Solution Methods of Ill-Conditioned Power System State Estimation: a Comparative Study

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***Abstract*—Power system state estimation is based on an iterative process for solving the weighted least squares (WLS) algorithm via the so-called Normal Equations (NE). This process is prone to be numerically unstable if the power system is ill-conditioned. Several reasons contribute to creating an ill-conditioned state estimator. However, this paper focuses on the effect of the high *R*/*X* ratios. A review of the main approaches that have been used for avoiding or mitigating this problem of ill-conditioning in the state estimation is presented in this paper. Additionally, simulation tests using MATLAB are implemented on 5-Bus and IEEE 30-Bus systems for evaluating these methods' performance according to their condition number and other characteristics of each process.**

***Keywords*—Condition number;Distribution systems; Ill-Conditioning; R/X ratio; State estimation.**

I. INTRODUCTION

Power systems monitoring is an essential activity for maintaining electrical service and avoiding a regional blackout. This process is implemented in power system control centers via the Energy Management System (EMS). The state of a power system, which includes the voltage magnitudes and the phase angles of the buses, can be obtained by the solution of the State Estimation (SE). The SE solution is used for other studies such as the contingency analysis and economic dispatch studies [1, 2].

The state estimator receives the telemetered measurements from different locations in the power system to produce the state vector. The measurements contain errors. Hence,the state estimator using an iterative weighted least square (WLS) formulation that is solved conventionally via the so-called Normal Equations (NE) [3] must process them. The SE using NE approach is very sensitive to any erroneous input data, i.e., a small erroneous measurement would result in a significant deviation in the estimated states. In this case, an ill-conditioned SE is declared. This is a serious problem because "If the system is ill-conditioned, then no amount of effort, trickery, or talent used in the computation can produce an accurate answer except by chance" [4]. The previous statement of Rice [4] indicates that this corruption threatens the whole process of estimating the fundamental quantities of power systems. Essentially, numerical sensitivity/instability in the state estimation arises from round-off errors, and other reasons related to the measurements types, numbers, and locations [5].

The problem of ill-conditioning in power systems had diagnosed early in the 1970s [6], [7]. Later, different methods were developed for overcoming this problem. Nevertheless, most of the earliest studies were devoted only to power flow studies such as the Newton-Raphson method [8]. However, the measurement Jacobian matrix of the SE solution has a different structure to the Jacobian of the power flow studies [9]. Therefore, their responses to the numerical stability problems are different. Additionally, though many studies have dealt with the ill-conditioning problem in state estimation, they have designed and tested on high-voltage transmission systems that have different characteristics to thedistribution systems [10], [11]. Nevertheless, the characteristics of the distribution grids have no considerable investigations in the previous comparative studies. Therefore, the validation of the SE solution methods should concern other features that are associated with the modern power networks such as the limited power measurements and the high *R*/*X* ratio of the distribution grids.

Accordingly, this paper provides an overview of the impacts of employing different types of the measurements on the numerical stability of the state estimator. Regarding the distribution grids, this paper considers the presence of short power lines and the high *R*/*X* ratio of the distribution feeders as reasons of the ill-conditioning problem. Numerical comparisonsrelevant to the performance of the common solution methods are implemented in this paper. The comparison considers the distribution systems such as the measurements deficiency and the effect of *R*/*X* ratio.

The next section presents the mathematical explanation of the NE method and the assessment of the numerical stability. In section III, a review of the alternative solution methods and a brief procedure of each one are delivered. In Section IV, five solution methods are tested using two power systems. A discussion of the simulation results is presented in Section V. Finally; the conclusions are presented in Section VI.

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II. WLS-BASED STATE ESTIMATION

It is not feasible to install all the required measuring devices at each bus and each power line since it increases the total cost of the system and complicates the coordination of these measuring devices and their communication channels. Therefore, the SE aims to use the available set of measurements to estimate the power system states as accurately as possible [1], [2]. When a measurement set in a power system is sufficient to provide a unique solution to the SE problem, the system is declared observable [12], [13].

In the state estimation, the state vector, which includes the voltage magnitudes and phase angles of all the buses, is estimated using the real-time measurement set. The conventional real-time measurement set includes these measurements: bus-bars voltages, injected real and reactive power in substations, power flows and currents in power lines in addition to positions of transformers tap changer and circuit breakers status [13]. In addition to the real-time measurements, there are other unmeasurable quantities can be added for enhancing the measurements availability. These non-real measurements are virtual measurements and pseudo-measurements [12], [14]. Virtual measurements are based on network constraints. The zero injection buses with no load or generation are commonly used in this type. Pseudo-measurements are based on historical data such as the forecasted loads and scheduled generation.

