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A review of optimal power flow studies applied to smart grids and microgrids

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ABSTRACT

The term smart grid refers to a modernization of the electrical network consisting in the integration of various technologies such as dispersed generation, dispatchable loads, communication systems and storage devices which operates in grid-connected and islanded modes. As a result, traditional optimization techniques in new power systems have been seriously influenced during the last decade. One of the most important technical and economical tools in this regard is the Optimal Power Flow (OPF). As a fundamental optimization tool in the operation and planning fields, OPF has an undeniable role in the power system. This paper reviews and compares the OPF approaches mainly related to smart distribution grids. In this work, the main OPF approaches are compared in terms of their objective functions, constraints, and methodologies. Furthermore, computational performances, case study networks and the publication date of these methods are reported. Finally, some basic challenges arising from the new OPF methodologies in smart grids are addressed.

1. Introduction

Bulk power generators are directly connected to the transmission system in a complicated manner, while distribution networks with their simple topology have enabled many consumers to easily connect to them. Generation Companies (GenCos) search for optimum utilization of the available generation utilities using proper load distribution. Moreover, distribution companies and consumers look for lower prices with higher supply reliability. The Transmission Companies (TransCos), however, tend to maintain standard operating conditions in terms of low transmission line congestion, high value of minimum bus voltage and low level of transmission loss, which are considered in Optimal Power Flow (OPF) problems of transmission systems [1] .

In the current distribution systems, an operator has no real-time monitoring capability related to the network and consumers. In other words, the operator contains no feedback in this case. However, the purpose of a power system is to deliver the power to consumers based on their momentous and changing demands. Fig. 1 shows the evolution of power systems.

In current power systems, electrical losses are significant in the distribution of electrical energy, especially at lower voltage levels. Loss reduction can be achieved through the appropriate control of Distributed Generation (DG) resources in the distribution systems [2], or more generally, through the control of dispatchable resources (DG, load, storage), which can be effectively assessed by using tools such as OPF-like software. Connecting the new power sources to the current distribution systems leads to some technical and economic challenges [3]. A possible vision for the solution of modern distribution systems consists in the creation of more or less independent cells which can interact in an internet-like structure. Microgrids can constitute the single element of this cellular structure in a large interconnected power system or be the natural answer to power supply in remote areas. In this regard, considering multi-microgrids as a system of microgrids would lead to different economic effects on the future smart grids [4]. Also, preserving privacy of OPF models in this system is an important aspect which is discussed in [5].

The term microgrid refers to a set of loads, power resources, and energy storage devices [6] in the lower voltage levels which can be operated as a single controllable load or a generator unit and provides heat and power for a designated area. This concept introduces a new paradigm in order to exploit DGs in the distribution level. Thus, a microgrid has high control capability and flexibility in terms of system reliability and power quality [7,8].

Generally, the operational modes of microgrids can be classified as islanded mode or grid-connected. In the islanded mode, a microgrid must be stable while it is disconnected from the main grid. Furthermore, the role of DERs is critical [9]. In other words, in the grid-connected mode, the public grid operates as a supporter which

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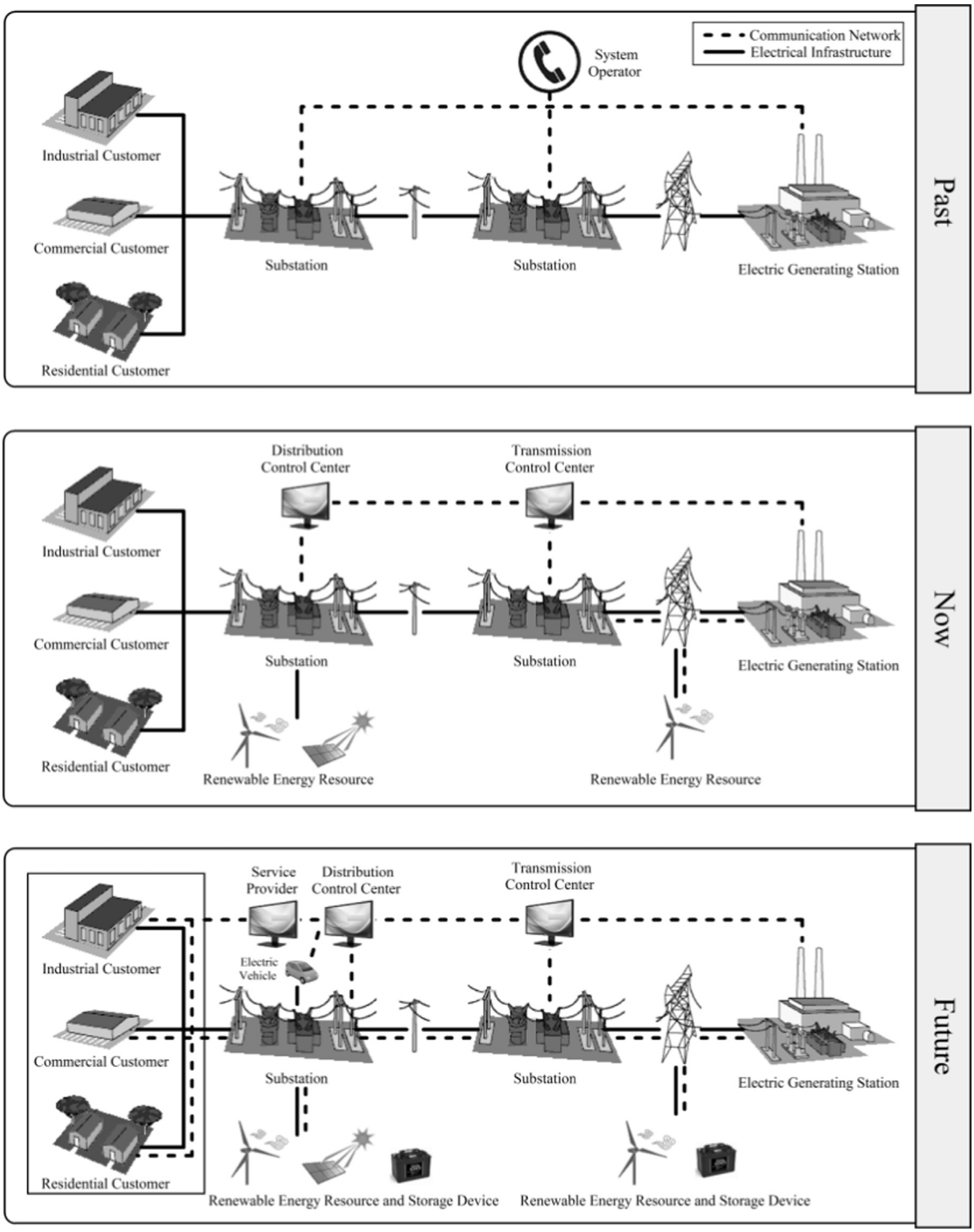

Fig. 1. The evolution of power systems.

microgrid can receive/send electrical power from/to it. Microsource Controller (MC) and Central Controller (CC) [10] manage and control the grids at different modes. Accordingly, changing from the grid-connected mode to islanded mode can be performed in two ways: full isolation of the national grid and isolation of each individual feeder. A typical configuration of a microgrid is shown in Fig. 2. In this system, the main purpose of MC is direct control of power flow and voltage level of connected loads to the grid at any conditions. Direct action indicates that MC can be operated separately from CC if required. Further, MC can participate in Economic Dispatch (ED), load management, and Demand-Side Management (DSM) through controlling the energy storage sources. In this context, CC applies control commands through MC [11]. In this regards, one of the most important commands is the optimal operation of microgrid. Since, one of the main objectives among the system operators is to minimize the microgrid cost, so, they should be able to consider and compare the energy cost of the main utility and the generation cost of the microgrid units while satisfying all constraints in the grid-connected mode.

The implementation of Advanced Metering Infrastructure (AMI) [12], real-time information systems, improved communication capabilities [13], greater number of sensors, and improved infrastructure for control systems transform the conventional distribution system into

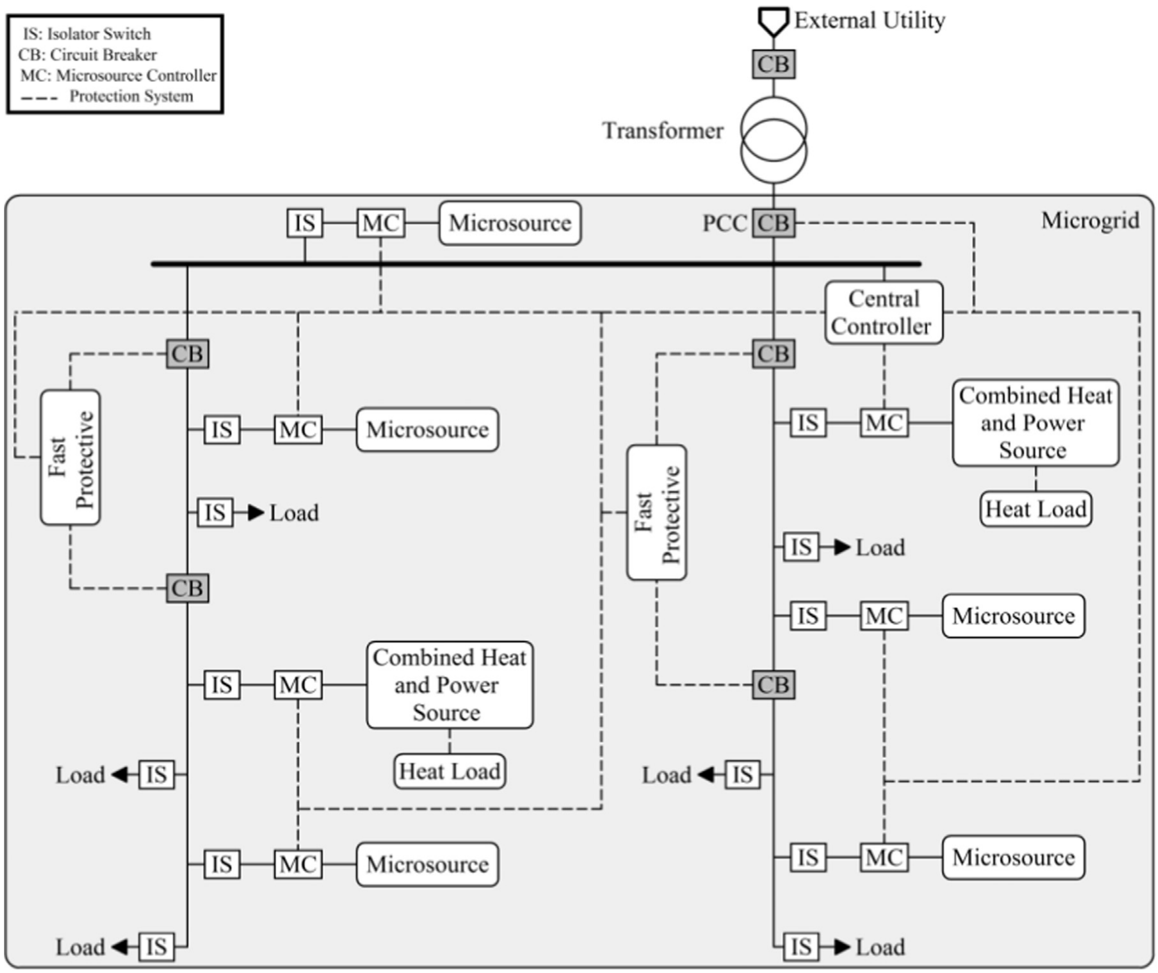

Fig. 2. A typical microgrid.

a "smart grid" [14]. It brings flexibility to distribution system operations via centralized control of distribution components [15].

The term smart grid is generally used to define many different features in modern power systems oriented to provide an affordable, reliable and sustainable supply of electricity [16]. The majority of industrial interests and research activities basically emphasize on distribution grid and DSM. Nevertheless, smart transmission grids have also been addressed as an industrial grid in order to ensure the same benefits promised by the smart grid paradigm [14]. In this vision, smart transmission grids are regarded as integrated systems consisting in the interaction of smart components such as smart control centers, smart transmission networks, and smart substations.

The term smart grid refers to a modernization of the electrical network consisting in the integration of various technologies such as dispersed generation, dispatchable loads, communication systems and storage devices which is used to efficiently deliver sustainable, economic and secure electricity. The smart grid concept is naturally associated with the integration of significant levels of Distributed Energy Resource (DER), including DGs, Demand Response (DR) [17,18], energy storage devices, and other energy sources into the electric grid. The smart grid scenario uses two-way flow of electricity and information between power network and consumers in order to create an automated and widely distributed energy delivery network. The most important features of smart grids are as following: increasing the penetration of renewable resources; increasing the participation of consumers in the network operation; decreasing the transmission and distribution losses, and the energy cost for customers, subsequently; decreasing the electrical power consumption, and the emission of fossil fuels, concurrently; enabling consumers and electrical companies to control the demand.

Smart grid or microgrid drivers are great in number, and linked to various factors such as the necessity of controlling dispersed generation, ensuring power supply in remote areas, improving demand-side management, increasing energy efficiency, and creating self-healing electrical networks. The main objectives of smart grids are to increase supply reliability and improve power system security against a series of contingencies or malicious attacks. Another important driver towards smart grids is the higher penetration of renewable energy resources in the distribution system level which has increased substantially control problems in these systems [19,20].

Smart microgrids have undeniable role in the evolution of the smart grids [8]. In this point of view, the smart grid can be divided into a system of integrated smart microgrids [21–23]. In fact, the smart microgrid can be considered and exploited as the main building block of the smart grid [24]. Therefore, smart grids and smart microgrids have common aspects such as interconnection with utility, interruptible loads, the use of different sources, employment of energy storage devices, optimal control based on customer requirements, optimal operation, the use of communications bandwidth for fast applications and GPS, time-tagging, and cyber security [25]. From these considerations, it follows that facing the problems of smart grids, and in particular of smart microgrids, needs some revisions in traditional power system studies which should be adopted in a similar perspective.

1.1. Motivation

One of the most important technical and economic tools in power systems is OPF. Using this piece of software, control variables related to the power system operation, at a specific time, are determined in order to achieve a particular objective and guarantee technical feasibility of the steady-state control actions. The use of OPF in smart grids and microgrids is regarded as a new development in power system studies. Hence, a coherent classification of these approaches needed at this stage of technology maturity. Moreover, no comprehensive research has been so far conducted on smart grids.

1.2. Scope

This paper reviews and compares the OPF approaches in smart grids from different perspectives. As it is reported, the main approaches are compared in terms of Objective Functions (OFs), constraints, algorithms, as well as computational performances. Moreover, the selected published papers are classified from different aspects such as system type, operational state, network topology, control strategy, solution algorithm, programing and realistic descriptions. Also, Energy Management System (EMS), DSM, energy storage devices, Phasor Measurement Units (PMUs), and multi-carrier energy systems as five key strategies/devices are introduced in this research. In fact, some basic issues of smart grids/microgrids such as OPF have been affected seriously by these strategies/devices. Furthermore, case study networks, date of publication, and main challenges arising from the new and complex OPF methodologies are investigated.

