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Highlights

- We integrate tactical supply chain considerations in a strategic forest management model.
- We employ a two stage linear programming formulation to solve the integrated planning problems.
- We propose acceleration strategies to solve practical large-scale forest management problems.
- A clear gain in profit could be achieved when planning is conducted in an integrated approach.
- Integrated forest planning shows superior performance compared to the non-integrated approach.

Integrated optimization of strategic and tactical planning decisions in forestry

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Abstract

The traditional approach to plan the forest products value chain using a combination of sequential and hierarchical planning phases leads to suboptimal solutions. We present an integrated planning model to support forest planning on the long term with anticipation of the impacts on the economic and logistic activities in the forest value chain on a shorter term, and we propose a novel optimization approach that includes acceleration strategies to efficiently solve large-scale practical instances of this integrated planning problem. Our model extends and binds the models implemented in two solver engines that have developed in previous work. The first system, called Logilab, allows for defining and solving value chain optimization problems. The second system, called Silvilab, allows for generating and solving strategic problems. We revisit the tactical model in Logilab and we extend the strategic model in Silvilab so that the integrated planning problem can be solved using column generation decomposition with the subproblems formulated as hypergraphs and solved using a dynamic programming algorithm. Also, a new set of spatial sustainability constraints is considered in this model. Based on numerical experiments on large-scale industrial cases, the integrated approach resulted in up to 13% profit increase in comparison with the non-integrated approach. In addition, the proposed approach compares advantageously with a standard LP column generation approach to the integrated forest planning problem, both in CPU time (with an average 2.4 factor speed-up) and in memory requirement (with an average reduction by a factor of 20).

Keywords: OR in natural resources, large scale systems, forest industry, strategic and tactical planning, integrated planning, dynamic programming

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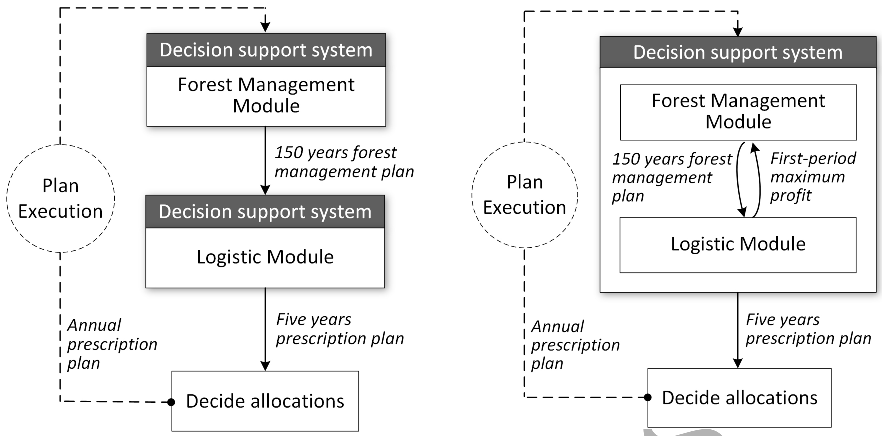
1. Introduction

The forest value chain is commonly associated with its timber-production networks. Ideally, the managers of these networks should be able to plan harvesting operations and silviculture treatments, deployment and maintenance of road networks, transportation of trees, logs and residues, manufacturing activities and distribution over extremely large and variable geographical areas and within a very long planning horizon. In practice, however, and in order to reduce the complexity of the task, the problem is typically viewed as two interrelated problems: the strategic problem that deals with long-term and large-scale non-spatial decisions, and the tactical problem that examines more specific considerations like the exact spatial location of the harvest, road building and adjacency constraints. The objective in the strategic problem is often to maximize the net present value of timber over a planning horizon covering one or several full forest rotation (In Canada for instance, forests can be managed over a 200 years planning horizon), and under a set of harvest volume restrictions, for example, non-declining yield constraints and limits of harvesting levels between consecutive years, also known as the Annual Allowable Cut (AAC) levels, that ensure stability of production at the short and long run (Paradis et al., 2013). On the other hand, the objective in the tactical problem is typically to maximize the actualized profits constrained by the availability of the logs over shorter horizons (typically 5 years).

Historically, the strategic and tactical problems have been solved either together in an integrated (or monolithic) model (Weintraub and Navon, 1976) or sequentially in what is known as a hierarchical model using either one (Smith, 1978) or many (Weintraub and Cholaky, 1991) iterations of the



(a) Hierarchical planning process (b) integrated planning process

Figure 1: Global forest management planning. The hierarchical process (a) finds maximum long-term species-wise even-flow AAC levels and management plan then maximizes the first-period profit of the value-creation network. The integrated process (b) iteratively finds improved maximum long-term species-wise even-flow AAC levels and management plan that maximize first-period profit of the value-creation network.

sequence (see Figure 1a). Hierarchical models are known to be easier to solve and often to better represent the real administrative context of the problem, however they might ignore the capacity of the value chain to take advantage of the anticipated net present value, thus the resulting solution may be suboptimal. In practice, the linkages between the strategic and tactical forest management planning levels do not ensure the coherent disaggregation of the long term supply allocations as input for short-term demand-driven harvest planning. This was clearly illustrated in a case study by Paradis et al. (2013). Based on several simulations made up of two-phase (strategic, tactical) rolling-horizon re-planning iterations, these authors found that this hierarchical forest planning process, that according to their understanding reflects the current situation in several jurisdictions where public forests are managed by government stewards for production of timber resources, is incoherent and dysfunctional: it fails to demonstrate long-term sustainability of government-endorsed short-term harvest levels, fails to reliably meet industrial fiber demand over time, and exacerbates incoherence between wood supply and fiber demand over several planning iterations. Since economic gains are determined by the decisions in the forest and their anticipated impacts on the value chain, the strategic and tactical levels of optimization must be closely entangled, which implies an integrated approach.

The literature shows few optimization models that integrate forest and mills decisions. Troncoso et al. (2015) proposed a mixed integer programming model that integrates forest management (25 years) and tactical logistic planning (5 years) using the characteristics of a vertically integrated company operating with forest plantation in Chile. For the longer term, they used estimated prices on log products and for the shorter term they used estimates on demand and product prices. The case study was very simplistic and aimed simply to show the superiority of an integrated strategy compared to hierarchical strategy. Troncoso et al. (2015) emphasized that with a more detailed description, including a less aggregated number of products and more detailed demand requirements in terms of log definitions (lengths, diameters), it is likely that the integrated model will give higher quality solutions. However, integrated forest planning problems become very complex to solve with the increase of the number of forest zones products, treatments and processes. Furthermore, Troncoso et al. (2015) did not explicitly consider spatial sustainability constraints in their integrated forest planning model. In forest management, long term sustainability of the timber resources is often achieved by considering non-declining yields constraints (Davis et al., 1987; Gunn, 2007). However, when a model assumes a more comprehensive value chain, spatial considerations are taken into account and there is a risk that the wood closest to the mills is predominantly harvested during the first periods.

In operationally sized problems, it is not uncommon for a forest to be represented by more than 300 000 stands and 50 000 different states. In many commercial optimization packages for forest planning problems, the time to solve the problem and the memory usage are major limiting factors on the scope of usability of the software. It is not uncommon that some of the LP problems generated by those software use more than 24 gigabytes of RAM and take up to 24 hours to solve. Things become even more difficult when value chain components are included in the formulation. In order to get some of the advantages of hierarchical planning in the context of an integrated model, Pittman et al. (2007) used a Dantzig-Wolfe decomposition approach (Dantzig and Wolfe, 1960). Inspired by the work of Schneeweiss (2003) on distributed planning, these authors considered the strategic and the tactical planning problems as independent and distributed, the mathematical model links them together by using a column generation approach (Dantzig and Wolfe, 1960; Desaulniers et al., 2005) This is also the approach we used.

