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A game equilibrium model of a retail electricity market with high penetration of small and mid-size renewable suppliers

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ABSTRACT

Game theory has provided a practical tool to model players' strategic behavior in electricity markets, particularly as the world moves towards a more competitive market. A game theoretic approach can be used to find the clearing electricity price in a retail electricity market with a high penetration of small and mid-size renewable suppliers.

1. Introduction

Traditionally regulated electric power markets have undergone massive changes due to environmental and economic incentives. Therefore, deployment of a market structure that favors more competitive and less regulated models, such as the retail competition model, has been a worldwide trend over the last few decades. Market models can be classified into centralized and decentralized versions (Barroso et al., 2005). Considering the wide range of varieties in electricity market structures, several methods have been introduced to analyze and optimize different aspects of deregulated electricity markets. These models vary significantly at the level of competition (Bompard et al., 2010) and can be ranged from the most uncompetitive situation, Stackelberg (Day et al., 2002) to the most competitive model, Bertrand Competition (Haraguchi and Matsumura, 2016; Ma et al., 2015; Younes and Ilic, 1999). There is a rich literature in modeling strategic interactions in electricity markets. Yang at al. (Peng et al., 2013) obtained the Nash equilibrium using backward induction to model the costs to utility companies arising from fluctuations in user demand. Song et al. (Song et al., 2002) employed the Nash equilibrium to analyze bidding strategies in a bilateral market. Market clearing prices within a hydrothermal power exchange market were found by developing a Nikaido-Isoda function to achieve the Nash-Cournot equilibrium in (Molina et al., 2011). In (Kiani and Annaswamy, 2010) authors analyzed the energy market in the presence of renewable energy resources and demand response. Chen et al. (Chen et al., 2010) developed a distributed demand response algorithms and achieved the equilibria in both a competitive and oligopolistic market. A non-cooperative game was employed by Sikdar et al. in (Sikdar and Rudie, 2014) to model a trade mechanism through the example of electricity trade at an electric vehicle charging facility to help create decentralized markets. Despite these scholarly efforts in finding the market equilibrium, the retail sector of the electricity market has not been extensively studied to the best of the author's knowledge. It is necessary to comprehend how the retail market responds to recent technological developments that allow the high integration of small renewable suppliers in a competitive context. In the proposed electricity market, the end users of electricity are actively engaged in the market either through generation or load management. This study covers the challenges at the intersection of the foreseeable future technologies, namely smart grids, and the concept of game theory from an economic point of view.

Introducing competition in a deregulated market structure gives rise to a high penetration of renewable resources, particularly wind and solar energy, and thus enables distributed generating units that are economically efficient (Negrete-Pincetic et al., 2015). Integration of these units at the residential level into the power grid can alleviate concerns regarding anticipated high load demand and sustainability issues. Small renewable suppliers, if employed at a large scale in the residential sector, can compensate for the high costs of operating reserve capacity in the utility grid. The resulting financial advantage will be shared among both consumers and the utility grid. Consumers will be financially incentivized by selling their generated electricity. The utility grid will benefit from both the reduced costs of the reserved capacity and increased ancillary services which are required by these small suppliers. Searching for possible market equilibria has been an objective for market participants (Pozo et al., 2011), since it is the most beneficial strategy for all the agents. It empowers all players to make an optimal decision, based on their competitors' choices. Due to the

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multitude of both suppliers and consumers in this market structure, this article employs a game theoretic method to clear hourly electricity market prices in a deregulated retail electricity market. The game modeling and the market framework in this study are unique and necessary for a better understanding of the future of the electricity markets.

The remainder of this article is organized as follows: Section 2 introduces the main features of the market model and formulates the problem based on a game theoretic approach. Section 3 describes the main features of game theory and Nash equilibrium and illustrates the modeling of the games in this problem. Section 4 presents the simulation results, including the optimized behavioral pattern of each of the market participants and the clearing electricity prices. Section 5 concludes this article's findings and discusses future directions. A detailed description of the mathematical formulation of the problem can be found in Appendix A.

