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Two-period pricing and quick response with strategic customers

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This research examines the impact of the strategic customer behavior in two-period pricing and the inventory decisions in a quick response system. A model with a differentiated value period of product is developed when customers are strategic and heterogeneous. Interestingly, the unique equilibrium is proven to exist if and only if the degree of customer strategic behavior is sufficiently high. Otherwise, the dynamic pricing strategy in one selling season is not a suitable choice for a firm. Moreover, the impact of strategic consumers on pricing and inventory strategies is investigated in the case where the clientele’s taste for product value follows a uniform distribution. Surprisingly, contrary to previous studies, we found that strategic consumers may yield more revenues in specific scenarios. An extended analysis on Beta distribution is also presented, showing that there is greater chance to obtain the highest profit in the supply chain when all customers are strategic and if more people prefer low-value products.

Key words: strategic customer; quick response; game theory; inventory and dynamic pricing decisions
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1. Introduction

Many products, such as electronic items and fashion apparel, decrease in value in one selling season. Some firms use a dynamic pricing strategy to promote the expansion of product demands in order to obtain greater profit. Based on this strategy, a firm dynamically adjusts the prices during different selling periods. It charges high when the product’s value is elevated, and later, when the product’s value has decreased, such items are sold at a marked down price. Advantages of dynamic pricing are well-known in the fashion apparel industry, and in various other Online-to-Offline (O2O) markets. In O2O markets, customers can more readily obtain information on the product’s price than in the traditional offline model. This difference further enhances the implementation of the dynamic pricing approach. For example, famous international companies such as Zara, Nike, Gap and Uniqlo, apply a dynamic pricing strategy which affects the consumers’ selections. Uniqlo, a Japanese fashion retailer, implements these tactics more successfully. Uniqlo has established that it is more beneficial to reduce the prices at the beginning of the year and sell items at a 50 percent discount during a season, than raise the prices by 30 percent at the end of the year. To execute this strategy, Kana (2016) claims that prices at the Uniqlo online and offline stores are reduced on a weekly
basis in a bid to overhaul steep discounts on the weekends, allowing the business to improve its gross profit margin in the last quarter (Kana, 2016).

A general finding of existing research is that the dynamic pricing strategy can be an effective choice for a firm to maximize its profits (Elmaghraby et al., 2008, Zhang et al., 2015, Papanastasiou and Savva, 2016). However, this study finds that this dynamic pricing strategy does not always engender an increase in revenue for the concerned products considering that this strategy is ineffective under some special circumstances. We prove that if, and only if, the level of customers’ strategic behavior is sufficiently high could this dynamic pricing strategy bring higher profit. So-called strategic customers are buyers who are always prudent and forward-looking in the market. When making a purchasing decision, they consider expectations of future prices and will either immediately choose to buy or delay their purchase. Under such circumstances, firms must consider customer strategic behavior in pricing decisions. Nonetheless, the level of customer strategic behavior is rarely considered in the choice of dynamic pricing strategy in extant literature, until now.

Another key component that we addressed in this research is the aspect of a quick response system, which has recently received considerable attention and is widely used in practice due to uncertainty in customer demands and appropriate quantity to produce. Therefore, numerous firms attempt to reduce the lead times, such that if they run out of stock during the selling season, they can immediately respond since there is an available alternative to quickly fill the orders. This pattern is called a quick response system. Although such a system may increase firms’ cost, it allows them to mitigate demand uncertainty and gain greater profit during the selling season. Cachon and Swinney (2009, 2011) modeled a firm’s joint-pricing and inventory decisions in a two-period setting with a quick response system. However, they either analyzed the inventory only in the second period pricing decision (an exogenous price in first period) (Cachon and Swinney, 2009), or analyzed the inventory in the first period pricing decision (an exogenous price in second period) (Cachon and Swinney, 2011). The present work relaxes the assumption of exogenous price in their research study and provides a more in-depth research on that basis.

For these reasons, we considered the situation wherein the product value changes in a selling season that is divided into two periods: the higher product value period and the lower product value period. Consumers will immediately purchase or delay buying according to their individual evaluation of the product value and the degree of their patience to wait for a lower price. This study investigates the effects of strategic customer behavior in such a dynamic pricing policy and addresses the following concerns:
1) Whether or not a dynamic pricing policy helps a firm benefit in the presence of strategic customers
2) How the degree of customers’ patience influences firm pricing and inventory decision-making
3) Impact on firm profit relative to the degree of customer patience, and in which scenarios will strategic consumers add more revenue.

