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Highlights

- We take into account the strategic weight manipulation in multiple attribute decision making
- We reveal the conditions to manipulate a strategic attribute weight
- We analyze the performance of MADM in defending against the strategic weight manipulation
- Simulation experiments are used to justify the validity of our models

Strategic weight manipulation in multiple attribute decision making[1]

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Abstract: In some real-world multiple attribute decision making (MADM) problems, a decision maker can strategically set attribute weights to obtain her/his desired ranking of alternatives, which is called the strategic weight manipulation of the MADM. In this paper, we define the concept of the ranking range of an alternative in the MADM, and propose a series of mixed 0-1 linear programming models (MLPMs) to show the process of designing a strategic attribute weight vector. Then, we reveal the conditions to manipulate a strategic attribute weight based on the ranking range and the proposed MLPMs. Finally, a numerical example with real background is used to demonstrate the validity of our models, and simulation experiments are presented to show the better performance of the ordered weighted averaging operator than the weighted averaging operator in defending against the strategic weight manipulation of the MADM problems.

Keywords: multiple attribute decision making, strategic weight manipulation, the ordered weighted averaging operator, ranking

1. Introduction

Multiple attribute decision making (MADM) refers to the problem of ranking alternatives based on the evaluation information of alternatives associated with multiple attributes [9, 10, 16, 25, 31]. The MADM has been widely used in engineering, technology, economy, management, and military, and many other fields

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[12, 15, 18, 22, 40].

The attribute weights play an important role in MADM problems. In the existing literature, there are several approaches to obtain the attribute weights that can be classified into three categories: the subjective approach, the objective approach and the integrated approach.

(1) The subjective approach determines the attribute weights in terms of the decision maker’s preference information on attributes [2, 8, 28]. Doyle et al. [8], for example, proposed direct rating and point allocation methods. Meanwhile, several ordinal ranking methods are investigated in [1, 26, 29], and recently, Danielson et al. [5] provided an augmenting ordinal method for obtaining attribute weights.

(2) The objective approach determines the weights of attributes using objective decision matrix information. This approach includes the entropy method [40], the TOPSIS-based method [20, 41] and some mathematical programming based methods (e.g. [3]).

(3) The integrated approach determines the weights of attributes using both decision makers' subjective information and objective decision matrix information. Within these approaches, Cook and Kress [4] proposed the preference-aggregation model based on the use of the Data Envelopment Analysis. Moreover, Fan et al. [11], Horsky and Rao [14] and Pekelman and Sen [23] constructed some optimization-based models to assess the attribute weights based on the use of decision maker’s preference information on alternatives.

Generally, in a process of decision making, the decision makers may express their opinions dishonestly to obtain their own interests, which is referred to as strategic manipulation or non-cooperative behavior. The strategic manipulation has been analyzed in-depth with respect to the aggregation function [24, 37, 38], the consensus reaching process [6, 13, 30], and also in large-scale group decision making [7, 21, 36]. It is natural to assume that the process of setting attribute weights in MADM problems is not immune to strategic manipulations, and that a decision maker may strategically set attribute weights in order to obtain her/his desired ranking of the alternative(s). In this study we refer to this kind of strategic manipulations in MADM as the strategic weight manipulation problem.

As mentioned above, there exist different (subjective, objective and integrated) approaches to attribute weights setting. Within these approaches, the decision maker is assumed to be honest, and aims to obtain "best" attribute weights to get a ranking of alternatives. We need to highlight that this paper focuses on the strategic weight manipulation problem in which the decision maker is assumed not to be honest, and she/he aims to strategically set attribute weights to obtain her/his desired ranking of the alternatives.

Although there exist numerous methods to set attribute weights, these approaches do not always consider the general theoretical framework that governs the strategic weight manipulation.

In order to fill this gap, several research challenges are proposed for analysis in this paper:

(1) How to determine the range of the ranking of alternatives when a decision maker strategically set the attribute weights in MADM problems.

(2) When a decision maker wishes to manipulate the ranking of alternatives with a predetermined purpose, how to design a strategic weight vector to achieve this purpose.

(3) How to analyze the performances of two different average operators, the weighted averaging (WA) and the ordered weighted averaging (OWA), in defending against strategic weight manipulation in MADM problems.