2. Materials and methods

2.1. Market framework

In the past few years, environmental concerns, increasing penetration of electric vehicles and subsequent concerns regarding high load demand, smart metering and energy storage needs, as well as the urgent need for a more efficient and reliable electricity network, have necessitated a more complicated and intelligent construct within the electricity market. The smart grid, defined by the Smart Grids European Technology Platform as: "electricity networks that can intelligently integrate the behavior and actions of all users connected to it – generators, consumers and those that do both – in order to efficiently deliver sustainable, economic, and secure electricity supplies" (Smart Grids Advisory Counsel, 2010) has captured great interest as a reliable and secure grid. Smart grid technologies enable the integration of small renewable resources at the residential level. These small suppliers are equipped with various generation and storage units, such as wind turbines, solar panels, diesel generators, and distributed energy storage devices (DESD) and are able to communicate and exchange information with other agents. This communication not only results in maximizing profits but also improves market stability and reliability. As a result, substantial innovations and cost reductions in the future of the electricity market can be expected. Plug-and-play technology enables customers to connect to the utility grid at any time, in order to buy or sell electricity. It provides an interface for all agents to be easily recognized as soon as they connect to the grid and collects information regarding loads, storage, and generating units of that agent. This feature is facilitated by the bidirectional flow of electricity (Bae et al., 2014) in smart grids. In this system, electricity suppliers become independent of their conventional role (Su and Huang, 2014). While the main focus of literature in the last few years has been on the distribution power operations of utility companies, increasing consumers' active engagement in a deregulated competitive market calls for an urgent attention to further address the technical concerns regarding distributed energy generation and storage.

In a traditional electric market, electricity price is set by regulations. In a retail competitive model, however, consumers' active involvement in the market will eventually result in a market with controlled lower prices. In this article, we employed an inverse-demand function to obtain the hourly electricity prices. These prices are a function of the aggregate load demand. Thus, by managing their dispatchable load demand, consumers are constantly involved in setting the market prices. The electricity cost function is based on the well-known Cournot model, which is widely used to approximate competition in the electricity market (Kwang-Ho Lee and Baldick, 2003; Siriruk and Valenzuela, 2011). Since suppliers are infinitesimal, they have no effect on the market price (Novshek, 1980). In this market structure, the role of utility grid is considerably different than in traditional models. The utility grid no longer monopolizes the whole market. In fact, it appears as a complementary unit to compensate for the deficiency of power from small suppliers. It is responsible for implementing the necessary infrastructure to enable secure communication among market participants. It can make a profit not just by selling electricity but by providing ancillary services to various consumers including small or mid-size suppliers. This article excludes the role of the utility grid as an active player in the game. Its main focus is on the interactions among a large number of suppliers and consumers. Thus, the grid is not considered a separate player, though its role is conspicuous when suppliers prefer to buy electricity from the grid rather than switching on a diesel generator with a high cost function.

In every market, participants strive to achieve maximum profit. At the same time, they are very well aware of the fact that their competitors' decisions will influence their results. Game theory provides a tool to analyze the strategic interactions among market participants (Singh, 1999). Depending on market characteristics, various game approaches can be employed to find a market's equilibrium. Since the proposed model is highly reliant on the active participation of multitudes of small or mid-size renewable suppliers, the market structure would be complex and dynamic. In the first step, since consumers are separate entities, a non-cooperative game is employed to find the Nash Equilibrium among consumers. The interactions among suppliers are modeled by a cooperative game. From suppliers' perspective, collaboration is not only possible, but can result in a more stable market as various suppliers share information. This coalition is facilitated by the utility grid, which enables small suppliers to have access to the necessary information for this cooperation to take place. Finally, a non-cooperative game among consumers and suppliers is taken into consideration to find the Nash equilibrium. In a Nash equilibrium, all the market participants can achieve the highest possible outcome. By employing a design of experiment approach, the rational reaction set of market participants can be obtained. These rational reaction sets are used to model the interactions among the consumers and the suppliers to find the Nash equilibrium. It reveals the clearing price at each hour, as well as the optimum behavior of each of the participants. Several well-known equilibrium models have been introduced and applied to electricity markets in the last few decades. For the proposed model, the same assumptions and features as the Cournot model are taken into consideration (Allaz and Vila, 1993; Vives, 1984):

- All units produce a homogeneous product.
- The market price is influenced by the total supply and therefore is fixed for all units.
- Each firm's output decision affects the market price.
- The number of firms is fixed during the market clearing price.
- Firms compete in quantities and act simultaneously.
- Each player is considered to be rational.

2.2. Problem formulation

This section formulates the mathematical model and key concepts in a highly competitive retail electricity market. This model allows for a high penetration of distributed generators (DG) and distributed energy storage devices (DESD). Market participants can be categorized into three groups: small suppliers, consumers, and the utility grid. However, the utility grid is not an active player in the game modeling of the problem. The objective function and constraints for each entity is formulated in mathematical terms.

2.2.1. Objective functions

For suppliers, the objective function of the ith player is defined as the summation of differences between revenue and cost over 24 h in one-hour intervals. This set of players seeks to maximize their objective function. This goal can be achieved whether by minimizing costs or maximizing revenue at each hour.

