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Optimal distributed generation allocation for reactive power control

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Insu Kim[1] ✉
[1]*Department of Electrical Engineering, Alabama A&M University, Normal, AL 35762, USA*
✉ *E-mail: insu.kim@aamu.edu*

Abstract: One of the main concerns of utilities is to minimise voltage variations along their feeders when they enhance their networks by various distributed generation (DG) systems that have uncertainty in their output. Furthermore, such DG systems can provide reactive power to the grid. Thus, the objective of this study is to optimally allocate locations and capacities of DG systems, particularly photovoltaic systems in this study, on distribution networks that host DG inverters with the capability of reactive power control, in other words, Volt/Var management and control. For this purpose, this study proposes a hybrid method based on optimal reactive power control and genetic algorithms. The proposed method is verified in well-known test feeders.

1 Introduction

Nowadays, one of the main goals of utilities is to enhance their networks by various distributed generation (DG) systems with capacities in the range of several kW to hundreds of MW. In spite of the relatively small individual capacities of DG systems, their cumulative effects on the distribution network may change the steady-state and transient behaviours of the network to which they are connected. Furthermore, such DG systems can control reactive power, referred to as 'Volt/Var control'. Therefore, during the last decades, many studies have proposed optimal solutions that minimise the total loss, the failure duration, and the costs of distribution networks and maximise the capacity of hosting DG and profits [1]. One study analysed the impact of the locations and the capacities of DG systems on the IEEE 37-bus test feeder either with or without a fault [2]. The study used an objective function that minimises voltage deviations, set the capacities of DG systems to 1/3, 1/2, and 2/3 of the total load, and set the locations of DG systems to buses 742, 709, 734, and 741. Using a Chebyshev norm that determines optimal reactive power control, another study minimised the voltage changes in the IEEE 30-bus test system at a worst-case analysis strategy [3]. Another study proposed a Volt/Var control algorithm in which all partial derivatives of an objective function (that minimises power loss, demand, and the number of control steps) with respect to control variables started from initial points to the largest derivatives of the next points [4]. Using the energy management system (of the State Electricity Commission of Victoria), one study calculated the optimal power flow of southeastern Australian transmission networks, minimised transmission loss, and examined the operational benefits of optimal Volt/Var scheduling [5]. In 2003, one study proposed a genetic algorithm (GA) that minimised distribution losses by optimally controlling (a) the tap position of load tap changers, (b) the switching capacities of capacitor banks, (c) the settings of the local controller, and (d) the voltage amplitudes of DG systems [6]. Another study presented a transition-optimised approach that controls reactive power and voltage by adaptively classifying load profiles, typically in hourly intervals, and minimising real power losses and the variance of the sequence of load profile data [7]. Then, another study proposed a hybrid algorithm of the ant colony algorithm and the GA that determined (a) the optimal reactive powers of DG systems and static Var compensators and (b) the settings of the local controllers for Volt/Var control. The algorithm minimised energy loss in the distribution system and the costs of the reactive power generated by the DG systems [8]. Recently, one study optimised the under-load tap-changing of transformers and capacitors for Volt/Var control, using the particle swarm optimisation method with a fuzzy multi-objective function that minimises loss, reactive power, and the switching operation of tap changers [9]. Another study optimally scheduled automatic voltage regulators and capacitor banks for Volt/Var control with the GA with a fuzzy logic to minimise loss [10]. Using particle swarm optimisation, one study solved an optimisation problem of the daily coordination of DG systems, an under-load tap changer, and capacitors for Volt/Var control while minimising daily energy and energy loss costs [11]. After that, another study presented the optimal settings of capacitor banks and voltage regulators to maximise life cycles using dynamic programming [12]. A more recent study optimised the reactive power dispatch of solid-state transformers with a Lagrangian function that minimises loss, solved by a Brute force method and an improved rule-based search method [13, 14]. Then, to maximise energy savings using neural network model selection, another study predicted day-ahead hourly energy at substations with an optimal Volt/Var strategy [15]. Next, one study analysed the impact of high-capacity photovoltaic (PV) systems at predefined power factors for reactive power control [16]. Another study proposed a reactive power control system for wind farms with a centralised microprocessor-based automation controller that regulates power factors and voltage [17]. In 2014, one study scheduled reactive power resources such as voltage regulators, capacitor banks, and load tap changers for Volt/Var control that minimises the switching operations of these devices by dynamic programming and branch-and-bound methods [18]. Another study suggested a GA with an objective function that minimises reactive power and tap operation and finds the optimal reactive power of DG systems [19].

None of these previous studies, however, have optimally allocated the location and the capacity of DG systems able to optimally control reactive power in order to maintain the voltage of a bus to which DG systems are connected within a set voltage range. Therefore, this study proposes a hybrid method based on the reactive power control algorithm and the GA. The method optimally allocates the locations and the capacities of DG systems with an objective function that (a) minimises voltage variations caused by the reactive power control of DG systems and (b) maintains the voltage of a bus to which DG systems are connected within a set voltage range. To maintain the bus voltage within a set value range, this study proposes a method that represents a bus to which DG systems are connected as a *P*–*V* bus. Then, to solve an optimisation problem on the allocation of DG systems, the GA integrated to the proposed *P*–*V* bus representation method is

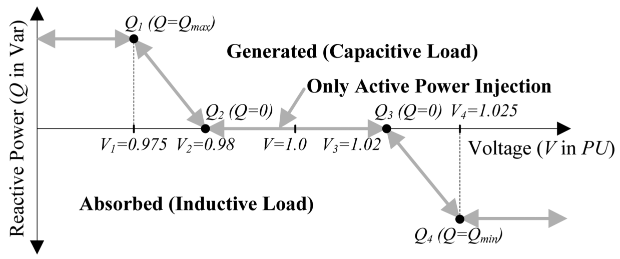

Fig. 1 *Volt/Var control*

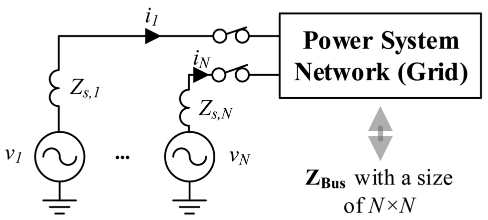

Fig. 2 *Power system grid represented by the bus impedance matrix ($\boldsymbol{Z}_{Bus}$)*

presented. The hybrid method is useful to investigate the effect of the Volt/Var control of various DG systems on voltage regulation.