2. Definition of OPF: variables, constraints and objective functions

2.1. Original OPF

Firstly, OPF was introduced by Carpentier in 1962 [26]. Generally, the OPF is a nonlinear and non-convex problem including an OF which must be optimized (maximized or minimized), a set of equality and inequality constraints which must be satisfied, and a problem solving method [27,28]. In other words, OPF optimizes a given OF controlling power flow within an electrical system without violating power flow constraints or operational limits [29,30]. In fact, it determines the optimal operation state for the system. Unlike the conventional power flow, OPF works with an under-constrained network [31]. Also, it can provide a useful support to the operator to overcome many difficulties in the planning, operation and control of power networks [32]. A variety of extended OPF versions has been reported so far. Some of them are as follows:

- Static OPF: This type of problem optimizes OF under various constraints at a certain time of interest. In other words, it can only handle a single load level at a particular time [33].
- Dynamic OPF: This type is an extended version of the static OPF and determines the optimal operating point over a time horizon. In fact, it covers multiple time periods [34,35].
- Transient stability-constrained OPF: This problem considers static and dynamic constraints of the power network during the optimization process simultaneously [36]. Under this condition, the system can withstand severe contingencies [37].
- Security-constrained OPF: This is another extended version of the OPF which involves constraints arising from the operation of the system under a set of postulated contingencies [38–40].
- Deterministic OPF: This widely used type does not consider stochastic factors.
- Stochastic OPF: This type considers uncertainties in power system parameters [41–43]. In fact, it regards the uncertainty as a part of the constraints and objective models. Hence, uncertain factors affect the optimization process as well as the final OPF results [44].
- Probabilistic OPF: It estimates the probability distribution functions of dependent variables based on the probability distributions of loads and other uncertain factors through using Monte-Carlo Simulation [45], Cumulant method [46], Point Estimate Method (PEM) [47], customized Gaussian mixture model [48], and etc. In this type of OPF, the uncertain factors do not affect the final OPF results [44].
- AC OPF: This is associated with the AC power networks and is based on the natural power flow characteristics of the system [49]. Consequently, the results obtained by this type of OPF are more accurate [50,51].
- DC OPF: This type does not consider the reactive power and transmission losses [49].
- Mixed AC/DC OPF: it is associated with OPF in both AC and DC grids [52,53].

These types cover only some extensions of OPF. Note that, some approaches can be combined with others to make new versions of OPF (for a specific type of OPF problem), e.g. probabilistic transient stability-constrained OPF [54], dynamic stochastic OPF [55], and etc. The other relevant types to smart grids and microgrids, will be introduced in the following sections.

Note that, several papers have reviewed the OPF studies, e.g. papers [56–59] which were published in 1990s. Later on, a lot of research interest was focused on this topic. For instance, authors in [60,61] surveyed several optimization algorithms applied to the OPF problems. Another instance is [62] in which the common deterministic OPF approaches are summarized and categorized. Refs. [29,63] reviewed the OPF studies in terms of formulations and deterministic methods [29] as well as non-deterministic and hybrid models [63]. Recently, a critical review of recent advances and further developments needed in AC OPF has been reported in [64].

2.2. Solution categories

Generally the feasible solution set can be achieved in one of three categories as followings (Fig. 3):

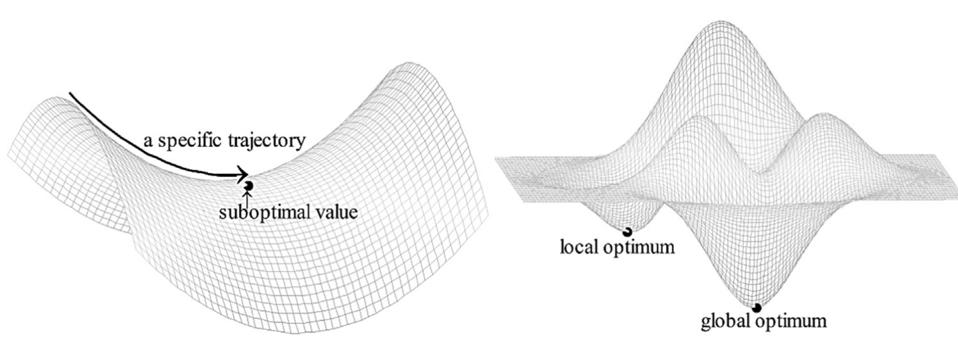

Fig. 3. Optimal Solution Concepts.

- Global optimum: It is the best solution for OF among all other feasible solutions.
- Local optimum: Nonlinear and non-convex problems have multiple optimum points. In this case, no better feasible solution can be found in the immediate neighborhood of the obtained solution.
- Suboptimal: This solution is no better than the global optimum. It can be illustrated as a saddle point. In this condition, the obtained OF value is optimum for a specific trajectory. However, it might be maximum (or minimum) for another trajectory.

2.3. Variables

Generally, there are two relevant variables in an optimization problem: independent (control or decision) variables and dependent (state) ones. Firstly, the optimal value must be determined for control variables and then, based on it, state variables should be calculated. In the OPF problem, control variables may include active power generation of all generator buses except slack bus, voltage of all generator buses, tap setting of all transformers, reactive power injection of shunt capacitor banks, etc. Moreover, state variables may also include active power output of the slack bus, load bus voltages, reactive power generation of generators, transmission line loadings, etc. It should be noted that the number of control variables determines the solution space. In fact, a problem with n-control variables, results in an n-dimensional solution space.

2.4. Constraints

Constraints are normally classified into two types, equality and inequality constraints. These conditions introduce the feasible region of the OPF problem. In fact, any solution to the problem must be within this region to satisfy all constraints.

Generally, power flow equations are included in the equality constraints. Ref. [65] avoids the explicit use of power flow equations by linearizing the network equations and using an approximation of loads as impedance. Moreover, [66] proposed an angular reference to represent a phase shift–e.g. 120 degrees–of a three-phase busbar (at least one node of the electrical system must be chosen) as a system reference. In general, inequality constraints represent the operational limits. Table 1 illustrates a taxonomy of the inequality constraints applied in [1,4,11,20,65–87]. The main inequality constraints are listed below:

- Active power constraints (generation or power supply to the load): these constraints reflect the physical restrictions (in this case, generating units and demand loads) related to stable operation in the power system.
- Reactive power constraints: these constraints limit the injected reactive power into the power system as lower and upper limits.
- Voltage constraints: in order to ensure system security, each bus (generation and load buses) in the grid is restricted by lower and upper limits.
- Current constraints (also known as thermal rate or maximum capability): due to the stability considerations, the power flow over a line must not exceed a specified maximum limit.
- Voltage angle constraints: these constraints represent operating limits on voltage angles.
- Tap position constraints: tap changing of the transformer Load Tap Changer (LTC) or Step Voltage Regulator (SVR) is a discrete variable which can be adjusted between maximum and minimum tap ratios through a certain step size.
- Capacitor bank switching constraints: capacitor banks are used as reactive power sources in power systems and their capacity constraints are determined by lower and upper limits. These constraints are from discrete type.
- Curtailment constraints: the demand elasticity of price has an important role in the OPF problem [88]. In this context, load curtailment can be modeled by specifying a demand curve and accommodated in the optimization process by the assistance of smart metering in order to achieve distinction in the operating condition [73]. Therefore, similar to generation, load demand is also scheduled e.g. based on the DR scenario– and the system operator can curtail a part of demand load in some conditions (such as peak load) [80] –e.g. according to DR curve of consumers [73]– for security limit violation. So, the demand is price dependent [89]; however, in order to further rationalize the curtailment, the generation surplus of GenCos can also consider the load as price dependent [1]. For instance, customers can modify their behavior in the power market and make ISO adjust curtailment depending on consumers' willingness to pay.
- Reserve constraints [90]: in the security rules, reserve structure includes: 1) amount of reserve for each control area or zone, 2) period of reserve power as a percentage of load being served, 3) appropriate geographic distribution to harness the reserve [91–93], and 4) reserve contract to participate in the reserve markets [92,93]. In this context, the time related constraints such as the ramp rate of each unit and reserve contract are known as inter-temporal constraints. As a result, an hourly operational decision may affect operational decisions during next hours and therefore, makes the OPF problem dynamic.
- Flowing AC power to DC grid and vice versa: This reflects constraints on power flows between hybrid AC/DC microgrids [77].
- Other inequality constraints: These constraints are related to various operational constraints associated with devices such as battery [4,94–96], fuel cell [78,97], conventional DG [79], the purchased and sold powers [4,96], photovoltaic shedding [81], and the like.

In the conventional distribution network operation, transformer tap changers and switched capacitors are the main controllable devices. Therefore, this operation can be treated as a volt/var control problem. Controlling these devices is based on local measurements; however, measurement and wide-area controls are evolving concepts [70]. Smart grids brings flexibility to distribution network operations through centralized control of distribution components such as LTCs, switched capacitors, and switches [15].

In this context, coordination of discrete variables (such as LTC and SVR) with the DG control is significant. The system voltage can be affected by DGs when it operates in constant power factor mode. The line drop compensation is installed on a SVR to calculate line voltage drop based on the active power flow from substations to loads (forward mode) or vice versa (reverse mode). So, it regulates the SVR output voltage to maintain proper voltage at the load location. The DG can affect the proper operation of the voltage regulator, if it is immediately located downstream of a SVR. This situation occurs when the feeder is heavily loaded and a significant fraction of the load is generated by DG. In this condition, the voltage regulator has relatively low load, causing SVR to regulate its voltage level, lower than required to maintain adequate voltage at the end of the feeder, while the line voltage drop from DG to the load center still reflects heavy loading. In this condition, the regulator output voltage is not increased, because of the low loading observed in SVR. As a result, low voltage conditions occur at the load location. In addition, when a DG is located downstream of SVR and, for example, at the end of the feeder, the SVR operation can be affected while DG is connected. If DG generates the active power less than the feeder load located downstream of the voltage regulator, SVR will be in forward mode and thus, regulates voltage level on the DG side. However, if the active power generated by DG exceeds the load demand between SVR and DG, the regulator will be in reverse mode, which regulates the voltage on the substation side. If substation side voltage (source side voltage) is greater than the SVR set-point voltage, SVR will tap down in an attempt to lower the voltage. Since the source voltage is fixed, the net effect is to increase the voltage on DG side. This sequence

Table 1
Taxonomy of the Reviewed Inequality Constraints.

Reference Number	Active Power(Generation Power (Generation or Supplied to the Load)–SC[q]	Reactive Power–SC	Voltage–SC	Current (or Thermal Rate or Maximum Capability or Transmission Constraint)–SC	Voltage Angle–SC	Tap Position and Capacitor Bank Switching–SC	Curtailment/Load Adjustment Factor–SC	Reserve–TRC[q]
[1]	✓	✓	✓	✓			✓	
[20]	✓	✓	✓					
[65]	✓	✓	✓	✓				
[66]	✓	✓	✓	✓	✓			✓
[67]	✓	✓	✓	✓	✓			
[68]	✓	✓						
[69]	✓			✓				
[70]	✓	✓	✓			✓		
[71]	✓	✓	✓	✓		✓		
[72]			✓	✓			✓	
[73]	✓	✓	✓				✓	
[74][a]	✓	✓	✓	✓	✓			
[75][b]	✓			✓		✓		
[76]	✓		✓	✓				
[77]	✓			✓[c]				
[78]	✓[d]							✓[e]
[79]	✓[f]	✓	✓				✓	
[4]	✓[g]							
[96]	✓[h]		✓	✓				
[11]	✓	✓	✓	✓				
[102]	✓[i]							
[80][j]	✓	✓	✓			✓		
[81][k]	✓						✓	
[82][l]	✓						✓	
[83][m]	✓							
[84][n]	✓							
[85][o]	✓							
[86][p]	✓			✓				
[87][q]	✓	✓	✓	✓				

[a] In [74], there are several inequality constraints such as limitations on stored power, charge process, and etc.
[b] In [75], there is limitation on the droop regulator of droop bus generator.
[c] In [77], there are some limitations on the flowing of AC power to DC network and vice versa.
[d] This constraint covers a wide range of limitations in [78].
[e] Fuel cell has been considered as emergency reserve due to its expensive cost.
[f] The stored energy in the battery is considered, too.
[g] In [4], this limitation covers the purchased and sold powers by microgrids.
[h] In [96], this limitation covers the purchased and sold powers by microgrid, battery, and etc.
[i] See [102].
[j] There are different inequality constraints on DG, controllable loads, etc.
[k] This reference considered many constraints such as grid time-of-use tariff, grid access limits, storage capacity and life cycle, load shedding and photovoltaic shedding together.
[l] This reference considered many constraints like [81].
[m] See [83].
[n] See [84].
[o] See [85].
[p] Different constraints have been modeled. See [86].
[q] SC and TRC denote the Security Constraint and Time Related one, respectively.

results in an over voltage on DG side of SVR, which is unacceptable for the DG connected system operation [98].

Another coordination between DG and on-load tap changers is addressed in [99], which allows a higher penetration of the DGs. In this regards, the coordination between DG and switched capacitors is investigated in [100]. By using this strategy, the voltage level increased by DG does not lead to any overvoltage condition when the capacitors are switched on. Coordination between DG and other traditional voltage and reactive power control equipment is proposed in [101].

All of the above mentioned constraints can be seriously affected by the choice of OPF methodologies which is discussed in the following sections.

2.5. Objective function

2.5.1. Definition

One feasible solution should be selected as a desirable solution through an OF definition. In smart grids and microgrids, different OFs are considered in OPF. Some parts of OFs are as follows:

- Active power generation cost
- Reactive power generation cost
- Power supplied to the grid from an external utility
- Active power losses
- Carbon emission
- Load curtailment
- Tap position and capacitor bank switching
- Social welfare
- Reserve cost
- Load adjustment.

Classification of OFs are summarized in Table 2 as they are reported in [1,4,11,20,65–87]. As pointed out in [102], energy saving (utilizing renewable energy sources [103]) is used to reduce carbon emission [20]. Therefore, in [20], the power generation of all available renewable resources is considered as an independent variable, fixed on the basis of the actual availability, and moreover, the carbon emission does not contribute to the OF. Consequently, the fuel cost of nonrenewable energy sources is used as the OF. In [69], the active power load

Table 2
Taxonomy of the Objective Function Functions.

Reference Number	Active Power Generation Cost	Reactive Power Generation Cost	Power Supplied from an External Utility to the Grid	Active Power Losses	Carbon Emission	Load Curtailment/ Adjustment	Tap Position and Capacitor Bank Switching	Social Welfare	Reserve Cost	Type of Objective Function
[1]	✓			✓		✓		✓[a]		Single
[20]	✓				✓[b]					Single
[65]	✓		✓							Multi
[66]	✓	✓							✓	Multi
[67]	✓	✓	✓[c]	✓						Multi
[68]	✓		✓	✓						Multi
[69]		✓	✓	✓		✓				Multi
[70]			✓				✓			Multi
[71]	✓		✓	✓						Multi
[72]						✓				Single
[73]	✓					✓				Multi
[74]	✓		✓			✓[d]				Multi
[75]				✓						Single
[76]				✓						Single
[77]	✓[e]			✓						Single
[78]	✓[f]									Single
[79]	✓[g]			✓		✓				Multi
[4]	✓[h]		✓							Single
[96]	✓[i]		✓		✓					Multi
[11][j]	✓									Multi
[102]	✓[k]									Single
[80]				✓						Single
[81][l]	✓		✓			✓				Single
[82][m]	✓		✓			✓				Single
[83]	✓		✓							Single
[84][n]	✓		✓							Single
[85][o]	✓		✓							Single
[86]	✓[p]									Single
[87]	✓[q]									Single

[a] In [1], Social Welfare is the objective function.
[b] In [20], the Carbon Emission is modeled as Active Power Generation Cost (unique objective function).
[c] In [67], the Power Supplied from an External Utility to the Grid is modeled as Active Power Generation Cost.
[d] In [74] , this term is related to generation curtailment power. Also, the costs associated with not-supplied demand, battery, and voltage deviations have been investigated.
[e] In [77] , this term also related to battery, wind power, flowing of AC power to DC network and vice versa, and considering an integer variable associated to the status of diesel generator.
[f] For more details see [78].
[g] This reference has modeled battery, too.
[h] In [4], operational and maintenance costs are considered, too.
[i] In [96], operational and maintenance and battery costs are modeled, too.
[j] In this reference, total voltage variation and voltage stability index have been mentioned in the objective function.
[k] This reference minimized the global energy cost. See [102].
[l] This reference considered a DC microgrid. The storage energy and photovoltaic shedding costs have been modeled, too.
[m] This reference considered a DC microgrid. The storage energy and photovoltaic shedding costs have been modeled, too.
[n] See [84].
[o] See [85].
[p] Costs of DR, wind spillage, and etc. have been applied.
[q] See [87].

controlling technique is handled applying special discounted tariffs. The active loads are re-dispatched in [104,105] according to a load curtailment scheme. Particularly, in [105], load curtailment is applied together with the active and reactive resources dispatching. In [106], active and reactive resources are dispatched together with discrete variables such as switching of circuit breakers and disconnectors by applying simulated annealing and OPF-like techniques. Another strategy to control dynamic operation in the smart microgrids is to apply large storage devices as an additional DER [65]. Levron *et al.* in [68] used storage devices as an auxiliary P–V source when the voltage magnitude of the storage devices is specified. Ref. [1] introduced the "willingness to pay" of consumers to regulate curtailment by the Independent System Operator (ISO). Ref. [73] employed the virtual generator concept to model load curtailment and implement Direct Load Control (DLC) program through auto-DR infrastructure. In some conditions, such as the peak load, ISO can curtail a part of the load based on the demand response curve of a consumer where the process is price dependent [89,107]; as a consequence, GenCos suffer from revenue loss, because, this process does not take them into account as a main concern.