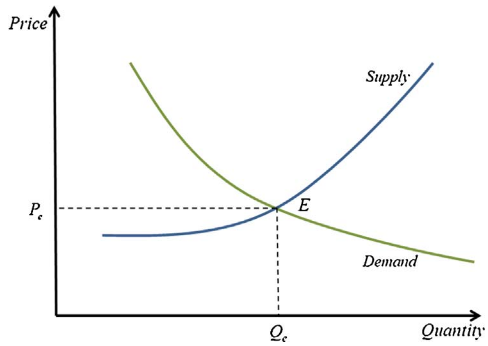

Fig. 1. Market Equilibrium.

$$\mathrm{Max}F_i = \sum_{t=1}^{t=24} (R_{i,t} - C_{i,t}) \tag{1}$$

Where Ri,t and Ci,t are the revenue and cost functions, respectively, of the ith player at tth hour. The electricity price is a function of the aggregate load demand. Demand in the market at each hour is characterized by an inverse-demand function and is presented with a negative slope.

$$\lambda(Pd) = -\propto \times Pd + \beta \tag{2}$$

Where Pd is the total load demand, λ is the electricity price in \$/kW h, and α and β are load demand curve coefficients. The retail electricity price is assumed to be identical for the whole residential distribution system. This electricity price function is based on the basic notion of supply and demand. Two different groups, electricity suppliers and consumers, are taken into consideration to determine the price of electricity at each time interval. This price is the intersection of supply and demand curves and is called the market equilibrium. The law of demand highlights the inverse proportionality of price and demand for the same quality of goods. This explains the downward sloping of the demand curve in Fig. 1. The supply curve demonstrates the relationship between the electricity price and the quantity that they can offer. Therefore, as the price increases, the quantity of the good supplied will also go up. In this model, small suppliers are equipped with wind turbine, solar panels, diesel generators, and storage devices. The electricity demand of each individual supplier is considered to be negligible compared to their generation. Although there are multitudes of factors which play an active role in the price of electricity, in this article, the cost of electricity is mainly a function of operating costs (Brinckerhoff, 2012). Finally, the cost function associated with diesel generators can be approximated as a quadratic function.

The fuel consumption of a diesel generator is a function of its capacity, as well as the load at which it is operating. The exact values for these coefficients are available for DGs with a high power rating (Djurovic et al., 2012; Mohamed and Koivo, 2010; Park et al., 1993; Zhai et al., 2009). However, since in this article a great number of suppliers are considered to own small or mid-size generators, an estimate of how much fuel a generator consumes is approximated as a quadratic function (Diesel Service and Supply, 2014). A detailed mathematical model for revenue and cost functions is provided in Appendix A. According to the defined revenue and cost functions, Eq. (1) can be written as:

$$MaxF_i = \sum_{t=1}^{t=24} (R_{i,t} - C_{i,t}) = \sum_{t=1}^{t=24} \lambda(Pd) \times [Pw_{i,t} + Ps_{i,t} + Pdg_{i,t} + Pde_{i,t}] - [a_i Pdg_i^2(t) + b_i Pdg_i(t) + c_i] \tag{3}$$

For the electricity consumers, the objective function is to minimize the cost by managing their own dispatchable load. For player i at tth hour, the objective function can be expressed as:

$$MinF_i = \sum_{t=1}^{t=24} \lambda(Pd) \times Pd_{i,t} \tag{4}$$

Where Pdi,t is the load demand of the ith player at tth hour. In this type of energy cell, each player has the ability to manage and control its hourly load demand, subject to a local constraint. As mentioned before, the main role of the utility grid in this model is to secure the critical load. In addition, suppliers can sell electricity to the grid at any time. However, a local constraint regarding the amount of tradable electricity of each unit is imposed on suppliers. The limited flexibility of the utility grid in this market structure makes it unable to exercise market power and thus, a price taker (Bompard et al., 2010). The utility grid doesn't act as an active player; it doesn't have the monopoly of the electricity market. It is considered as an infinite source of energy with the capability to compensate for power deficiencies in the market.

