, equations giving the system P-V and the Q-V curves are formulated as;

$$\left(R^2 + X^2\right)\left(1 + \tan^2\phi\right)P_L^2 + 2\left(R + X\tan\phi\right)V_2^2 P_L + \left(V_2^4 - V_1^2 V_2^2\right) = 0 , \qquad (7)$$

$$\left(R^2 + X^2\right)\left(1 + \tan^2\phi\right)Q_L^2 + 2\left(R + X\tan\phi\right)V_2^2 \tan\phi\, Q_L + \left(V_2^4 - V_1^2 V_2^2\right)\tan^2\phi = 0 , \qquad (8)$$

with $\dot{Z}_0 = R + jX$. Therefore, it is important to note that eqs. (7) and (8) include $V_1(t)$, $V_2(t)$ and the power factor $\cos\phi(t)$, and the system P-V and Q-V curves depend on time.

III. Load Modeling

In general, the active power P_L of load has been expressed by the following types;

Exponential-type $\quad P_L = P_0 V_2^n \quad$ (9)

Polynomial-type $\quad P_L = a_0 + a_1 V_2 + a_2 V_2^2 \quad$ (10)

with constant coefficients $\{P_0, a_0, a_1, a_2\}$ and an integer n. Here, the dynamic load model [6] can be rewritten as;

$$P_L = a_0 + a_1 V_2 + a_2 (G(t) V_2^2 - 1) , \qquad (11)$$

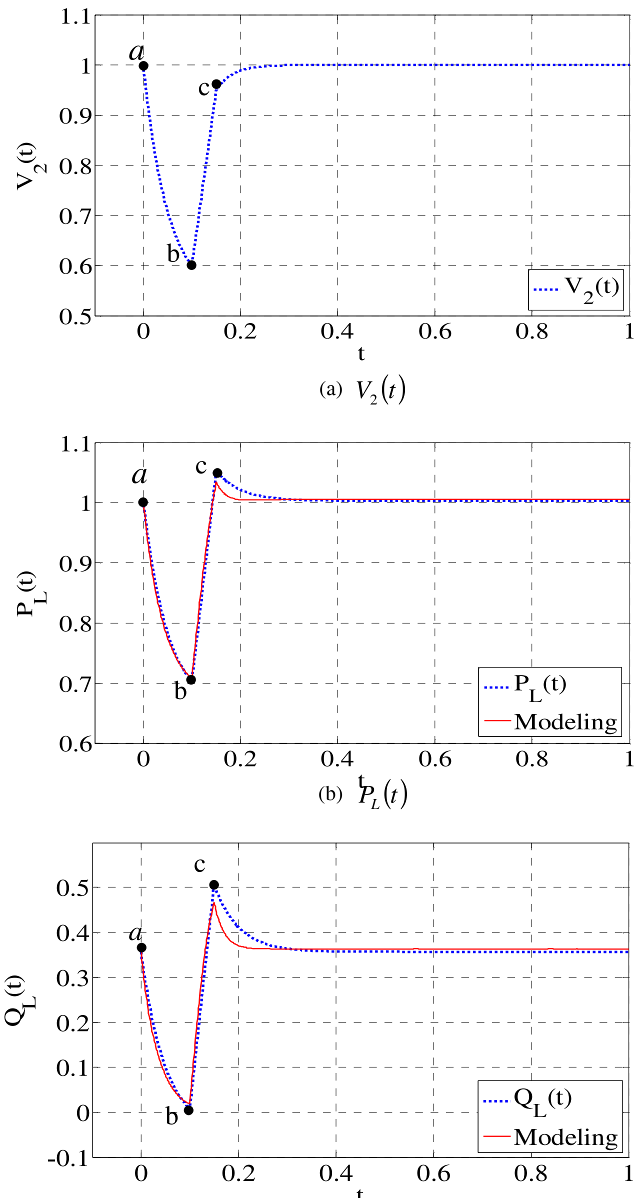
$$dG(t)/dt = (-1/T)(G(t)V_2^2 - 1) , \quad G(0) = 1 , \qquad (12)$$

$$Q_L = b_0 + b_1 V_2 + b_2 (B(t) V_2^2 - 1) , \qquad (13)$$

$$dB(t)/dt = (-1/T)(B(t)V_2^2 - 1) , \quad B(0) = 1 , \qquad (14)$$

where $G(t)$ is the conductance of the load, $B(t)$ the susceptance, and T the time constant. The first and the second terms of the rhs in eqs. (11) and (13) give a static load, and the third term denotes a dynamic one.