This paper is organised as follows: Section 2 presents the brief problem statement. Section 3 describes the method of modelling the Volt/Var control of DG inverters and the representation of a DG bus as a *P–V* bus. Section 4 indicates a GA that optimally allocates the location and capacity of DG systems. Then, two case studies presented in Section 5 verify the proposed hybrid algorithm. Finally, Section 6 summarises the major conclusions and contributions.

2. Problem statement

For the optimal allocation of DG systems capable of controlling Volt/Var on voltage regulation, this study takes the following uncertainties into account:

i. *Location on the distribution network*. DG systems may be connected anywhere across the network. This study particularly examines PV systems, but it can be extended for other types of DG systems.
ii. *Capacity of a DG system*. The capacity of a DG system is unknown.
iii. *Volt/Var*. The DG system can provide reactive power to the grid at any set points of Volt/Var control operation.

To maintain the voltage of a distribution network within the desired range, typically within ±5% of the rated voltage [20], DG systems can participate in reactive power control under the mutual agreement of DG system owners (or operators) and the utility [20–23]. In addition, much more DG systems, including PV systems, wind farms, and other inverter-based DG systems, able to control reactive power are continuously connected to the distribution network. Thus, DG systems can affect the behaviour of the distribution network despite of their small capacity. Thus, the optimal allocation of DG systems should be solved for DG systems capable of controlling Volt/Var.

3 Volt/Var control

The state-of-the-art inverter-based DG systems are often able to control reactive power, referred to as Volt/Var management and control. Furthermore, the amendment to the IEEE standard that interconnects DG systems to electric power systems allows reactive power injection at planned or limited situations [21]. Fig. 1 illustrates a method that a DG inverter controls Volt/Var. For example, if the voltage of a bus to which a DG system is connected is kept within the normal voltage of V_2 (e.g., 0.98 pu in Fig. 1) to V_3 (e.g., 1.02 pu), the inverter injects only active power (at a power factor of 1.0 pu). However, if the bus voltage exceeds V_3 (e.g., 1.02 pu), the inverter absorbs reactive power like an inductive load. On the contrary, it decreases below V_2 (e.g., 0.98 pu), the inverter generates reactive power like a capacitive load.

The ratio of reactive power deviation to voltage deviation, which can be defined by the slopes of the lines in Fig. 1, can be optimally determined by the following method of a *P–V* bus representation.

3.1 Positive-sequence impedance matrix

If a bus to which DG systems are connected participates in reactive power control and the bus voltage is maintained within a positive-sequence target value, a DG bus should be modelled by a *P–V* bus, not a *P–Q* bus. To maintain the positive-sequence voltage of the DG bus, this study initially proposes the positive-sequence impedance matrix that represents a power system network in Fig. 2 with a relationship between the positive-sequence current and the voltage. That is, the proposed matrix is

$$\boldsymbol{V}^{+}_{\text{Bus}} = \boldsymbol{I}^{+}_{\text{Bus}}\boldsymbol{Z}^{+}_{\text{Bus}}, \tag{1}$$

where $\boldsymbol{V}^{+}_{\text{Bus}}$ is the positive-sequence voltage of each node, $\boldsymbol{I}^{+}_{\text{Bus}}$ is the positive-sequence current flowing on each node, and $\boldsymbol{Z}^{+}_{\text{Bus}}$ is the positive-sequence impedance matrix.

If each phase impedance of lines is given by a size of 3×3 (after removing the neutral components from the phase impedance matrix by Kron reduction), the positive-sequence impedance (e.g., Z_{11}) can be derived by

$$\boldsymbol{Z}_{012} = \boldsymbol{T}\boldsymbol{Z}_{abc}\boldsymbol{T}^{-1} = \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix}, \tag{2}$$

where $\boldsymbol{Z_{abc}}$ is the phase impedance (with a size of usually 3 × 3), $\boldsymbol{T}$ = 1/3 × [1 1 1;1 a a^2;1 a^2 a], $a = 1\angle 120.0°$.

After each positive-sequence impedance of lines of a power system network is calculated, to build the positive-sequence impedance matrix for the network, the following rules are proposed:

(1) *Adding a node to the ground*. For example, in Fig. 3, an initial node (e.g., node 1) is added to the ground with a positive-sequence branch impedance of z_{g} (e.g., z^{+}_{10}). The impedance matrix is

$$\boldsymbol{Z}^{+(k)}_{\text{Bus}} = \left[z_{\text{g}}\right] = \left[z^{+}_{10}\right], \tag{3}$$

where k is the number of the current steps (e.g., $k = 1$).
(2) *Adding a new node to the existing node*. New node q (e.g., node 2) is added to existing node p (e.g., node 1) with a positive-sequence branch impedance of z (e.g., $z = z^{+}_{12}$)

$$\boldsymbol{Z}^{+(k)}_{\text{Bus}} = \begin{bmatrix} \boldsymbol{Z}^{+(k-1)}_{\text{Bus}} & \boldsymbol{Z}^{+(k-1)}_{p} \\ (\boldsymbol{Z}^{+(k-1)}_{p})^{\text{T}} & \boldsymbol{Z}^{+(k-1)}_{pp} + z \end{bmatrix} = \begin{bmatrix} z^{+}_{10} & z^{+}_{10} \\ z^{+}_{10} & z^{+}_{10} + z^{+}_{12} \end{bmatrix}, \tag{4}$$