2.2.2. Constraints

Each agent is subject to a number of local constraints as well as a global constraint. The technical features of any wind turbine, solar panel, diesel generator, or any storage devices, impose some constraints on the objective function. These inequality constraints, which determine the boundaries of our solution space, are illustrated in detail in Appendix A. They are all imposed in an hourly manner upon every single unit. The required data regarding wind and solar power output were obtained through the System Advisor Model (SAM), developed by the National Renewable Energy Laboratory (NREL, 2012). SAM enables users to simulate models of renewable energy projects and was used in this article to simulate the power output of different wind and solar systems. Storage units are primarily constrained by their state of charge (SOC). To avoid any overcharging or over discharging of a battery, the statement of charge for each energy storage device must be within the safe range (Li et al., 2015; Maharjan et al., 2009; Rajasekharachari et al., 2013; Su and Huang, 2014). Consumers are limited by the lower and upper bounds on their manageable load demand, which is highly dependent on their habitual consumption behavior. Finally, according to the concept of conservation of energy, the amount of generated power is equal to the consumed power. Due to the small and mid-size capacity of the suppliers in this paper, the amount of power loss is considered negligible. This balance can be expressed as:

$$\sum_{i \in N} [Pw_i(t) + Ps_i(t) + Pdg_i(t) + Pde_i(t) + Pg_i(t)] = \sum_{j \in N} Pd_j(t) \tag{5}$$

The left hand side of this constraint indicates the generated power in the market. $Pg_i(t)$ refers to the amount of electricity which is bought or sold from the electricity grid. The right hand side shows the total consumption by consumers. Therefore, this equality constraint must be satisfied for the whole market model for any given hour.

3. Theory

In modeling an electricity market, strategic interactions among players must be taken into consideration. These interactions are modeled with the objective of maximizing the profit for each player. Power suppliers and consumers choose strategies to gain the maximum payoff. The payoff function for suppliers amounts to the power sold to consumers or utility grid. For consumers, the payoff function is represented by minimizing the electricity bill. Game theory provides a tool to model this context. This article considers a case in which some suppliers communicate and share information with each other to form a coalition. However, no cooperation is considered among consumers. After that, a Design of Experiment – Response Surface method (DOE-RSM) method is employed to find the Nash equilibrium for the game between suppliers and consumers.

The Nash equilibrium provides the best possible strategy for any player, given the strategies of other players. In other words, in a Nash equilibrium, there is no incentive for players to unilaterally deviate from their current strategy (Pavel, 2012). There is no need to mention that all players are assumed to act reasonably in order to increase their payoff functions. A game consists of the following three elements: a set of players, a set of actions available to each player, and a payoff function available to each player. An action profile is a list of actions available to each player and a payoff function represents players' preferences over action profiles. Considering the action profile ai of every player i in a strategic game, a* is the Nash equilibrium if a* is at least as good for player i as the action profile (a_i, a_{-i}^*); aiwhere every other player j chooses a_j^* while player i chooses

Thus:

$$U_i(a^*) \geq U_i(a_i, a_{-i}^*) \quad (6)$$

This means that if all players choose their equilibria profiles, no action profile generates a more preferable outcome for player i than the Nash equilibrium. In order to find the Nash equilibrium, the rational reaction set (RRS) for each type of players should be obtained. The intersection of these sets provides the Nash Equilibrium. One approach to estimate the RRS would be a sensitivity-based approach (Ghotbi and Dhingra, 2012). The other approach would be a Design of Experiment (DOE) to estimate RRS. Although sensitivity based approach is more accurate than DOE (Ghotbi et al., 2014), a DOE approach was employed due to simplicity. DOE techniques enable designers to scrutinize simultaneously the effects of many different factors that could influence the final output. Factorial experimentation is a method in DOE, in which the effects of each factor and combination of factors are estimated (Telford, 2007). Fig. 2 demonstrates a two- and three-factorial design. Each point represents a unique combination of factors. In this article, a factorial design method is employed to find the sensitivity of each generating unit to Pd. In addition, in finding the Nash equilibrium for consumers, factorial design assists with finding the rational reaction set of each consumer as a function of other consumers' load demand.

4. Results and discussion

This section illustrates the result of market simulation. All simulations were performed on an Intel(R) Core(TM) i5-3470 CPU @ 3.20 GHz with 8.00 GB of installed memory in a 64-bit Operating system and on MATLAB R2014a software. All supplier units are considered to own a single wind turbine, solar panels, and diesel generators and storage devices. However, each of these components has different features for each supplier. The figures in this section are obtained through Microsoft PowerPoint and OriginPro software. Wind and solar data are approximated by conducting a simulation in System Advisor Model (SAM) (NREL, 2012), developed by the National Renewable Energy Laboratory. In this article, the strategic interactions among market participants are simulated with two consumers and two small-scale electricity suppliers. Supplier 1 is considered to have an 11 kW wind turbine. Supplier 2 operates with a 5 kW wind turbine. Table 1 summarizes diesel generator cost function coefficients, as well as load demand curve coefficients. The entire problem can be separated into three parts. First, the cooperative game among suppliers was modeled. Applying DOE to the game among the suppliers, the rational reaction set of each player could be approximated as a function of the total load demand. In this context, the primary factor is considered to be Pd. Thus,

Table 1
Cost and price coefficient.