In order to do so, the rest of this paper is organized as follows. Section 2 provides the basic knowledge regarding MADM problems and introduces the proposed strategic weight manipulation problem. Then, in Section 3, mixed 0-1 linear programming models are proposed to obtain the ranking range of an alternative under the conditions that the attribute weights being strategically changed, and several desired properties of the ranking range of alternatives are studies. In section 4, mixed 0-1 linear programming models are used to analyze how to design a strategic weight vector to manipulate the ranking of alternative(s) to achieve a desired purpose. Section 5 presents a numerical example to illustrate the proposed models, and simulation experiments are presented to compare the performances of the WA and OWA [32, 39] operators in defending against strategic weight manipulation in MADM problems. Concluding remarks and future research agenda are provided in Section 6.

2. Background

This section introduces the MADM problem and the concept of ranking range of an alternative, which will provide a basis to study the strategic weight manipulation problem in MADM.

2.1 MADM problem

Let $A=\{A_1,A_2,\ldots,A_m\}$ be the set of alternatives, $C=\{C_1,C_2,\ldots,C_n\}$ the set of predefined attributes, and $w=(w_1,w_2,\ldots,w_n)^T$ the associated weight vector of the attributes, such that $w_j\geq 0$ and $\sum_{j=1}^{n} w_j=1$. Let $V=(v_{ij})_{m\times n}$ be the decision matrix given by the decision maker, where v_{ij} denotes the preference value for the alternative $A_i\in A$ with respect to the attribute $C_j\in C$, representing how well alternative A_i verifies attribute C_j.

Generally, the resolution process of MADM problems includes three steps:

(1) Normalization of the decision matrix

In MADM problems, attributes are classified into two categories: benefit attributes and cost attributes. The decision maker's decision matrix $V=(v_{ij})_{m\times n}$ needs to be normalized into a corresponding standardized individual's decision matrix $\bar{V}=(\bar{v}_{ij})_{m\times n}$, where

$$\bar{v}_{ij}=\frac{v_{ij}-\min_i(v_{ij})}{\max_i(v_{ij})-\min_i(v_{ij})} \tag{1}$$

if $C_j\in C$ is a benefit attribute, and

$$\bar{v}_{ij}=\frac{\max_i(v_{ij})-v_{ij}}{\max_i(v_{ij})-\min_i(v_{ij})} \tag{2}$$

if $C_j\in C$ is a cost attribute.

(2) Aggregation of the standardized decision matrix

Let $F(A_i)$ be the decision evaluation value of the alternative A_i, which is obtained by aggregating its associated attribute preference values using Eq. (3) and an appropriate aggregation operator F:

$$D(x_i) = F(\bar{v}_{i1}, \bar{v}_{i2}, ..., \bar{v}_{im}) \quad (3)$$

In MADM problems, the aggregation operators frequently used are the WA operator and the OWA operator [32, 39].

When F is a WA operator with an associated weight vector $w = (w_1, w_2, ..., w_m)^T$, Eq. (3) can be rewritten as follows:

$$D(x_i) = WA_w(\bar{v}_{i1}, \bar{v}_{i2}, ..., \bar{v}_{im}) = \sum_{j=1}^{m} w_j \bar{v}_{ij} \quad (4)$$

While, when F is a OWA operator with an associated weight vector $w = (w_1, w_2, ..., w_m)^T$, Eq. (3) can be rewritten as follows:

$$D(x_i) = OWA_w(\bar{v}_{i(1)}, \bar{v}_{i(2)}, ..., \bar{v}_{i(m)}) = \sum_{j=1}^{m} w_j \bar{v}_{i(j)} \quad (5)$$

where $\bar{v}_{i(j)}$ is the jth largest value in $\{\bar{v}_{i1}, \bar{v}_{i2}, ..., \bar{v}_{im}\}$.

(3) Ranking of alternatives

Let $\tilde{O}_k = \{x_i | D(x_i) > D(x_k), i = 1, 2, ..., n\}$ be the set of the alternatives whose decision evaluation value is greater than that of the alternative x_k, and $|\tilde{O}_k|$ be its cardinality. Clearly, for $\tilde{O}_k$, because $x_k \notin \tilde{O}_k$, then alternative x_k such that $D(x_k) = \max\{D(x_1), ..., D(x_n)\}$ might verify as well that $|\tilde{O}_k| = 0$, while alternative $x_l \notin \tilde{O}_l$, such that $D(x_l) = \min\{D(x_1), ..., D(x_n)\}$ might as well have $|\tilde{O}_l| = n - 1$, and therefore this alternative will be ranked in 1-st and n-th positions, i.e., it is justified the following definition of the ranking position of an alternative in terms of $|\tilde{O}_k|$: $r_k = |\tilde{O}_k| + 1$, i.e.,

$$r(x_k) = \left| \{ x_i \,|\, D(x_i) > D(x_k), \quad i = 1, 2, ..., n \} \right| + 1 \quad (6)$$

Based on the ranking of alternatives, we can easily obtain the following results.