2.5.2. Mathematical formulation

The OF is differently defined in relevant references, some of which are selected and summarized in the following paragraphs.

- **Total active generation cost**

Ref. [20] used the total active generation cost as the OF:

$$O.\,F. \quad = \sum_{i \in G} \alpha_i P_{gi}^2 + \beta_i P_{gi} + \gamma_i \tag{1}$$

where $O.\,F.$ denotes the objective function; P_g is the active power generation; G denotes the total number of nonrenewable energy generation buses (with controllable active power); and α_i, β_i, and γ_i are the cost coefficients.

The cost function in [65] is as following:

$$O.\ F.=\sum_{i\in N_{CC}\cup N_{EI}} f_i\,(P_{gi}) \tag{2}$$

where f_i can be defined as:

1) Linear function

$f_i\,(P_{gi})=\beta_i P_{gi}$

2) Quadratic function

$f_i\,(P_{gi})=\alpha_i P_{gi}^2+\beta_i P_{gi}$

3) Convex piecewise linear function

$$P_{gi}\in(P_{pice,i,k-1},\quad P_{pice,i,k}),\quad k=1,\quad 2,\quad \ldots,\quad K_n$$

and where N_{CC} is a set of Points of Common Couplings (PCCs); K_n represents the number of slack variables; N_{EI} is the set of Power Electronic Interfaces (PEIs). The cost function of PEI ith has a is as follows:

$f_i\,(P_{gi})=\beta_{i,k}P_{gi}$

In the above definition of f_i, the linear form perfectly includes a wide range of functions such as line loss minimization by setting β_i=1 for all PCCs and PEIs, or PCC cost minimization by setting β_i=1 for PCCs and β_i=0 for PEIs, or other linear functions. Moreover, quadratic function is equal to (1) in (2) when γ_i is equal to 0 in (1). Furthermore, an accurate cost function may require a piecewise polynomial form (mostly, linear or quadratic). Generally, the piecewise linear is used in linear programming, while most nonlinear programming techniques use a quadratic form.

- **Considering not supplied demand**

A similar OF considering battery, not supplied demand, and other resources for the next day has been investigated in [74]. Furthermore, another function is as follows:

$$O.\ F.\ =\sum_{t=1}^{Period}\left[\left(\sum_{\text{for all bus}}|V_{\text{ref}}^t-V^t|\right)+\text{Penalty}\times(Cost_{\text{NSD}}^t+Cost_{\text{GCP}}^t)\right] \tag{3}$$

where V_{ref}^t denotes the voltage reference of slack bus at period t; Penalty means the penalization factor for costs of non-supplied demand ($Cost_{\text{NSD}}^t$) and generation curtailment power ($Cost_{\text{GCP}}^t$) at period t.

- **Considering load curtailment costs**

Ref. [74] optimized a multi-objective problem constructed with two mentioned cost functions through using the weighted sum method. In [73], the load curtailment cost has been added to (1) based on the virtual generators in which each virtual unit supplies the curtailed loads. Using this method, implementing the DLC program and evaluating the associated costs would be possible. In [11], the voltage stability index, is considered in addition to the total voltage variation and fuel cost.

Bruno *et al.* in [69] proposed a load curtailment strategy to reduce the unbalanced overload on the HV/MV interface line (HV interconnecting line) as:

$$O.\ F.=\alpha_0\sum_{i\in CL}\left(\frac{P_{li}-P_{li}^0}{P_{li}^0}\right)^2 \tag{4}$$

where CL is the set of curtailable loads; P_l denotes the active power supplied to the load; P_l^0 is the initial active power demand; and α_0 represents the scalar weight of the OF.

- **Considering active power losses**

Araujo *et al.* in [67] used the total active and reactive power generation costs as well as active power losses as two separate OFs:

$$O.\ F.=\sum_{i\in G}[\alpha_i P_{gi}^2+\beta_i P_{gi}+\gamma_i]+\frac{1}{2}\sum_{l\in gQ}[\delta_l Q_{gl}^2] \tag{5}$$

$$O.\ F.=\sum_{\substack{rs\in all\,\text{branches}\\ sr\in all\,\text{branches}}}[P_{rs}+P_{sr}] \tag{6}$$

where Q_g is the reactive power generation; δ denotes the reactive power generation cost coefficient; gQ is the set of controllable reactive power sources; and P_{sr} and P_{rs} are the active power flow in lines $s\to r$ and $r\to s$, respectively.

A similar approach to calculate the power loss in the distribution network is used in [80]. Ref. [75] optimized the active power losses based on Kron's loss formula, as follows:

$$O.\ F.=\sum_{\substack{i\in\text{all}\\ \text{generation}\\ \text{units}}}\ \sum_{\substack{j\in\text{all}\\ \text{generation}\\ \text{units}}} P_{gi}B_{ij}P_{gj}+\sum_{\substack{i\in\text{all}\\ \text{generation}\\ \text{units}}} B_{0i}P_{gi}+B_{00} \tag{7}$$

where $\{B_{ij},\ B_{0i},\ B_{00}\}$ are loss coefficients.

Another OF in [69] is as follows:

$$O.\ F.=\alpha_0\left(\frac{\text{Total Losses}}{\sum_{i=1}^{N}P_{Li}}\right)^2 \tag{8}$$

where P_L is the absorbed active power; and N denotes the total number of buses.

Ref. [71] minimized the active power losses as (4) and the cost of supplied power as (12).

$$O.\ F.=\alpha_c\sum_{\phi\in\mathcal{P}_0}V_0^\phi(I_0^\phi)^*+\sum_{s\in\mathcal{S}}\beta_s\sum_{\phi\in\mathcal{P}_s}P_{G,s}^\phi \tag{9}$$

where $\mathcal{S}$ is the subset of nodes that includes DG; $P_{G,s}$ are active power supplied by unit $s\in\mathcal{S}$; V_0^ϕ and I_0^ϕ are the complex voltage and complex current of phase ϕ, respectively; α_c is the cost of import power from the PCC where α_c>0; β_s represents the one incurred by the use of DG unit $s\in\mathcal{S}$ as β_s>0; $\mathcal{P}$ denotes the set of phases; subscript 0 denotes the PCC; and $(\cdot)^*$ denotes the complex conjugation.

The first section of (9) considers the power input from an external network as a part of the OF.

- **Considering cost of energy import**

Ref. [69] optimized the cost of energy imported from the utility for a single-phase or a balanced three-phase system as:

$$O.\ F.=\int_0^T P_1(t).\ PS(t)dt \tag{10}$$

and for an unbalanced three-phase system as:

$$O.\ F.=\int_0^T (P_a(t)+P_b(t)+P_c(t)).\ PS(t)dt \tag{11}$$

where P is the power at PCC, T is the entire period, and PS is the price signal.

It should be noted that (10) can be employed to minimize the overall cost in presence of storage devices as discussed in [68].

- **Considering tap changers and capacitor units**

Paudyal *et al.* in [70] proposed the OF as below:

$$O.\ F.=\rho\sum_{h=1}^{24}Pinput_h+\sum_p\sum_t\left(\sigma_t\sum_{h=2}^{24}|tap_{p,t,h}-tap_{p,t,h-1}|\right)+\sum_p\sum_{Cb}\left(\mu_C\sum_{h=2}^{24}|cap_{p,Cb,h}-cap_{p,Cb,h-1}|\right)+\sum_{nii}K_{nii}(w_{nii}-round\,(w_{nii}))^2 \quad (12)$$

where $Pinput$ is the input power from the utility; tap is the tap position; t represents the Controllable Tap Changers (CTCs) with $t = 1,2,\ldots,Ntap$ where $Ntap$ denotes the total number of CTCs; cap is the total number of capacitor unit switched in the controllable capacitor bank Cb with $Cb = 1,2,\ldots,NCap$; where $NCap$ denotes the total number of controllable capacitor banks; w denotes the continuous variables (tap and cap); h is the hours of operation; p represents the phases with $p = a,\ b,\ c$; K_{nii} is the parameter that should be carefully chosen depending on various conditions [108,109]; nii denotes the integer variables; ρ, σ, and μ_C are the scalar weights of the OF.

- **Considering optimal operation of the smart grid**

Optimal operation of the smart grid is ensured only when demand response, load curtailment [110–112], congestion cost, generation cost, and voltage and loss profile management costs [113–115] are standardized to maximize the social welfare objective [1,116–118]. In this context, [111] incorporated the voltage stability margin criteria into the load shedding procedure and showed that price responsive demand can improve the system reliability [113]. Also, [1] minimized the overall price without compromising power market stability and social welfare. So, the OF which should be maximized is as follows:

$$O.\ F.=\left(\sum_{j=1}^{n_d}r_jP_{dj}^2+s_jP_{dj}\right)-\left(\sum_{i\in G}\alpha_iP_{gi}^2+\beta_iP_{gi}+\gamma_i+P_cPf_1+T_LPf_2+V_{min}Pf_3+P_{lmax}Pf_4\right) \quad (13)$$

where P_{dj} is the load catered to the jth load; P_c, T_L, V_{min}, and P_{lmax} denote the load curtailed, total transmission loss, minimum bus voltage, and maximum line flow, respectively; Pf_1, Pf_2, Pf_3, and Pf_4 are the penalties to limit violations which will be set by ISO; $\{r_j,\ s_j\}$ are the coefficients of jth demand cost; and n_d denotes the set of loads in the demand cost function.

Ref. [66] tried to decouple two following concepts as a result of the optimal day-ahead hedge for the system operator:

- Day-ahead programmed dispatch,
- Day-ahead contracted quantity.

Also, to extract the sensitivity information important to microeconomics, the day-ahead energy and reserve market allocation [91,92] have been completely formulated in a single stage; in fact, [66] combined and considered several standard problems in a single comprehensive mathematical programming.

Ref. [66] selected the OF as follows:

$$O.\ F.=f_P(P)+f_Q(Q)+f_{RP}(R_P)+f_{RQ}(R_Q) \quad (14)$$

where

$$f_P(P)=\sum_{k=0}^{n_c}\pi_k\sum_{i\in G^k}[C_{Pi}(p_{ik})+C_{Pi}^+(p_{ik}^+)+C_{Pi}^-(p_{ik}^-)];$$

$$f_Q(Q)=\sum_{k=0}^{n_c}\pi_k\sum_{i\in G^k}\left[C_{Qi}(q_{ik})+C_{Qi}^+(q_{ik}^+)+C_{Qi}^-(q_{ik}^-)\right]$$

$$f_{RP}(R_P)=\sum_{i=1}^{n_g}[C_{RPi}^+(r_{Pi}^+)+C_{RPi}^-(r_{Pi}^-)];$$

$$f_{RQ}(R_Q)=\sum_{i=1}^{n_g}\left[C_{RQi}^+\left(r_{Qi}^+\right)+C_{RQi}^-\left(r_{Qi}^-\right)\right]$$

and where $f_P(P)$, $f_Q(Q)$, $f_{RP}(R_P)$, and $f_{RQ}(R_Q)$ represent the costs of active power, reactive power, active reserve, and reactive reserve, respectively; subscript ik denotes the ith generator in the kth contingency; $C_{Pi}(p_{ik})$, $C_{Pi}^+(p_{ik}^+)$, and $C_{Pi}^-(p_{ik}^-)$ are the generation cost, incremental cost on upward deviations from day-ahead contract, and additional cost on downward deviations from day-ahead contract, respectively, for active injections; similarly, $C_{Qi}(q_{ik})$, $C_{Qi}^+(q_{ik}^+)$, and $C_{Qi}^-(q_{ik}^-)$ for reactive injections; $\{C_{RPi}^+(r_{Pi}^+),\ C_{RQi}^+(r_{Qi}^+)\}$ and $\{C_{RPi}^-(r_{Pi}^-),\ C_{RQi}^-(r_{Qi}^-)\}$ represent the cost functions for upward and downward reserve purchased from ith injection where evaluated at upward and downward active and reactive reserves ($\{r_{Pi}^+,\ r_{Qi}^+\}$ and ($\{r_{Pi}^-,\ r_{Qi}^-\}$), respectively; $\{p_{ik},\ q_{ik}\}$, $\{p_{ik}^+,q_{ik}^+\}$, and $\{p_{ik}^-,q_{ik}^-\}$ denote the active and reactive injections, and upward and downward deviations from contracted amount, respectively, in the kth post-contingency state; n_c is the number of considered contingencies; n_g denotes the number of generators and dispatchable or curtailable loads initially available; G^k is the set of generator indices presented in the kth contingency; $k = 0$ denotes the base case or no contingencies; π_k is the probability of transition from the day-ahead base case to the kth contingency.

Ref. [72] applied OPF to evaluate and maximize the network benefits of DSM. The objective is to minimize the total amount of load adjustment required to satisfy grid constraints:

$$O.\ F.=\sum_{n=1}^{N_{DSM}}C_nP_{n,\ \ init}(1-\Psi_n) \quad (15)$$

where N_{DSM} is the number of network load buses where DSM can be applied; $P_{n,init}$ denotes the initial active power of nth load bus; Ψ_n represents the load adjustment factor, or the portion of the initial load at the nth bus which is available for deferral; C_n is the cost of load adjustment assigned to the DSM enabling load at nth bus.

Refs. [81] and [82] optimized the following OF. Also, a similar OF can be found in [119].

$$O.\ F.=\frac{1}{3.\ 6\times10^6}\times\sum_{t_i=t_o}^{t_f}[(Cost_G(t_i)\times\Delta t\times(-p_{G-I}(t_i)-p_{G-S}(t_i)))+(Cost_{SE}(t_i)\times\Delta t\times(p_{S-C}(t_i)-p_{S-D}(t_i)))+(Cost_{PV}(t_i)\times\Delta t\times p_{PV-S}(t_i))+(Cost_{LS}(t_i)\times\Delta t\times p_{L-S}(t_i))] \quad (16)$$

where $Cost_G$, $Cost_{SE}$, $Cost_{PV}$, and $Cost_{LS}$ are the specific tariffs for grid energy, storage energy, photovoltaic shedding, and load shedding, respectively; p_{G-I} and p_{G-S} denote the grid injection and supply powers, respectively; p_{S-C} and p_{S-D} represent storage charging and discharging power, respectively; p_{PV-S} is the shed photovoltaic power and p_{L-S} denotes the shed load power by load shedding; discrete time instant t_i starts from initial time t_0 to final time t_f, with the time interval Δt.