where $\boldsymbol{Z}^{+(k-1)}_{p}$ is the pth column of the positive-sequence impedance matrix at step $(k-1)$ (e.g., $k = 2$ and $p = 1$), $\boldsymbol{Z}^{+(k-1)}_{pp}$ is the element on the pth row and column of the positive-sequence matrix at step $(k-1)$. Similarly, new nodes 3 and 4 in Fig. 3 are added to the existing nodes 2 and 3. If repeating this rule, the positive-sequence impedance matrix with a size of 4×4 can be found as the following:

$$\boldsymbol{Z}^{+(4)}_{\text{Bus}} = \begin{bmatrix} z^{+}_{10} & z^{+}_{10} & z^{+}_{10} & z^{+}_{10} \\ z^{+}_{10} & z^{+}_{10} + z^{+}_{12} & z^{+}_{10} + z^{+}_{12} & z^{+}_{10} + z^{+}_{12} \\ z^{+}_{10} & z^{+}_{10} + z^{+}_{12} & z^{+}_{10} + z^{+}_{12} + z^{+}_{23} & z^{+}_{10} + z^{+}_{12} + z^{+}_{23} \\ z^{+}_{10} & z^{+}_{10} + z^{+}_{12} & z^{+}_{10} + z^{+}_{12} + z^{+}_{23} & z^{+}_{10} + z^{+}_{12} + z^{+}_{23} + z^{+}_{34} \end{bmatrix}. \tag{5}$$

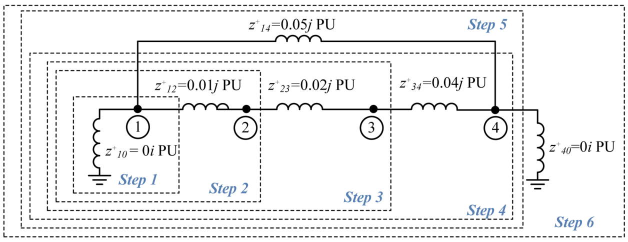

Fig. 3 *Example of a power system network with four nodes*

(3) *Adding a branch between the existing two nodes*. A new branch with a positive-sequence impedance of z_r (e.g., z_{14}^{+} in Fig. 3) is added between the existing nodes p and q (e.g., $p = 1$ and $q = 4$)

$$r = (z_r + \boldsymbol{Z}_{pp}^{+(k-1)} + \boldsymbol{Z}_{qq}^{+(k-1)} - 2\boldsymbol{Z}_{pq}^{+(k-1)})^{-1}, \tag{6}$$

$$\boldsymbol{Z}_{\rm Bus}^{+(k)} = \boldsymbol{Z}_{\rm Bus}^{+(k-1)} - r(\boldsymbol{Z}_{p}^{+(k-1)} - \boldsymbol{Z}_{q}^{+(k-1)})(\boldsymbol{Z}_{p}^{+(k-1)} - \boldsymbol{Z}_{q}^{+(k-1)})^{\rm T}, \tag{7}$$

where $k = 5$.

(4) *Connecting an existing node to the ground*. An existing node (e.g., node 4 in Fig. 3) is connected to the ground through a positive-sequence branch impedance of $z_{\rm g}$ (e.g., z_{40}^{+}). In this case, after connecting a new $(r+1)$ node (e.g., $[r+1] = 5$) to the existing node (e.g., node 4) based on rule (2) and setting the voltage of the new node to the ground voltage (e.g., zero), which is known as the Kron reduction, by

$$\boldsymbol{Z}_{pq}^{+(k)} = \boldsymbol{Z}_{pq}^{+(k-1)} - \frac{\boldsymbol{Z}_{pr}^{+(k-1)}\boldsymbol{Z}_{rq}^{+(k-1)}}{\boldsymbol{Z}_{ii}^{+(k-1)} + z_{\rm g}}, \tag{8}$$

where $k = 6$, i = the existing node connected to the ground (e.g., i = 4), $(r+1)$ is the temporary node connected to the ground and set to the ground voltage, $r = 4$, p, $q = 1$ to r.

For example, the power system network in Fig. 3 can be represented by the following the bus impedance matrix is:

$$\boldsymbol{Z}_{\rm Bus}^{+} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.0086i & 0.0057i & 0 \\ 0 & 0.0057i & 0.0171i & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{9}$$

3.2 P–V bus modelling

If a DG system participates in reactive power control and maintains a terminal voltage within a set range, a DG bus should be modelled by a P–V bus, not a P–Q bus. To represent a bus to which a DG system is connected as a P–V bus, this study proposes the following positive-sequence voltage regulation method. Let a set voltage of a P–V bus be the positive-sequence voltage. Then, the change of the positive-sequence voltage caused by the reactive power injection of DG systems can be derived by (1), using the positive-sequence impedance matrix:

$$\Delta V_j^{+(k)} = \Delta I_{q,j}^{+(k)} |Z_{{\rm Bus},j}^{+}|, \tag{10}$$

where $\Delta V_j^{+(k)}$ is the change in the positive-sequence voltage of P–V bus j at iteration k, $\Delta I_{q,j}^{+(k)}$ is the change in the positive-sequence reactive current of P–V bus j at iteration k, $\left|Z_{{\rm Bus},j}^{+}\right|$ is the absolute value of the driving-point impedance of PV-bus j, which can be found in the positive-sequence impedance matrix determined by the four rules presented in the previous section, $j = 1,2,3,\ldots,M$ if M is the number of P–V buses.

Equation (10) reveals that the terminal voltage of a P–V bus can be determined by the reactive current injected by DG systems if the positive-sequence impedance matrix is known. However, although the impedance matrix can be found by the four rules presented in the previous section, since the reactive current injected by DG systems is based on initial nominal voltage, the following iteration is required:

(1) *Calculate the positive-sequence voltage mismatch*. The mismatch between the positive-sequence set value and the voltage calculated at the present iteration is determined by

$$\Delta V_j^{+(k)} = V_{{\rm set},j}^{+} - V_j^{+(k)}, \tag{11}$$

where $V_{{\rm set},j}^{+}$ is the positive-sequence set voltage of P–V bus j, $V_j^{+(k)}$ is the positive-sequence voltage of P–V bus j at iteration k.