(1) Let $x_i \succ x_l \Leftrightarrow r(x_i) < r(x_l)$, then we have $x_i \succ x_l \Leftrightarrow D(x_i) > D(x_l)$.

(2) Let $x_i \sim x_l \Leftrightarrow r(x_i) = r(x_l)$, then we have $x_i \sim x_l \Leftrightarrow D(x_i) = D(x_l)$.

2.2 The proposed research problem: Strategic weight manipulation

Let $r_w(x_k)$ be the ranking of alternative x_k when setting the associated weight vector of the attributes $w=(w_1,w_2,\ldots,w_m)^T$. Clearly, $r(x_i)$ can change when the weight vector $w=(w_1,w_2,\ldots,w_m)^T$ is changed, in other words, the manipulation of the weight vector can lead to a change in the ranking order of the alternatives. The following example clearly illustrates this issue.

Example 1: Assume three alternatives x_1,x_2,x_3 and four attributes u_1,u_2,u_3,u_4 with the following standardized decision matrix $\bar{V}=(\bar{v}_{ij})_{3\times 4}$ is:

$$\bar{V}=\begin{bmatrix} 0.59 & 1 & 0.8 & 0.63 \\ 0.6 & 0.8 & 1 & 0.46 \\ 1 & 0.5 & 0.4 & 1 \end{bmatrix}$$

Different $w=(w_1,w_2,\ldots,w_m)^T$ lead to different rankings $r(x)=(r(x_1),r(x_2),r(x_3))$ of the alternatives $X=\{x_1,x_2,x_3\}$. Indeed,

1) If we set $w=(0.3,0.2,0.1,0.4)$, then we have $r(x)=(3,2,1)$;

2) If we set $w=(0.1,0.45,0.24,0.21)$, then it is $r(x)=(1,2,3)$;

3) While if we set $w=(0.3,0.1,0.5,0.1)$, then $r(x)=(2,1,3)$ is obtained.

Because different attribute weights yield different ranking of alternatives, in this paper, we give the definition of ranking range of an alternative as follows:

Definition 1: In MADM problems, $R(x_i)=[\underline{r}(x_i),\bar{r}(x_i)]$ is known as the ranking range of the alternative x_i, with $\underline{r}(x_i)=\min_{w\in W} r(x_i)$ and $\bar{r}(x_i)=\max_{w\in W} r(x_i)$ being the best and worst rankings of alternative x_i, respectively, and $W=\{w=(w_1,w_2,\ldots,w_m) \mid \sum_{j=1}^{m} w_j=1,\ \ 0\le w_j\le 1\}$.

In addition, in this paper, we introduce the concept of attribute ranking and attribute ranking range to analyze the properties of the ranking range of an alternative.

Let $U_j(x_i)=\{x_k \mid \bar{v}_{kj}>\bar{v}_{ij}\}$ (i=1,2,…,n; j=1,2,…,m) be the set of alternatives whose decision evaluation value is greater than that of the alternative x_i associated with the attribute u_j, and $|U_j(x_i)|$ be its cardinality. Let $\bar{U}_j(x_i)=\{x_k \mid \bar{v}_{kj}\le\bar{v}_{ij}\}$ (i=1,2,…,n; j=1,2,…,m) be the set of alternatives whose decision evaluation value is not greater than that of the alternative x_i associated with the attribute u_j, and $|\bar{U}_j(x_i)|$ be its cardinality.

Based on the sets $V_j(x_k)$ and $\bar{V}_j(x_k)$, the concept of attribute ranking and attribute ranking range can be formally presented as follows:

Definition 2: In MADM problems, $c_j(x_k) = |V_j(x_k)| + 1$, i.e.,

$$c_j(x_k) = \left|\{x_i \left| \bar{v}_{ij} > \bar{v}_{kj}\}\right.\right| + 1, \quad (j = 1, 2, ..., m) \tag{7}$$

is the attribute ranking of the alternative x_k associated with the attribute c_j. Then, let $\underline{c}(x_k) = \min_j c_j(x_k)$ and $\bar{c}(x_k) = \max_j c_j(x_k)$, $C(x_k) = [\underline{c}(x_k), \bar{c}(x_k)]$ is the attribute ranking range of the alternative x_k.

