Controller Design for Cuk Converter Using Model Order Reduction

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Abstract: Cuk converter contains two inductor and two capacitor hence it is fourth order dc-dc converter. It provides output voltage both higher as well as lower than the input voltage. Design of feedback compensator for fourth order system is quite complex. In this paper, model order reduction technique is used for controller design of Cuk converter. First small signal dynamic model for Cuk converter using state space analysis (SSA) is obtained which provides fourth order transfer function. Then this fourth order transfer function is reduced to second order using Pade approximation.

Index Terms- Cuk Converter, Model Order Reduction, Compensator, State-Space Averaging.

I. INTRODUCTION

The Ćuk converter is a type of dc-dc converter that has an output voltage magnitude that is either greater than or less than the input voltage. Cuk converters have excellent properties like capacitive energy transfer, full transformer utilization and good steady-state performances such as wide conversion ratio, smooth input and output currents.

The dynamic response, however, is affected by the fourth order characteristic, which generally calls for closed-loop bandwidth limitations in order to ensure large-signal stabilization. Moreover, stability may require big energy transfer capacitors in order to decouple input and output stages.

For the purpose of optimizing the converter dynamics, while ensuring correct operation in any working condition, robust multivariable controllers could be used. These, however, may involve considerable complexity of both theoretical analysis and control implementation. To remove this difficulty first we reduce the order of transfer function of Cuk converter then design controller.

The Cuk converter is made up of two capacitors, two inductors, a power switch and diode thus it is fourth order non-linear system. For the feedback control design linear model is needed. The linear model of the converter is derived by the replacement of switch and diode of converter by small signal averaged switch model [7]. The desired transfer function is obtained using state space averaging technique. This paper presents Cuk converter operating in continuous conduction mode (CCM). In continuous conduction mode inductor current never falls to zero during one switching period. The state space averaging technique (SSA) [3] is used to find small signal linear model and its various forms of transfer functions. Depending on control-to-output transfer function, the PWM feedback controller [10] is designed to regulate the output voltage of the Cuk converter. This transfer function is found to have two pair of complex pole in left half plane and three zeros in RHP. RHP zero is undesirable for controller design because it provides extra 900 phase lag. Also, higher order system increases controller complexity, to remove these difficulties model order reduction technique is used. In this paper Pade approximation [13] method is used for model order reduction. The reduced order system has one RHP zero and one pair of complex pole.

II. SSA TECHNIQUE

The power stage of closed loop system is a non-linear system. Since non-linear system is difficult to model and their behavior is also difficult to predict, therefore, it is common practice to approximate non-linear system to a linear system. For linearized power stage of dc-dc converter Bode plot can be used to determine suitable compensation in feedback loop for desired steady state and transient response. For this state space averaging technique is used.

In dc-dc converter operating in continuous condition mode, there exist two states one when switch is on and other when switch is off.

During switch on;
\[ \dot{X} = A_1X + B_1V_d \]

During switch off;
\[ \dot{X} = A_2X + B_2V_d \]

\[ V_o = C_1X \]

\[ V_o = C_2X \]

To produce an average description of the circuit over a switching period, the equations corresponding to the two foregoing states are time weighted and averaged, resulting in the following equations-

\[ \dot{X} = [A_1d + A_2(1-d)]X + [B_1d + B_2(1-1-d)]V_d \]

\[ V_o = [C_1d + C_2(1-1-d)]X \]

III. SYSTEM ANALYSIS

Cuk converter is a switching regulator which yields a variable output voltage from a constant dc supply. The state-space averaged model used to derive the steady-state and the dynamic models of the Cuk converter based on its state space averaged model [3].
A. Modeling of Cuk converter by state space technique

The Cuk converter contains two inductors $L_1$ and $L_2$ with equivalent series resistances $r_{L1}$, $r_{L2}$ respectively, two capacitors $C_1$ and $C_2$ with equivalent series resistances $r_{C1}$ and $r_{C2}$ respectively, switch (MOSFET) $S$ and a diode $D$ as shown in fig.1a. The resistance $R$ is representing the load. The input voltage $V_d$ is fed into the circuit via inductor $L_1$. When switch is on as shown in fig 1.b, current $i_{L1}$ builds the magnetic field of the inductor in the input stage. The diode $D$ is reverse biased, and the energy dissipates from the storage elements in the output stage.

When the switch is turned-off as shown in fig.1.c, the inductor $L_1$ tries to maintain the current flowing through it by reversing polarity and sourcing current as its magnetic field collapses. It thus provides energy to the output stage of the circuit via capacitor $C_1$.