(2) *Calculate the reactive current injected.* The reactive current to be injected is determined by the positive-sequence voltage mismatch:

$$|\Delta I_{q,j}^{+(k)}| = \frac{\Delta V_j^{+(k)}}{|Z_{{\rm Bus},j}^{+}|} = \frac{V_{{\rm set},j}^{+} - V_j^{+(k)}}{|Z_{{\rm Bus},j}^{+}|}, \tag{12}$$

$$\Delta \boldsymbol{I}_{abc,j}^{+(k)} = \Delta \boldsymbol{I}_{abc,j}^{+(k-1)} + \begin{bmatrix} |\Delta I_{q,j}^{+(k)}| \angle(\gamma + \delta_{V_{a,j}^{(k)}}) \\ |\Delta I_{q,j}^{+(k)}| \angle(\gamma + \delta_{V_{b,j}^{(k)}}) \\ |\Delta I_{q,j}^{+(k)}| \angle(\gamma + \delta_{V_{c,j}^{(k)}}) \end{bmatrix}, \tag{13}$$

$$\gamma = {\rm Sign}\left(\Delta V_j^{+(k)}\right) \times \frac{\pi}{2}, \tag{14}$$

where $\Delta \boldsymbol{I}_{abc,j}^{+(k)}$ is the reactive current to be injected in phases a, b, and c of P–V bus j at iteration k, Sign $(x) = +1$ if $x > 0$ and -1 for otherwise, and $\delta_{V_{a,j}^{(k)}}$ is the voltage angle of phase a of P–V bus j at iteration k.

(3) *Check the constraints and convergence.* If the reactive power determined by the previous step exceeds the feasible reactive power operating range of the DG inverter, the reactive power amount to be injected is set to the maximum feasible limit. Then, the proposed method iterates these steps until convergence.

4 Genetic algorithm

The strategic placement (e.g., their optimal location and capacity) of DG systems able to control reactive power, which can be modelled by the representation of a DG bus as a P–V bus, can be seen as an optimisation problem. To solve such an optimisation problem, this study proposes a GA combined with the proposed reactive power control method because of the broad size of search space. The GA, originates from the natural selection of randomly generated trait variations (e.g., crossover and mutation), approaches to an optimal solution, or the best offspring, with the following objective function.

4.1 Objective function of GA

The objective function of the proposed GA is to minimise variations of each bus voltage from the unity value and the installation costs of DG systems with each weighting factor:

$${\rm Cost}_{V,i} = \sum_{p \in \{\text{All phases}\}} \left|1.0 - \left|V_{i,p}^{(k)}\right|\right|, \tag{15}$$

$${\rm Cost}_{\$,i} = C_{\rm C} \times C_{{\rm DG},i}, \tag{16}$$

$${\rm Objective} = {\rm Minimise}\left(W_{\rm V}\frac{\sum_{i=1}^{M}{\rm Cost}_{V,i}}{C_{V,\max}} + W\frac{\sum_{i=1}^{N}{\rm Cost}_{\$,i}}{C_{\$,\max}}\right), \tag{17}$$

subject to

$$0.95 \le \left|V_{i,p}^{(k)}\right| \le 1.05 \text{ PU} \quad \text{for} \quad i = 1, \ldots, M \text{ and } p \in \{\text{allphases}\},$$
$$0 \le P_{\text{DG},i} \le P_{\text{DG},i,\max} \quad \text{for} \quad i = 1, \ldots, N,$$
$$Q_{\text{DG},i,\min} \le Q_{\text{DG},i} \le Q_{\text{DG},i,\max} \quad \text{for} \quad i = 1, \ldots, N,$$
$$\text{PF}_{\min} \le \text{PF}_{\text{DG},j}, \quad j = 1, \ldots, \text{ and } N,$$

where $V_{i,p}^{(k)}$is the voltage of phase p of bus i at iteration k in pu, C_C is the coefficient of DG installation costs in \$/W, $C_{DG,i}$ is the total capacity of DG system i in kW, W_V is the weighting factor of the voltage variation term in %, $C_{V,max}$ is the maximum variation in pu when the voltages of all buses show the allowable maximum variation (e.g., 5%) in the worst case, $W_\$$ is the weighting factor of the DG installation cost term in %, $C_{\$,max}$ is the maximum cost in \$ when DG systems are connected to all buses in the worst case, M is the number of all buses, N is the number of all DG buses, $P_{DG,i}$ is the active power of DG system i in kW, $Q_{DG,i}$ is the reactive power of DG system i in kVar, and $\text{PF}_{DG,i}$ is the leading or lagging power factor of DG bus i.

4.2 Implementation of the GA

(1) *Initialisation*. The proposed GA initialises population members, referred to as 'offspring' or 'solutions,' with uniform random numbers. Population member i, S_i, is defined by

$$S_i = \{[x_{i,j}\ y_{i,j}] \mid x_{i,j} \in \{\text{allbuses}\}, \quad C_{\text{Min}} \le y_{i,j} \le C_{\text{Max}}, \quad i = 1, \ldots, M, j = 1, \ldots, N_i\} \tag{18}$$

where $x_{i,j}$ is the location of DG system j of population member i, $y_{i,j}$ is the capacity of DG system j of population member i, $C_{\min}$ and $C_{\max}$ are the minimum and maximum capacities in kVA, respectively, N_i is the number of DG systems of population member i, and M is the number of population members.