The integrated forest planning model presented in this paper operationalizes the global decision process illustrated in Figure 1b. It is an iterative two-phase model simulating interaction between the long- and short-term planning processes. The objective of the optimization problem consists in finding the maximum long-term species-wise even-flow AAC levels and forest management plan that maximizes simultaneously first-period profit of the value-creation network. Considering that the problem has to be solved many times in a column generation approach, a dynamic programming algorithm is proposed to speed up the solution process. The paper proposes a decomposition approach and strategies to reduce solution time and memory needs. In addition, this decomposition enables the consideration of a new set of spatial sustainability constraints.

To build the proposed integrated model, we used the models built in two solver engines that we have developed in previous work. The first solver engine, called Logilab, is a logistic planning module that allows for the definition and solving of the value chain optimization problem. The second solver engine, called Silvilab, is a forest management module that allows the generation and solving of the forest planning problem. The paper demonstrates how the strategic model in Silvilab and the tactical model in Logilab can be linked together efficiently using a column generation approach. The paper shows also how hypergraphs (a generalization of graphs that are used to represent complex systems) can be used in order to increase the resolution efficiency even further. Finally, several numerical experiments built from real forest and industrial data are presented. It is noted that a first version of the model we present here was tested by Paradis et al. (2013) to illustrate the potential value of explicitly anticipating certain aspects of the tactical planning process (e.g. demand satisfaction) within a strategic planning model, however no detailed description was included. These authors simulated the entire loop depicted in 1b in order to show potential benefit (on stability of wood supply and value-creation potential) of integrating a simple demand-anticipation mechanism into the long-term planning process. The implementation of the model was however very simplistic (small forest area and number of species, one harvesting treatment and two silviculture treatments) and performance regarding computation and memory requirements was not an issue. Furthermore, Paradis et al. (2013) did not explicitly consider spatial sustainability constraints in their integrated model.

2. The Strategic and tactical planning models

Like many existing logistic planning module and forest planning modules, the Logilab and Silvilab solver engines that we used to build the integrated decision process depicted in Figure 1b can be also used to build the hierarchical scheme of Figure 1a. We start by revisiting the tactical model in Logilab and we extend the strategic model in Silvilab so that the forest planning problem can be solved using a dynamic programming approach based on hypergraphs.

2.1. The tactical model

This model considers essentially the spatial distribution of the harvest and the forest value chain objectives. Figure 2 illustrates an example of a network representation of the value chain optimization problem. The divergent transformation process begins with a given forest inventory scattered across the territory, then the logs are transported to a sawmill, pulp and paper mill, energy plants, or any other processing unit. At any given mill, there could be one or many possible transformation processes. The flow is ended by a sale to a customer. The monetary value associated with a given harvest plan is the sum of all sales it has generated minus the total cost incurred in all the steps of the supply chain. This value is determined using the assumption that the choice of operations is made in order to maximize the global value for the whole value chain.

Many aspects of this planning problem were integrated into previous harvesting models except for the mills capacities and market opportunities (Weintraub and Navon, 1976). One of the reasons for this is that harvest planning is essentially a strategic decision while product manufacturing and sale is more on a tactic, even operational level. In order to integrate the

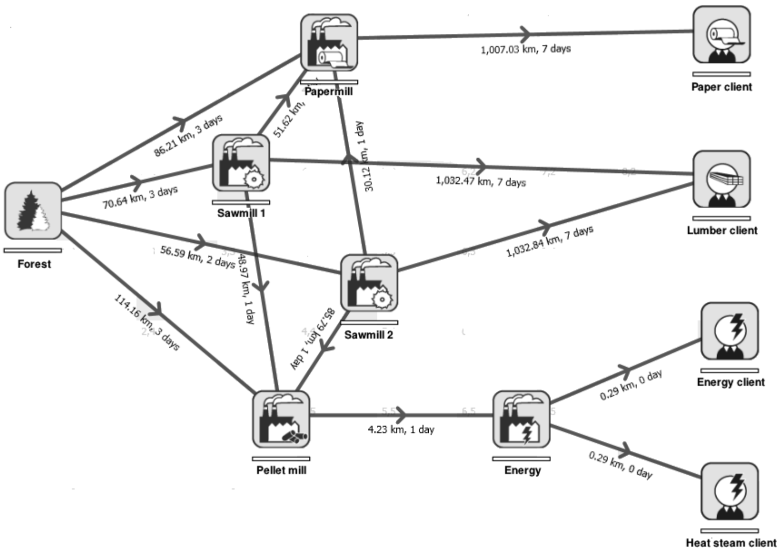
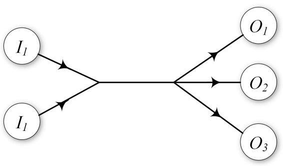


Figure 2: Network representation of the value chain optimization problem. The distances between the production units and the transportation time are indicated.

mills capacities and market opportunities, we must aggregate them as much as possible without making them trivial. In fact, there is no advantage to model the internal processes of the mills using more detailed data than rough estimates of the forestry production divided into major families of products.



I_1 : *10* m^3 of pine logs
I_1 : *2 hours* of sawmill capacity

O_1 : *8* m^3 of 8 foot 2x4 boards
O_2 : *1* m^3 of chips
O_3 : *1* m^3 of sawdust

Figure 3: A typical transformation process in the value chain optimization model.

In the model, a mill is modelled as a set of transformation processes and a set of different types of capacities with given limits. One transformation process requires a quantity of different raw materials, produces various quantities of output products, and consumes a fraction of the available capacities. For example, Figure 3 presents a sawmill transformation process where 10 m^3 of pine logs are transformed to 8 m^3 of 2x4 boards, 1 m^3 of chips and 1 m^3 of sawdust while utilizing 2 hours of sawmill machine capacity. Thus, the set of processes are chosen in such a way as to represent the operations at the mill. To limit the size of the model, the operations are grouped into broad classes. The capacities considered for a mill are associated with potential bottlenecks. For example, the processing lines at the sawmill may be limited to the processing of 8-foot or 16-foot logs.

For the market opportunities, the model can only be as detailed as the underlying data and assumptions. Since, in practice, it is difficult to obtain enough data to model markets beyond a demand limit per customer product, we did not limit our model by a specific demand; we can sell any amount of any product. However, we define a piecewise linear capacity function to include a market saturation effect, meaning that the price of a product decreases as the quantity sold rises. In some cases, a price of \$0 is assigned past a certain quantity to allow for over production, while a negative price can be associated to a harmful waste that needs to be managed.

The value chain optimization model formulates a generic supply chain problem with essentially two types of objects: **generic products (set P)**, and **generic processes (set W)**. A generic product is a triplet of product/resource, location and period. On the other hand, a generic process takes generic products as input and give generic products as output. The following parameters are needed to define the mathematical formulation:

x_i : amount by which generic process i is used

c_i : loss ($-$) or gain ($+$) associated to the use of generic process i

α_{ij} : quantity of generic product j used ($-$) or produced ($+$) by generic process i

s_j : amount of generic product j supplied $(-)$ or required $(+)$ by external components of the system

l_i : lower bound on the quantity of generic process i

u_i : upper bound on the quantity of generic process i

In a very abstract form, the mathematical formulation of the value chain optimization model can be as follows :

$$\max \sum_{i \in W} c_i x_i \tag{1}$$

$$\text{s.t.} \sum_{i \in W} \alpha_{ij} x_i = s_j \quad \forall j \in P \tag{2}$$

$$l_i \leq x_i \leq u_i \quad \forall i \in W \tag{3}$$

In this model, the generic processes represent the transformative capacity of the value chain. The generic products can be, at a given location in time, either a physical product or a resource. In our context, a physical product could be a raw material (log), a products undergoing processing or a final product. Resources are consumed by processes and are of limited capacity. A resource can be a machine or an employee. A process could be, for example, the transportation of a given type of physical products from one location to another at a given time, or the transformation of a given type of logs at a given mill using a specific sawing recipe at a certain period. A detailed version of this model is described in (Jerbi et al., 2012).