The remaining parts of this paper are organized as follows: Section 2 provides a literature review; Section 3 presents the model description, which consists of notations and the firm and consumer decision procedure; Section 4 presents the modeling details and analyzes the effect of consumers’ strategic behavior; Section 5 provides an extended analysis of whether customers’ taste for product value follows the Beta distribution; and Section 6 presents the conclusions of this paper.

2. Literature review

The literature on strategic consumer purchasing behavior has a long history beginning with the inspiration from Coase (1972) that customers will wait for lower price in a durable goods monopolist market. In recent years, the strategic consumer behavior has been incorporated into the traditional operational models. In a pioneering paper in QR system by Iyer ad Bergen (1997), formal models of the inventory decisions of manufacturers and retailers were built to study the impacts on choices of production and marketing variables in the apparel industry under the QR system. In many studies, firm decision-making can be affected by strategic consumer behavior. For example, Liu and van Ryzin (2008, 2011), Lai et al. (2010) and Jerath et al. (2010) focused on how strategic consumers anticipate future surplus driven by firm inventory decisions. Moreover, Aviv and Pazgal (2008), Yin et al. (2009) and Su (2010b) examined firms’ pricing decisions under certain conditions where consumers anticipate future markdown of prices. Moreover, Su and Zhang (2008) found that when the customers are forward-looking in the news vendor model and the price is set exogenously, the seller’s stock level is lower because of consumers’ expectations of future prices. These authors also considered the situation where stocks are very costly to consumers, and further examined two strategies intended to enhance profits. Su (2010a) developed a model with speculators and strategic consumers and found that speculators tend to increase firm profits and lower capacity investments. Additionally, Liu and Zhang (2013) examined a heterogeneous market with two firms and differentiated products, and emphasized the role of product quality and the value of price commitment. Several researchers, such as Anderson and Wilson (2003), and Aviv and Pazgal (2008) considered that the strategic consumers may
negatively impact firm revenues. Conversely, Li et al. (2014) used a structural estimation model to analyze strategic customers, and found that the presence of strategic customers does not necessarily hurt firm profit. Based on this research, the present paper develops a theoretical model and ascertains the conditions under which the presence of strategic customers yield more revenues.

Other related stream of literature focuses on the quick response (QR) system. Some research including Fisher and Raman (1996), Fisher et al. (2001), and Goyal and Netessine (2007), examined the firm’s decision under the QR system given non-strategic customers. These authors also studied the performance of the QR system in competition. Li and Ha (2008), Caro and Martínez-de-Albéniz (2010) and Lin and Parlaktürk (2012) inspected the competitive value of the QR system. Krishnan et al. (2010) highlighted a potentially damaging impact of the QR system on retailer effort. Wang et al. (2014) found that competing firms choose responsiveness to be favorable under the conditions of demand uncertainty or of weak product competition. Several studies focused on the influence of the QR system to the period of inventory decision, such as Serel (2012) who analyzed a single-period inventory model for multiple products in a QR system with a budget constraint, and revealed that this can lead to the increase in order size. More recent works focused on multi-period inventory decisions. Choi (2013) analyzed the carbon footprint taxation scheme by examining both the single-ordering and the dual-ordering QR system, and found that a properly designed carbon footprint taxation scheme can enhance environmental sustainability. Gong et al. (2014), on the other hand, compared a quick-response supplier and a regular supplier in two inventory periods, and uncovered that supply source diversification or high supplier reliability increases optimal profit while lowering the selling price. Calvo and Martínez-de-Albéniz (2015) observed that general dual-sourcing does not always lead to higher supply chain efficiency or buyer profits than single-sourcing. Further, Choi and Sethi (2010) provided useful information in helping academicians and practitioners to effectively design, control, and implement QR programs by classifying the literature into three major areas: supply information management, demand information management, and the values of information and supporting technologies.