The primary fuel cost of wind turbine and solar photovoltaic is zero. Therefore, only the Operation and Maintenance (O & M) costs are considered [4]. Moreover, this reference, fuel cell cost is similar to micro turbine cost. So, in probabilistic analysis, OF for sample s is formulated as follows [4]:

$$O.\ F.=\underbrace{\sum\left(\frac{C_{MT,s}}{L_{MT,s}}\times\frac{P_s}{\eta_{MT,s}}\right)}_{\text{Micro turbine cost}}+\underbrace{\sum\left(\frac{C_{FC,s}}{L_{FC,s}}\times\frac{P_s}{\eta_{FC,s}}\right)}_{\text{Fuel cell cost}}+\underbrace{\sum\left(\frac{C_{MT,s}}{L_{MT,s}}\times\frac{P_s}{\eta_{MT,s}}\right)\times\left(1-\frac{\varepsilon_{\text{rec}}(\eta_{T,s}-\eta_{e,s})}{\eta_b}\right)}_{\text{CHP cost}}+\underbrace{Cost_{\text{O \& M},\ \ s}}_{\text{Wind tutbine, Micro turbine, Photovoltaic, Fuel cell, CHP}}+\sum Cost_{\text{Trans},\ \ s} \quad (17)$$

where P_s represents the output power of wind turbine or fuel cell; η_{MT} and η_{FC} are the wind turbine and fuel cell efficiencies, respectively; further, C_{MT} and C_{FC} denote the gas price values for the two mentioned units; L_{MT} and L_{FC} are low-hot values; ε_{rec} represents the heat recovery factor; η_e denotes the electrical efficiency of micro turbine and η_T and η_b

are the total efficiencies of Combined Heat and Power (CHP) and boiler, respectively; O & M costs of wind turbine, micro turbines, photovoltaic, fuel cell, and CHP are described through $Cost_{\text{O\&M}}$ (see [4]); and finally, the cost of transaction of powers between microgrid and external grid is stated by $Cost_{\text{Trans}}$.

Note that, due to using micro turbine in the CHP units, the cost of micro turbine is appeared in the CHP cost.

In [96], an optimal strategy is suggested in order to optimal operation of a typical microgrid including renewable DGs (wind and solar), conventional DGs (micro turbine and diesel generator) and batteries. In this reference, sum of operation, emission, and reliability costs are considered as OF; In [96], reliability cost is modeled by the energy not served; but, the battery cost is modeled as below:

$$Cost_{battery}=Cost_{\text{O\&M}}+Cost_{\text{inv}}+Cost_{\text{PEI}}+Cost_{\text{wear}} \tag{18}$$

where $Cost_{\text{inv}}$, $Cost_{\text{PEI}}$, and $Cost_{\text{wear}}$ denote investment, PEI, and wear costs of battery.

2.5.3. Single–objective versus multi–objective

The optimization problem is considered to be single-objective if just one OF is to be optimized. It is in contrast to multi-objective problems in which several functions are to be simultaneously, optimized. These two functions are shown in Table 2.

3. OPF methodology

The objective functions and relevant constraints may be affected by the choice of the optimization procedures for solving under smart grids and smart microgrids. Also, the system type (such as balanced/ unbalanced, smart distribution/transmission grids/microgrids) has a significant effect on the OPF methodologies. So, the OPF approaches could be categorized as follows.

3.1. Different approaches–Comprehensive mathematical representation

Hereafter the most important mathematical approaches proposed in relevant references for OPF are presented.

- **Distributed and Parallel OPF (DPOPF)**

Authors in [20] proposed a Distributed and Parallel OPF (DPOPF) for smart grid transmission systems which uses Processing Unit (PU) at each bus. As previously discussed, like a supercomputer, the smart grid provides an infrastructure for the DPOPF algorithm. This method minimizes the unconstrained optimization sub-problem at each bus by using PUs. A similar approach, namely Distributed OPF (DOPF), is presented in [76] for islanded microgrids in which an approximate solution of the OPF is reached without a central controller.

Complete decomposition is achieved by combining two methods:

- The Recursive Quadratic Programming (RQP) method,
- The Lagrange Projected Gradient (LPG) method.

The OPF problem is:

$$\min_{\boldsymbol{y}} c(\boldsymbol{y});\ \text{s. t.}: \begin{cases} h_i(y_i, y_{J_i})=0, & i=1, 2,\ldots, N,\ J_i \subseteq L_i \\ \underline{y_i}\le y_i\le \overline{y_i}, & i=1, 2,\ldots, N \end{cases} \tag{19}$$

where

$$\boldsymbol{y}=[\boldsymbol{x}^T, \boldsymbol{u}^T, \boldsymbol{z}^T, \boldsymbol{v}^T]$$
$$\boldsymbol{x}=[x_1^T,\ldots, x_N^T]^T;\quad x_i=[V_{Rei}, V_{Imi}]^T,\quad i=1, 2,\ldots, N$$
$$\boldsymbol{u}=[u_{i_g}];\quad u_{i_g}=\left[P_{g_{i_g}}, Q_{g_{i_g}}\right]^T,\quad i_g\in G$$
$$\boldsymbol{z}=[P_{sr}];\quad P_{sr}=p_{sr}(x_s, x_r),\quad (s, r)\in L$$
$$\boldsymbol{v}=[v_i];\quad v_i=\sqrt{V_{Rei}^2+V_{Imi}^2}$$
$$\underline{P_{sr}}\le P_{sr}\le \overline{P_{sr}},\ \underline{v_i}\le v_i\le \overline{v_i},\ \underline{u_i}\le u_i\le \overline{u_i}, -\infty\le x_i\le +\infty$$

V_i represents i's complex voltage; v_i represents the voltage magnitude at bus i; L denotes the set of all transmission lines; $c(\boldsymbol{y})$ is the OF; $h_i(y_i, y_{J_i})$ with J_i being L_i, represents the nonlinear power flow equations, $P_{sr}=p_{sr}(x_s, x_r)$ and $v_i=\sqrt{V_{Rei}^2+V_{Imi}^2}$; the inequality constraints $\underline{P_{sr}}\le P_{sr}\le \overline{P_{sr}}$, $\underline{v_i}\le v_i\le \overline{v_i}$, $\underline{u_i}\le u_i\le \overline{u_i}$, and $-\infty\le x_i\le +\infty$ are as $\underline{y_i}\le y_i\le \overline{y_i}$; $\overline{(\cdot)}$ and $\underline{(\cdot)}$ denote the maximum and minimum of $(\cdot)$, respectively; $(\cdot)_{Re}$ and $(\cdot)_{Im}$ denote the real and imaginary parts of $(\cdot)$, respectively; and L_i is the set of all buses connected to bus i. Bold phrases (e.g. $\boldsymbol{x}$) denote the vectors and matrices.

The RQP method converts the nonlinear equality and simple bounded constraints to the linear equality and simple bounded constraints so that a quadratic approximation problem of (19) can be solved iteratively, where the iterations of the RQP method can be stated as follows:

$$y_i^{(k+1)}=y_i^{(k)}+\alpha_{lk}\,\Delta y_i^{(k)},\quad i=1,\ 2,\ \ldots,\ N \tag{20}$$

where k and α_{lk} denote the iteration index and the step size, respectively. The optimal solution, $\Delta y_i^{(k)}$, can be obtained using the following Quadratic Programming Problem (QPP).

$$\min_{\Delta y_i,\ i=1,\ \ldots,\ N} \sum_{i=1}^{N}\left[\frac{1}{2}\Delta y_i^T\nabla^2 c_i(\boldsymbol{y}^{(k)})\Delta y_i+\nabla c_i(\boldsymbol{y}^{(k)})^T\Delta y_i+\eta\,\Delta y_i^T\Delta y_i\right]$$
$$\text{s. t.}: \begin{cases} h_i\left(y_i^{(k)}, y_{J_i}^{(k)}\right)+\nabla_{y_i}h_i^T\left(y_i^{(k)}, y_{J_i}^{(k)}\right)\Delta y_i \\ \quad +\nabla_{y_{J_i}}h_i^T\left(y_i^{(k)}, y_{J_i}^{(k)}\right)\Delta y_{J_i}=0 \\ \underline{y_i}\le y_i^{(k)}+\Delta y_i\le \overline{y_i} \end{cases} \tag{21}$$

where $\nabla^2 c_i(\boldsymbol{y}^{(k)})$ is the diagonal submatrix of $\nabla^2 c(\boldsymbol{y}^{(k)})$ corresponding to y_i; $\nabla c_i(\boldsymbol{y}^{(k)})$ is the subvector of $\nabla c(\boldsymbol{y}^{(k)})$ corresponding to y_i; η is a positive constant that should be carefully chosen so that (21) becomes strictly convex.; and finally, ∇^2 denotes the Hessian matrix and ∇ represents the first-order derivative (the gradient).

The LPG method comprises two steps for complete decomposition. Dual problem of (21) is solved in the first step.

$$\max_{\lambda}\Phi(\lambda) \tag{22}$$

where

$$\Phi(\lambda)=\min_{\substack{\underline{y_i}-y_i^{(k)}\le\Delta y_i\le\overline{y_i}-y_i^{(k)}\\ i=1,\ \ldots,\ N}}\sum_{i=1}^{N}\left[\frac{1}{2}\Delta y_i^T\nabla^2 c_i(\boldsymbol{y}^{(k)})\Delta y_i+\nabla c_i(\boldsymbol{y}^{(k)})^T\Delta y_i+\eta\,\Delta y_i^T\Delta y_i\right]$$
$$+\sum_{i=1}^{N}\lambda_i^T\left[h_i\left(y_i^{(k)}, y_{J_i}^{(k)}\right)+\nabla_{y_i}h_i^T\left(y_i^{(k)}, y_{J_i}^{(k)}\right)\Delta y_i+\nabla_{y_{J_i}}h_i^T\left(y_i^{(k)}, y_{J_i}^{(k)}\right)\Delta y_{J_i}\right]$$

and $\lambda=[\lambda_1^T,\ldots,\lambda_N^T]^T$ where λ_i is the Lagrange multiplier vector for the equality constraints at bus i. The iterations of the LPG method are as:

$$\lambda_i^{(l+1)}=\lambda_i^{(l)}+\beta_l\nabla_{\lambda_i}\Phi(\lambda^{(l)}),\quad i=1,\ 2,\ \ldots,\ N \tag{23}$$

where l and β_l denote the iteration index and the step size, respectively; moreover, $\nabla_{\lambda_i}\Phi(\lambda)$ and $\Delta y=\Delta\hat{y}$ are evaluated at $\lambda=\lambda^{(l)}$.

The optimal solution can be obtained by solving the following N independent sub-problems:

$$\min_{\Delta y_i} \Delta y_i^T \left(\frac{1}{2}\nabla^2 c_i(y_i^{(k)})+\eta I\right) \Delta y_i + \nabla c_i(y_i^{(k)})^T \Delta y_i$$
$$+\lambda_i^T h_i\left(y_i^{(k)}, y_{J_i}^{(k)}\right)+\lambda_i^T \nabla_{y_i} h_i^T\left(y_i^{(k)}, y_{J_i}^{(k)}\right) \Delta y_i + \sum_{j\in J_i} \lambda_j \nabla_{y_i} h_j^T(y_j^{(k)}, y_{J_i}^{(k)}) \Delta y_i \tag{24}$$

In the second step of the LPG method, $\Delta\hat{y}_i$ for each i, can be obtained by the following formula:

$$\Delta\hat{y}_i=\begin{cases}\Delta\tilde{y}_i, \text{if } \underline{y}_i-y_i^{(k)}\le\Delta\tilde{y}_i\le\overline{y}_i-y_i^{(k)}\\ \underline{y}_i-y_i^{(k)}, \text{if } \underline{y}_i-y_i^{(k)}\ge\Delta\tilde{y}_i\\ \overline{y}_i-y_i^{(k)}, \text{if } \Delta\tilde{y}_i\ge\overline{y}_i-y_i^{(k)}\end{cases} \tag{25}$$

Finally, unconstrained optimization sub-problems can be solved by PUs at each bus. To maintain the computational synchronization of the DPOPF algorithm under the asynchronous data arrival, [20] proposed a Petri Net (PN) control model [120] which is used to describe a Discrete Event System (DES). Moreover a directed convergence tree to implement a termination condition has been described in [20].

The DPOPF algorithm is applied as the data uncertainty increases in deregulated power systems. The main uncertainties are consisting of time varying power generation of renewable energy sources with different power factors, time varying load demand with different power factors, and time varying cost coefficients of nonrenewable energy resources [20]. Further, numerical results demonstrated that the DPOPF algorithm can consider fast variations of the power generated by renewable energy sources and other data uncertainties in the smart grid. However, the power output of renewable energy sources can be considered as a timely constant power output. In this case, the simple bounded constraints on the generated power of these sources can be neglected [20].

- **Multiphase OPF (MOPF)**

Ref. [67] optimized an n-conductor electrical network by proposing a Multiphase OPF (MOPF). This methodology was developed by using two methods including:

- Primal-dual interior point method,
- n-conductor current injection method.

The MOPF problem is as:

$$\min f(z);\ \text{s. t.}:\begin{cases}\boldsymbol{I}_{Re}(z)=0, \quad \boldsymbol{I}_{Im}(z)=0\\ \boldsymbol{g}(z)=0, \quad \boldsymbol{h}(z)\le 0\\ \underline{z}\le z\le\overline{z}\end{cases} \tag{26}$$

where $f(z)$ is the OF; $\boldsymbol{I}$ denotes the sum of the currents injected to all nodes; $\boldsymbol{g}(z)$ and $\boldsymbol{h}(z)$ represent the equality and inequality constraints, respectively; and z denotes the state and control variables.

In this methodology, individual input contributions of each element can be written by using the contribution of the injected currents to the Lagrange function, allowing for more complete analyses through the use of a detailed representation of electrical power networks. In addition, by using slack variables, inequality constraints are converted to equality ones. The Lagrange function is presented as:

$$L(z,\lambda,\boldsymbol{\pi})=f(z)-\sum_{i=1}^{N}\lambda_{Im,i}\boldsymbol{I}_{Re,i}(z)-\sum_{i=1}^{N}\lambda_{Re,i}\boldsymbol{I}_{Im,i}(z)-\sum_{i=1}^{ni}\lambda_i g_i(z)$$
$$-\sum_{j=1}^{nd}\pi_{j.1}(z_j-z_{j.min}-sl_{j.1})-\mu b\sum_{j=1}^{nd}\log(sl_{j.1})$$
$$-\sum_{j=1}^{nd}\pi_{j.2}(z_j-z_{j.max}-sl_{j.2})-\mu b\sum_{j=1}^{nd}\log(sl_{j.2}) \tag{27}$$

where ni, and nd denote the total number of constraints (equality + inequality), and the number of inequality constraints and minimum/maximum variable limits, respectively; λ_{Re}, λ_{Im}, π_1, and π_2 are the dual variables (Lagrange multipliers); sl is the set of slack variables; and $\mu b > 0$ is the barrier parameter.

The linear system is represented as follows:

$$\nabla^2 L(z,\lambda).\Delta(z,\lambda)=-\nabla L(z,\lambda) \tag{28}$$

where $\nabla^2 L(z,\lambda)$ and $\nabla L(z,\lambda)$ are the Hessian matrix and the first-order derivatives of the Lagrange function, respectively.