As mentioned above, in MADM problems, a decision maker could strategically set an attribute weight vector to obtain her/his desired ranking of alternative(s), which in this paper is referred to as the strategic weight manipulation in MADM.

In the following, based on the concept of ranking range, we investigate some issues on the strategic weight manipulation of the MADM to deal with the challenges presented in the introduction section.

In order to improve readability, the main notation used in this paper is listed as follows.

X: The set of alternatives;

A: The set of attributes;

$D = (v_{ij})_{n \times m}$: Decision matrix;

$\bar{D} = (\bar{v}_{ij})_{n \times m}$: Standardized decision matrix;

W: The set of attribute weight vectors;

$V(x_i)$: The evaluation value of the alternative x_i;

$r_w(x_k)$: The ranking of the alternative x_k under the attribute weight vector w;

$\underline{r}(x_k)$: The best ranking of the alternative x_k;

$\bar{r}(x_k)$: The worst ranking of the alternative x_k;

$R(x_k) = [\underline{r}(x_k), \bar{r}(x_k)]$: Ranking range of the alternative x_k;

$R_{WA}(x_k) = [\underline{r}_{WA}(x_k), \bar{r}_{WA}(x_k)]$: Ranking range under the WA operator;

$R_{OWA}(x_k) = [\underline{r}_{OWA}(x_k), \bar{r}_{OWA}(x_k)]$: Ranking range under the OWA operator;

$C(x_k) = [\underline{c}(x_k), \bar{c}(x_k)]$: Attribute ranking range of the alternative x_k.

3. Ranking range

The ranking range of an alternative is used to provide the best and worst ranking of the alternative, which is a basis for strategically setting the attribute weights in MADM problems. In this section, we present mixed 0-1 linear programming models to obtain the ranking range of an alternative, and show several desired properties of the ranking range of an alternative.

3.1 Obtaining the ranking range via a mixed 0-1 linear programming

Let $y_i \in \{0,1\}$, M a large enough number, and $U(x_i)$ be defined as per Eq. (3). Then, we can easily obtain the following results.

(1) $x_i \succ x_k$ if and only if $y_i = 1$ under the conditions $U(x_i) > U(x_k) - (1-y_i)M$ and $U(x_i) \le U(x_k) + y_i M$.

(2) $x_i \preceq x_k$ if and only if $y_i = 0$ under the conditions $U(x_i) \le U(x_k) + y_i M$ and $U(x_i) > U(x_k) - (1-y_i)M$.

Based on the above results, Theorems 1 and 2 to obtain the ranking range $R(x_k) = [\underline{r}(x_k), \bar{r}(x_k)]$ of the alternative x_k under the WA and OWA operators are presented.

Theorem 1: Let $R_{WA}(x_k) = [\underline{r}_{WA}(x_k), \bar{r}_{WA}(x_k)]$ be the ranking range of alternative x_k when the WA operator U_{WA} is used to compute the decision evaluation function as per Eq. (4). Then,

(1) The best ranking of alternative x_k, $\underline{r}_{WA}(x_k)$ can be obtained via the mixed 0-1 linear programming models (8)-(13).

$$
\begin{cases}
\underline{r}_{WA}(x_k) = \min \sum_{i=1}^{n} y_i + 1 & (8) \\
s.t. \begin{cases}
\sum_{j=1}^{m} w_j \bar{v}_{ij} > \sum_{j=1}^{m} w_j \bar{v}_{kj} - (1-y_i)M, \quad (i=1,2,\dots,n) & (9) \\
\sum_{j=1}^{m} w_j \bar{v}_{ij} \le \sum_{j=1}^{m} w_j \bar{v}_{kj} + y_i M, \quad (i=1,2,\dots,n) & (10) \\
\sum_{j=1}^{m} w_j = 1 & (11) \\
0 \le w_j \le 1, \quad (j=1,2,\dots,m) & (12) \\
y_i = 1 \ \ or \ \ 0, \quad (i=1,2,\dots,n) & (13)
\end{cases}
\end{cases}
$$

(2) In models (8)-(13), replace the objective function (8) by

$$\overline{r}_{WA}(x_k) = \max \sum_{i=1}^{n} y_i + 1 \tag{14}$$

Then, the worst ranking of alternative x_k, $\overline{r}_{WA}(x_k)$, can be obtained via the mixed 0-1 linear programming models (9)-(14).