The observed data V_2, P_L and Q_L at a sub-station are illustrated by approximated functions in Figure 4 (dotted lines). Also, the solutions $G(t)$ and $B(t)$ of eqs. (12) and (14) can be obtained numerically. Substituting the data $G(t)$, $B(t)$, V_2, P_L and Q_L into eqs. (11) and (13) estimates coefficient parameters $\{a_0, a_1, a_2, b_0, b_1, b_2\}$ by the least squares method. Solid lines in Figure 4 show P_L and Q_L obtained by the estimated parameters, and are compared with the data. Then, it is found that solutions $G(t)$ and $B(t)$ of eqs. (12) and (14) can be expressed by the following simple polynomial function $V_2(t)$;



(a) $V_2(t)$

(b) $P_L(t)$

(c) $Q_L(t)$

Fig. 4. Modeling of $V_2(t)$, $P_L(t)$ and $Q_L(t)$.

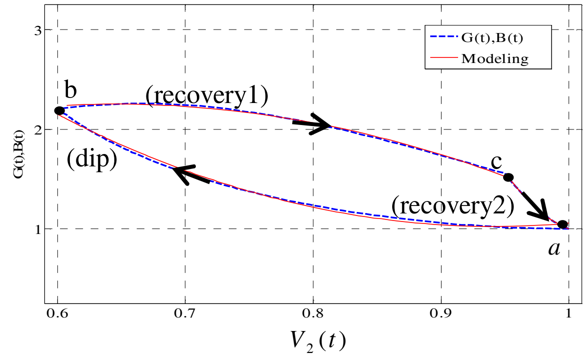


Fig. 5. Modeling of $G(t)$ and $B(t)$.

TABLE Ⅰ
ESTIMATED PARAMETERS

P_L		Q_L	
a_0	0.3405	b_0	-0.3249
a_1	0.6654	b_1	0.6882
a_2	0.1521	b_2	0.3398
m_1	9.5685	r_1	9.5685
m_2	-0.9829	r_2	-0.9829
m_3	136.1427	r_3	136.1427
n_1	-18.1512	s_1	-18.1512
n_2	10.0329	s_2	10.0329
n_3	-265.4507	s_3	-265.4507
p_1	9.6378	u_1	9.6378
p_2	-7.7811	u_2	-7.7811
p_3	130.3079	u_3	130.3079

$$G(t) \approx m + nV_2 + pV_2^2 , \tag{15}$$
$$B(t) \approx r + sV_2 + uV_2^2 , \tag{16}$$

with parameters $\{m, n, p, r, s, u\}$. In Figure 5, the modeling of $G(t)$, $B(t)$ and $V_2(t)$ is illustrated. Therefore, substituting eq. (15) or (16) into eq. (11) or (13) gives;

$$P_L = (a_0 - a_2) + a_1V_2 + a_2mV_2^2 + a_2nV_2^3 + a_2pV_2^4 , \tag{17}$$
$$Q_L = (b_0 - b_2) + b_1V_2 + b_2rV_2^2 + b_2sV_2^3 + b_2uV_2^4 . \tag{18}$$