In this method, a linear system should be solved in each iteration. The proposed algorithm is convergent and useful in unbalanced systems. Unbalanced systems have a great influence on the final result of the optimization process. However, this influence is not so great for the problem convergence.

- **OPF approach based on linearization and approximation**

Erseghe, and Tomasin in [65] proposed an OPF approach based on the linearization of grid equations, assuming the loads as impedances, and the optimal injection currents of DERs and PCC. This approach considers the system nodes as belonging to three categories (PCC, PEI and Loads) and uses a linear network, in achieving a Quadratic Constrained Quadratic (QCQ) problem. Then, the optimization problem is converted to a convex one, using the slack variables and convex constraints. The semi-definite and convex OPF problem is presented as:

$$\min \operatorname{trace}(\boldsymbol{CX});\ \text{s. t.}:\begin{cases}\operatorname{trace}(\boldsymbol{C}_{kc}\boldsymbol{X})\le 0\\ [\boldsymbol{X}]_{2N_{EI}+1,\ 2N_{EI}+1}=1\\ \boldsymbol{X}\succcurlyeq 0\end{cases} \tag{29}$$

where $kc = 1,2,\ldots,ni$; and trace, $\boldsymbol{X}$, $\boldsymbol{C}$, and $\boldsymbol{C}_{kc}$ are defined in [65]. Moreover, in this equation, $\boldsymbol{X}\succcurlyeq 0$ denotes the positive semidefinite matrix.

Initial points are provided by the Semi-definite Programming (SDP) relaxation technique. Ref. [65] showed that in the grid-connected mode:

- Increasing the number of active constraints or the PCC cost, leads to increasing the total cost.
- By activating the voltage constraints, the relationship between the total cost and the PCC cost becomes approximately linear and the loss is dramatically reduced.
- Activating the line current constraints leads to slight reduction in losses.
- As the voltage decreases, the constraint on voltages becomes more stringent and the total smart microgrid cost will be increased.

and in the islanded mode:

- The total cost of current solution (applying a full optimization) is much higher when the PCC is replaced by a PEI and the PEI enforces the voltage constraint neglecting its cost.

- **OPF approach based on considering storage devices**

Ref. [68] proposed an OPF considering storage devices for a generic network. This approach covers the network-storage problem in both network and time domains. Moreover, to achieve a numerical efficient solution, a robust combination of dynamic programming recursive search with power flow solver has been proposed in [68]. In this reference, the optimization problem is actually regarded as an allocation problem; in fact, the power supplied at PCC is optimized by using a dynamic programming algorithm and energy is allocated to the time domain. The presented approach combines two mentioned domains. This algorithm computes the optimal energy management of the storage devices in the grid-connected mode in microgrids.

The M-dimensional OPF problem is formulated as (the Bellman equation) follows:

$$\min_{\substack{E_1(t+dt)\\ \dots\\ E_M(t+dt)}} \left\{ \begin{array}{l} \Delta V\left(\begin{array}{c} E_1, E_1(t+dt), E_2, E_2(t+dt), \\ \dots, E_M, E_M(t+dt) \end{array} \right) \\ + V(t+dt, E_1(t+dt), E_2(t+dt), \\ \qquad \dots, E_M(t+dt)) \end{array} \right\} \tag{30}$$

where $V(\cdot)$ is defined as (7) and (8); with initial condition:

$$E_i(t)=E_i, \quad i=1, \quad 2, \quad \dots, \quad M$$

where $E_i(t)$ is the stored energy; Moreover, calculation of the differential cost ΔV is computed by using [68] for M-D points $\{t, E_1(t), \dots, E_M(t)\}$ and $\{t+dt, E_1(t+dt), \dots, E_M(t+dt)\}$.

For computing the output power of the both storage devices, the computation involves evaluation of M derivatives as:

$$\frac{d}{dt}E_i \approx \frac{E_i(t+dt)-E_i(t)}{dt}, \quad i=1, \quad 2, \quad \dots, \quad M \tag{31}$$

The main disadvantage of the presented approach in [68] is the growing numerical complexity in power law with the number of (different) storage devices.

- **Unbalanced Three-phase OPF (TOPF)**

Bruno *et al.* in [69] proposed an unbalanced Three-phase OPF (TOPF) for Distribution Management System (DMS) based on Distribution Load Flow (DLF). This methodology is useful for unbalanced distribution networks. In [69], authors have assumed that in a near future, Distribution Network Operators (DNOs) will be able to develop online smart functions in the extended real-time framework of daily system operation. Also, they manage interaction of microgrids with the main grid and offer both grid and market functionalities [121]. Ref. [69] used the curtailment approach based on Automatic Meter Reading (AMR) and AMI devices; in fact, they are already able to limit the maximum demand through receiving a signal. Moreover, this approach can be used for the optimal control of reactive resources.

In [75], authors proposed a TOPF for islanded AC microgrids in order to provide a minimum losses operating point for the purposes of increasing the lifetime of lines and components as well as evaluating the contextual adjustment of the droop parameters used for primary voltage and frequency regulation of inverter interfaced units.

The TOPF problem is formulated as:

$$\min_{\mathbf{u}} C_{obj}(\boldsymbol{x}, \boldsymbol{u}); \text{ s. t. :} \begin{cases} \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u})=0 \\ \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}) \le 0 \\ \underline{\boldsymbol{u}} \le \boldsymbol{u} \le \overline{\boldsymbol{u}} \end{cases} \tag{32}$$

where C_{obj} is the objective function; $\boldsymbol{x}$ denotes the state variable vector so that $\boldsymbol{x} \in \Re^n$; and $\boldsymbol{u}$ is the control variable vector so that $\boldsymbol{u} \in \Re^m$.

Authors in [122] formulated the inequality constraints as the penalty functions through the following manner:

$$\min_{\mathbf{u}} C(\boldsymbol{x}, \boldsymbol{u}); \text{ s. t. : } \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u})=0 \tag{33}$$

where

$$C(\boldsymbol{x}, \boldsymbol{u})=C_{obj}(\boldsymbol{x}, \boldsymbol{u})+\sum_{i \in PF} C_p^i(\boldsymbol{x}, \boldsymbol{u})$$

and where PF is the set of penalty functions; and C and C_p are the overall objective and penalty functions, respectively.

By using the implicit function theorem conditions [123], an unconstrained optimization problem as (34) can be achieved and solved by the quasi-Newton method (35), iteratively.

$$\min_{\mathbf{u}} C(\gamma(\boldsymbol{u}), \boldsymbol{u}) \tag{34}$$

$$\boldsymbol{u}^{(k+1)}=\boldsymbol{u}^{(k)}+\lambda^{(k)}.\ \mathbf{I}.\ \mathbf{F}^{(k)} \tag{35}$$

where

$$\lambda^{(k)}=\frac{(\Delta\mathbf{F}^{(k-1)})^T.\Delta\boldsymbol{u}^{(k-1)}}{(\Delta\mathbf{F}^{(k-1)})^T.\Delta\mathbf{F}^{(k-1)}}$$

and where γ: $N(\overline{\boldsymbol{u}}) \to E^n$ and $\gamma \in C^k[N(\overline{\boldsymbol{u}})]$ with $\gamma(\overline{\boldsymbol{u}})=\overline{\boldsymbol{x}}$ and $\boldsymbol{g}(\gamma(\overline{\boldsymbol{u}}), \overline{\boldsymbol{u}})=0$ for all $\overline{\boldsymbol{u}} \in N(\overline{\boldsymbol{u}})$ when $\boldsymbol{g}$: $E^{n+m} \to E^n$ is k times continuously differentiable (C^k class) and $N(\overline{\boldsymbol{u}}) \subset E^m$; $\mathbf{F}^{(k)}$ denotes $F(\boldsymbol{u}^{(k)})$; $\lambda^{(k)}$ is a scalar; $\mathbf{I}$ is the $m \times m$ identity matrix; Δ denotes the forward difference operator defined as follows:

$$\Delta(\cdot)_i=(\cdot)_{i+1}-(\cdot)_i \tag{36}$$

The single-phase OPF is faster than the proposed TOPF methodology in [69]. It is mainly due to the fact that a greater number of equations and variables must be treated in the TOPF approach. Ref. [69] showed how the three-phase formulation is more effective in distribution systems; since it takes into account natural unbalances among different phases and allows to better exploit active/reactive resources just on one phase. They are mostly used to solve problems related to a specific phase where technical constraints are violated.

Ref. [70] proposed a three-phase Distribution OPF (DOPF) algorithm. This algorithm exploits detailed representation of electrical distribution network components that can be used by Local Distribution Companies (LDCs) for unbalanced networks in order to integrate the relevant distribution feeders (individual feeders can be optimized separately) into a smart grid. Also, it uses a weighted sum with a penalty function in order to construct the OF. Since the capacitor switching and LTC operations are discrete, a Mixed Integer Nonlinear Programming (MINLP) problem is converted to a Nonlinear Programming (NLP) applying methods presented in [13,109]. Furthermore, [70] introduced a novel local search method considering a quadratic penalty term to reduce the search space significantly.

Authors in [71] proposed an unbalanced TOPF for the distribution network based on SDP relaxation. Applying the relaxed SDP for the OPF problem was initially proposed for balanced transmission systems in [124] and [125]. This technique was then extended to balanced distribution systems in [126] and [127]. ; However, [71] developed the SDP relaxation for unbalanced smart microgrids. In this method, the main non-convex problem is converted to a convex one. In [71], the OPF problem has been stated as:

$$\min_{\mathbf{V}} \widetilde{C}_m(\mathbf{V}); \text{ s. t. :} \begin{cases} \mathrm{Tr}(\mathbf{\Phi}_{P,n}^{\phi}\mathbf{V})+P_{L,n}^{\phi}=0, \\ \quad \forall\phi, \forall n \in \mathcal{NS} \\ \mathrm{Tr}(\mathbf{\Phi}_{Q,n}^{\phi}\mathbf{V})+Q_{L,n}^{\phi}-y_{C,n}^{\phi}\ \mathrm{Tr}(\mathbf{\Phi}_{V,n}^{\phi}\mathbf{V})=0, \\ \quad \forall\phi, \forall n \in \mathcal{NS} \\ \underline{P}_{G,s} \le \mathrm{Tr}(\mathbf{\Phi}_{P,n}^{\phi}\mathbf{V})+P_{L,n}^{\phi} \le \overline{P}_{G,s}, \\ \quad \forall\phi, \forall n \in \mathcal{S} \\ \underline{Q}_{G,s} \le \mathrm{Tr}(\mathbf{\Phi}_{Q,n}^{\phi}\mathbf{V})+Q_{L,n}^{\phi} \le \overline{Q}_{G,s}, \\ \quad \forall\phi, \forall n \in \mathcal{S} \\ (\underline{V}_n)^2 \le \mathrm{Tr}(\mathbf{\Phi}_{V,n}^{\phi}\mathbf{V}) \le (\overline{V}_n)^2, \\ \quad \forall\phi, \forall n \in \mathcal{N} \\ [\mathbf{V}]_{\mathcal{P},\mathcal{P}}=\mathbf{v}_0\mathbf{v}_0^{\mathcal{H}} \\ \mathbf{V} \succcurlyeq 0 \end{cases} \tag{37}$$

where the Hermitian matrices are formulated as:

$$\mathbf{\Phi}_{P,n}^{\phi} \coloneqq \frac{1}{2}(\mathbf{Y}_n^{\phi}+(\mathbf{Y}_n^{\phi})^{\mathcal{H}}); \mathbf{\Phi}_{Q,n}^{\phi} \coloneqq \frac{j}{2}(\mathbf{Y}_n^{\phi}-(\mathbf{Y}_n^{\phi})^{\mathcal{H}}); \mathbf{\Phi}_{V,n}^{\phi} \coloneqq \bar{\mathbf{e}}_n^{\phi}(\bar{\mathbf{e}}_n^{\phi})^T$$

and where

$$\mathrm{Tr}(\mathbf{\Phi}_{P,n}^{\phi}\mathbf{V})=P_{G,n}^{\phi}-P_{L,n}^{\phi}; \mathrm{Tr}(\mathbf{\Phi}_{Q,n}^{\phi}\mathbf{V})=Q_{G,n}^{\phi}-Q_{L,n}^{\phi}+y_{C,n}^{\phi}\ \mathrm{Tr}(\mathbf{\Phi}_{V,n}^{\phi}\mathbf{V}); \mathrm{Tr}(\mathbf{\Phi}_{V,n}^{\phi}\mathbf{V})=|V_n^{\phi}|^2$$

with $\mathcal{N} \coloneqq \{1,2,\ldots,N\}$; $P_{L,n}^{\phi}$ and $Q_{L,n}^{\phi}$ are the active and reactive loads at node n on phase ϕ, respectively; $P_{G,n}^{\phi}$ and $Q_{G,n}^{\phi}$ represent active and reactive powers supplied at node n on phase ϕ, respectively; $P_{G,s}^{\phi}$ and $Q_{G,s}^{\phi}$ denote active and reactive powers supplied through unit $s \in \mathcal{S}$, respectively; V_n is the voltage magnitude; the voltages $\mathbf{v}_0 \coloneqq [V_0^a, V_0^b, V_0^c]^T$ for phasorial representation are taken as reference; $y_{C,n}^{\phi}$ represents the susceptance of a capacitor block connected at node n and phase ϕ; $\mathbf{Y}_n^{\phi}(\coloneqq \bar{\mathbf{e}}_n^{\phi}(\bar{\mathbf{e}}_n^{\phi})^T\mathbf{Y})$ denotes the admittance related matrix per node n and phase ϕ in which $\mathbf{Y}$ is a symmetric block admittance matrix; and finally, $\bar{\mathbf{e}}_n^{\phi}(\coloneqq [\mathbf{0}_{|\mathcal{P}_0|}^T,\ldots,\mathbf{0}_{|\mathcal{P}_{n-1}|}^T, \mathbf{e}_{\mathcal{P}_n}^{\phi,T}, \mathbf{0}_{|\mathcal{P}_{n+1}|}^T,\ldots,\mathbf{0}_{|\mathcal{P}_N|}^T]^T)$ and $\{\mathbf{e}_{\mathcal{P}_n}^{\phi}\}_{\phi}\in\mathcal{P}_n$ denote the canonical basis of $\mathbb{R}^{|\mathcal{P}_n|}$; $\mathcal{P}_n\subseteq\{a_n, b_n, c_n\}$ with the phase of node $n \in \mathcal{N}$.

- **Alternating Direction Method of Multipliers (ADMM)**

Authors in [71] used an Alternating Direction Method of Multipliers (ADMM) [128–130], where an optimization sub-problem must be solved by each Local Controller (LC), which then each LC exchanges simple messages with its neighboring LCs. Numerical results show that the global optimal solutions of the original OPF problem are always reached. This algorithm ensures scalability with respect to the microgrid size~~,~~ and robustness to communication outages, and also preserves data privacy and integrity. The approach presented in [87], unlike the SDP based distributed algorithms, allows to handle non-quadratic convex OFs. Moreover, it uses ADMM to accomplish the distributed implementation of its algorithm among the electrical buses.