(2) *Natural selection process*. This step determines the fitness of population members by calculating the objective function defined by (17). In this step, this study proposes a scaled roulette method. That is, it randomly throws darts on a roulette with scaled slots, counts the number of darts stuck on the roulette, and then reproduces population members according to the number of counted picks. The higher probability of the scaled roulette, the more offspring will be produced. To determine each slot size of a scaled roulette wheel, the probability that originates from a geometric progression is used. That is, the probability (P_i) of each slot size is

$$P_i = \frac{f_i}{\sum_{j=1}^{M} f_j} = \frac{p(1-p)^{r_i - 1}}{1 - (1-p)^M}, \tag{19}$$

$$\sum_{i=1}^{M} P_i = 1.0, \tag{20}$$

where P_i is the probability of each slot size of a scaled roulette wheel, p is the probability that selects the best solution (or population), r_i is the normalised geometric rank of population member i in natural number set (e.g., $1 \le r_i \le M$, in which 1 indicates the best population member, in other words, a population member with the lowest objective function value), and M is the number of population members.

(3) *Crossover of traits*. The proposed GA performs the following arithmetic crossover of traits between two population members by

$$S_i' = rS_i + (1-r)S_j, \tag{21}$$

$$S_j' = (1-r)S_i + rS_j, \tag{22}$$

where r is uniform(0,1), S_i and S_j are the population members i and j, respectively.

(4) *Mutation of traits*. The proposed GA experiences the random mutation of traits from one generation to another. In fact, it randomly produces a new trait (e.g., a new location and capacity) to appear in population members. Therefore, the convergence to local minimums can be avoided.

(5) *Convergence*. It iterates steps 2–4 until the proposed GA selects a single best population member, referred to as a 'solution'.

The detailed procedure of the proposed GA is presented in Fig. 4. The GA initially examines data related to the objective function, constraints, and parameters. Using (3)–(8), it determines the positive-sequence sensitivity impedance matrix of the test feeder. Then, it initialises the population members of the first generation with uniformly distributed random variables. Note that each population member includes the size and location of DG systems. Next, it determines the reactive power to be injected from the DG systems in each population member, based on the presented *P–V* bus modelling method, or using (10)–(14). Then, it calculates the power flow of the test feeder enhanced by the DG systems able to inject reactive power to maintain the voltage of a bus to which a DG system is connected within a set voltage range. It estimates the installation costs of DG systems and evaluates the objective function of population members of the first generation. The GA repeats the fitness calculation of population members, natural selection, crossover, and mutation processes in the next generation until convergence.

5 Case study

5.1 Voltage regulation example

This study initially implemented a power-flow calculation algorithm that uses the backward and forward sweep method presented in [24], the power-flow analysis results of which revealed the accurate results compared to the IEEE solutions published in [25]. Using the backward–forward sweep power-flow algorithm developed in [24], this study models the IEEE 37-bus test feeder in Fig. 5. Then, the positive-sequence impedance matrix of the test feeder is determined by (2)–(8). For example, Table 1 shows the power-flow results (e.g., line-to-line voltages) of two buses, bus 732 experiences an increase in voltage compared to 1.0 pu and bus 740 experiences a decrease in voltage. To verify the proposed method for representing a *P–V* bus, this study adds a 300-kVA DG system (e.g., DG 1 in Fig. 5) to bus 740 that injects the reactive power to the grid. Therefore, DG 1 can maintain the voltage of bus 740 to a set voltage of 1.0 pu. In fact, it functions as a capacitive load that injects optimal reactive power.

Fig. 6 indicates the convergence curve of positive-sequence voltage of bus 740, which converges from 0.993 pu (e.g., without controlling reactive power) to a set voltage of 1.0 pu. Note that Fig. 6 is plotted by the backward–forward sweep power-flow algorithm developed in [24]. Figs. 7 and 8 show the convergence curves of the magnitude and angle of the reactive current injected by DG 1. Before adding DG 1, since bus 740 consumes a power of 85 + j40 kVA, phase *a* of the bus experiences a decrease in phase voltage, or 0.993 pu. After reactive power control (e.g., after six iterations), the reactive current converges to 35.54∠89.96° A and the consumed power is changed to 82.92 − j255.51 kVA. In other words, to increase the terminal voltage to a set voltage of 1.0 pu, DG 1 injects the much more reactive power while functioning as a capacitive load. Note that a positive value in the power corresponds to the power consumed at the bus, a negative value corresponds to the power injected by the bus, and a positive sign in the angle of the reactive current follows the sign determined by (11).

For the scenario that decreases the terminal voltage, this study adds a 300-kVA DG system (e.g., DG 2 in Fig. 5) to bus 732 that absorbs the reactive power from the grid and decreases the voltage of bus 732 up to 1.0 pu. For example, before adding DG 2, bus 732 consumes a power of 41.95 + j21 kVA. However, to maintain a set voltage of 1.0 pu, the consumed reactive power is changed to 41.95 + j78.15 kVA. In other words, DG 2 functions as an inductive load by absorbing the much more reactive power. Note that the reactive power limit is not set in this verification, but the limit is