2.2. The strategic model

The strategic model describes the trajectory of the forest through time. It considers various forest related outputs and constraints that deal with spatial distribution, natural growth, succession related to natural and silvicultural treatment. The objective is guided by the marginal value of the output generated by the forests. From a mathematical perspective, the model is based on the assumption that the forest cover in every zone is divided into different forest states, not necessarily contiguous, that share the same age (or same age distribution) and various forestry attributes (species content, rotation, history of previous treatments). This aggregation is based on the information available and is as fine as possible, that is, any further subdivision would not change the solution of the problem. The typical treatments considered are essentially, plantation, various harvest techniques, and commercial and pre-commercial thinning. A treatment has two effects. Firstly, it produces outputs of different kinds in precise amounts. Secondly, it transforms all areas in the same state to a set of sub areas (that cover the same surface) with new forest states. We call the transformation associated to a treatment a state transition. In a state transition, the exact location of the sub areas is not considered, and only fractions of the original area are evaluated. For every output produced, we have a precise value of the marginal impact by unit of this output on the global problem. The only constraint is the state transition of the forest that is imposed as time passes and treatments are performed. Figure 4 presents a total harvest treatment that translates, for example, a 100 hectare territory covered by a 90 year-old pine forest at period 1into a 95 hectare territory covered by a 0 year-old pine forest and a 5 hectare un-forested territory at period 2. Also, the output of the treatment in the example is 15 000 m^3 of pine logs at period 1 and a contribution of 15 000 m^3 to the constraint, associated with period 1, requiring an even level of total harvested volume during the planning horizon.

As pointed out by Gunn (2007), there are three separate modelling ap-

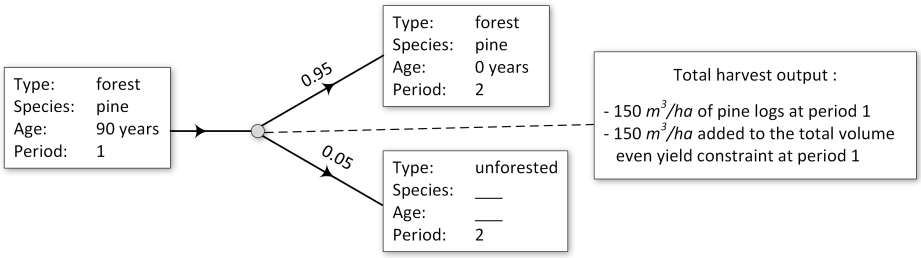


Figure 4: An example of the state transition.

proaches to forest growth and management. They are known as Model I, Model II and Model III. The three approaches represent the process of growth and harvesting of the forest as the flow through a network. The crucial difference lies in the level of aggregation on the treatments performed in each arc of the flow network. Model I and Model II are the most widely known (see Davis et al. (1987)). In model I, each arc is a complete set of treatments for a stand or an aggregation of stands, covering the entire duration of the planning horizon. The decision variables represent a sequence of actions on a given forest unit for the entire planning period. Model II is more detailed; in this family of models, the arcs are a succession of treatments that cover a large period, typically a complete rotation, thus the decision variables represent a sequence of actions on an even-aged forest unit from its beginning to the moment when it is cut or to the moment that it dies. In Model III, each arc is a specific treatment (or group of few treatments) over a specific period, thus the variables represent individual actions (or groups of few actions) on a given forest unit (see Garcia (1990)). Our strategic model formulation is similar to model I linear programming formulation. However, as it will be shown in the following section, the proposed integrated model is also inspired by Model III representation.

To construct our linear programming model, we define the following:

D : set of all the different forest states describing the forest land;

T : interval $1 \ldots |T|$ = the number of periods;

Z : set of state-spatial zone, i.e. spatial zone subdivided by forest states;

A : set of all possible forest treatments, a treatment is associated to a specific forest state;

A_d : the set of all treatments that can be applied to state d;

O : set of outputs;

σ_{ad} : fraction of territory in state d' converted to state d by treatment $a \in A_{d'}$;

γ_{aoz} : quantity of output o produced by treatment a in zone z;

μ_{ozt} : pricing on the output o in zone z at period t ;

d_z : initial state of spatial-state zone z;

c_{azt} : cost of treatment a on state-spatial zone z at time t. In our hypothesis the cost of a treatment is a linear function of area treated.

x_{azt} : fraction of the initial area of zone z on which treatment a is applied at period t.

The linear formulation of the problem (**F**):

$$\max \sum_{z \in Z} \sum_{tinT} \sum_{a \in A} (\sum_{o \in O} \mu_{o,z,t} \gamma_{a,o,z} - c_{a,z,t}) x_{a,z,t} \tag{4}$$

s.t.

$$\sum_{a \in A_{d_z}} x_{az1} = 1 \quad \forall z \in Z \tag{5}$$

$$\sum_{a \in A} \sigma_{ad} x_{az,t-1} = \sum_{a \in A_d} x_{azt} \quad \forall z \in Z, d \in D, t = 2 \ldots |T| - 1 \tag{6}$$

$$x_{azt} \geq 0 \quad \forall a \in A, z \in Z, t = 1 \ldots |T| \tag{7}$$

The first constraints states that for every spatial-state zone z, the sum of all the treatments at the first period covers the whole area (leaving a zone in its natural evolution is considered a treatment). Note that the only treatments considered are those that can be applied to state d_z. The second constraint states that in every zone z at every period t (except the first and last period), the fraction of the area concerned with treatments to state d be equal to the fraction of the area in state d resulting from treatment done in the previous period. One can see that the preceding model can be split into several similar independent models, one for every spatial-state zone z. We call $\mathbf{F(z)}$ the model $\mathbf{F(z)}$ limited to zone z.

The matrix A, defined by the constraints of $\mathbf{F(z)}$, can readily be seen to have the following properties: (i) Each column has exactly one positive element since every action acts on exactly one forest state (note that it can however convert the state into various different output forest states); and (ii) there exists a $\bar{x} \geq 0$ such that $A\bar{x} > 0$. Consider the solution x that simply consists of having $x_{az1} = 1$ for $a \in A_{d_z}$ and 0 everywhere else. A matrix A having these two properties is said to be Leontief and $Ax = b, x \geq 0$ is said to be a Leontief substitution system. A Leontief substitution system can be associated with a generalization of a flow problem on general graphs which is a flow problem on a hypergraph, (see Jeroslow et al. (1992)). In section 3.2.1, we present the forest planning problem as a flow problem on a hypergraph, and in section 3.2.2, we develop an algorithm that permits the generation of optimal solutions that are also integral. But first, we explain how this strategic model is integrated with the tactical model via a decomposition approach, namely the column generation approach.

3. The proposed integrated planning model

Our goal is to integrate value chain considerations into the forest management problem. The fact that we deal with two different problems has led us to a decomposition approach, namely the column generation approach in which the global problem is divided into a master problem and one or many subproblems. The master problem corresponds to the original problem with only a subset of variables being considered. The subproblems are new problems created to identify new variables or columns using information provided by the master problem as pricing on the different generated outputs. These variables are then added to the master problem which is subsequently is re-solved. As such, this process is iterative. At each iteration and until convergence criteria are met, the master problem is solved to produce a solution and new pricing information, and the subproblems are solved to generate new columns. In our case, the master problem is a combination of the tactical planning model and a set of sustainability constraints, and is solved using Logilab. On the other hand, the subproblems use the strategic planning model to produce new columns that represent a complete feasible set of treatments, and are solved using Silvilab (see Figure 5).

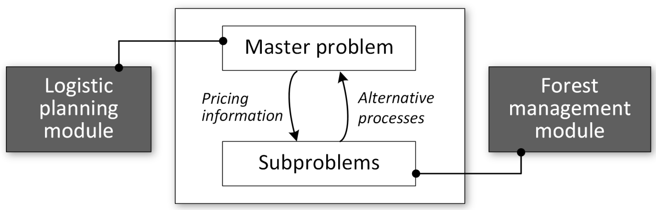


Figure 5: Schematic of the column generation decomposition.