To extract the price of $(h + 1)$th hour from the price of hth hour, [1] developed the state space model of the power system as a dynamic problem as below:

$$\mathbf{x}(h+1)=\boldsymbol{A}\mathbf{x}(h)+\boldsymbol{B}\boldsymbol{u}(h); yy(h+1)=\boldsymbol{CC}\mathbf{x}(h)=C(d) - C(glv) \tag{38}$$

where

$$\mathbf{x}=[P_{s1}P_{s2}, \quad \ldots, \quad P_{sm}L_{s1}L_{s2}, \quad \ldots, \quad L_{snn}P_cT_LV_{min}P_{lmax}]^T\ \boldsymbol{A} =\begin{bmatrix} A_{11} & \cdots & A_{1(m+nn+4)} \\ \vdots & \ddots & \vdots \\ A_{(m+nn+4)1} & \cdots & A_{(m+nn+4)(m+nn+4)} \end{bmatrix}; A_{ij}=\frac{b(i,1)_{nn+1} - b(i,1)_{nn}}{b(j,1)_{nn+1} - b(j,1)_{nn}}$$

$$\boldsymbol{CC}=[-C_{g1}-C_{g2}, \quad \ldots, \quad -C_{gm}C_{L1}C_{L2}, \quad \ldots, \quad C_{Lnn}-C_{Pc}-C_{TL}-C_{Vmin}-C_{Plmax}]\ C(d)=\sum_{j=1}^{nd} r_jP_{dj}^2+s_jP_{dj}; C(glv)=\sum_{i\in G}\alpha_iP_{gi}^2+\beta_iP_{gi}+\gamma_i+P_cPf_1+T_LPf_2+V_{min}Pf_3+P_{lmax}Pf_4$$

and where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{CC}$ are the sensitivity, contingency (or state modification), and output matrices, respectively; $\mathbf{x}$ denotes the price sensitive state variables of the system; yy is the social welfare with the desired operating condition; $C(d)$ and $C(glv)$ are the demand cost benefit and generation cost with limitations on the violated constraints, respectively; P_{si} and L_{si} are the ith generation bus for a particular schedule and the load catered to the ith bus for a particular load schedule, respectively; C_{gi}, C_{Lj}, C_{Pc}, C_{TL}, C_{Vmin}, and C_{Plmax} are cofactors of the ith generation bus, the jth load bus, the curtailed load, the line loss, the minimum bus voltage, and the maximum line flow, respectively; m and nn are the total number of generators and loads, respectively.

On the basis of this approach, ISO implements OPF and determines $\mathbf{x}(h + 1)$. So, the OPF formulation is as follows:

$$\max C(d)-C(glv);\ \text{s. t. :}\begin{cases} \text{Power balance equations} \\ \text{Active and reactive power generation limits} \\ \text{Bus voltage limits} \\ \text{Transmission line flow and line loss constraints} \\ \text{Curtailment limits as} 0<P_c < \left(P_{max}-\frac{P_{max}(S-S_{min})}{S_{max}-S_{min}}\right) \end{cases} \tag{39}$$

where P_{max} is defined as the difference between maximum and minimum limits of the requested demand; S is the generation surplus, and consequently, S_{max} and S_{min} represent the maximum and minimum generation surpluses, respectively. Note that, ISO can set load curtailment limits or may use the presented approximated curtailment-surplus relationship as mentioned above.

Additionally, the eigen values of matrix $\boldsymbol{A}$ are used in [1] to determine the system stability and the power market price equilibrium. To estimate the system stability by using the Newton's formula in the steady-state condition, the Jacobian matrix $J^{(r)}$ at the rth iteration should be:

$$J^{(r)}=\boldsymbol{A}-\mathbf{I} \tag{40}$$

Due to the convex nature of the proposed algorithm in [1], a stochastic optimization technique such as Particle Swarm Optimization (PSO) [131] is selected to demonstrate the feasibility and effectiveness of the algorithm.

- **OPF based on simultaneous formulation of the post-contingency flows**

Some approaches consider the simultaneous formulation of the post-contingency flows with additional constrains [91–93,132–138]. They bound the injections deviations from the relevant base case deviations in the post-contingency flows. A general tree structure to represent transitions has been proposed in [66], in which the operational cost is weighted by its probability of occurrence making a constrained stochastic optimization. In a tree with one root, the problem formulation is as follows:

$$\min_{\substack{\theta,\ V,\ P,\ Q,\\ P^+,\ P^-,\ Q^+,\ Q^-,\\ P_c,\ Q_c,\ R_P,\ R_Q}} \text{Equation (14) s. t. :}$$

$$\begin{cases} \text{Active and reactive power flow constraints for all post} \\ \quad - \text{contingency state} \\ \text{Transmission line, voltage, generation and other limits in post} \\ \quad - \text{contingency state} \\ p_{ik}^+\geq 0, p_{ik}^+\geq p_{ik}-p_{ci},\ q_{ik}^+\geq 0, q_{ik}^+\geq q_{ik}-q_{ci},\ R_{Pi}^{max+}\geq r_{Pi}^+\geq p_{ik}^+ \\ \quad ,R_{Qi}^{max+}\geq r_{Qi}^+\geq q_{ik}^+ \quad \forall\ i,\ k \\ p_{ik}^-\geq 0, p_{ik}^-\geq p_{ci}-p_{ik},\ q_{ik}^-\geq 0, q_{ik}^-\geq q_{ci}-q_{ik},\ R_{Pi}^{max-}\geq r_{Pi}^-\geq p_{ik}^- \\ \quad ,R_{Qi}^{max-}\geq r_{Qi}^-\geq q_{ik}^- \quad \forall\ i,\ k \\ \Delta_{Pi}^+\geq p_{ik}-p_{i0}\geq -\Delta_{Pi}^-,\ \Delta_{Qi}^+\geq q_{ik}-q_{i0}\geq -\Delta_{Qi}^- \quad \forall\ i,\ k = 1,\ldots,n_c \\ \quad \alpha \geq p_{i0}-p_{ci}\geq -\alpha,\ \alpha \geq q_{i0}-q_{ci}\geq -\alpha \quad \forall\ i \end{cases} \tag{41}$$

where $\{\Delta_i^+, \Delta_i^-\}$ represents the physical ramp rate of each unit; α is a parameter which should be selected so that the contracted quantity can be specified to be equal to the base case dispatch if desired; and p_{ci} and q_{ci} are purchase amounts specified in the day-ahead contract for active and reactive powers from the ith injection, respectively.

For sake of simplicity, [66] assumed that there are no reactive power offers in the treatment and there is only one generation unit at a given bus. So, the Lagrangian function for an active-only problem is as follows:

$$
\begin{aligned}
&L(\theta, V, P, Q, P_c, P^+, P^-, R_P^+, R_P^-, \lambda, \mu)\\
&=f_P(P)+f_{RP}(R_P)+\sum_{k=0}^{n_c}\sum_{i\in G^k}\lambda_{ik}(g_{Pgik}-p_{ik}+P_{Di})+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{P_{ik}^{max}}(-P_{ik}^{max}+p_{ik})\\
&\quad+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{P_{ik}^{min}}(P_{ik}^{min}+p_{ik})+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{ik}^{(1)}(-p_{ik}^+)\\
&\quad+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{ik}^{(2)}(p_{ik}-p_{ci}-p_{ik}^+)+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{ik}^{(3a)}(p_{ik}^+-r_{Pi}^+)\\
&\quad+\sum_{i=1}^{n_g}\mu_i^{(3b)}(r_{Pi}^+-R_{Pi}^{max+})+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{ik}^{(4)}(-p_{ik}^-)\\
&\quad+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{ik}^{(5)}(p_{ci}-p_{ik}-p_{ik}^-)+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{ik}^{(6a)}(p_{ik}^--r_{Pi}^-)\\
&\quad+\sum_{i=1}^{n_g}\mu_i^{(6b)}(r_{Pi}^--R_{Pi}^{max-})+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{ik}^{(7a)}(-p_{ik}+p_{i0}-\Delta_{Pi}^-)\\
&\quad+\sum_{k=0}^{n_c}\sum_{i\in G^k}\mu_{ik}^{(7b)}(p_{ik}-p_{i0}-\Delta_{Pi}^+)+\sum_{i=1}^{n_g}\mu_i^{(8a)}(p_{ci}-p_{i0}-\alpha)\\
&\quad+\sum_{i=1}^{n_g}\mu_i^{(8b)}(p_{i0}-p_{ci}-\alpha)+\lambda_{other}^T g_{other}+\mu_{other}^T h_{other}
\end{aligned}
\tag{42}
$$

where λ_{ik} denotes the multipliers on the active power flow constraints; μ is Karush-Kuhn-Tucker (KKT) multiplier on additional inequality; here, superscript a (or b) denotes the first (or second) inequality; the numbered superscripts on μ are just to make the distinction; g_{other} and h_{other} are other equality and inequality constraints, respectively; and $\{P_c, R_P^+, R_P^-, Q_c, R_Q^+, R_Q^-\}$ are the optimal day-ahead contract quantities.

Ref. [75] used a gradient method to solve the Lagrangian function (similar to [66]) with load-dependent frequency and voltage.

- **Uncertainty-based OPF models**

Uncertainty can be defined as the probability of difference between the forecasted and real values. Smaller probability will result in less cost of the power system operation. This purpose necessitates modeling of system random variables (such as the output power of renewable resources and the load demand) with appropriate and practicable methods [96]. In [4], an optimal power dispatch problem related to multi-microgrids is described which considers uncertainties [139] in load and probabilistic modeling of generated power by renewable small-scale energy resources. In this reference, power exchanging in microgrids has been implemented to satisfy power balance between generation and load sections so that different costs including power generation cost in each microgrid and power exchanging cost between microgrids and main grid are optimized. In [4], the proposed probabilistic optimal power dispatch problem is solved by PSO algorithm.

In [4], the load demand is modeled as a normal distribution function as follows [140]:

$$f(P_l)=\frac{1}{\sigma\times\sqrt{2\pi}}\exp\left(-\frac{(P_l-\mu)^2}{2\sigma^2}\right) \tag{43}$$

where μ is the mean value and σ denotes the standard deviations to illustrate a probabilistic load demand.

In fact, to model the load variation (uncertainty) in a given period, a predefined number of load samples are generated based on the normal distribution [4].

Power generation of a wind turbine depends on wind speed which varies minutely, hourly, daily and seasonally. To describe the distribution of wind speed, Weibull Probability Density Function (PDF) is used as follows [4]:

$$f_\nu(\nu)=\begin{cases}\frac{\beta}{\alpha}\times\left(\frac{\nu}{\beta}\right)^{\rho-1}\times\exp(\frac{\nu\beta}{\alpha}), & \text{if}\nu\geq 0\\ 0, & \text{Otherwise}\end{cases} \tag{44}$$

where α and β are the shape and scale parameters of Weibull function, respectively; and ν represents the wind speed.

Converting wind speed as the primary energy source to electrical power can be stated as follows [4,140]:

$$P_g^{\mathrm{WT}}(\nu)=\begin{cases}P_r\times\left(\frac{\nu_{\mathrm{cut-out}}^n-\nu^n}{\nu_{\mathrm{cut-in}}^n-\nu_r^n}\right), & \text{if}\nu_{\mathrm{cut-in}}\leq\nu\leq\nu_r\\ P_r, & \text{if}\nu_r\leq\nu\leq\nu_{\mathrm{cut-out}}\\ 0, & \text{Otherwise}\end{cases} \tag{45}$$

where ν_r, $\nu_{\mathrm{cut-in}}$, and $\nu_{\mathrm{cut-out}}$ are the rated, low cut, and high cut speeds of wind turbine, respectively; $P_g^{\mathrm{WT}}(\nu)$ and P_r denote the power generation at speed ν; and finally, parameter n in the above equation describes the rate of characteristic curve between $\nu_{\mathrm{cut-in}}$ and ν_r.

Power generation of a solar photovoltaic is affected by solar radiation and air temperature [4]. In [4], irradiance and air temperature are modeled by normal distribution function for a given time. The power generation of a solar photovoltaic module (i.e. P_g^{PV}) at irradiance G_{ING} can be determined as follows (see [4,141]):

$$P_g^{\mathrm{PV}}=P_{\mathrm{STC}}\times\frac{G_{\mathrm{ING}}}{G_{\mathrm{STC}}}\times(1+k(T_c-T_r)) \tag{46}$$

where P_{STC} is the rated power at standard test condition; T_r and T_c represent the air and cell temperatures, respectively; and k denotes maximum power temperature coefficient.

In [96], PEM has been employed to model wind and solar power uncertainties according to Weibull and Beta PDFs, respectively. Further, robust optimization has been used to model the load demand uncertainty. Finally, the short-term energy management scheme as an optimization problem is solved using PSO algorithm under different technical constraints.

Paper [83] proposed a stochastic framework based on the scenario production technique to consider the uncertainty associated with the load forecast error, wind turbine/photovoltaic power generation, and market price. This approach consists of two phases: 1) a specific PDF is considered for each of the input variables. Then, using roulette wheel mechanism, different scenarios with different probabilities are generated. Afterwards, a number of dissimilar scenarios with the highest occurrence probability are selected to reduce the number of scenarios. 2) for each of the selected scenarios, the microgrid operation management is separately solved. Then, the aggregation step is applied to the obtained solutions in order to detect the optimal aggregated solution.

In [84], the stochastic behavior of uncertain variables is investigated through using two-PEM. In fact, each uncertain variable is replaced by two deterministic points located on each side of the mean value of the relevant distribution function. One of the main advantages of this framework is low computational cost; since$2n$ deterministic analysis is required for n uncertain variables. Therefore, this method can be applied to photovoltaic, wind turbine, load forecast error, and market bid changes. A similar approach is also proposed in [85].

3.2. System type: three phases versus single phase – System balance: balanced versus unbalanced

Distribution networks are inherently unbalanced, because of [70,71]:

- Unequal single-phase loads on each phase.
- Unbalanced n-phase loads for n-phase systems where $n \geq 2$.
- Unequal conductor spacing of m-phase line segments where $m \geq 3$.
- Un-transposed m-phase feeders.
- Existence of single-phase laterals.

From the system balance viewpoint the previous methodologies can be categorized into two main parts:

Table 3
Classifying Based on System Type and System Balance.

Reference Number	Useful for Smart Microgrid/ Grid Distribution System	Useful for Smart Microgrid/Grid Transmission System	Useful for Balanced Smart Microgrid/ Grid	Useful for Unbalanced Smart Microgrid/ Grid
[1]		✓	✓	
[20]	✓		✓	
[65]		✓	✓	
[66]		✓	✓	
[67]	✓	✓		✓
[68]	✓			✓
[69]	✓			✓
[70]	✓			✓
[71]	✓			✓
[72]	✓	✓[a]	✓	
[73]	✓	✓	✓	
[74]	✓		✓	
[75]	✓			✓
[76]	✓		✓	
[77][b]	✓			
[78][c]	✓			
[79]	✓		✓	
[4]	✓		✓	
[96]	✓		✓	
[11]	✓		✓	
[102]	✓		✓	
[80]	✓		✓	
[81][d]	✓			
[82][e]	✓			
[83]	✓		✓	
[84]	✓		✓	
[85]	✓		✓	
[86]	✓	✓	✓	
[87]		✓	✓	

[a] Applicable to meshed distribution network.
[b] This reference used a hybrid AC/DC microgrid.
[c] This reference considered an isolated load area supplied through a DC microgrid renewable energy park.
[d] This reference used a DC microgrid.
[e] This reference optimized a DC microgrid.

- Balanced approaches,
- Unbalanced approaches.

Generally, unbalanced approaches (n-phase OPF) are not suitable to be applied to balanced networks. This is mainly due to significant increase in processing time. In fact, in unbalanced approaches, a greater number of equations and variables must be treated [69]. The methods presented in [4,11,20,68–71,74–85,96,142] are useful for application in smart grid and microgrid distribution networks. Also, the approaches exhibited in [1,65–67,72,73,86,87] are useful to be applied to smart grids and microgrids with transmission and distribution structures. A classification based on system types (balanced/ unbalanced smart distribution/transmission microgrids/grids) is described in Table 3.