Units	a \$/kW^3 h	b \$/kW^2 h	c \$/kW h	α \$/kW^2 h	β \$/kW h
Supplier 1	−0.0067	0.3333	0	0.001	0.24
Supplier 2	−0.0085	0.4972	0	0.001	0.24

20 levels of values for Pd, each composed of data for 24 h, were integrated into the suppliers problem. These values must satisfy constraint (12) in Appendix A. In the next step, the problem was run for each set of Pd and the optimized values for Pwi, Psi, Pdgi, Pdei and Pgi were achieved. Finally, a linear equation as a function of Pd was obtained for each hour through regression for each of the generating units. Eq. (13) in Appendix shows some of these RRS.

On the other hand, finding the Nash equilibrium among consumers also requires a factorial design method. In this context, every player's load demand was divided into 20 levels and used when solving the other players' problem. Every player solves its own problem for every level of the other players' load demand. Finally, each consumer's load demand could be modeled through regression as a linear equation, which is a function of the other players' load demand in an hourly manner. Finding the intersection of the hourly linear equations provides us with the Nash equilibrium among consumers. Eq. (14) in Appendix A shows the RRS for consumer 1 at the 9th hour.

Finally, as the third part of the problem, optimum demand values of consumers were substituted into the suppliers' equations to acquire the Nash equilibrium. It is noteworthy to mention that at Nash equilibrium no player can obtain a higher payoff function through changing its own strategy unilaterally. Given the equilibrium solution, the clearing price of the proposed restructured electricity market could be found for each hour in \$/kW h and is represented in Table 2.

Fig. 3 indicates the share of small suppliers and the utility grid in securing the load demand in 24 h at the Nash equilibrium. If adequately implemented, small suppliers have a significant role in securing the demand from consumers and thus, relieving the utility grid from high load demands. Fig. 4 demonstrates the market behavior of one of the suppliers in an hourly manner. According to this figure, the bars below the horizontal axis represent the electricity power that was either sold to the grid or used to charge the storage units. Moreover, due to the high costs of operating a diesel generator, supplier 1 makes a strategic decision to not switch on this unit and instead relies on the utility grid to secure the load demand. The market behavior of this supplier is basically influenced by the intermittent nature of renewable resources, as well as consumers' strategic decisions in managing their load demand. According to Eq. (4) consumers try to minimize their electricity bill by managing their load demand. The market behavior of consumers is presented in Fig. 5.

Consumers' active involvement in the market, which is enabled through the electricity price function, is demonstrated in Fig. 6. Empirical evidence suggests that demand increases in response to a short-term price increase (Faruqui and George, 2002; Yusta and Dominguez,

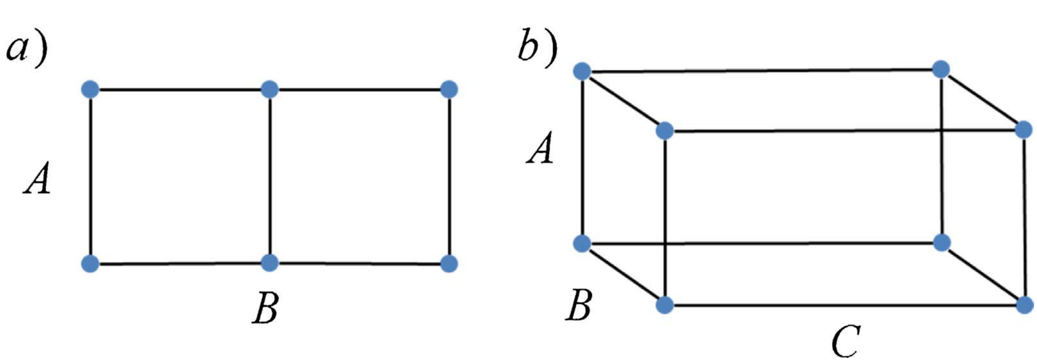

Fig. 2. a) Two levels of factor A, three levels of factor B. b) Two levels of factor A, B and C.