Sum of both currents $i_{L1}$ and $i_{L2}$ must be zero in the steady state, with the assumption that voltage $v_{C1}$ is essentially constant (given that the voltage across a capacitor cannot change instantaneously and the switching speed of the circuit is high). This provides following energy conservation relation-

$$V_d = \frac{d}{1-d}$$

Where $d$ is the duty cycle of the switch. This equation shows that by controlling the duty cycle of the switch output voltage $V_o$ can be controlled and output voltage can be higher or lower than the input voltage $V_d$. By using a controller to vary the duty cycle during operation, the circuit can also be made to reject disturbances.

B. State space equation of Cuk converter

The state space equation for Cuk converter during switch on and off are

During switch is ON

$$\frac{di_{L1}}{dt} = -\frac{r_{L1}}{L_1}i_{L1} + \frac{V_d}{L_1}$$

$$\frac{di_{L2}}{dt} = \frac{V_{C1}}{L_2} + \frac{(r_{C2} + r_{L2})}{L_2}i_{L2} - \frac{r_{C2}}{R + r_{C2}}\frac{V_{C2}}{C_2}$$

$$\frac{dv_{C1}}{dt} = -\frac{i_{L2}}{C_1}$$

$$\frac{dv_{C2}}{dt} = -\frac{R}{(r_{C2} + R)C_2} - \frac{V_{C2}}{(R + r_{C2})C_2}$$

During switch off

$$\frac{di_{L1}}{dt} = \frac{(r_{L1} + r_{C1})}{L_1}i_{L1} - \frac{V_{C1}}{L_1} + \frac{V_d}{L_1}$$

$$\frac{di_{L2}}{dt} = \frac{(r_{L2} + R)(r_{C2} + r_{L2})}{L_2}i_{L2} + \frac{(r_{C2} + r_{L2})}{L_2} - 1\frac{V_{C2}}{L_2}$$

$$\frac{dv_{C1}}{dt} = \frac{i_{L1}}{C_1}$$

$$\frac{dv_{C2}}{dt} = \frac{R}{(r_{C2} + R)C_2} - \frac{V_{C2}}{(R + r_{C2})C_2}$$

The averaged matrices for the steady-state and linear small-signal state-space equations can be written according to above equations.

$$A_1 = \begin{bmatrix} -\frac{r_{L1}}{L_1} & 0 & 0 & 0 \\ 0 & -(r_{L2} + r_{C2} + r_{L2}) & \frac{1}{L_2} & \frac{r_{C2}}{R + r_{C2}} - 1 \\ 0 & 0 & 1 \frac{C_1}{C_1} & 0 \\ 0 & \frac{R}{(r_{C2} + R)C_2} & 0 & -1 \frac{1}{(r_{C2} + R)C_2} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{r_{L1} + r_{C1}}{L_1} & 0 & -1 \frac{1}{L_2} & 0 \\ 0 & -(r_{L2} + r_{C2} + r_{L2}) & \frac{1}{L_2} & \frac{r_{C2}}{R + r_{C2}} - 1 \\ 0 & 0 & 1 \frac{C_1}{C_1} & 0 \\ 0 & \frac{R}{(r_{C2} + R)C_2} & 0 & -1 \frac{1}{(r_{C2} + R)C_2} \end{bmatrix}$$
\[
\begin{pmatrix}
1 & 0 \\
0 & 0 \\
0 & 0
\end{pmatrix}
\]

\[
B_1 = B_2 = B = C_1 = C_2 = C = (0 \ 0 \ 0 \ 1)
\]

\[
E_1 = E_2 = E = (0)
\]

**C. Transfer function**

With the state space matrices defined above, the control -to -output transfer function can be calculated as

\[
G_{vd} = C (S I - A)^{-1} B_d + E_d
\]  
(11)

Where

\[
B_d = (A_1 - A_2) X + (B_1 - B_2) V_d
\]  
(12)

Output to input transfer function

\[
G_{vi} = C (S I - A)^{-1} B
\]  
(13)

\[
X = -C A^{-1} V_d
\]  
(14)

**IV. CONTROL OF CUK CONVERTER**

**A. PWM feedback control**

Fig. 2(a) shows a Cuk converter with PWM feedback control [8]. The output voltage \( V_0 \), is fed back and compared with the reference voltage, \( V_{ref} \). Which produces error voltage \( V_e \) which is applied to the compensator, \( G_c(s) \), which produces the control voltage, \( V_c \), to compare with the saw tooth voltage of amplitude \( V_M \) at the PWM comparator. As depicted in Fig. 2(b), the MOSFET is turned on when \( V_c \) is larger than \( V_{saw} \), and turned off when \( V_c \) is smaller than \( V_{saw} \). If \( V_0 \) is changed, feedback control will respond by adjusting \( V_c \) and then duty cycle of the MOSFET until \( V_0 \) is again equal to \( V_{ref} \).