The decision support systems in Logilab and Silvilab were extended using the improvement strategies described in the following subsections. Notice that the forest planning variables used in the master problem, which are generated by the forest planning subproblems, are clearly of the same nature as Model I arcs discussed previously. Because we assumed that a treatment could split an aggregated area of one forest type into many sub areas of different types, then our subproblems cannot be modelled by a simple directed graph. This explains why we present a model based on hypergraphs that is very similar in spirit to Model III representation. The fundamental difference between hypergraphs and graphs is that in a graph, an arc links two nodes, while in a hypergraph, an arc (hyperarc) links two sets of nodes (hypernodes). Thus, a treatment can be modelled by a hyperarc going from the set containing the hyper node representing the initial state to the set of hypernodes representing the transformed states. A pretreatment is done in the hypergraph that aggregates some hyperpaths into hyperpedges that are somehow similar to Model II arcs.

The forest planning problem as a subproblem generates alternative processes used in the master problem that represent the wood that could be obtained through harvesting and silvicultural treatments. A subset of the generic products used in the master problem will be part of the set of outputs considered in the forest planning problem. At a given column generation iteration, the dual value associated with the flow conservation constraints of that generic product at a given time defines the marginal benefit or cost. Another set of outputs used in the forest planning problem comes from the set of sustainability constraints added to the value chain optimization model to form the master problem. In this case as well, the dual values associated with those constraints, define the marginal cost of the associated output.

Using the column generation approach, the integrated planning problem is solved to optimality. This contrasts with the sequential heuristic procedure used to solve the hierarchical planning problem that in general provides an inferior solution. Thus, if we consider, for instance, a forest region producing species A and B, and we would like to revise the strategic plan of this forest region in order to maximize the volume of specie A with respect to volume of specie B, then this can only be achieved using integrated planning approach.

3.1. The master problem

Our master problem (**MP**) is formulated as follows:

$$\max \quad \sum_{i \in W} c_i x_i + \sum_{\theta \in \Omega} c_\theta x_\theta \tag{8}$$

s.t.

$$\sum_{i \in W} \alpha_{ij} x_i + \sum_{\theta \in \Omega} \beta_{\theta j} x_\theta = s_j \quad \forall j \in P \tag{9}$$

$$l_{pj} \leq \sum_{\theta \in \Omega} \beta_{\theta j} x_\theta \leq u_{pj} \quad \forall j \in B \tag{10}$$

$$(1 - \gamma_j) v_j \leq \sum_{\theta \in \Omega} \beta_{\theta j} x_\theta \leq v_j \quad \forall j \in S \tag{11}$$

$$\sum_{\theta \in \Omega_z} x_\theta = 1 \quad \forall z \in Z \tag{12}$$

$$l_{wi} \leq x_i \leq u_{wi} \quad \forall i \in W \tag{13}$$

where

sets

S : set of sustainable products

B : set of bounded products

Z : set of forest zone

Ω : set of feasible complete forest plans

Ω_z : set of feasible complete forest plans for the zone z

variables

x_i : quantity of generic process i used

x_θ : fraction of the zone z where the plan $\theta \in \Omega_z$ is applied

v_j : targeted amount of sustainable product j

parameters

$\beta_{\theta j}$: quantity of generic product j used $(-)$ or produced $(+)$ by forest plan θ

α_j : maximal absolute variation of the targeted volume of j allowed as a fraction

l_{pi} : lower bound on the quantity of generic bounded product i

u_{pi} : upper bound on the quantity of generic bounded product i

l_{wi} : lower bound on the quantity of generic process i

u_{wi} : upper bound on the quantity of generic process i

The model is similar to the value chain optimization model where there are two different types of processes: value chain processes (x) and forest planning processes (θ). Also, three constraints have been added: (i) constraints [10] set lower bound and upper bound on some bounded products; (ii) constraints [11], in conjunction with target variable v, impose even levels on sustainable products; and (iii) constraints [12] assure that the complete area of each zone received a treatment (natural growth is considered a treatment). The sustainable products and bounded products restriction applies exclusively to forest products and are used to impose restrictions and guides to the forest exploitation.

3.2. The subproblems

In this section, we formulate the subproblems using hypergraphs. In order to get a high level of performance, it is paramount that each subproblem is solved swiftly, since the subproblems have to be solved many times. To achieve this, we propose a dynamic programming algorithm to solve the forest planning problem.

3.2.1. Hypergraph formulation

The following definitions are related to hypergraphs:

Weighted directed hypergraphs : These are generalizations of graphs. They are represented by $H = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of hypernodes, and $E = \{e_1, e_2, \ldots, e_m\}$ is the set of hyperarcs. A hyperarc is a triplet (t_e, H_e, W_e), where $H_e \subset V$ is the head of e, $t_e \in V \setminus H_e$ is its tail and $W_e \in \Re^+$ is its weight set: every member v of H_e is given a weight w_{ev} in W_e. This definition is closely related to the one given by (Cambini et al., 1997), a more general one, that considers multiple tails can also be found in (Gallo et al., 1993). The set of arcs that have v as its tail is denoted by $\delta_+(v)$ and the set of arcs that have w in its head set is called $\delta_-(w)$.

Flow(d,Q) : A flow from source node d to termination nodes set Q, is a function $f : E \rightarrow \Re^+$ that respects the two following conditions:

1. *intial condition*: there is exactly one unit of flow going out of node d : $\sum_{e \in \delta_+(d)} f(e) = 1$.
2. *conservation condition*: For every node $v \notin T \cup \{d\}$, the incoming flow in a node equals the outgoing flow from the node, i.e.,

$$\sum_{e\in\delta_-(v)} w_{e,v} f(e) = \sum_{e\in\delta_+(v)} f(e).$$

Hyperpath : A flow from d to Q that has no cycle and where the support hypergraph (the hypergraph restricted to hyperarcs where $f(e) > 0$) defines a hyperarboresence (i.e. for every node $v \in T$, there is at most one hyperarc $e \in \delta_+(v)$ such that $f(e) \neq 0$).

The forest planning problem can be modelled using hypergraph. Let $v_{dt} \in V$ be a node corresponding to the state d at period t. Also, let $e_{at} \in E$ be an arc (v_{dt}, H_{at}, W_{at}) corresponding to doing treatment a at period t. Note that the space d corresponding to v_{dt} is the only state that a can be applied to, hence we have $a \in A_d$. The set H_{at} consists of all the node $v_{d',t+1}$ such that $\sigma_{ad'} \neq 0$ and the weight $w_{e_{at},v_{d',t+1}}$ associated with $v_{d',t+1}$ is equal to $\sigma_{ad'}$. To every state-spatial zone z, we associate a problem and a cost function $p_z : E \rightarrow \Re$ that attributes a cost to every arc. The cost of arc e_{at} is $c_{azt} + \sum_{o\in O} \mu_{ozt}\gamma_{aoz}$. The maximal flow in a hypergraph problem associated with state-spatial zone z is to find the flow x from $v_{d_z 1}$ to $Q = \{v_{dt} | t = T\}$ that maximizes $\sum_{e\in E} f_z(e) p_z(e)$. Since every arc goes from a node associated with a certain time t to a node associated with time $t + 1$, the hypergraph is acyclic by design so every solution is a connected directed hypergraph which contains no cycles (referred to as hypertree). In fact, as it will be shown later every optimal solution to the maximal flow in a hypergraph problem associated with state-spatial zone z is a hyperpath and we will sometimes refer to this problem as the maximal hyperpath problem in a hypergraph.