Unlike the studies mentioned, this research looks into the QR system with strategic consumer behavior since very few papers focus on this problem. Cachon and Swinney (2009) considered that the value of quick response can be enhanced from strategic consumer behavior. They developed a model in a single season which is divided into two periods and our model has a similar assumption. However, in their model, the retailer sells the product at a fixed and exogenous full price. They (2011) further analyzed four potential operational systems with
strategic consumers and compared their performance. These four potential operational systems include: a traditional system, a quick response system, an enhanced design system, and a fast fashion system. In their research, a product was sold at a fixed price during selling season and had an exogenous salvage. Unlike their research, this work studies the dynamic pricing and inventory problem with strategic consumers. In addition, several papers studied the dynamic pricing or markdown problem with strategic consumers. For example, Elmaghraby et al. (2008) analyzed the optimal markdown pricing mechanism with preannounced prices in the presence of rational or strategic buyers. Aviv and Pazgal (2008) studied the optimal pricing of a finite quantity of a fashion-like seasonal good in the presence of strategic customers. Further, Levin et al. (2009) built a stochastic dynamic game model for a monopolistic company selling a perishable product to a finite population of strategic consumers. Wu et al. (2015) considered a retailer’s markdown pricing and inventory decisions in multiple seasons where consumers can learn from reference prices to decide when to purchase. In these studies, dynamic pricing always bring more profit than a fixed pricing strategy. However, the present paper demonstrates that the dynamic pricing strategy in one selling season is not a suitable choice for a firm if insufficient myopic customers exist. Some scholars explored the O2O environment with a QR system and strategic customers such as Huang and Mieghem (2013) who used a newsvendor framework to evaluate the value of online click-tracking of strategic customers by comparing with quantity commitment, availability guarantees, and quick response. Moreover, Swinney (2011) demonstrated that the value of the QR system is lower with strategic customers than with non-strategic customers when they have uncertain and heterogeneous valuations for a product. As discussed, such a viewpoint is common. The present paper demonstrates that the viewpoint is incorrect in some conditions. Yang et al. (2015) analyzed the impact of quick response on supply chain performance for various supply chain structures with strategic customer behavior and found that the value of the QR system would be greater in centralized supply chains systems than in decentralized systems when the extra cost of quick-response was relatively low. They focused on the impact of quick-response on supply chain performance. However, the present paper focuses more on the impact of strategic customers.

To summarize, our model is the first, to our knowledge, which considers the relation between a firm’s joint dynamic pricing and inventory decisions under a QR system and the strategic and heterogeneous consumers’ behavior when the value of product changes in the selling season.
3. Model description

In our model, the firm’s decision-making period is divided into two periods: the selling season period and the period prior to the selling season. The selling season includes high-value and low-value periods. The firm’s decisions consist of regular price, markdown price, and initial stocking quantity, which are usually announced before the sales season (Elmaghraby et al., 2008). Furthermore, regular-season prices and markdown prices are executed respectively in high-value and low-value periods, and the initial stocking quantity is executed prior the selling season. The consumers and the firm participate in a game where the consumers decide when to buy the product (during high value or low-value periods), and the firm decides whether to use the dynamic pricing strategy. If the dynamic pricing strategy could bring more benefit, then the firm chooses the early production quantity executed prior to the season and what price to charge executed during the selling season.

3.1 Notations

The following notations are used in this present study:

\( H \) The first period in a selling season
\( L \) The second period in a selling season
\( v_i \) The product’s value in period \( i \) \( (i = H, L) \)
\( q \) Initial stocking quantity, decision variable
\( N \) Market size
\( F(\cdot) \) Distribution function of market size \( N \)
\( f(\cdot) \) Density function of market size \( N \)
\( \mu \) Mean value of market size \( N \)
\( c \) Unit cost in early production (Before the selling season)
\( c_{qr} \) Unit cost in the postponed production (During the selling season)
\( \theta \) Customers’ heterogeneous tastes on value \( v_i \)
\( G(\cdot) \) Distribution function of customers’ tastes \( \theta \)
\( g(\cdot) \) Density function of customers’ tastes \( \theta \)
\( p_i \) The price in period \( i \) \( (i = H, L) \), decision variable
\( \delta \) Discount factor (or equivalently, a waiting cost, Li et al. 2014), \( 0 \leq \delta \leq 1 \)
\( D_i \) Expected demand during the selling period \( i \) \( (i = H, L) \)
\( \theta_i \) Expected ratio of customers who purchase during the selling period \( i \) \( (i = H, L) \)
\( \Theta = \theta_H + \theta_L \)
\( \Pi^{\omega} \) The firm’s expected profit with a quick response system
A