3.3. Operational state: the islanded mode versus the grid-connected mode

A smart distribution grid can be supplied (fully or partially) by PCC. Hence, the grid-connected mode refers to an operational one in which the smart grid is connected to the main system and supplied through it. When a distribution power system suffers from the outage events at the upstream grid due to some events such as faults in the main power system, a part of grid can be de-energized from the main grid and supplied through available DERs. This operational mode denotes the islanded one.

As is shown in Table 1, most of the studies have considered the network in the grid-connected mode. This is mainly due to the fact that most of the approaches assume the power to be supplied from an external utility to the grid as a part of OF.

3.4. Network topology: distribution versus transmission

Generally, the generated power from the electric power sources is distributed through the transmission, sub transmission, and distribution lines to the loads. The first two levels are interconnected and operated as mesh structure. However, distribution systems are operated radially. From this viewpoint, the operational conditions may be different. Various OPF approaches based on the network topology are presented in Table 3.

3.5. Programming model: dynamic versus static

The tremendous numerical complexity of the OPF problem with storage devices is because of both the network domain and the time domain, related to storage devices. In this regard, traditional gradient based solvers such as the Newton-Raphson method are extremely useful in the network domain; but inadequate in the time domain. Hence, such solvers cannot be applied to the combined network-storage problem [68]. Ref. [68] integrates the storage devices to compute the globally OPF in both the time and network domains. For this purpose, it combines a load flow solver with a Dynamic Programming (DP) recursive search. It is in contrast to the static programming which mainly considers the network domain. In fact, a static OPF is carried out for a single time step, i.e., none of the variables are time-dependent [72]. In [142], a step by step optimization is introduced based on basic DP and an original self-adaptive DP is developed.

3.6. Control strategy: centralized versus decentralized control – Multi agent versus central agent

Effective control of a network of microgrids has an important role in the optimal operation of such networks. In general, there are two strategies to control a network of microgrids: centralized control and decentralized control [143]. In the centralized configuration, the optimal control strategy is applied via a central agent, which needs to communicate with other microgrids. The main disadvantage of this manner is that any failure in the central agent leads to an entire failure in the global network [144]. In the decentralized approach, a multi agent system [145] is used to control a network of microgrids in which each microgrid is related to an agent controlling some conditions (such as power flow to/from the external world) [146–149]. The main advantage of this approach is that it reduces the need to manipulate large quantities of data related to the complex system of microgrids; because, all agents are independent decision makers and can exchange their knowledge based on the type of the control strategy [143]. In this context, from a control viewpoint, a smart microgrid is completely autonomous and can significantly affect the OPF problem. On this basis, it controls the power exchanging between the smart microgrid and other smart microgrids or the utility.

High computational capability is required at CC for the centralized approaches [150–152]. In addition, a centralized EMS [153] requires CC to gather DERs and loads information for the purpose of optimization [79]. However, this information may not be available due to privacy [79,154]. A review of energy management by strategic deployment of DERs has been presented in [155]. Authors in [156] employed multi agent systems for the energy management of DGs in networked microgrids in such a way that different entities can participate in the market. A survey of multi agent systems to control microgrids can be found in [157]. In [158], an agent-based EMS is proposed to control the operation of a system of microgrids, in which customers can participate in DR. Optimal coordinated control of networked microgrids considering operational and economical objectives in the distribution grids is discussed in [159]. Furthermore, a review of classification of control strategies in the hybrid AC/DC microgrids can be found in [160].

3.7. Solution algorithm: mathematical approach versus heuristic algorithm

The OPF problem can be solved by some available optimization techniques. In general, they may be classified as mathematical and heuristic approaches. Mathematical methods refer to the mathematical representation of the problem. Some of these methods include calculus methods (in which both the OF and constraints are continuous and differentiable), LP [161] (in which both the objective function and constraints are linear functions of variables), NLP (in which the objective function and/or constraints are nonlinear functions of variables), dynamic programming [162] (it is a multistage decision problem in which optimal decisions have to be made over some stages), Integer Programming (IP) (in which the control variables have integer values), etc. Most of these approaches can guarantee reaching an optimal solution (local optimum), while do not necessarily guarantee reaching a global optimum (it should be noted that global optimum can be guaranteed for special cases such as LP problems as they are convex nature). Moreover, mathematical methods may never solve highly complex problems.

Heuristic algorithms eliminate the main drawbacks of mathematical approaches and can sometimes solve highly complex problem in a reasonable computational time. Most of these algorithms are based on biological behaviors. Basically, they start from either a point or a set of points (population), moving through a guided search towards a better solution.

In this regard, conventional OPF methodologies have been applied for decades to the bulk power systems. These methodologies are mainly based on gradient method [75], the usage of Lagrange multipliers, KKT conditions, and LP [163]. Some other approaches such as sequential quadratic programming [161], and approximate dynamic programming [162] have been presented in relation to smart grids. Because of the nonlinear nature of OPF, which is mainly related to discrete control variables and some continues variables, the conventional methods are not so effective and therefore, are not greatly emphasized. As a result, several meta-heuristic approaches have been proposed. Some of which include applying PSO and Differential Evolution (DE) to reactive power control [164], hybrid approaches with Artificial Neural Networks (ANNs) [110], and Evolutionary Algorithms (EAs) techniques [165]. However, due to the emergence of smart grids, as a new conceptual approach, significant changes in OPF approaches in terms of new constraints and different objective functions adopted for these types of studies are observable. As an example of new OPF methodologies in smart grid, this type of grids is mostly equipped with the most advanced communication and information technologies [166,167], where a PU is used at each bus. PUs use high speed optical fibers between buses in order to exploit the smart grid as a supercomputer and provide an infrastructure for distributed and parallel computation. Therefore, the computation speeds up in the smart grid to meet the real-time requirement in the smart grid (such as power generation of renewable energy sources). In this case, the information on the most updated electrical generation in distributed renewable energy sources can be collected by each PU to corporate with the distributed computation in solving OPF problems [20]. In [76], buses only exchange information with their neighbors, and no information is needed about the network's topology.

In [11], a hybrid optimization method, namely Harmonic Search-Genetic Algorithm (HS-GA), is proposed to plan operation of an autonomous microgrid in terms of optimal sitting, maximum capacity, and droop parameters of DGs. Paper [168] reported a GA-based OPF for DC microgrids. In [4,80,96], PSO algorithm is employed to find the best solution. Mixed Integer LP (MILP) is used in [81,82] to optimize a DC microgrid power flow under a supervision control. Moreover, in [83], an adaptive modified firefly algorithm is suggested for the purpose of solving problems. Authors in [169], introduced an approach based on the matrix real-coded genetic algorithm to find a three-phase smart strategy (including forecasting, storage, and management modules) for optimizing the microgrid operation. In [84], a self-adaptive modification technique is applied to θ-PSO algorithm. Ref. [85] employed a firefly algorithm. In [170], glowworm swarm optimization [171] is used to solve multi-objective optimization problems.

3.8. Realistic description: deterministic versus uncertainty

Deterministic analysis does not consider the influence of the uncertainties arisen from high penetration of RESs in the new power grids, load demand forecasting errors, and etc. In this condition, due to the inaccurate data, the total operation can be affected seriously. Furthermore, the final optimal solution may not be the best real operating point. In this context, the utilization of the stochastic frameworks can be useful [84]. In a technical categorization, there are three main methods to consider the uncertainty effects [84,172,173]: 1) Monte-Carlo simulation, 2) analytical methods, and 3) approximate methods. The first method is accurate to handle complex uncertain variables; but, it is computationally expensive [84]. Analytical techniques have been introduced through employing some mathematical assumptions to simplify the problem and overcome the deficiency of Monte-Carlo simulation [174]. Some approximate methods have been suggested in order to model both shortages of two mentioned methods e.g. the first-order second-moment method [45,175–177], Taylor series expansion method [176,178], Cumulant method [46], the common uncertain source method [179], Discretization method [179], and PEM [47,84,180]. In this context, a complete review on uncertainty modeling techniques in power system studies can be found in [181].

Uncertainty of input parameters can be represented through uncertainty sets. Robust optimization [36,77,96,181–183] ensures that the obtained decisions remain optimal for the worst-case realization of the uncertain parameter within a given set [181,184]. In fact, this is a new method to solve optimization problems affected by uncertainty specially in case of lack of full information on the nature of uncertainty [181,182]. A short description of robust optimization has been illustrated in [181].

4. Different equipment and strategies

Smart grids are equipped with different strategies and devices. Among them renewable energy, storage devices, DSM, automation based on a high penetration of Information and Communication Technology (ICT) [69] can be mentioned. For the purpose of optimal power flow in smart grids, the main strategies are briefly discussed here.

4.1. Energy storage devices

These devices match energy production to consumption and facilitate a smooth and robust energy balance within the system [185]. Stored energy is controlled to balance power production of renewable sources such as wind power so that overall power consumption at the PCC is optimized. Assume that a renewable source such as wind farm [186], is coupled with an energy storage device such as a battery. During high winds, energy is stored in the battery. This energy is released when wind is low while smoothing total power injected to the system. In this context, an optimal energy storage control strategy for grid-connected microgrids can be found in [187]. In [188] optimal sizing of battery energy storage for microgrid management is presented. Also, optimal allocation and economic analysis of energy storage system in microgrids is reported in [189]. Assuming that the voltage magnitude of a device is specified, wind power may be replaced by an auxiliary P–V unit, with known power and voltage values [68].

4.2. Demand-side management

DSM as an open strategy to improve balancing generation-consumption and regulation load profiles, affects overall demands, while cannot control demands in specific areas or locations in the system. Moreover, this strategy has significant potential for demand-responsive loads to alleviate network contingencies and manage constraints. Three are a lot of combined and coordinate DSM actions from many highly-distributed users in order to achieve the demand quantities required to participate in balancing mechanism and make a significant contribution to grid ancillary services. The “aggregator” [190] and “virtual power plant” concepts [191,192] are two examples in this regard. In order to realize DSM, ICT technologies should be used in the residential and commercial load sectors. This results in a new market mechanism as discussed in [193,194]. Note that, one of the key challenges of designing DSM models includes the need for modeling customer behavior; because, in the smart grid, demands are expected to be an integral part of system [195].

4.3. Energy management system

In microgrids, an Energy Management System (EMS) is essential to manage power flow through the network. There are two main methods to develop EMSs [77]:

- Rule-based: In this approach, power flow management is implemented according to some predefined rules based on operating modes [196]. However, it cannot provide an OPF solution [77].
- Optimization-based: This method manages power flow through optimizing an OF according to performance expectations of the microgrid while satisfying operational constraints.

Among the above strategies, the optimization-based EMS has attracted more attention for AC, DC, and hybrid AC/DC microgrids. Accordingly, [197,198] proposed an optimization-based EMS for a DC microgrid [199] for the purpose of limiting the battery charge current and setting the wind subsystem as the primary resource while satisfying the load demand. A similar approach to optimize the running cost has been considered for AC microgrids in [200,201] and for DC ones in [81,82,202]. Another optimization-based EMS is considered in [78] for AC microgrids to minimize running and fuel costs as well as start/stop frequency of DG. In this context, [203] optimized the solar power profit in a wind-battery microgrid while [204] minimized the energy bill of the owner of a PV-battery system. In another study by [183], a robust EMS for grid-connected mode has been introduced to apply uncertainties (related to the generation prediction) to the design procedure. In [205], a rule-based EMS for hybrid AC/DC microgrids is introduced considering a wind generator as an AC source and a PV as a DC source. This method has been used for 15 distinct operation modes in [206]. An optimization-based EMS, namely Robust Optimal Power Management System (ROPMS), for such microgrids has been presented in [77]. A distributed EMS based on convex formulation and dual decomposition can be found in [183]. A characterization of EMS has been presented in [207]. Economic analysis and optimal energy management models for a specific microgrid are presented in [208]. A generalized formulation for intelligent energy management of a microgrid based on artificial intelligence techniques is introduced in [209]. A real-time EMS for islanded microgrids is introduced in [210]. Paper [211] proposed an EMS for enhanced resiliency of islanded microgrids. In [212], EMS is applied to smart grids. Moreover, a review of existing optimization objectives, constraints, solution approaches and tools used in microgrid energy management has been presented in [213]. Authors of [214] reviewed EMSs in microgrid operations. A review related to optimal control techniques for energy management and control in microgrids can be also found in [215].

4.4. Phasor measurement unit

Phasor measurement unit or PMU as it is more commonly referred to, is the most accurate and advanced time-synchronized technology available to provide voltage and current phasors and frequency information [216] (synchronized phasor measurements [217] or synchrophasors [218]), synchronized with high precision to a common time reference provided by the Global Positioning System (GPS) [219]. Further, real-time state information is provided by phasor data concentrators [218]. PMU operation is based on numerical measurement algorithms and the collected data to support state estimation [220] and enable multiple applications including distributed wide area control, protection, wide-area situational awareness, post-event analysis [218]. Nonetheless, its particular application is for dynamic-response-type applications [219]. Initially, PMUs were considered for transmission systems; but, today, their applications have been extended to distribution networks [221–225] to improve the observability of such grids [218]. In this context, micro-synchrophasors (μPMUs [226]) are high-precision measurement units that can work well in distribution grids [227]. Therefore, a fast communication network is needed for safety- and time-critical applications [228].

Different PMU placement methods have been proposed e.g. based on convex relaxation in [229], binary linear program in [230], binary quadratic program in [231], and probabilistic approach in [232]. Optimal PMU placement for identification of multiple power line outages in smart grids has been introduced in [233]. Employment of PMUs in the optimal operation of smart microgrids has been addressed in [234].

Fig. 4 illustrates the real time application of PMU information in the OPF studies [235]. As it was depicted, the entire data are provided by PMU [236] and/or state estimation. Then, the base case solution for linearization of constraints is obtained through the power flow analysis. Finally, the OPF model is formulated and an optimization method is applied as well as different constraints are checked.

Ref. [237] focused on the Standing Phase Angle Difference (SPAD) for power system restoration. If an excessive SPAD is detected across lines, then the restoration process may be delayed. As a result, the economic and social costs will be increased. So, an inequality constraint should be applied to OPF formulation to ensure that the SPAD is smaller than a predefined maximum value as follows:

$$|\delta_i - \delta_j| \le |\delta_i - \delta_j|_{max} \tag{47}$$

where δ_i and δ_j are the measured voltage phases at buses i and j by PMUs, respectively.