Figure 6 presents a small example of such an hypergraph. In this example there are 5 states d_1, d_2, d_3, d_4, d_5 and 3 periods. From state d_1, there are two possible treatments. One treatment will convert all the initial territory into

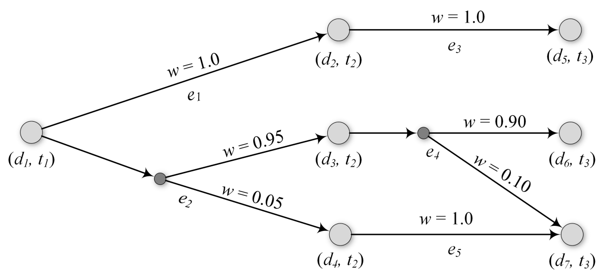


Figure 6: An example of a small hypergraph model.

state d_2 at period 2 represented by the arc $e_1 = ((d_1, t_1), \{(d_2, t_2)\}, \{1.0\})$. The second treatment will convert the initial territory into state d_3 on 95% of the territory and into state d_4 on 5% of the territory at period 2. This is represented by the arc $e_2 = ((d_1, t_1)\ , \{(d_3, t_2), (d_4, t_2)\}\ , \{0.95, 0.05\})$. At period 2, states d_2 and d_4 allow only one kind of treatment (which consist in doing nothing) that keeps the territory in the same condition but one period older (states d_5, d_7 and arcs e_3, e_5). The only treatment that can be applied to state d_3 at period 2, arc e_4, transforms 90% of the territory into d_6 and 10% into d_7 at period 3. There are two possible hyperpaths from (d_1, t_1) to the set $\{(d_5, t_3), (d_6, t_3), (d_7, t_3)\}$, the first is the path $f(e_1) = 1, f(e_3) = 1$ and the second is $f(e_2) = 1, f(e_4) = 0.95, f(e_5) = 0.05$.

In the preceding section, we established that the matrix A defined by the constraints [5]-[7] is a Leontief substitution system. Problems that can be modelled in whole or in part as an optimization problem over a Leontief substitution system can be solved using matrix iterative methods, with several computational advantages of the simplex method (Koehler et al., 1975). It is shown in Jeroslow et al. (1992) that Leontief substitution systems can be associated with flow problems on directed hypergraphs. The link between the Leontief substitution system associated with $(\mathbf{F(z)})$ and the flow problem defined on the directed hypergraph is quite straightforward: the flow on the arc e_{at} is associated with variable x_{azt}, the cost $p_z(e)$ is associated with the the cost $c_{azt} + \sum_{o\in O} \mu_{ozt}\gamma_{aoz}$ and the weight $w_{e_{at},v_{d',t+1}}$ is associated with the coefficient $\sigma_{ad'}$.

3.2.2. Dynamic programming approach

The dynamic programming approach is based on a generalization of the Dijkstra algorithm (Bondy and Murty, 1976). The fact that there is no loop in the graph allows for a very simple and efficient implementation of the Dijkstra algorithm in the context of hypergraphs. We define $V_i, i = 1 \dots n$ to be a partition of the node of a hypergraph $H = (V, E)$ such that for any nodes v in V_k we have for all $e = (t_e, H_e, W_e) \in \delta_+(v)$ that $H_e \subseteq \cup_{i>k} V_i$. Since, to our knowledge, there is no succinct description of the generalization of the Dijkstra algorithm to hypergraphs in the literature, we provide one in Figure 7. It presents a generic description of the algorithm that can be used to find the maximal hyperpath starting at node v_0 for an arbitrary price function ϕ. The algorithm has two main parts, the first one (the first loop) defines the hyperarcs used in the hyperpath solution, and the second (the second loop) computes the only feasible flow on the hyperarcs of the generated hyperpath. For the hypergraph formulation of the forest planning problem for zone z, we have $V_i = \{v_{dt} | t = i\}$ and $\phi(e) = p_z(e)$.

3.2.3. Improving the subproblems

The subproblems consist in solving the hypergraph problem for every zone $z \in Z$. The cost function $p_z : E \to \Re$ that attributes a cost to every

Let $e(v)$ be the hyperedge going out of v in the solution
Let $v(e)$ be the tail of edge e in the solution
$c(v) \leftarrow 0, \forall v, \quad f(v) \leftarrow 0, \forall v \neq v_0, \quad f(v_0) \leftarrow 1$
$D \leftarrow \{v_0\}$
# Define the hyperarcs used in the hyperpath
for $i = n-1$ to 1 **do**
  for all $v \in V_i$ **do**
    for all $e \in \delta_+(v)$ **do**
      if $\phi(e) + \sum_{w \in H_e} w_{ew} c(w) \geq c(v)$ **then**
        $c(v) \leftarrow \phi(e) + \sum_{w \in H_e} w_{ew} c(w)$
        $e(v) \leftarrow e, \quad v(e) \leftarrow v$
      end if
    end for
  end for
end for
# Compute the flow on the visited hyperarcs
while $|D| > 0$ **do**
  Let v be any node in D
  $D \leftarrow D \setminus \{v\}$
  $F \leftarrow F \cup \{v\}$
  for all $w \in H_{e(v)}$ **do**
    if $w \notin F$ **then**
      $D \leftarrow D \cup \{w\}$
    end if
    $f(e(w)) \leftarrow f(e(w)) + w_{e(v)w} f(e(v))$
  end for
end while

Figure 7: A dynamic programming algorithm to solve the maximal hyperpath problem in a loopless hypergraph.

arc is defined for the arc e_{atz} by:

$$c_{azt} + \sum_{p \in P} \mu_p \gamma_{apz} + \sum_{b \in B} \lambda_b \gamma_{abz} + \sum_{s \in S} \pi_s \gamma_{asz}$$

$\gamma_{a,o,z}$ is the amount of output o (generic product p, bounded product b or sustainable product s) generated by treatment a in zone z. c_{azt} is the cost of treatment a in zone z at period t. Also, μ_p is the dual value associated with constraints [9] and product p, λ_b is the dual value associated with constraints [10] and bounded product b, and π_s is the dual value associated with constraints [11] and sustainable product s. Once, a given subproblem is solved, a new variable θ is added to the column set Ω with coefficient value:

$$\beta_{\theta j} = \begin{cases} \sum_{z\in Z}\sum_{t\in T}\sum_{a\in A}\gamma_{apz}f(e_{atz}) & \forall j \in P \\ \sum_{z\in Z}\sum_{t\in T}\sum_{a\in A}\gamma_{abz}f(e_{atz}) & \forall j \in B \\ \sum_{z\in Z}\sum_{t\in T}\sum_{a\in A}\gamma_{asz}f(e_{atz}) & \forall j \in S \end{cases}$$

To limit the size of the subproblems and improve the performance of the algorithm, we limit the number of forest states by aggregating all states that are undistinguishable as far as the optimization process is concerned. For example, two forest states could belong to different watersheds, but be exactly the same on other aspects. If the distinction has no effect on their yield, growth or any other parameter considered during the optimization, then considering them as one state has no impact on the optimal solution. Since the possibility to aggregate two states depends on the detail of the problem, the aggregation is recalculated just before the optimization process.

To reduce solution time and memory needs, we implement the following two strategies. The first strategy (Strategy 1) is based on the observation that the structure of the hypergraph used in the subproblems is completely captured by the parameter σ_{ad} which depends only on the treatment considered. So instead of keeping in memory the whole structure of the hypergraph which has many copies of the same forest state and treatment information (as much as $|L||T|$ copies, where L is the number of different spatial zones), the hypernode and hyperarc structure can be represented by keeping only one layer. This layer should be representative for every spatial zone and time period. The second strategy (Strategy 2) is implemented when solving the subproblems using the dynamic programming algorithm presented in Figure 7, and is based on some of the specificities of the problem at hand. Since all subproblems shared the same hypergraph structure, they only differed by the pricing on their arcs. The pricing of an arc e_{atz} depends on the dual value for constraints associated to the products $\mu_p, \forall p \in P$, the sustainable products $\pi_s, \forall s \in S$ and the bounded product $\lambda_b, \forall b \in B$. In the problem we solved, the sustainable products are not related to the spatial zone. Also, the supply chain is considered only for the first period, so the generic products are only related to the first period. Hence the cost label $c(v)$ used in the dynamic programming algorithm for node associated to periods other than the first period are the same for all the zone subproblems. We used this fact in our acceleration strategy by grouping the zone into one large set and executing the dynamic programming algorithm only once for all the partition $V_i, i > 1$ and then differentiating the zone only for the partition V_1. The complexity of the algorithm is proportional to the number of arcs and is not influenced by the reduction done in the previous strategy.