Table 2
Hourly electricity prices at Nash equilibrium.

t	Price ($/kW h)	t	Price ($/kW h)	t	Price ($/kW h)	t	Price ($/kW h)
1	0.230963	7	0.230153	13	0.229435	19	0.211093
2	0.231907	8	0.220645	14	0.229715	20	0.211530
3	0.232246	9	0.224304	15	0.229793	21	0.213386
4	0.232319	10	0.229209	16	0.229290	22	0.215374
5	0.232301	11	0.226743	17	0.220764	23	0.219154
6	0.231877	12	0.229112	18	0.214184	24	0.227644

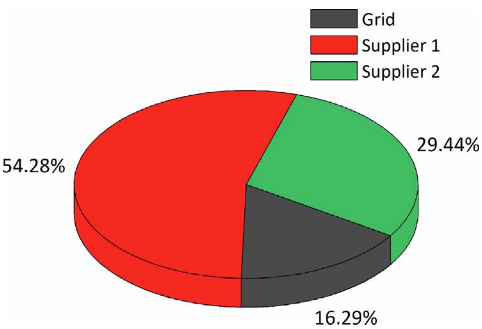

Fig. 3. Share of small suppliers and the utility grid in securing load demand.

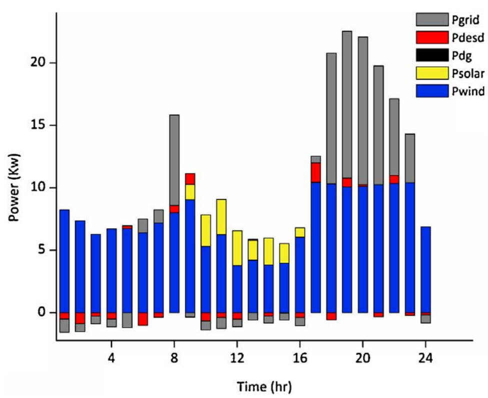

Fig. 4. Market behavior of supplier 1.

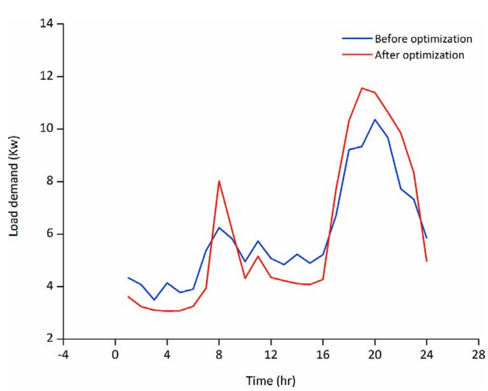

Fig. 5. Market behavior of consumer 1.

2002), mainly because the level of convenience provided by electricity is so ingrained in consumers' lifestyle that they probably don't reduce their level of comfort to cut the electricity bill (Kirschen and Member, 2003). However, employing an inverse-demand function, which was initially inspired by improving the role of consumers in the market, highlights the inverse proportionality of load demand and electricity

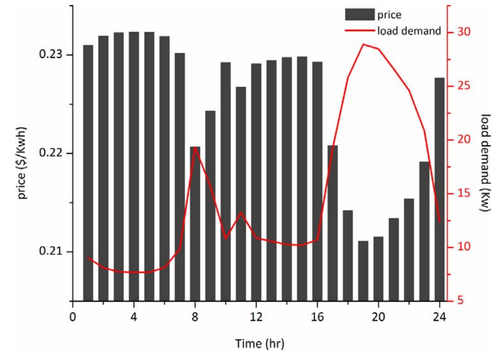

Fig. 6. Hourly electricity prices and load demand.

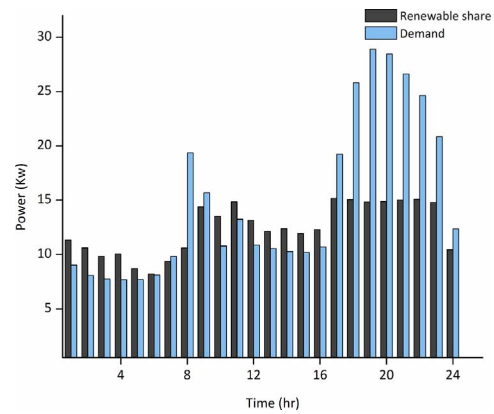

Fig. 7. Hourly share of renewable resources.

prices. According to Fig. 6., consumers' collective effort is directed towards increasing the load demand at peak periods in order to decrease the electricity prices at those hours. It is imperative to notice the undeniable role of renewable energies in the future of electricity markets. Fig. 7 shows the hourly share of renewable energies in this market. At a Nash equilibrium, suppliers' dependency on renewable resources and electricity grid is displayed in Fig. 8. This figure signifies the contribution of small renewable suppliers in generation sector.