A. Formulation of WLS State Estimator

If m is the number of measurements and n is the number of state variables including the bus voltage magnitudes |*V*| and phase angles δ, then the principal formula of the SE solution will be as follows [14]:

$$z = h(x) + \varepsilon$$

where z is the ($m\times 1$) measurement vector, x is the ($n\times 1$) state vector, $h(x)$ is a nonlinear vector function relating measurements to states, and ε is the measurement error vector. The vector ε can be written as r for representing residuals of measurements, i.e., the difference between the estimated and measured values for the ith measurement is:

$$r_i = z_i - h_i(x) = \Delta z_i$$

where: Δz_i refers to the residual of the ith measurements. Now, the objective function should be based on minimizing the difference between the estimated state vector and the measured values , i.e. obtaining minimum residuals. For this purpose, the weighted sum of squares of the residuals should be minimized for the whole set of m measurements [1],[2]:

$$\text{Minimize } \sum_{i=1}^{m} W_{ii}\, r_i^2 \qquad (1)$$

$$\text{Subject to } z_i = h_i(x) - r_i\,, \qquad i = 1, 2, ..., m.$$

where: W_{ii} represents the diagonal elements of a weighting factors matrix. The W_{ii} represents the inverse of the measurement variances, i.e, $W_{ii} = 1/\sigma_{ii}^2$, for measurement i. Accordingly, different values are relating to the diagonal elements of this matrix depending on the accuracy of each measurement.

The solution of (1) is the WLS estimation of x. The aim is to obtain the value of estimated state x that minimize$J(x)$. Hence, the objective function can be rewritten as follows:

$$J(x) = \sum_{i=1}^{m} \frac{[z_i - h_i(x)]^2}{\sigma_i^2}$$

$$J(x) = [z_i - h_i(x)]^T W\, [z_i - h_i(x)]$$

This solution is subjected to the first-order optimality condition, which is:

$$H_x^T W\, [z_i - h_i(x)] = 0$$

where$H_x = H(x) = \partial h(x)/\partial x,$ is the ($m\times n$) Jacobian matrix of $h(x)$. Then, by neglecting terms that contain higher-order derivatives, an iterative solution of Gauss-Newton scheme is resulted in as shown below:

$$H_x^T W H_x \Delta x^k = H_x^T\, W\, [z_i - h_i(x^k)]$$

$$G(x^k)\Delta x^k = H_x^T\, W\, [z_i - h_i(x^k)] \qquad (2)$$

where x^krefers to the value of state x at the kth iteration, $\Delta x^k = x^k - x^{k-1}$ and $G(x^k) = H_x^T W H_x$, is known as the *gain matrix* for the kth iteration. The coefficient matrix of (2) i.e. the gain matrix should be square, sparse, positive definite and symmetrical for an observable system[15]. However, if the weight factors of all measurements are equal to unity, the gain matrix will be equal to $H_x^T H_x$, which squares the Jacobian. For simplicity, H will refer to the Jacobian matrix.

B. Numerical stability of the NE method

The SE solution by NE requires more computational efforts than the conventional load flow studies. The additional burdenis created because the gain matrix is less sparse than the bus admittance matrix that is used in Newton-Raphson load flow method. Another challenge arises from the numerical stability as the NE-based state estimator is vulnerable to be ill-conditioned. In the ill-conditioned systems, a small error in the coefficient/gain matrix or the R.H.S of (2) leads to a considerable error in the resulting state vector. Consequently, the SE solution is prone to be divergent, unstable, or even unsolvable if the G matrix becomes singular.

The decision whether a vector-based system such as the WLS state estimation is a well-conditioned system is based on the value of a factor that is called the condition number. The condition number is a measure of thenumerical sensitivity of the system. The condition number of the SEis defined as [4]:

$$\kappa(G) = ||G|| \cdot ||G^{-1}|| \qquad (3)$$

where $||G||$ refers to the 2-norm of the gain matrix. The condition number is unity or close to one for a well–conditioned matrix; whereas, it is infinity if the matrix is singular. For κ values between unity and infinity, the system could be unstable, and the solution may diverge if there is any noise in the measurements data [4], [16]. An extreme case can happen when the gain matrix has no weight matrix, i.e., a square of the Jacobian matrix which leads to squaring the condition number $\kappa(H^T H) = [\kappa(H)]^2$[17]. Thus, there would be a salient ill-conditioning case.