**B. Example[11]**

Parameters of the Cuk converter:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage ( V_{in} )</td>
<td>12 Volts</td>
</tr>
<tr>
<td>Output Voltage</td>
<td>24 Volts</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>100 kHz</td>
</tr>
<tr>
<td>Load</td>
<td>12 Ohm</td>
</tr>
<tr>
<td>PWM Gain</td>
<td>1/5</td>
</tr>
<tr>
<td>( L_1 ) ( \mu \text{H} )</td>
<td>68.7 ( \mu \text{H} )</td>
</tr>
<tr>
<td>( L_2 ) ( \mu \text{H} )</td>
<td>2.2mH</td>
</tr>
<tr>
<td>( C_1 ) ( \mu \text{F} )</td>
<td>3.7 ( \mu \text{F} )</td>
</tr>
<tr>
<td>( C_2 ) ( \mu \text{F} )</td>
<td>984 ( \mu \text{F} )</td>
</tr>
<tr>
<td>Output ripple</td>
<td>5%</td>
</tr>
</tbody>
</table>

The transfer function of the converter is obtained from (11) is as follows:
This is the fourth order transfer function. It has two pair of complex pole and three zero in the RHP. Zeros and poles of the converter are as given as:

Zeros are-
- 4995.0229 +35864.4541i
- 4995.0229 - 35864.4541i
- 20156.5294

Poles
- -11.1685 +22175.8678i
- -11.1685 - 22175.8678i
- -63.5356 +637.7841i
- -63.5356 - 637.7841i

C. Model Reduction:
Using Pade-Approximation method [13], the reduced order transfer function of the converter is obtained as follows:

\[ G_{1vd} = \frac{-814.85^3 + 2.456 \times 10^7 s^3 - 1.232 \times 10^{12} s + 2.154 \times 10^{16}}{s^4 + 149.4 s^3 + 4.922 \times 10^8 s^2 + 6.25 \times 10^{10} s + 2.02 \times 10^{12}} \]

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- -63.5356 - 637.7841i

Zeros are-
- 4995.0229 +35864.4541i
- 4995.0229 - 35864.4541i
- 20156.5294

D. Feedback controller design
In this paper voltage-mode linear averaged feedback controllers for dc–dc converter is designed in frequency domain. The main objective of the controller design is to obtain stable operation of the converter by varying the duty cycle. Following points are taken care while designing of the compensator.

1) The averaged mathematical model is accurate up to one tenth of switching frequency. Here the switching frequency is taken as 100 kHz therefore the bandwidth (0 dB cross over frequency of closed loop system) should be near 10kHz.

2) High gain at low frequency region provides good output voltage regulation. And phase margin determines the transient response to sudden change in input voltage. The suitable phase margin is between 45° to 60° degree.

Fig.5 shows Bode plot without compensator that has both gain margin and phase margin negative. To make the gain margin and phase margin positive suitable poles and zeros of compensator is selected.

Fig.5 shows complex pair of pole that occurs at 640 rad/sec which provide 180° phase lag. To overcome, this problem two zeros is added at 640 rad/sec in compensator. To minimize the effect of noise at high frequency one pole is added at high frequency. To provide good output regulation one pole is added at very low frequency.

With these considerations the designed compensator is

\[ G_c = \frac{400(\frac{s}{640} + 1)^2}{s(\frac{s}{2 \times 10^7} + 1)} \]

and therefore, the overall open-loop transfer is

\[ T(s) = \frac{-0.4904 s^3 + 7923 s^2 + 1.074 \times 10^7 s + 3.502 \times 10^9}{5 \times 10^{-6} s^4 + 1.001 s^3 + 129.1 s^2 + 410600 s} \]

Fig.4 Step response of reduced and full order system

Integral Square Error (ISE) between original system and reduced order system is 0.003249

Fig.5

Figure 5 shows the Bode plot of uncompensated open loop system which has gain margin -11.9dB and phase margin -7.42 deg.
This paper deals with modeling and control of Cuk converter operating in continuous conduction mode (CCM). The state space averaging technique is applied to find out the linear model of Cuk converter and the desired transfer function in terms of duty ratio to output voltage \( G_{vd} \) is obtained which is a fourth order transfer function. Designing a compensator for the fourth order system is very difficult. Therefore, fourth order transfer function of Cuk converter is reduced to second order and it is found that step response of reduced order model closely follow the original system. The compensator designed for second order system gives quite satisfactory response with the original system.

**REFERENCES**