Expected lost sales function

3.2 The firm and the consumers

Consider a firm selling a product in a single selling season. The selling season is divided into two consecutive periods: the first is period $H$, while the second period is $L$. The product is valued by the customers at $v_i$ in different periods. We assume $v_H > v_L$ throughout. Therefore, the product has higher value in period $H$ than in period $L$. Two potential production opportunities exist for a firm: early production and postponed production. Early production (initial stocking quantity, $q$) is far in advance of the selling season and the market size is unknown. We assume $N$ to be a random variable with positive support, distribution function as $F(\cdot)$ and density as $f(\cdot)$. The postponed production is during the selling season, and the market size is known perfectly (Robert Swinney 2011). Early production incurs a unit cost $c$, whereas the postponed production incurs a higher unit cost as $c_q > c$. We assume the production has a short enough lead time during the latter opportunity. This is one reason why we assume that $c_q > c$.

All customers arrive at the beginning of the selling season. Customers have heterogeneous tastes on value ($v_i$) as denoted by $\theta$, and which is a private information of each customer (Tirole 1988, Liu and Zhang 2013, Wu 2015). We assume that $\theta$ follows a distribution function $G(\cdot)$ and density $g(\cdot)$, which is a common knowledge. The prices offered in period $H$ is denoted by $p_H$ while in period $L$ is denoted by $p_L$. At the end of the season, the remaining inventory’s salvage is zero. The firm determines the prices ($p_H / p_L$) and quantity ($q$) simultaneously at the beginning of the selling season to maximize their respective profits collected over the selling season. To simultaneously model both strategic and myopic customers, we introduced a discount factor denoted by $\delta$ ($0 \leq \delta \leq 1$). We can interpret $\delta$ as the degree of patience to wait for the markdown or the level of strategic behavior. A higher $\delta$ implies more patient or strategic consumers. If $\delta = 0$, then all customers do not anticipate the opportunity to purchase in period $L$, whereas if $\delta = 1$, then all customers do without discount of future purchases (Su and Zhang 2008).

If a customer with tastes $\theta$ purchases product at price $p_H$ in period $H$, then she earns a surplus of $\theta v_H - p_H$. The expected surplus from a delayed purchase is $\delta(\theta v_L - p_L)$. In some research, the surplus of a delayed purchase is $\delta \theta v_L - p_L$. Cachon and Swinney (2011) considered the latter results in slightly higher full prices. She can also choose not to purchase and earn zero surplus. The sequence of events is depicted in Figure 1 below.
3.3 The consumer decision: wait, buy or not buy?

A customer with taste $\theta$ purchases in period $H$ if it leads to positive surplus that is higher than purchasing in period $L$, i.e., $\theta v_H - p_H > \delta (\theta v_L - p_L)$. Similarly, she purchases in period $L$ if $0 < \delta (\theta v_L - p_L) \geq (\theta v_H - p_H)$. Otherwise, if $\delta (\theta v_L - p_L) \leq 0$, she cannot obtain surplus through purchasing and will choose not to buy.

From equation $\theta v_H - p_H = \delta (\theta v_L - p_L)$, we obtain $\theta = \frac{p_H - \delta p_L}{v_H - \delta v_L}$. Further, from equation $\delta (\theta v_L - p_L) = 0$, we obtain $\theta = \frac{p_L}{v_L}$. Hence, in period $H$ if customer’s taste on value is greater than $\frac{p_H - \delta p_L}{v_H - \delta v_L}$ (i.e., $\theta > \frac{p_H - \delta p_L}{v_H - \delta v_L}$), then she chooses to purchase. Otherwise (i.e., $\theta \leq \frac{p_H - \delta p_L}{v_H - \delta v_L}$), she will delay purchasing during period $L$. Further, in period $L$, if the customers’ taste on value is greater than $\frac{p_L}{v_L}$ (i.e., $\theta > \frac{p_L}{v_L}$), then she chooses to purchase. Otherwise (i.e., $\theta \leq \frac{p_L}{v_L}$), she will choose not to buy.

Therefore, given the market size $N$, we can get the expected demand in period $H$ as

$$D_H = N\theta_H$$

where $\theta_H = 1 - G_1(p_H - \delta p_L) = 1 - G_1$ (Denote $G_1 = G(p_H - \delta p_L)$, $g_1 = g(p_H - \delta p_L)$).
The expected demand in period $L$ is given by

$$D_L = N\theta_L$$

(2)

where $\theta_L = G_L\left(\frac{p_H - \delta p_L}{v_H - \delta v_L}\right) = G_L - G_{\delta}$ (Denote $G_2 = G_L\left(\frac{p_L}{v_L}\right), g_2 = g_L\left(\frac{p_L}{v_L}\right)$).