The numerical stability of WLS-based state estimator is influenced negatively by the following situations [11, 18]:

1. In the case of using high numbers of injection power measurementsareemployed in the measurements set.

2. When usinghigh-weighted measurements are used in the state estimation. This is noticed for the virtual measurements that are required for supporting the measurements availability.
3. If the power system has a long line and a short line at the same bus,i.e., lines with different impedances.
4. The case of power measurements deficiency in the distribution grids, and the high R/X ratio of the distribution feeders. The deterioration of the diagonal entries of the gain matrix in state estimator of distribution systems is mainly due to the high R/X ratios. This situation can be noticed in low-voltage grids due to their low reactance [19]. In contrast, the Jacobian of the transmission systems state estimation has dominant diagonal entries. On the other hand, the limitation of the power measurements in the distribution grids results in dependence on current measurements (Ampere measurements) which cannot provide the phase angles of the system's buses [20].

C. Using State Estimation Solution Methods

Many algorithms have been developed for the state estimation solution. Thosesolution algorithms can be broadly classified into five main approaches which are the Normal Equations (NE) method, Orthogonal Factorization technique, Augmented matrix approach (Hatchel's matrix) Normal equations with Equality-Constraints (NE/C), and Blocked formulation method. All the other methods, indeed, represent modified versions of the above methods. The SE solution methods aim to mitigate the ill-conditioning and enhancing the SE numerical stability. For achieving this goal, these methods (except the NE) try to avoid using the same gain matrix or treating the virtual measurements in a different way. The alternative coefficient matrices that replace the gain matrix in the solution methods are shown in Table I [21]-[29].

III. NUMERICAL SIMULATION RESULTS

Two power systems have been employed to evaluate the performance of SE solution methods. Three numerical tests are implemented for examining the numerical stability of the SE solution methods, theircomputational complexity, the storage size required to the state estimator, and the response to the high R/X ratios.

The first power system is a small and simple system (only five buses and six branches) andlowR/Xratios, whichare equal to 0.25 for all the power lines[30]. Fig. 1 illustrates the 5-bus system. This power systemis a small-scale grid with equal R/X ratios for all its power lines, which is required for examining the influence of R/Xratio. Additionally, the 5-bus systemhas a relatively well-condition state estimator. For more realistic and a larger system, the IEEE 30-bus system has been used as well [31]. The 30-bus network has relatively low R/X ratios and high condition numbers. The tests have been implemented using MATLAB R2016a.

TABLE I. THE JACOBIAN AND THE COEFFICIENT MATRICES OF THE SE SOLUTION METHODS

Methods	Jacobian matrices	Coefficient matrices
NE	Jacobian matrix is composited based on the measurements.	The gain matrix is built as follows: $G(x^k) = H^T W H$
QR	Jacobian matrix is decomposed to: $H=Q{\cdot}R$ where Q is a $(m{\times}m)$ orthogonal matrix, and R is a $(m{\times}n)$ upper triangular matrix.	The coefficient matrix is Q. However, R matrix needs to be saved as well which increase the required storage size.
NE/C	The virtual measurements are treated as equality constraints with a separate set $c(x)$.	The coefficient matrix is: $\begin{bmatrix} H^T W H & C^T \\ C & 0 \end{bmatrix}$ where $C = \partial c(x)/\partial x$
Hatchel's matrix	The virtual measurements are treated as equality constraints with a separate set $c(x)$.	The coefficient matrix is the Hatchel's matrix: $\begin{bmatrix} \alpha W^{-1} & H & \mathbf{0} \\ H^T & 0 & \boldsymbol{C^T} \\ \mathbf{0} & \boldsymbol{C} & \mathbf{0} \end{bmatrix}$ where α = 1/max (W_{ii}) is a scaling factor.
Blocked formulation	Measurement setsare divided into two groups: injection set (I) and set (F) that contains the remaining ordinary sets. The resulting Jacobian matrix contains: H_I& H_F	The coefficient matrix is: $\begin{bmatrix} R_I & 0 & \mathbf{0} \\ H_F^T & -H_F^T W_F H_F^T & \boldsymbol{C^T} \\ \mathbf{0} & \boldsymbol{C} & \mathbf{0} \end{bmatrix}$