4. Numerical experiments

The planning problems used for the experiments were built from real forest and industrial data and, together, they define an actual set of industrial forest planning problems. Two types of models were used to solve the industrial cases: (1) a non-integrated model where the forest planning problem with conservation constraints is first solved, and then the value chain optimization problem is solved with a fixed forest supply, (2) an integrated model where both the forest planning problem with conservation constraints and value chain optimization problem are solved together as one big problem. Three sets of experiments were conducted. The first set of experiments presented a comparison between several integrated and non-integrated models and analyzed the cherry picking effect in integrated models. This effect corresponds to the tendency for integrated models to harvest near the mills and to favour a species (in this case softwood) over another (hardwood). The second set of experiments aimed at demonstrating that the approach presented is efficient in solving large-scale problems. The last set of experiments studied carbon sequestration and demonstrated how the proposed approach, by its generality, can tackle non-traditional considerations within harvest planning.

4.1. Forest areas and test instances design

The data used in this research was pertaining to two actual Forest Management Units (FMU 031-53 and FMU 097-51) located in the boreal forest region in the province of Quebec, Canada. In FMU 031-53, the total productive area is 102 040 hectares, and the majority of the initial growing stock is softwood (88%) with presence of hardwood species (12%). Most recently published official AAC for this area (determined by government planners) is 100 600 m^3 for softwood and 9600 m^3 for hardwood. In FMU 097-51, the total productive area is 1 562 238 hectares, and the majority of initial growing stock is softwood (78%) with presence of hardwood species (22%). For both areas, some pure softwood stands occur naturally, and plantations are generally pure spruce. Also, a significant proportion of the forest cover is made up of mixed wood stands (33%) containing different proportions of hardwood mixed in with the softwood. For the two forest areas, we consider one harvesting treatment (clearcut) and two silviculture treatments (planting and pre-commercial thinning). No species-wise selective cutting is possible in mixed wood stands (i.e. hardwood must be harvested if present). The initial forest inventory, silviculture treatment eligibility and operability, yield curves, and state transition matrix were all compiled by analysts working for the government of Quebec.

We have tested our optimization model against an actual wood supply model used by the government, and confirmed that it perfectly replicates the source model structure. Our simplification of the objective function and constraints structure relative to full government regulation aside (our model is based on assumptions such as the linearity in the relationship between the cost of a treatment and the area treated, and it does not consider details on insects, diseases, and fire management), our model perfectly replicates the long-term wood supply model. Table 1 presents the important parameters used in our model for the two forest areas used for the various problem instances. These are the number of land zones ($|L|$), the number of different forest states ($|S|$), the number of different initial forest states ($|S_0|$), the initial number of forest zones ($|Z_0|$), the total number of forest zones ($|Z|$), the number of forest products (P), the number of individual stands (Stands), the number of hypernodes ($|V|$) and the number of hyperarcs ($|E|$) in the complete hypergraph.

Table 1: The principal model characteristics of the instances forest areas.

FMU	$\|L\|$	$\|S\|$	$\|S_0\|$	$\|Z_0\|$	$\|Z\|$	$\|P\|$	Stands	$\|V\|$	$\|E\|$
031-53	30	22 377	880	4164	671 310	8	75 502	7 117 230	9 297 630
097-51	16	395 892	13 596	21 596	6 334 272	8	161 391	101 757 552	133 548 752

The industrial network considered was composed of four sawmills selling their lumber products on the Canadian market, and a single paper mill that sells its paper products on the Canadian, American and European markets.

The species considered are SPF (spruce, pine and fir), paper birch, and yellow birch. The network has a limited capacity to process hardwood, and thus softwood is preferred. The value chain optimization model associated with this industrial network has 5 periods, 36 locations, 216 processes, 505 transportation links and 172 products and resources. This produces a total of 3566 generic processes (variables) and 1045 generic products (constraints).

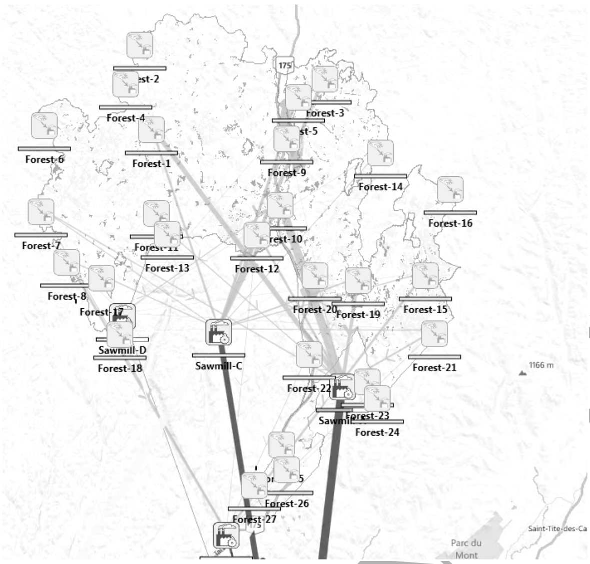
A virtual computing machine offering a mix of eight Intel Xeon 2.6 or 3.06 GHz processor cores and 96 Gb of RAM was used for all the computations.

4.2. Comparison between integrated and non-integrated approaches

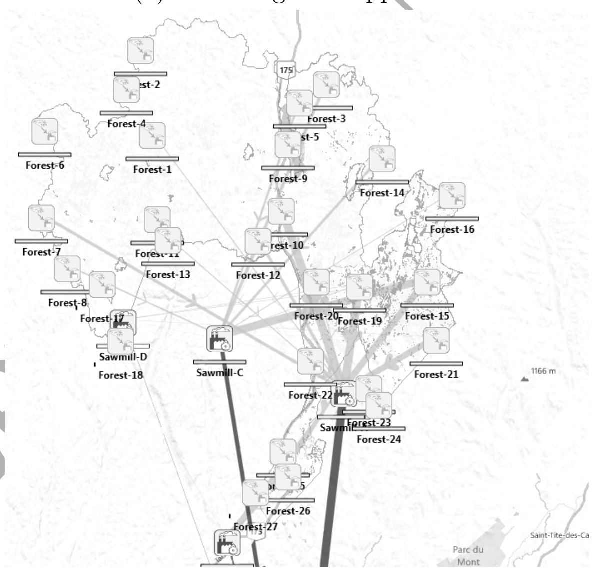
For this numerical experiment, the forest area FMU 031-53 was used in conjunction with industrial network described previously. Table 2 compares the profit, transportation cost, harvested volume and proportion of hardwood versus softwood resulting from the integrated and non-integrated approaches. We chose those parameters because we wanted to evaluate the influence of two cherry picking effects. The results show a clear gain in profit when planning is conducted using the integrated approach. Reduced transportation cost accounts for almost half the gain, but a noticeable decrease in the harvest of hardwood might also be of importance. In this experiment both cherry picking effects (the trend to harvest near the mills and to favour softwood over hardwood) were clearly present (see Figure 8a and Figure 8b).

Table 2: Integrated-non-integrated comparison for FMU 031-53.

	Integrated	Non-integrated
Profit ($)	17 614 383	19 905 315
Transportation cost ($)	21 839 367	20 974 401
Harvested volume (m^3)	544 621	514 625
Proportion of hardwood (%)	11.6	7.0



(a) Non-integrated approach.



(b) Integrated approach.

Figure 8: Harvest plan and network flow for the first 5 years. The light gray lines indicate the flow of logs from the forest areas to the sawmills. The dark line indicate the flow of chips from the sawmills to the paper mill (not shown in the graphs). The thickness of the lines is proportional to the volumes transported. When the harvest plan is generated using the integrated approach, much of the wood is harvested in areas in close proximity to the sawmills.

4.2.1. Subproblems improvement strategies

In this section, we present evidence of the effectiveness of the two strategies proposed to improve the subproblems. Table 3 presents the experimental results of the hypergraph generation approach on two integrated forest planning problems with the improvement strategies (Strategy 1 and Strategy 2) implemented or not. Since the product derived from the multiplication of the number of land zones by the number of periods ($|L||T|$) corresponds to the number of copies of the forest state and treatment information that would be kept in memory if the first improvement strategy was not implemented (see section 3.2.3), then it becomes possible to quantify the reductions in terms of the number of hypernodes and the number of hyperarcs in the hypergraph (see the third and second last rows in Table 3). It can be deduced that the memory size of the problem was reduced by a factor that ranges from 244 to 318 which corresponds to 34% to 64%. The speedup generated by improvement strategies is given in the last row of Table 3. It can be noticed that the solution time is expected to decrease by a factor varying from 147 to 158.