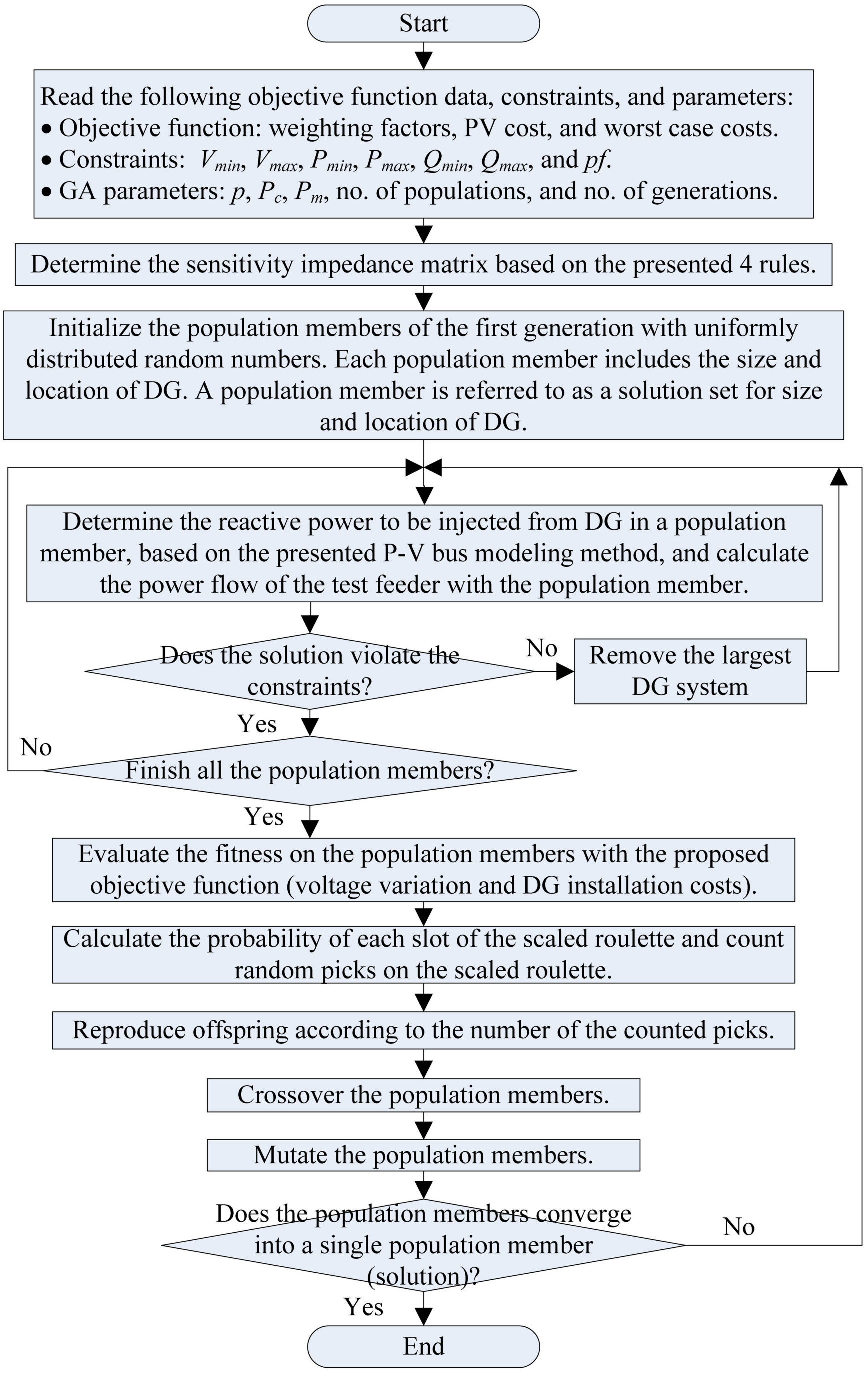

Fig. 4 *Flowchart of the proposed method*

set in the following case for practical application of the proposed method.

5.2 Optimal DG allocation

To verify the proposed hybrid method based on optimal reactive power control and GAs, the IEEE 37-bus test feeder in Fig. 5 is reused. To determine the optimal location and capacity of DG systems, this study assumes the following constraints:

i. A single DG system should have a capacity of 0–100% of the test feeder rating.
ii. All DG systems participate in Volt/Var control at a leading or lagging power factor of higher than 0.9 [26].
iii. If one of the bus voltages exceeds ±5% of the rated voltage, the solution is ignored.
iv. The PV system is used as an example of the DG systems at an installed cost of \$1.95/W–\$6/W [27, 28].
v. The weighting factors of the voltage variation and installation cost terms in (17) are 50 and 50%, respectively.

Table 2 summarises the parameters for the proposed GA. The following scaled roulette, arithmetic crossover, and uniform mutation parameters are determined by the trial and error optimisation method. The proposed GA is implemented in MATLAB. The power flow of the test feeder is calculated by OpenDSS, an open-source power distribution system analysis program.

To verify the GA, this study plots the standard deviation of objective functions of population members of each generation in Fig. 9. Note that a standard deviation of 0 means that all the population members has the same solution for finding optimal location and size of DG systems. In other words, Fig. 9 indicates the proposed method converges to a single optimal solution for finding optimal location and size of DG after 19 generations. Therefore, the proposed method successfully finds the optimal allocation of DG systems, while minimising the voltage variations