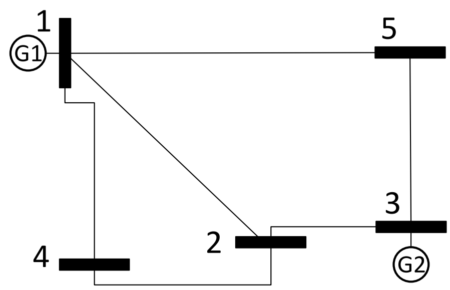

Fig. 1. The test system of the 5-Bus network.

A. Structures of the coefficient matrices

The first test is for analyzing the structure of the gain matrix and comparing it with the coefficient matrices of other solution methods. These coefficient matrices are constructed for both 5-bus and IEEE 30-bus systems according to Table I. The structure of these coefficient matrices, and their densities are shown in Fig. 2and Fig. 3. Firstly, six power flows measurements (one for each branch), and unity weight factorshave been used for all the measurements. This situation is required for demonstrating the coefficients matrices intrinsically without the effect of the weights and the injection power measurements. Therefore, matrices of NE/C and the Blocked formulation are not involved in this test since they have the same structure of the conventional NE approach.

In Fig. 1, the dark blue dots refer to the nonzero elements in those matrices with the numbers and percentages of these elements. For numerical demonstration, a summary of the coefficient matrices of the 5-bus system is shown in Table II.

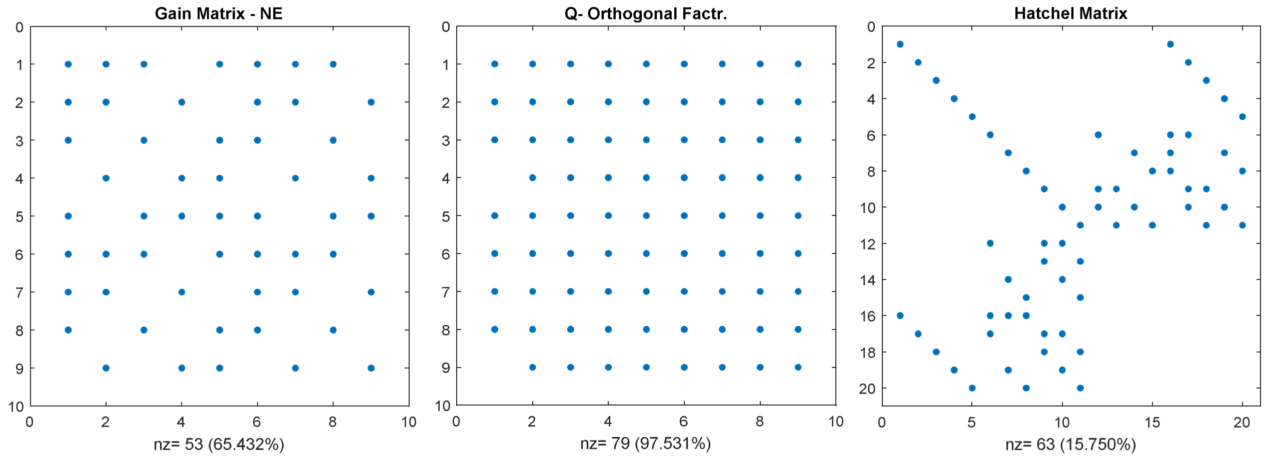

Fig. 2. The scatter pattern of the coefficient matrices (gain matrix, Q matrix, and Hatchel's matrix) of three different methods for 5-bus test system.