Here, the estimated parameters are shown on TABLE Ⅰ, where subscripts 1, 2 and 3 of parameters $\{m, n, p, r, s, u\}$ denote the dip and the recovery of $V_2(t)$ in Figure 4. Also, it should be emphasized that the estimated parameters, especially $\{ m_3, n_3, p_3, r_3, s_3, u_3 \}$ on TABLE Ⅰ, are realistic only in the data region $0.6 < V_2(t) < 1.0$ as shown in Fig 5. Thus, the load model given by eqs. (17) and (18) can be expressed in the fourth order polynomial, which implies that the dynamic load can be replaced by higher order terms of V_2. Since P_L and Q_L are given as the observed data, the power factor is calculated as;

$$\cos\phi(t) = \cos\left\{\tan^{-1}\left(\frac{Q_L(t)}{P_L(t)}\right)\right\}, \tag{19}$$

and is shown in Figure. 6. Therefore, it is found that the power factor is lag, and corresponds to the system P-V curve (lag) discussed in Section IV.

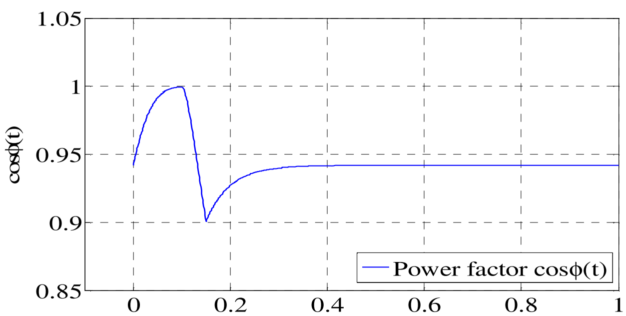


Fig. 6. Power factor $\cos\phi(t)$.

IV. Voltage Stability Analysis

As indicated in Section II, the system P-V and Q-V curves can be derived from the one-machine one-load decentralized system as eqs. (7) and (8). In this Section, the Japanese standard system model illustrated in Figure 7 [15] is treated as the decentralized one.

On the other hand, the load P-V and Q-V curves including dynamic characteristics are given by eqs. (17) and (18), and are shown in Figure 8 (dotted lines I, II and III). Also, the system P-V and Q-V curves (broken and solid lines) in Figure 8 are obtained by resetting parameters ($R = 0.0126 \rightarrow 0.0020$, $X = 0.739 \rightarrow 0.241$) of the standard system model in order to match the system curve to the load curve. Then, the intersection of the system and the load P-V or Q-V curves in Figure 8 gives an operation point, which moves as $a \rightarrow b \rightarrow c \rightarrow a$ corresponding to Figure 4.

In the Figure 9, operation points { a, b, c } are indicated as the intersection of the system curves at t=0, 0.10, 0.15, and the load curves I (dip), II (recovery 1) and III (recovery 2)

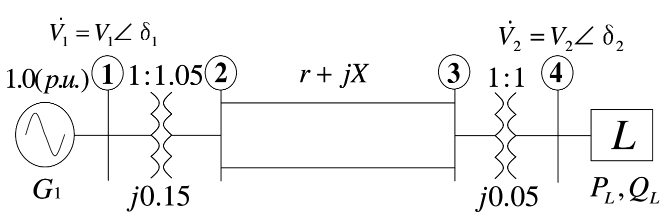
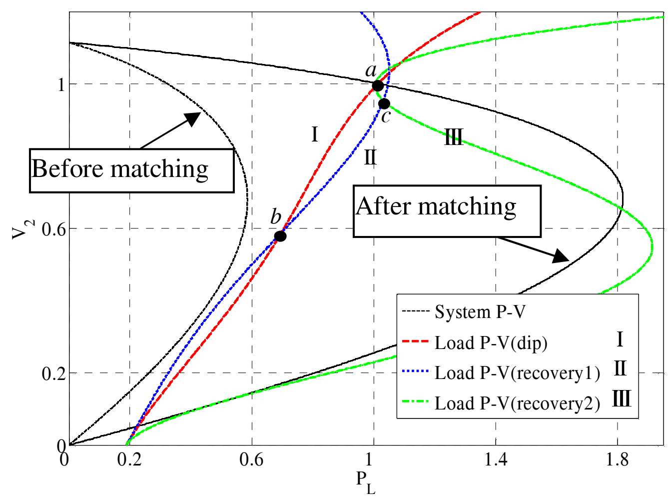