The proof of Theorem 1 is provided in Appendix A.

To simplify the notation, models (8)-(13) and models (9)-(14) are both called M_1 in this paper.

Theorem 2: Let $[\underline{r}_{OWA}(x_k), \overline{r}_{OWA}(x_k)]$ be the ranking range of alternative x_k when the OWA operator F is used to compute the decision evaluation function as per Eq. (5). Then,

(1)The best ranking of alternative x_k, $\underline{r}_{OWA}(x_k)$ can be obtained via the 0-1 linear programming models (15)-(20).

$$\begin{cases} \underline{r}_{OWA}(x_k) = \min \sum_{i=1}^{n} y_i + 1 & (15) \\ s.t. \begin{cases} \sum_{j=1}^{m} w_j \overline{v}_{i(j)} > \sum_{j=1}^{m} w_j \overline{v}_{k(j)} - (1-y_i)M, \quad (i=1,2,...,n) & (16) \\ \sum_{j=1}^{m} w_j \overline{v}_{i(j)} \le \sum_{j=1}^{m} w_j \overline{v}_{k(j)} + y_i M, \quad (i=1,2,...,n) & (17) \\ \sum_{j=1}^{m} w_j = 1 & (18) \\ 0 \le w_j \le 1, \quad (j=1,2,...,m) & (19) \\ y_i = 1 \ \ or \ \ 0, \quad (i=1,2,...,n) & (20) \end{cases} \end{cases}$$

(2) In models (15)-(20), replace the objective function (15) by

$$\overline{r}_{OWA}(x_k) = \max \sum_{i=1}^{n} y_i + 1 \tag{21}$$

Then, the worst ranking of alternative x_k, $\overline{r}_{OWA}(x_k)$, can be obtained via the mixed 0-1 linear programming models (16)-(21).

The proof of Theorem 2 is provided in Appendix A.

To simplify the notation, models (15)-(20) and models (16)-(21) are both called M_2 in this paper. In both models M_1 and M_2, y_i $(i=1,2,...,n)$ and w_j $(j=1,2,...,m)$ are decision variables.

21	30.8	14.1	41.9	48.6	70.8	28.8
22	34.4	0.0	56.2	41.3	75.9	25.6
23	14.5	35.8	44.2	34.5	62.0	38.0
24	30.8	34.8	40.2	35.7	62.5	24.6
25	19.9	17.2	38.8	38.6	79.1	29.3
26	31.6	37.2	33.6	32.6	59.0	23.5
27	28.1	31.9	35.2	40.3	56.2	23.3
28	15.4	22.1	50.3	37.0	58.1	28.8
29	29.9	36.2	34.9	32.4	55.5	28.6
30	29.5	16.3	47.3	32.5	64.0	26.2
31	15.4	14.9	50.2	39.0	61.7	23.9
32	22.9	24.9	40.7	41.8	50.5	26.0
33	17.0	59.8	30.3	40.9	19.0	39.6
34	12.6	34.1	37.1	36.0	45.0	34.2
35	21.8	18.8	28.0	34.0	63.2	39.2
36	33.6	27.4	25.8	29.8	59.2	23.9
37	16.2	16.3	38.6	37.5	56.0	26.6
38	14.5	39.1	38.7	27.5	37.3	38.0
39	8.9	16.3	39.8	33.5	61.2	25.6
40	15.4	18.8	32.8	32.0	63.0	24.2
41	18.5	32.6	27.1	26.1	56.2	25.6
42	30.3	54.3	16.8	17.4	47.3	26.6
43	19.2	20.0	33.0	31.6	52.7	26.5
44	17.0	13.3	28.6	25.3	66.9	30.2
45	18.5	34.5	31.1	35.6	35.8	22.4
46	19.9	25.3	23.7	29.4	51.7	34.2
47	20.5	24.9	25.8	30.5	51.9	25.9
48	22.4	26.6	24.8	23.5	50.8	37.4
49	0.0	31.7	35.6	22.1	53.2	20.3
50	0.0	29.3	34.3	28.9	45.3	27.5