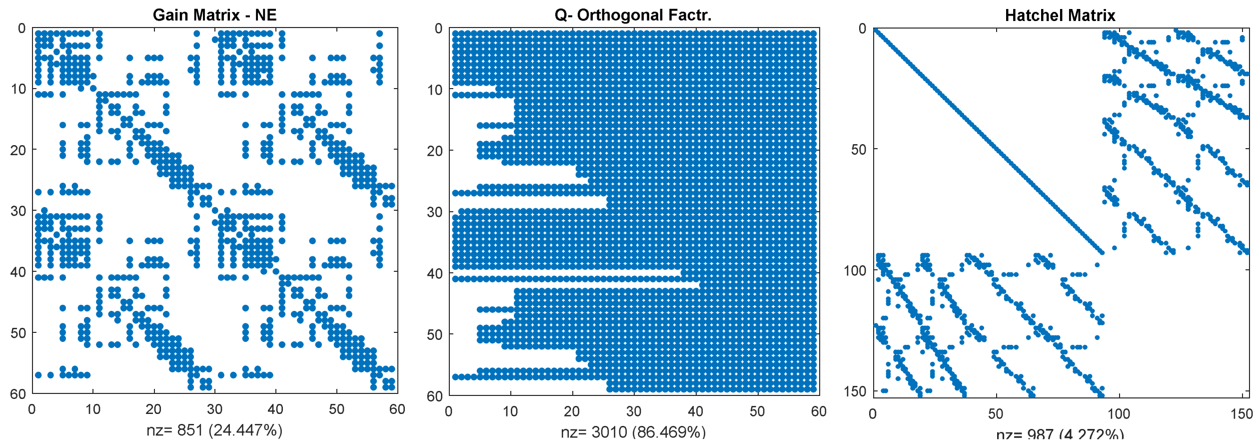

Fig. 3. The scatter pattern of the coefficient matrices (gain matrix, Qmatrix, and Hatchel's matrix) of three different methods for IEEE 30-bus test system.

TABLE II. COEFFICIENT MATRICES ENTRIES OF 5-BUS TEST SYSTEM

Features	Gain Matrix	*Q* Matrix	Hatchel's Matrix
Total entries	81	81	400
Numberand Percentages of Nonzero entries	53 65.432%	79 97.53%	63 15.75%

Based on Fig. 2, 3, and Table II, the$Q$ matrix is denser than other coefficient matrices including the gain matrix that has the same size of Q matrix. Hatchel's matrix has the lowest density (sparser), but it has a larger size and a larger number of nonzero entries than that of thegain matrix and Q matrix. Accordingly, the Hatchel's matrix has a size drawback.

Regarding the dominance of the diagonal elements, Hatchel's matrix has a relatively dominant diagonal, especially for its first quarter. In contrast, G and Q matrices have nearly a scatter distribution.

The storage size can be a problem when dealing with large-scale power system since the size of the coefficient matrices belongs to the 30-bus state estimation, and their non-zero elements refers

B. Comparing the condition numbers

In this test, four different cases relevant to the measurements type and system configuration have been considered. The condition numbers that are associated with the coefficient matrix of each solution method has been calculated for a comparison purpose. The four cases that have been applied to the IEEE 30-bus system are as follows.

1. Using 76 power flow measurements for all branches.
2. Adding injection measurements to case 1, above, at buses 2 and 3.
3. Same with case 2, but with weights of 1000 tothe power injection measurements.
4. Same with case 2 with a different length of lines (line 1-2 becomes 100 times shorter).

Table VI provides the condition numbers of the coefficient matrices of three solutions methods for the above four case studies.

TABLE III. THE CORRESPONDING CONDITION NUMBERS FOR EACH MEASUREMENT CASE OF 30-BUS SYSTEM

Cases	NE	Blocked	Hatchel
1-Six power flows	106.48	106.48	19.01
2-Injection measurements (regular) : buses 1&3	304.7	119.2	31.2
3-Null injection measurements	$1.8*10^5$	131	37
4-Short power line	$1.2*10^6$	$6.8*10^5$	980

It can be noticed that Hatchel's augmented method is the lowest sensitive approach, and thereby the most stable one. However, the real practice for numerical stability is the case of including injection measurements as the condition numbers of all methods tend to grow up. Nevertheless, the blocked factorization method still has a good performance and a low increasing rate for its condition number. The worst case in this test happens when one of the grid branches is a relatively short line. The case of power lines with various lengths is common in distribution networks in addition to the situation of high *R*/*X* ratio.

C. Impact of R/X ratio

The case of high R/X ratio can be observed mainly in the distribution systems, and hence, this test is applied to transmission grids that have a very low *R*/*X* ratio. Table IV illustrates the influence of increasing the feeder's resistance relative to their reactance. The original case for the 5-bus network is that branches' reactance equals four times of their resistances, and then three additional ratios have been tested. These four cases show that even an efficient algorithm like the Hatchel's could be deteriorated in the situation of thehigh ratio of feeders' resistance. That is because the diagonal elements of Hatchel's matrix would not be dominant in this case. In case 3 and further, the difference between NE condition number and that of Hatchel's matrix is reduced and then it is reversed in the 4th case. The methods of NE/C and Blocked formulation are excluded from this test because it is implemented for only flat start case with only power flows measurements, and thus the NE/C and blocked formulation results are the same of the NE's.