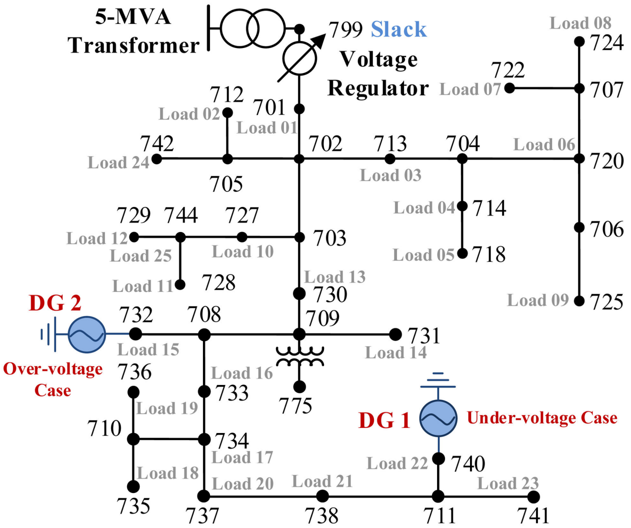

Fig. 5 *IEEE 37-bus test feeder that hosts a DG system*

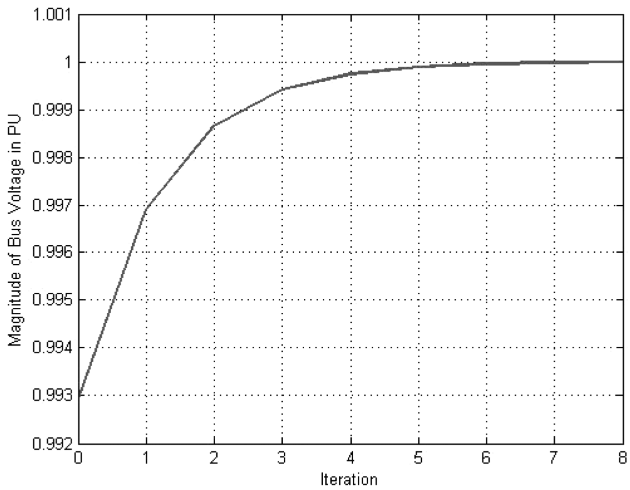

Fig. 6 *Convergence curve of positive-sequence voltage of bus 740*

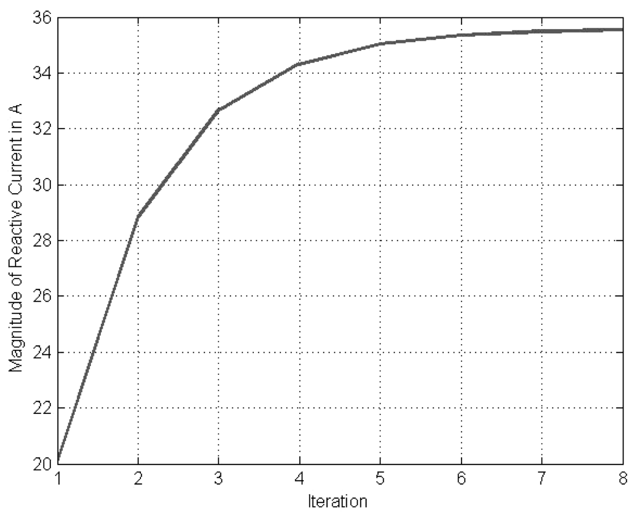

Fig. 7 *Magnitude of phase A of reactive current injected by DG 1*

and the installation costs, as shown in Table 3. The optimal capacity of DG systems is 25% of the feeder rating on buses 733, 734, and 738. To examine the effect of the optimal DG allocation on the voltage variations, this study plots the voltage profile along the bus in Fig. 10. The magnitudes of the voltages of all buses are sorted by the voltage magnitudes of the reference scenario (which is the scenario without DG systems) in descending order. Fig. 10 shows that the DG systems able to control reactive power provide the less variations to the case of without DG systems.

To verify the proposed GA on another test feeder, the IEEE 13-bus test feeder is used. The detailed descriptions and results are presented in the Appendix.

6 Conclusion

The objective of this study was to propose a GA that optimally allocates the locations and capacities of DG systems capable of generating real and reactive power upon the voltage regulation requirement. This study takes DG inverters with the capability of reactive power control, in other words, Volt/Var management and control, into account. For this purpose, this study has formulated the four rules that determine the positive-sequence impedance matrix of a power system network. In addition, using the impedance matrix, this study has presented a method for optimally controlling reactive power, in other words, representing a DG bus as a *P–V* bus. Then, this study has designed a GA that solves the strategic placement (e.g., their optimal location and capacity) of DG systems able to control reactive power. Finally, the hybrid method is applied to well-known test feeders, or the IEEE 13- and 37-bus test feeders.

From the three case studies, this study has not only verified the proposed reactive power control method but also successfully found the optimal location and capacity with an objective function that minimises the voltage variations and the installation costs. The proposed hybrid algorithm has an objective function that takes the installation costs of DG systems into account. Thus, the proposed method can be applied to find the maximum effect of other DG systems based on inverters such as wind farms, PV systems, microturbines, combined heat and power systems, and energy storage systems on the distribution system if they can control Volt/Var. Furthermore, the proposed reactive power control method can be applied for single- or two-phase DG systems. Next, the proposed objective function can be extended to take other functions such as loss, energy savings, and reliability into account. Thus, the proposed hybrid method can be useful for planning, designing, or upgrading DG-enhanced feeders.

This study, however, did not apply the proposed method to a sufficiently large distribution network with various types of DG systems. Furthermore, it did not apply load profile data that vary continuously according to customer demand for the proposed GA. However, the proposed hybrid method could be extended for such cases by modelling a sufficiently large distribution network and adding other types of DG systems to the networks. Therefore, the future work should entail the more accurate analysis of DG systems.

Table 1 Voltages of two buses of the IEEE 37-node test feeder

Bus	IEEE solution published in [25]		Proposed algorithm		IEEE solution published in [25]		Proposed algorithm	
	Voltage of phases *A* and *B* (line-to-line)				Voltage of phases *B* and *C* (line-to-line)			
	Magnitude, pu	Angle, deg.	Magnitude, pu	Angle, deg.	Magnitude, pu	Angle, deg.	Magnitude, pu	Angle, deg.
732	1.0086	−0.07	1.0085	−0.07	1.0001	−120.74	1.0001	−120.74
740	0.9981	0.07	0.9980	0.07	0.9961	−120.75	0.9961	−120.75

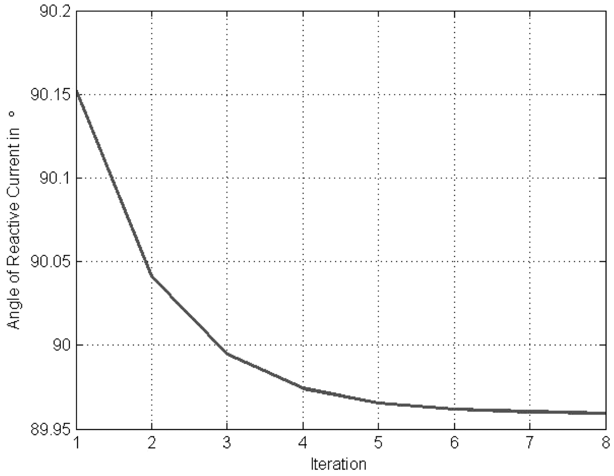

Fig. 8 *Angle of phase A of reactive current injected by DG 1 (on bus 740)*

Table 2 Parameters of the proposed GA

Operation	Method	Parameter
scaled roulette	probability for selecting the best population	$p = 0.001$
crossover	arithmetic crossover per population	$P_c = 1.0$
mutation	uniform mutation	$P_m = 0.1$
experiment data	total number of populations	5000
	total number of generations	100