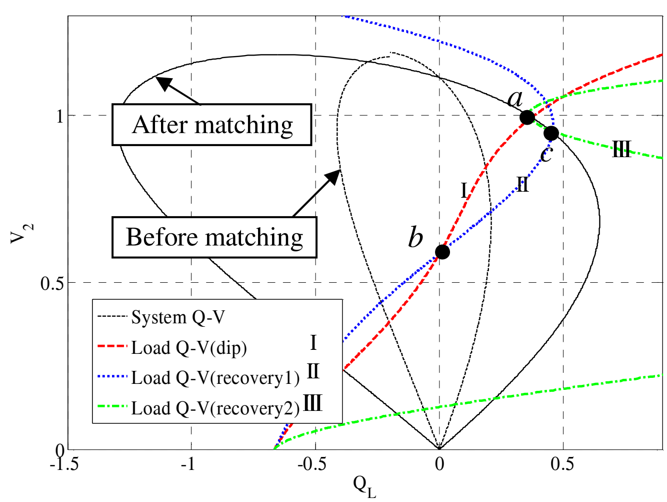
Fig. 7. The Japanese standard one-machine system model with load.

corresponding to Figure 5. Thus, the point a at where a fault occurred in the load and the point b at where the fault was removed, are in the stable region ($dV_2/dP_L < 0$ and $dV_2/dQ_L < 0$).

Supposing the increase of coefficients ($a_0 - a_2$) and ($b_0 - b_2$) for the constant active and reactive power terms in eqs. (17) and (18), the load P-V and Q-V curves are shown as I-1, I-2, and I-3, in Figure 10, where P-V curves (I-2; 50% increase, I-3; 160% increase) and Q-V curves (I-2; 5% increase, I-3; 17% increase). Then, the operation point b moves from b to b' and b'', and it is found that the point b'' is on the boundary between the stable and the unstable regions. Here, it should be noticed that the constant reactive power term of Q_L has a larger influence to the voltage stability than the active one of P_L.

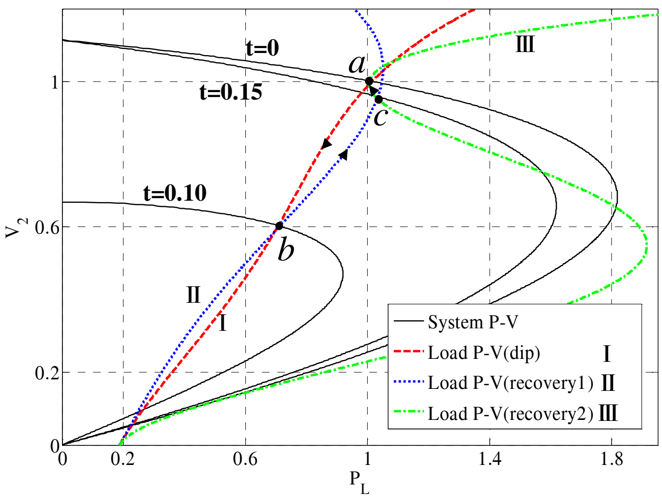


(a) System and load P-V curves

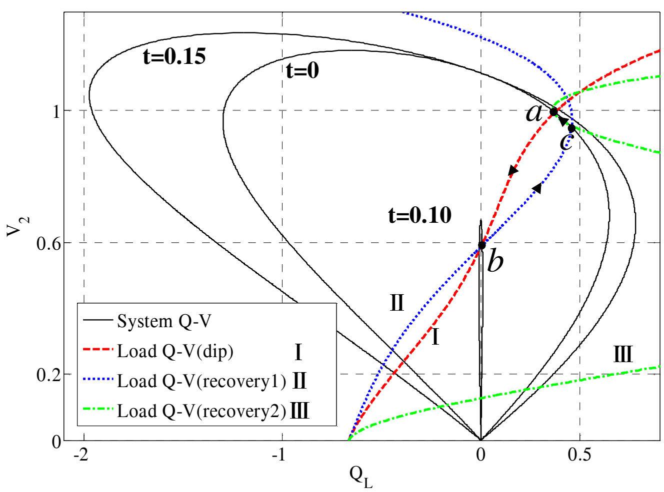


(b) System and load Q-V curves

Fig. 8. Matching the load to the system model.