4.3. Performance analysis

Table 4 presents a comparison of the computation times and memory requirements resulting from different instances of the integrated and non-integrated models resolved using the hypergraph column generation or the standard LP column generation. The master problem was solved with Cplex 12.4. Notice that, for all the instances, the same basic set of $|T|$ conservation constraints, which are even level constraints on the sustainable products associated with the total harvested volume at each period, were used for all the instances. Also, the additional set of more restrictive conservation

Table 3: Experimental results of the hypergraph column generation approach on two integrated forest planning problems. We compare the hypergraph size and the computational performance before and after reduction using the improvement strategies.

Forest Management Unit	031-53	097-51
Nb. of different spatial zones ($\|L\|$)	30	16
Nb. of periods ($\|T\|$)	30	25
Nb. of hyperarcs for the first period($\|E_0\|$)	1350	18 676
Nb. of hyperarcs before reduction ($\|E\|$)	9 297 630	133 548 752
Nb. of hyperarcs after reduction ($\|E_R\|$)	30 127	547 486
Nb. of hypernodes before reduction ($\|V\|$)	7 117 230	101 757 552
Nb. of hypernodes after reduction ($\|V_R\|$)	22 377	395 892
Reduction in the nb. of hypernodes ($\frac{\|V\|}{\|V_R\|\|L\|\|T\|}$)	0.35	0.64
Reduction in the nb. of hyperarcs ($\frac{\|E\|}{\|E_R\|\|L\|\|T\|}$)	0.34	0.61
Estimated speedup: $\frac{\|E\|}{\|E_R\|+\|L\|\times\|E_0\|}$	147	158

constraints contains $7 \times |T|$ new even level constraints on the sustainable products associated with the total harvested volume of seven species at each period. Note that we did not use bounded products in any of the instances and they are put into the model only for the sake of completeness. The results support the utility of the proposed formulation and accelerating strategies. In average, a noticeable 58% reduction in computation time and a much more impressive 95% reduction in memory requirement could be achieved when hypergraph column generation is used. This memory reduction accomplishment is an important gain since large-scale forest planning problems often use a vast quantity of memory. To appreciate this memory utilization, one can see that the highest memory peak in Table 4 is 76.20 Gb, even though a commercial implementation can reduce that amount of memory utilization by two or three it is still a very memory intensive operation and much bigger problems are routinely encountered.

Table 4: Comparison of the computation times and memory requirements resulting from different integrated and non-integrated forest planning problems resolved using the hypergraph column generation (HG) and the standard LP column generation; The instances differ according to the number of 5-year periods ($|T|$), the forest area (FMU), whether (1) or not (0) an additional set of more restrictive conservation constraints is present (MRC) and whether or not the problem is integrated.

Inst.	$\|T\|$	FMU	MRC	Integrated	HG		LP	
					Time(s)	Memory(Gb)	Time(s)	Memory(Gb)
#1	20	031-53	0	0	2.7	0.13	11.8	3,04
#2	30	031-53	0	0	3.6	0.13	15.4	3.63
#3	40	031-53	0	0	5.5	0.13	19.2	4.25
#4	20	031-53	1	0	5.8	0.15	19.2	3,76
#5	30	031-53	1	0	12.8	0.15	28.1	4.56
#6	40	031-53	1	0	27.7	0.17	48.7	5.41
#7	20	031-53	0	1	5.9	0.13	14.3	3.04
#8	30	031-53	0	1	7.5	0.13	17.9	3.74
#9	40	031-53	0	1	10.7	0.15	23.5	4.39
#10	20	031-53	1	1	18.3	0.23	21.1	3,96
#11	30	031-53	1	1	26.2	0.29	30.1	4.77
#12	40	031-53	1	1	71.3	0.44	54.3	5.55
#13	20	097-51	0	0	40.1	2.02	116.5	20.91
#14	30	097-51	0	0	55.6	2.17	178.5	31.33
#15	40	097-51	0	0	89.3	2.33	268.8	41.71
#16	20	097-51	1	0	64.1	2.61	241.0	27.21
#17	30	097-51	1	0	128.6	2.61	384.0	40.82
#18	40	097-51	1	0	265.7	2.61	567.4	54.22
#19	20	097-51	0	1	60.2	2.34	214.5	38.18
#20	30	097-51	0	1	73.8	2.35	305.6	48.61
#21	40	097-51	0	1	112.7	2.35	430.3	59.02
#22	20	097-51	1	1	111.4	2.61	422.0	49.12
#23	30	097-51	1	1	256.6	2.61	696.6	62.70
#24	40	097-51	1	1	339.3	2.61	861.3	76.20

4.4. New sustainability concerns

As forest planners are more and more concerned with the sustainability of the forest use, two experiments were conducted to explore how to tackle these new sustainability concerns. The first experiment aimed at generating harvest plans that are sustainable during the planning horizon in regard to the transportation capacity (i.e the distance $\times$ the volume). This is an important issue considering the tendency of integrated planning approaches to exploit forests nearest to the mills and by the fact that integrated models, like the one proposed here, only consider the consequences of the proposed long-term forest plan on the supply chain for the first period. The second experiment used the flexibility of the proposed model to propose how carbon sequestration can be included in long-term forest planning problem.

4.4.1. Sustainable transportation capacity

For this numerical experiment, forest area FMU 097-51 was used in conjunction with the industrial network described in section 4.1. Notice that, in this case, the FMU is actually used to supply the mills in the Côte-Nord region. This numerical experiment aimed at addressing the issue that may arise in integrated approach if the solution for the first period harvests too much wood in the area close to the mills. FMU 097-51 is a good choice for this experiment since the forest area is a narrow north-south band with all the mills located in the south, hence there is a major advantage in limiting harvest to the forest nearest to mills. Again, we ran two distinct experiments. The non-integrated case first plans the forest on a horizon of 25 periods of 5 years by maximizing volume, then the value chain is optimized to make the best of what has been planned for the first 5 years. In the integrated case, both the 125 years and the industrial network first 5 years were optimized simultaneously by our integrated approach. In both cases, the harvest was restricted by an even yield constraint with 5% flexibility. The extra set of conservation constraints was used for these instances. The comparison results are presented in Table 5. Here, we wanted to evaluate the tendency for integrated models to harvest near the mills (a 40% decrease in harvest distance).

Table 5: Integrated-non-integrated comparison for MU 097-51.

	Integrated	Non-integrated
Profit (\$)	40 099 001	41 004 562
Transportation cost (\$)	2 414 631	1 416 411
Harvested volume (m^3)	494 314	491 226
Average distance (km)	48.8	28.8

Considering this massive reduction in transportation distance, an obvious question to ask at this stage is: What is the effect of systematically harvesting zones located in proximity to mills on the sustainability of the harvest? To investigate this question we introduced a constraint, inspired by the flexible even-flow constraints, to limit the variation of the average harvest distance across time. In fact, we added a conservation constraint that imposes that the transportation capacity is within γ % of the maximum distance capacity encountered during all five-year periods. More specifically, the following constraint is added to the master problem $(1-\gamma)c \leq \sum_{\theta\in\Omega} c_\theta x_\theta \leq c$ where c_θ is the transportation capacity associated to column θ, c is a variable representing the target transportation capacity value, and γ is a flexibility parameter. This new constraint is an even level constraint on sustainable products and as such it is just a particular case of one of the group of constraints defined in the master problem.