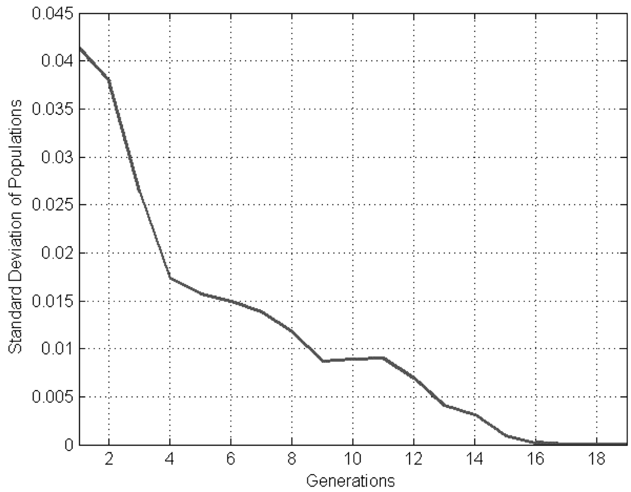

Fig. 9 *Convergence characteristic of population members of the GA (the standard deviation of the objective functions of population members of each generation)*

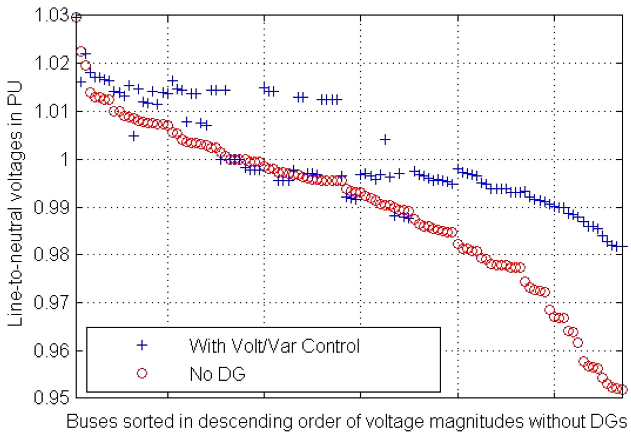

Fig. 10 *Voltage profile along the feeder*

Table 3 Optimal allocation of DG systems on the IEEE 37-bus test feeder

Scenario	Total power (slack bus) kW + kVar, pf	Bus	Capacity kVA	Objective function pu
no DG	2584.42 + j1537.31 (0.86)	—	0	0.1486
optimal Volt/Var	1782.28 + j1393.73 (0.79)	734	663.41 (22%)	0.0956
		738	60.31 (2%)	
		733	30.15 (1%)	

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9 Appendix

To verify the proposed hybrid method based on optimal reactive power control and GAs, the IEEE 13-bus test feeder in Fig. 11 is modelled [25]. The test feeder includes two three-phase transformers, a voltage regulator, capacitor banks, and several load types. Since the proposed GA optimally allocates three-phase DG systems, single-phase lines of the test feeder have been changed to three-phase lines. The constraints and the parameters for the proposed GA are the same as the IEEE 37-bus test feeder. The proposed method successfully finds the optimal allocation of DG systems, while minimising the voltage variations and the installation costs, as shown in Table 4. Fig. 12 shows that the DG systems able to control reactive power provide the less variations compared to the case of without DG systems.

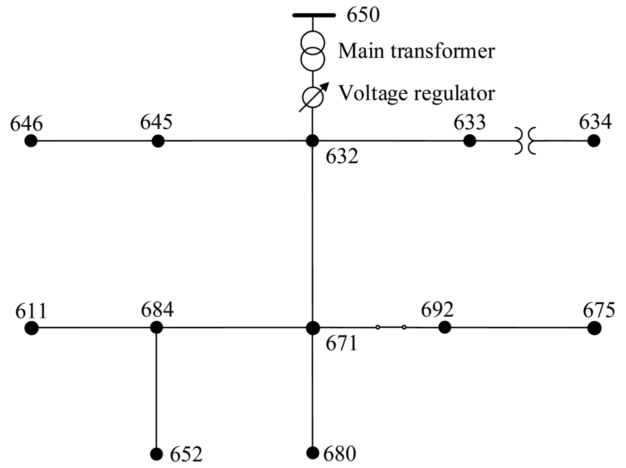

Fig. 11 *IEEE 13-bus test feeder*

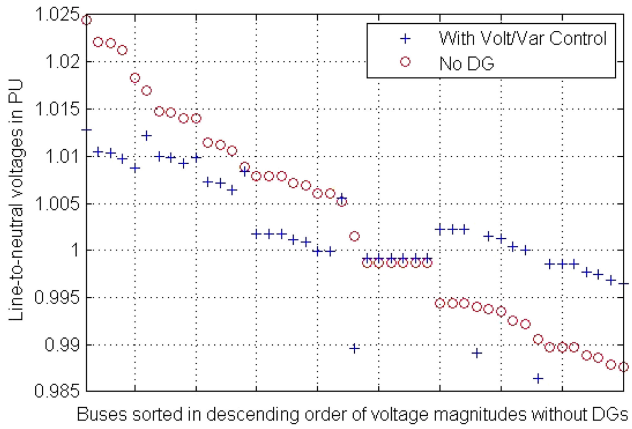

Fig. 12 *Voltage profile along the feeder*

Table 4 Optimal allocation of DG systems on the IEEE 13-bus test feeder

Scenario	Total power (slack bus) kW + kVar, pf	Bus	Capacity kVA	Objective function pu
no DG	3566.67 + j1703.42 (0.90)	—	0	0.1075
optimal Volt/Var	1083.51 + j1468.89 (0.59)	692	2411.06 (61%)	0.0762