4.5. Multi-carrier energy system

Todays, different industrial and commercial consumers require various forms of energy services provided by different energy infrastructures which are most often considered and operated independently [238–240]. For example, nearly all reviewed papers considered the electrical power only. In fact, they did not investigate an integrated view of energy systems including multiple energy carriers e.g. electricity, district heat, natural gas. Its main motivation is given by utilization converter elements. In these devices, power is converted between different carriers and makes a coupling of the corresponding power flows resulting in system interactions [238,241–243]. For example, a CHP unit [244] consumes natural gas to produce electricity and heat simultaneously. So, this device can affect power flow in three systems including gas, heat, and electrical networks [238]. Energy Hub (EH), as a unit, creates a connection point between different infrastructures and consumes various forms of energy to provide different

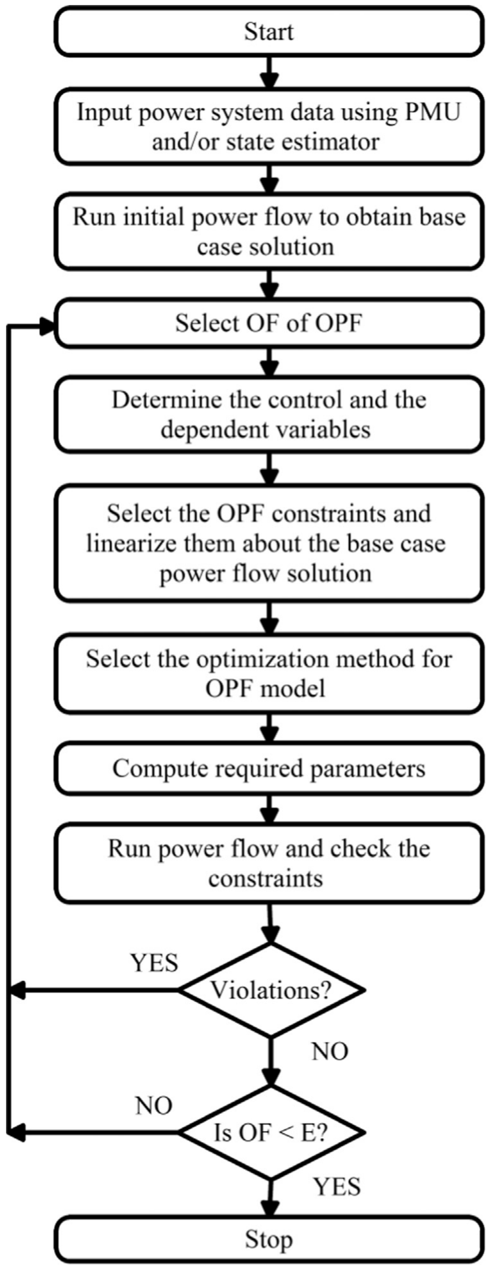

Fig. 4. Application of PMU in the OPF study [235].

carriers at its output [238,245–248]. This unit is constructed by different elements including converters (e.g. CHP, gas furnace, etc.), direct connections (e.g. electrical cables, pipelines, etc.), and storage devices (e.g. battery, etc.) [238]. Therefore, as mentioned earlier, by using EH, different systems affect each other [249].

The mentioned concepts affect various researches in power systems e.g. PF [250] and OPF [238] studies. The OPF has been extended to a new framework, namely Optimal Energy Flow (OEF) [251], to optimize energy flow through multi-carrier energy systems [238,251–254]. Also, EF refers to PF; except is applied to analyze the multiple energy carriers system [250]. Therefore, the presence of EHs in the future vision of energy networks can create an opportunity for electrical engineers to move toward more efficient energy grids. Moreover, it is envisioned that smart grids can cover the natural gas system. Smart EH as an upgraded model of conventional EH for smart grids has been introduced in [255]. An application of this model is real-time energy management [255]. Integration of smart EH with DSM and EMS can be found in [256,257]. Integration of DR in smart EH has been discussed in [258]. In [259], optimal operation of smart EH has been reported.

It should be noted that combining the mentioned concepts with OEF in the smart grids provides more efficient energy systems in the near future and will be the subject of future researches.

5. Other aspects of comparison

5.1. Computational performances

The purpose of this section is to compare the computational performance of the proposed method as identified in [1,4,11,20,65–87]. The computational performance such as CPU time and the maximum iteration at each case is compared for each case study network. Moreover, specific softwares/programs applied in each case are described in Table 4.

In Table 4, case study networks presented in [4,65,68,71,73,75–78,80,81,85,142,154,260–276] and some specific analysis programs presented in [275,277–282] are used to demonstrate the comparative analysis related to the feasibility and effectiveness of OPF approaches presented in [1,4,11,20,65–87].

It should be mentioned that all the previous approaches ensure convergence and their computational speed is dependent on a number of factors such as the number of buses, initial points, phase balance, network structure, objective function, and constraints [65,69,70]. This is mainly due to development of the state-of-the-art smart grids and arises from the support of bi-directional communication devices, processing information in real-time, as well as dynamic changes of the grid [283].

5.2. Accepted date and name of the method

Another useful classification is based on the accepted date of the papers shown in Table 5. This table shows that the number of studies on OPF in smart grids and microgrids has been seriously increased since 2009. Further, the table indicates that more than 80% of the papers have been accepted in 2013–2015.

6. Main challenges

The OPF problem solution in smart grids and microgrids is based on one or several ideas, generally causing various challenges in computational or technical fields. The main challenges related to OPF methodologies can be addressed as follows:

- Increase of the dimension of the problem especially in distribution systems at lower voltage levels and consequent increase of computational complexity [67].
- Dealing with unbalanced networks at lower voltage levels and adopting the TOPF [69].
- Integration of the storage devices which imply the representation of State of Charge (SOC) across the time domain [4,68,96,142]. Growth of the numerical complexity with a number of the different storage devices [68].
- Complete decomposition of the OPF problem into a set of subproblems [20].
- Computational synchronization of asynchronous data arrival [20].
- Implementation of termination conditions [20].
- Use of the SDP relaxation method to convert the non-convex OPF problem to the convex and semi-definite one for balanced networks in [65] and unbalanced networks in [71].
- Development of a decision-making framework for distribution systems operating with multi-faceted players with flexible operating possibilities [70].
- Conversion of MINLP problems into NLP problems [70].
- Applying MILP to the optimization problem [81,82].

Table 4
Computational Performances of Some Reported Studies.

Reference Number	Test subcase	Maximum CPU Time (s)	Maximum Iterations	Test Systems	Type of PC that is Used	Specific Programs and Windows
[1]	–	–	–	● IEEE 30-bus[256]	–	● PSO environment
[20]	Case 1[a] Case 2 Case 3	0.006 0.081 0.089	LPG/RQP=20/1 LPG/RQP=21/12 LPG/RQP=21/13	● 26-bus[257]	Single Core 2 Quad with 2.00 GB RAM PC	–
[65]	Large Network (60 nodes)	1.804	29	● Proposed Power System in[65] ● Power System of[258]	–	● MATLAB ● CVX with SeDuMi solver
[66]	–	–	–	● IEEE 30-bus ● IEEE 118-bus[256]	–	● MATLAB ● MATPOWER[271]
[67]	On NEV IEEE	< 0.1	17	● IEEE 14-bus[256] ● IEEE 34-bus[258] ● NEV System[258]	C2Q, 2.83 GHz, 8 GB RAM PC	● Windows XP ● C++
[68]	–	Over a 72-h period	–	● Power system of[259] ● Proposed Microgrid in[68]	–	–
[69]	Test B[b] Test C Test D	2.48 2.53 36.86	18 18 27	● IEEE 123-bus[260]	HP Compaq 8000 Elite CMT PC with Intel Core 2 Quad CPU Q 9650 at 3.00 GHz and 4.00 GB RAM	● MATLAB ● Open DSS[273]
[70]	Case 2 for IEEE 13-node[c] Case 2 for Hydro One distribution feeder	4.76 10.08	– –	● IEEE 13-node[260] ● Hydro One distribution feeder[261]	Intel machine with eight 2.83 GHz 32-bit virtual processors and 3 GB memory	● Windows Server 2003 ● MINOS solver[274]
[71]	–	6	–	● IEEE 37-node[260] ● 10-node Microgrid[71]	Machine with Intel Core i7–2600 CPU at 3.40 GHz	● MATLAB ● CVX with SeDuMi solver
[72]	–	–	–	● U.K. Radial Distribution System[262] ● U.K. Generic Distribution System (UKGDS)[263]	–	● PSS/E[275] ● Interior point method (IPM)
[73]	IEEE 30-bus	1.792	28	● 13-node Industrial System[73] ● IEEE 14-bus ● IEEE 30-bus	Intel Pentium Dual Core Processor T3200, 2.0 GHz with 2.0 GB RAM PC	● Windows 7 ● MATLAB
[74]	–	–	–	● 33-bus distribution network[264]	Two processors Intel® Xeon® W3520 2.67 GHz, each one with two cores, with 3 GB of RAM PC	● Windows 7 ● GAMS (general algebraic modeling system)[276]
[75]	–	–	–	● 6-bus test system[75]	–	● MATLAB
[76]	–	–	–	● 9-bus test microgrid system[76]	–	● MATLAB
[77]	–	Over a 200-s period	–	● Hybrid AC/DC microgrid[77]	Intel(R) Core(TM) i5–2410 M CPU 2.30 GHz with 4.00 GB of RAM	● MATLAB/Simulink package/ Optimization toolbox
[78]	–	0.09	30	● Budapest Tech Renewable Equipment (real case in Hungary)[78]	–	● GAMS (CPLEX[277])
[79]	–	Over a 24-h period	–	● A real microgrid in China[154]	–	–
[4]	–	Over 300-sample	–	● A system of microgrids[4]	–	● PSO environment
[96]	–	Over 24-d	–	● A low voltage microgrid[265]	–	● PSO environment
[11]	–	–	–	● 33-bus system[266] ● 69-bus system[267]	–	● MATLAB ● HS-GA
[102]	–	600[d]	–	● A smart microgrid[102]	–	–
[80]	–	Over a 24-h period	–	● A 15-node distribution system[80]	–	● PSO environment
[81]	–	3[e]	–	● A DC microgrid[81]	A normal computer with Core i5 processor	● CPLEX
[82]	–	10	–	● A DC microgrid[f]	A normal computer with Core i5 processor	● MATLAB-Simulink ● CPLEX
[83]	First status of operation	173.75[g]	–	● A low voltage microgrid[268]	–	● Adaptive modified firefly algorithm
[84]	First scenario	8.12[h]		● A low voltage microgrid[268]	–	● A modified version of θ-PSO
[85]	$2m + 1$ scheme[i]	0.126	–	● A low voltage network[85,269]	Pentium-IV, core i3, 2.13-GHz PC with 3 GB of RAM	● MATLAB ● Firefly algorithm
[86]	–	–	–	● 4-bus network[270] ● 39-bus New England test network[271] ● IEEE test networks consisting of 14, 30,	A single core 64 bit Linux machine with 8 GiB RAM	● CPLEX called from an AMPL[278]

(continued on next page)

Table 4 (*continued*)

Reference Number	Test subcase	Maximum CPU Time (s)	Maximum Iterations	Test Systems	Type of PC that is Used	Specific Programs and Windows
[87]	IEEE 118	48.62[j]	3000	57, 118, 300 buses[256] • 9- and 24-bus test cases in[271] • 3-bus system[272] • Case 9[271] • IEEE 14-bus system • IEEE 30-bus system • IEEE 118-bus system • IEEE 300-bus system	–	• MATLAB

[a] See [20].
[b] See [69].
[c] See [70].
[d] See [102].
[e] See [81].
[f] See [82].
[g] For stochastic environment.
[h] For deterministic environment.
[I] See [85].
[j] The reported time is related to parallel running time.

Table 5
Accepted Date and Name of Some Selected Methods.

Reference Number	Name of Method	Accepted Date
[78]	–	February 25, 2009
[69]	TOPF	December 16, 2010
[70]	TDOPF	January 19, 2011
[85]	–	September 12, 2012
[65]	LSDP	September 18, 2012
[68]	–	February 02, 2013
[71]	–	February 16, 2013
[84]	–	March 19, 2013
[20]	DPOPF	April 09, 2013
[66]	–	April 23, 2013
[67]	MOPF	May 10, 2013
[83]	–	August 8, 2013
[72]	–	December 20, 2013
[1]	–	January 11, 2014
[82]	–	January 18, 2014
[81]	–	March 2, 2014
[80]	–	May 11, 2014
[102]	–	July 19, 2014
[79]	–	November 16, 2014
[4]	–	December 30, 2014
[87]	ADMM-DOPF	2014
[75]	TOPF	January 24, 2015
[96]	–	February 11, 2015
[77]	ROPMS	February 15, 2015
[74]	–	March 14, 2015
[73]	–	June 16, 2015
[76]	DOPF	October 9, 2015
[11]	–	November 17, 2015
[86]	–	November 17, 2015

- Guarantee of a feasible solution space for NLP problem [70].
- Participation of ISO in limiting load curtailment [1].
- Consideration of the post-contingency state and reserve structure for day-ahead problems [66].
- Modeling load and generation uncertainties [4,84].
- Extraction of OPF for the networked micrograds [4].
- Optimizing a DC microgrid by multi-layer supervision control [81,82].
- Handling a non-quadratic convex OF [87].
- Applying robust optimization to an OPF problem [36,77,96,181–183].
- Implementation OPF in DC grids [77,78,81,82].
- Use of PMUs for optimal operation of smart microgrids [234].
- Implementation of EMSs [77,78,81,82].

7. Conclusion

Since the publication of the first OPF methodology for bulk power systems, numerous contributions to the development of basic idea of OPF have been proposed to suit the requirements of many applications. The advent of microgrids and then smart grids with their unique features and infrastructures to overcome most of the operational analysis such as OPF has added a new chapter to the field of power systems. As discussed, the concept of smartness in these grids helps to overcome the limitations of the traditional OPF and provides a novel optimal operation concept. The superior performance of OPF approaches in smart grids has attracted researchers and power system companies all over the world. In this field, various methodologies, objective functions, and constraints are suggested. Accordingly, a deep analysis of them (i.e. new OPF methodologies, objective functions, constraints related to the smart grids, etc.) was carried out in this study. In fact, this research reviewed and compared OPF approaches of smart grids and smart microgrids from different perspectives in order to provide an overall vision of this problem. This categorization and survey help researchers to comprehend all of them. The presented list is by no means complete, but it can be used as an essential guideline for anyone entering this research field. In general, regardless of the approach used to solve the OPF problem, the models so far developed have at least one of the following specifications.

- The comparison between objective functions shows that most of the proposed approaches consider the power to be supplied from the utility at PCC in the set of objective functions. In other words, most of the reported approaches, have considered the electrical smart grid in a grid-connected mode. A more flexible function to cover most of the objective functions such as line loss, PCC cost, quadratic active power generation cost, and piecewise polynomial forms are suggested by [68].
- Moreover, all methods have considered the power generation limits in the set of constraints. Further, a number of methods have also considered the important role of discrete variables (and limitations on them) and related optimization on the optimal operation of a power system. Moreover, these studies investigated the proper coordination among traditional control variables with DG controls.
- The review shows that most of the approaches have considered the smart microgrids/grids as unbalanced distribution systems. As previously discussed in this study, the reason is the inherently unbalanced nature of the distribution systems. So, a more effective optimization method should be employed. As a result, each power system type has a set of optimization approaches which is more suitable for its purpose.
- Most efforts were made to simplify the OPF approaches and reduce its computational complexity, especially when applied to lower voltage distribution systems. To this end, the authors attempted to convert a MINLP formulation to a NLP one, a non-convex problem to a convex one, a highly constrained problem into an unconstrained one, etc.

As the main drawbacks that are still in this field, following items should be addressed:

- Various aspects related to operation, reliability, security, and quality issues are not taken into account by the proposed algorithms as an integrated problem.
- Multiple contingencies are not considered.
- The change in the load curtailment costs related to various operation intervals is neglected.
- Some constraints such as topology and stability constraints are not discussed.
- The impact of the market practice on reactive cost is neglected.
- The role of multi-carrier energy systems as a potential strategy for the purpose of grid optimization has not been fully considered.
- New version of OPF, namely OEF, has not been employed in smart grids and microgrids.
- Impact of smart EHs on OEF has not been investigated.

and finally, challenges are mostly derived from:

- novel and comprehensive algorithms to solve real world OPF problems, mainly based on the heuristic approaches,
- new applicable and comprehensive analysis software,
- the modeling of new constraints (such as capacitor bank switching or tap position),
- the modeling of uncertainties,
- the use of new components such as storage systems and the reduction of the related numerical complexity,
- new applications, and consequently new objective functions (for example linked to the problem of storage devices allocation),
- main assumptions related to smart grid infrastructures such as PUs, AMIs, smart EH.

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