5. Conclusion

With the recurrent developments in the electricity market structure, there is a great desire among scholars to find a promising solution that not only can handle market complexities but also has the capability of securing high anticipated load demand. In this article, we proposed a highly dynamic market framework for the electricity market which is distinctly efficient due to the high participation of end users in electricity generation. We further approached this competitive market structure by employing game theory and analyzing market behavior at Nash equilibrium. By simulation, we have shown the promising role of consumers as active market participants and the significant share of renewable energies in securing the demand. Electricity consumers and utility grid are financially incentivized and will benefit from a more reliable and stable network. In the future, it is necessary to explore various pricing models in the proposed electricity market. While an inverse-demand function empowers end users to set the hourly price of electricity, it results in higher peaks and lower valleys in the consumption pattern of end users, which is not a favorable outcome in this context. The electricity price function is one of the most important features of every market and can channel consumer's behavior towards a more efficient conduct.

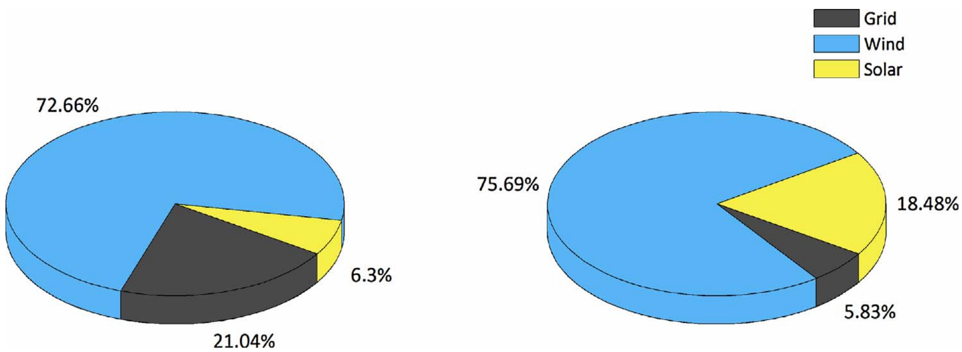

Fig. 8. a) Share of renewable resources for supplier 1 b) Share of renewable resources for supplier 2.

Appendix A

The revenue function for every single entity can be defined as the multiplication of the total generation and electricity price at each hour. Note that the electricity demand of each individual supplier is considered to be negligible compared to their generation. Thus:

$$R_{i,t} = \lambda(Pd) \times [Pw_{i,t} + Ps_{i,t} + Pdg_{i,t} + Pde_{i,t}] \tag{1}$$

Where Pwi,t is the wind power output, Psi,t solar power output, Pdgi,t diesel generator power output and Pdei,t distributed energy storage device power output of the ith energy cell at tth hour in kW. Pdei,t can be positive or negative, based on the charging or discharging status of each energy storage device. There are multitudes of factors that play an active role in the price of electricity. In this article, the cost of electricity is mainly a function of operating costs. The cost function of player i can be expressed as the following:

$$C_{i,t} = Cw(Pw_{i,t}) + Cs(Ps_{i,t}) + Cdg(Pdg_{i,t}) + Cde(Pde_{i,t}) \tag{2}$$

Where Ci,t is the total cost function of player i at tth hour. Cw and Cs represent the costs associated with maintenance and generation of power through the wind turbine and solar generating systems, respectively. In this article, we assume that the generation and maintenance cost of renewable resources are negligible in long term. Thus Cw = 0 and Cs = 0. The deterioration of storage systems is beyond the scope of this article. Therefore, the costs associated with storage devices are assumed to be insignificant and therefore, e = 0. Finally, the cost function associated with diesel generators can be approximated as a quadratic function:

$$Cdg_{i,t} = a_i Pdg_i^{\,2}(t) + b_i Pdg_i(t) + c_i \tag{3}$$

where ai, bi and ci are diesel generators' coefficients of cost function. The fuel consumption of a diesel generator is a function of its capacity, as well as the load it is operating at. In addition, we consider a negligible startup and shutdown time for small DGs.

A detailed illustration of the constraints imposed on this model is provided here. These constraints imply the technical features and limitations associated with the operation of any wind turbine, solar panel, diesel generator, or storage units. The power output of any wind turbine cannot exceed the rating of the turbine. Therefore:

$$Pw_{i,t} \le Pw_{i,max} \tag{4}$$

where Pwi,t is the power output and Pwi,max is the maximum power output of the wind turbine for the ith player at tth hour.