TABLE IV. THE CORRESPONDING CONDITION NUMBERS FOR EACH RATIO OF *R*/*X*

Cases	Ratios	NE	Hatchel
1	R/X = 0.25	106.48	19.007
2	R/X = 1	281.24	52.5
3	R/X = 5	1038	632.95
4	R/X = 10	4248	4443

IV. DISCUSSION

Based on the above tests, it can be deduced thatthe performance of the solution methods has its shortcomings in addition to its advantages. Accordingly, the decision of selecting the best method is a relative one. The decision should be based on the desired performance that is required for the SE, i.e., which is the preferable feature need to be available for a specific power application.

The spider net that is shown in Fig. 4 illustrates the weaknesses and strengthens of the methods have been discussed in this paper. The five vertices of the chart refer to the main numerical characteristics of the solution methods. The percentages in the Fig. 4 are approximate values that are estimated based on this paper tests, and on the previous studies as well.

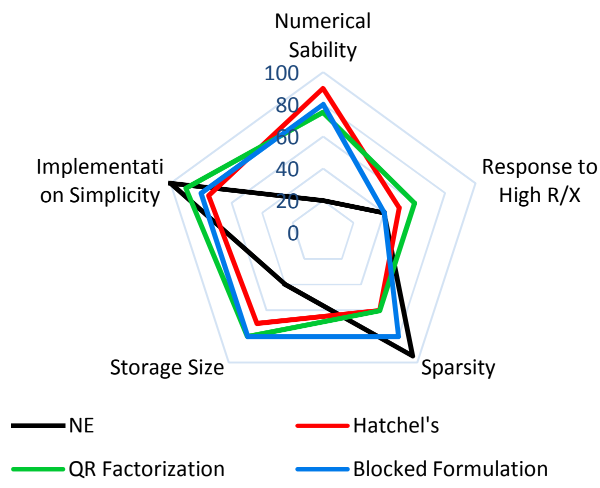

Fig. 4. Spider net chart for the characteristics of SE methods.

In Fig. 4, Hatchel's method and the QR method seem to have superior performance compared to other solution methods. Nevertheless, both Hatchel's and QR methods have their obvious weaknesses as they response inefficiently toward the high *R*/*X* ratio and the storage size status. It can be observed from the right-side vertex that all the methods have a low or a modest performance when responding to the case of high *R*/*X* ratio. On the other hand, the conventional NE solution is still considerable since it has a competitive response in all the aspects, except the numerical stability vertex, which is far awayfrom other methods' evaluation.

The methods are discussed in this paper target the algorithm of WLS solution for state estimation. However, other trends explore different horizons for improving state estimation reliability and accuracy. These other patterns represent the vertical extension of SE solution method while the methods of section II represent the horizontal expansion. That means the vertical extension approaches target either the base of state estimation process which simulates the load flow studies or tries to use untraditional way for SE solution. Studies relating to this recent trend can be divided into two categories: The first approach is to work with a modified Newton-Raphson method, which could be more suitable for distribution system requirements [8]. Consequently, this mitigates the divergence problem of the Jacobian matrix [32]. The second approach is to use an unconventional SE in the form of a linear SE process. This method is currently under development. Since linear SE is a non-iterative process, it avoids the problem of a divergent solution or an undefined Jacobian matrix[33]. However, this solution sacrifices the SE accuracy and needs more measurements.

Solution methods that are described in the above two categories can be considered as promising tools for state estimation of modern distribution grids and exceptionally for smart grids studies.

V. CONCLUSION

This paper discusses several methods that have been developed for avoiding this ill-conditioning case. Accordingly, comparative study of five methods has been implemented theoretically and numerically. This comparison is based on the main objectives of these methods characteristics such as numerical stability, sparsity, size, implementation complexity and their behavior in the case of thepower grid with high *R*/*X* ratios. Despite the adequate numerical stability of some solution methods, new techniques need to be developed for satisfying the new challenges of themodern distribution grids. Another processfor obtaining more stable and accurate SE solutionis the linearization of the state estimator which needs phasor measurement units (PMUs) to emulate the conventional nonlinear state estimator.

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