Figure 9 presents how much of the initial distance (48.8 km) obtained

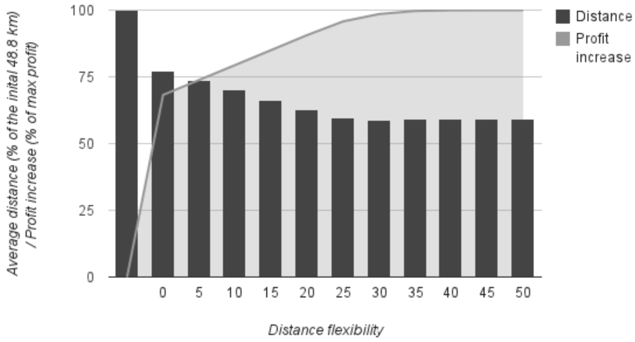


Figure 9: Average distance and profit increase vs distance flexibility.

in the non-integrated case can be reduced in the integrated case combined with a sustainable harvest distance constraint when the flexibility parameter value varies from 0% to 50%. The results show that in this case, a flexibility of at least 30% is necessary to get the minimal distance of 28.8 km in average. It is interesting to see that reductions of about 25% are still possible with a very strict average distance limitation. Remember that even with flexibility of 0%, there can still be some variation across time in the average distance since the total volume may vary. This is an encouraging result since it shows that an integrated approach to forest planning can generate a sustainable transportation economy and produce plans that better reflect the behaviour of the industry.

4.4.2. Carbon sequestration consideration in harvest planning

The objective here is to capture the kind of trade-off necessary to consider carbon sequestration during harvest planning. Since there is no carbon sequestration model available in the Silvilab software when doing the experiments, we used a very simple rule to determine the quantity of carbon sequestration: At a given period, the quantity of carbon captured is proportional to the forest increase in volume. The proportionality depends on an average content of carbon by cubic meter of wood. We stress that the only aim of these results aim at showing the capacity of our approach to easily tackle such extensions in forest planning and suggesting that one of the factors contributing to carbon sequestration, namely forest growth, can go hand in hand with sustainable forest planning.

Four scenarios were analyzed using forest area FMU 031-53. In the first scenario, the forest was left to grow naturally according to the growth curve model used in the simulation. The second scenario optimized the volume of wood that could be harvested with sustainable levels across time with 5% tolerance. The third scenario optimized the total carbon sequestration across time. Finally, the last scenario also optimized the total carbon sequestration across time but did not allow volume to fall under 99.999% of its maximum value. Table 6 presents the volume of wood harvested and the quantity of carbon captured for each of the four experiments.

Table 6: Variation of the quantity of CO_2 sequestered according to the objective.

	Volume (m^3)	CO_2 sequestration (T)
Natural growth	0	3 574 578
Max volume	18 244 114	4 910 863
Max sequestration	18 109 953	6 010 987
Max sequestration (with volume $\geq$ 99.999% of Max volume)	18 244 000	6 009 931

The results reveal that there is almost no trade-off between maximizing volume and maximizing carbon sequestration. This is in fact not very surprising considering that with our very simple carbon sequestration model, we seek in both cases to have the maximal steady volume increase. It is interesting to note that more than 22% increase in carbon sequestration could be achieved in the second and third scenarios (when carbon sequestration was optimized) in comparison with second scenario (when volume was maximized and carbon not considered). In our experimental setting, this increase could be at the detriment of old forest which, according to our growth curves used in the models, produce less volume (hence emitting carbon) as they get old.

5. Conclusion

Integrated planning consisting in considering tactical aspects in the long-term forest management problem brings new challenges and opportunities and enlarges the spectrum of sustainability concerns that need to be tackled. We presented a model to support forest planning on the long term with anticipation of the impacts on the economic and logistic activities in the forest value chain on a shorter term, and we proposed a novel optimization approach to efficiently solve large-scale practical instances of this integrated planning problem. The core of the approach is based on column generation decomposition and a dynamical programming algorithm to solve the maximal hyperpath in hypergraph subproblems. Two acceleration strategies were also introduced. The hypergraph structure and the acceleration strategies also allow for a reduction of the memory requirement.

We used data from a large-scale industrial case with a real forest and a real industrial network located in the Côte-Nord region of the province of Quebec, eastern Canada, in order to compare the proposed integrated approach with a non-integrated approach. Our results showed that a clear gain in profit could be achieved when planning is conducted in an integrated approach. This is a very critical result for the stakeholders in the Côte-Nord region as these stakeholders are currently studying their supply network. In fact, their mills are currently supplied from FMU 097-51 but they have the possibility to be supplied from FMU 031-53, and the stakeholders, including industrials and the government, are looking to develop a collaborative approach in managing their forests and supply chains. Finally, we used the data from our large scale industrial case to compare the performance of the proposed optimization approach with that of a mixed integer programming approach. We conducted several numerical experiments and found that improvements by average factors of 2.4 and 20 could be achieved in CPU time and memory requirement, respectively. These aspects are among the main limiting factors when solving large-scale forest planning problems.

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References

Bondy, J., Murty, U., 1976. Graph Theory with Applications. Elsevier North-Holland.

Cambini, R., Gallo, G., Scutell, M. G., 1997. Flows on hypergraphs. Mathematical Programming 78 (2), 195–217.
URL http://dx.doi.org/10.1007/BF02614371

Dantzig, G. B., Wolfe, P., 1960. Decomposition principle for linear programs. Operations Research 8 (1), pp. 101–111.
URL http://www.jstor.org/stable/167547

Davis, L., Johnson, K., Davis, K., 1987. Forest management. McGraw-Hill series in forest resources. McGraw-Hill.
URL http://books.google.ca/books?id=yzLxAAAAMAAJ

Desaulniers, G., Desrosiers, J., Solomon, M., 2005. Column generation. Cahiers du GERAD. Springer.
URL http://books.google.ca/books?id=ncD7qnkT-1EC

Gallo, G., Longo, G., Pallottino, S., Nguyen, S., 1993. Directed hypergraphs and applications. Discrete Applied Mathematics 42 (23), 177 – 201.
URL http://www.sciencedirect.com/science/article/pii/0166218X9390045P

Garcia, O., 1990. Linear programming and related approaches in forest planning. New Zealand Journal of Forestry Science 20, 307–331.

Gunn, E. A., 2007. Models for strategic forest management. In: Weintraub, A., Romero, C., Bjrndal, T., Epstein, R., Miranda, J. (Eds.), Handbook Of Operations Research In Natural Resources. Vol. 99 of International Series In Operations Research amp; Mana. Springer US, pp. 317–341.
URL http://dx.doi.org/10.1007/978-0-387-71815-6_16

Jerbi, W., Gaudreault, J., D'Amours, S., Nourelfath, M., Lemieux, S., Marier, P., Bouchard, M., 2012. Optimization/simulation-based framework for the evaluation of supply chain management policies in the forest product industry. In: Systems, Man, and Cybernetics (SMC), 2012 IEEE International Conference on. pp. 1742–1748.

Jeroslow, R. G., Martin, K., Rardin, R. L., Wang, J., 1992. Gainfree leontieff substitution flow problems. Mathematical Programming 57, 375–414.

Koehler, G., Whinston, A., Wright, G., 1975. Optimization over Leontief Substitution Systems. North-Holland.

Paradis, G., Lebel, L., Amours, S. D., Bouchard, M., 2013. On the risk of systematic drift under incoherent hierarchical forest management planning. Canadian Journal of Forest Research 43 (5), 480–492.

Pittman, S. D., Bare, B. B., Briggs, D. G., 2007. Hierarchical production planning in forestry using price-directed decomposition. Canadian Journal of Forest Reserach 37, 2010–2021.

Schneeweiss, C., 2003. Distributed decision making–a unified approach. European Journal of Operational Research 150 (2), 237–252.

Smith, S., 1978. A two-phase method for timber supply analysis. proceedings of IUFRO Operational forest management planning methods, Bucharest, Romania., 76–81.

Troncoso, J. J., D'Amours, S., Flisberg, P., Rönnqvist, M., Weintraub, A., 2015. A mixed integer programming model to evaluate integrating strategies in the forest value chain: A case study in the Chilean forest industry. Canadian Journal of Forest Research 45 (April), 937–949.

Weintraub, A. P., Cholaky, A., 1991. A hierarchical approach to forest planning. Forest Science 37 (2), 439–460.

Weintraub, A. P., Navon, D., 1976. A forest management planning model integrating silvicultural and transportation activities. Management Science 22 (12), 1299–1309.
URL `http://www.jstor.org/stable/2630143`