$$Ps_{i,t} \le Ps_{i,max} \tag{5}$$

where Psi,t corresponds to the power generation through solar energy for the ith player at tth hour. Psi,max is the maximum power output of the solar system. The required data regarding wind and solar power output were obtained through the System Advisor Model (SAM) developed by the National Renewable Energy Laboratory (NREL, 2012). SAM enables users to simulate models of renewable energy projects and was used in this article to simulate the power output of different wind and solar systems. Technical limitations of diesel generators must be taken into consideration when modeling this problem. The power output of any generator must not exceed its rating. Moreover, for a reliable operation, generators output must not drop below a certain value. Therefore, DGs, when active, must satisfy the following constraint:

$$Pdg_{i,min} \le Pdg_{i,t} \le Pdg_{i,max} \tag{6}$$

where Pdgi,t is the power output of the diesel generator of player i at tth hour. Pdgi,min and Pdgi,max are respectively the minimum and maximum power outputs of the diesel generator. High costs of operating a diesel generator, coupled with technical issues, restrict the suppliers from switching on the generator for any given output. In other words, the desired power output must be greater than Pdgi,min. Every storage device is subject to the following constraint:

$$Pde_{i,min} \le Pde_{i,t} \le Pde_{i,max} \tag{7}$$

where Pdei,t is the power output of the energy storage unit of the ith player at tth hour. Pdei,min and Pdei,max are respectively the minimum and maximum power output of the storage unit. Also, the battery state-of-charge imposes some constraints on any storage units. Battery state-of-charge (SOC) is the energy stored at the moment divided by the maximum energy that can be stored (Maharjan et al., 2009). A basic principle about state of charge must be taken into consideration. Units with higher SOC release more power when discharging, while units with lower SOC absorb more power when charging (Li et al., 2015). The statement of charge for each energy storage device must be within the safe range. To avoid any SOC imbalance which can result in overcharging or over-discharging of a battery (Rajasekharachari et al., 2013), the following constraints must be satisfied:

$$SOC_{i,min} \le SOC_{i,t} \le SOC_{i,max} \tag{8}$$

where SOCi,min is the minimum battery storage state-of-charge and SOCi,max is the maximum battery storage state-of-charge. If the SOC of a storage unit goes beyond the safe range, the energy storage unit will switch to a standby mode (Su and Huang, 2014). The battery state-of-charge or each hour is calculated through the following equation:

$$SOC_i(t+1) = SOC_i(t) - Pde_i(t)^{\Delta t}\Big/_{Ede_i} \tag{9}$$

where Edei is the battery capacity in kW h. Also, Δt refers to the time interval, which is considered 1 h. Pdei might be positive or negative depending on charging or discharging status. The following constraint ensures the availability of a certain amount of electricity stored in DESD at the beginning of the next day.

$$SOC_{i,end} \le SOC_{i,24} \tag{10}$$

Although this market structure allows suppliers to buy or sell electricity from the grid, every entity is subject to the following constraint when attempting to sell electricity to the grid. It is assumed that the number of consumers in the market is greater than small generating units. Small producers sell their generated power to the consumers first, and the excess will be sold to the utility grid. This constraint is to ensure that small suppliers consider selling their power to the consumers first before deciding to sell it to the utility grid.

$$|Pg_{i,t}| \le 0.1(Pw_{i,t} + Ps_{i,t} + Pdg_{i,t} + Pde_{i,t}) \tag{11}$$

where Pgi,t is the power sold to the utility grid by the ith player at tth hour. No need to mention that if Pgi,t is positive, it implies buying electricity from grid and if its negative, it implies selling electricity to the grid. Also, it is important to mention that in the case where the RHS of this constraint is negative, selling to the utility grid will not occur and therefore, this constraint will not be imposed. All suppliers are subject to the aforementioned constraints. Since consumers have the ability to manage and control their dispatchable loads at any given hour, the following constraint must be satisfied.

$$\sigma_1 Pb_i(t) \le Pd_i \le \sigma_2 Pb_i(t) \tag{12}$$

where σ1 and σ2 are the minimum and maximum percentage of the manageable load, respectively, and $Pb_i(t)$ is the basic load demand of the ith player at tth hour.

The following equations show a RRS of supplier 1 as a function of the total load demand at the 12th hour. The RRS for the entire 24 h for each supplier have been obtained.

$$\begin{aligned} Pw_{1,12} &= 0.219519 \times Pd + 1.394027 \\ Ps_{1,12} &= 0.189506 \times Pd + 0.72126 \\ Pdg_{1,12} &= 0 \\ Pde_{1,12} &= 0.146153 \times Pd - 2.11172 \\ Pg_{1,12} &= -0.05552 \times Pd - 0.00036 \end{aligned} \tag{13}$$

The following equation shows the RRS of consumer 1 at the 9th hour. It is a sample RRS to show how a RRS looks like for consumers at a specified hour.

$$Pd_{1,9} = 0.739359 \times Pd_{2,9} - 0.9095 \tag{14}$$

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