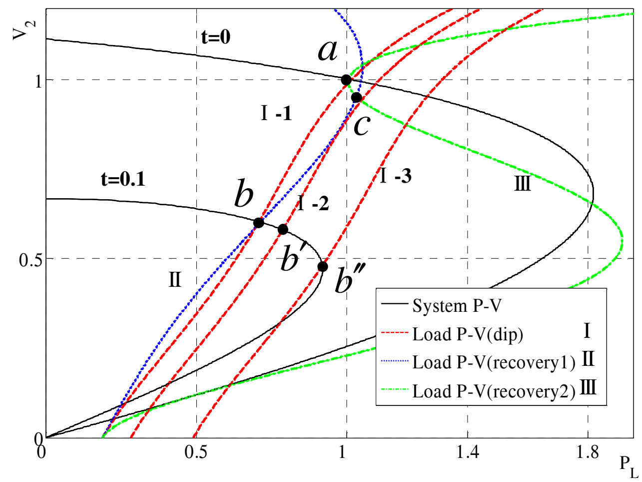


(a) System and load P-V curves

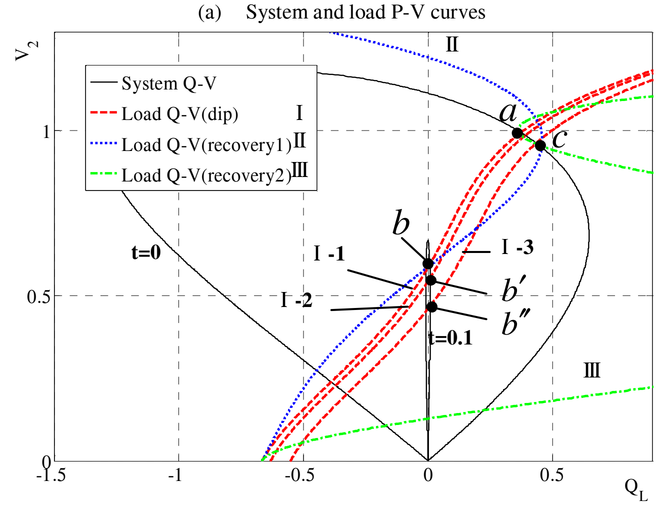


(b) System and load Q-V curves

Fig. 9. Operation points in the stable region.



(a) System and load P-V curves



(b) System and load Q-V curves

Fig. 10. Increase of the constant power term.

V. Conclusion

In this paper, firstly a bulk power system has been treated, and has been rewritten into a decentralized system. Next the Japanese standard one-machine system model with a dynamic load has been introduced as the decentralized system. In addition, the observed data V_2, P_L and Q_L at a sub-station were expressed by the approximated functions. Then, the P-V and Q-V curves have been constructed, and the voltage stability could be discussed by iterating the proposed method at each load-bus of bulk power system. As a result, the following is concluded:

1) A bulk power system can be rewritten without approximation into a nonlinear decentralized system at each load-bus.

2) The dynamic load model with a differential equation can be approximated in a higher order polynomial form of V_2.

3) The voltage stability can be analyzed by the system P-V and Q-V curves, and the load P-V and Q-V ones.

4) In the case of the data observed in Japan, the reactive power is more sensitive to the voltage stability than the active one by considering the increase of the constant power in load.

The application of the proposed analysis method to other data of V_2, P_L and Q_L, the sensitivity analysis and the matching of the P-V and the Q-V curves to the swing equation of upper system are future subjects.

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VIII. Biographies

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