Denote $\Theta = \theta_L + \theta_L = 1 - G_2$.

4. Model with two-period pricing strategy in quick response regime

4.1 Model development

In the quick-response regime, the firm can procure inventory before and after receiving a forecast update prior to the start of the selling season (Cachon and Swinney 2009, 2011). We define the equilibrium to pricing-inventory-purchasing game as follows:

**Model Definition.** An equilibrium with rational expectations and nonzero production to the game between strategic consumers and the firm satisfies the following condition:

1. The firm sets prices and inventory to maximize the expected profit, given that a part of consumers purchase in period $H$ and another part of consumers purchase in period $L$.
2. Consumers purchase in different periods, given the selling price.

The firm’s expected profit with quick response as a function of the initial stocking quantity $(q)$ and prices $(p_H, p_L)$ is, supposed as $p_H > p_L > c_v$,

$$\Pi^r(q, p_H, p_L) = E\left[ p_H D_H + p_L D_L - cq - c_v (D_H + D_L - q) \right]$$

(3)

$$= p_H \mu \theta_H + p_L \mu \theta_L - cq - c_v L(q).$$

Let $L(q)$ be the expected lost sales function (excess demand above $q$).

$$L(q) = E(D_H + D_L - q)^+ = \mu \Theta + \Theta \int_0^q F(x)dx - q.$$  

(4)

Thus, the equilibrium conditions with quick response are as follows:

1. $(q, p_H, p_L) = \arg \max_{q, p_H, p_L} \Pi^r(q, p_H, p_L)$;
2. $\theta_H = 1 - G_2$;
3. $\theta_L = G_2 - G_2$.

**Theorem 1.** (i) If $G(\cdot)$ is an increasing generalized failure rate (IGFR) distribution, then the firm’s profit function has local maximum value point if and only if $1 - \delta > \delta^*$ in a quick response regime. Here, $\delta^*$ is the minimum value that satisfies inequality (5).

$$(1 - \delta)^* < \frac{1}{g_2^2} \frac{v_H - \delta v_L}{v_L} \left(2g_2 + \frac{g_2'}{g_2} (\theta_H + \delta \theta_L) \left(2 \theta_H + \theta_L \frac{g_2'}{g_1}ight) \right)$$

(5)

(ii) If $\delta = 1$, then the local maximum value point always exists.
Proof. The first order conditions for maximizing $\Pi^\nu$ is

$$\frac{\partial \Pi^\nu}{\partial q} = -c - c_q \left(F(\frac{q}{\Theta}) - 1\right)$$  \hspace{1cm} (6)

$$\frac{\partial \Pi^\nu}{\partial p_H} = \mu \theta_H - \frac{\mu (p_H - p_L)}{v_H - \delta v_L} g_1 = \mu \bar{G}_1 \left(1 - \frac{(p_H - p_L) g_1}{v_H - \delta v_L} \right)$$ \hspace{1cm} (7)

$$\frac{\partial \Pi^\nu}{\partial p_L} = \frac{\mu (p_H - p_L)}{v_H - \delta v_L} g_1 + \mu \theta_L + \mu \frac{g_2}{v_L} (M - p_L)$$ \hspace{1cm} (8)

Where $M = \frac{c_q}{\mu} \left(\mu + \int_0^\frac{q}{\Theta} F(x)dx - \frac{q}{\Theta} F(\frac{q}{\Theta})\right)$.

From $\frac{\partial \Pi^\nu}{\partial q} = 0$, we have $F(\frac{q}{\Theta}) = 1 - \frac{c}{c_q}$. Therefore, $M$ only depends on the cost of quick response ($c_q$), the cost of early opportunity ($c$) and the distribution function of market size. Obviously, $\int_0^{F^{-1}(1 - \frac{c}{c_q})} F(x)dx - \frac{(c_q - c)}{c_q} F^{-1}(\frac{c_q - c}{c_q})$ is strictly decreasing in the cost of quick response ($c_q$). Therefore $M < c_q$ when $c_q > c$.

Next, note that the right-hand side of Eq.(7) is decreasing in $p_H$, if $G(\cdot)$ is an increasing generalized failure rate (IGFR) distribution. Therefore

$$\frac{\partial^2 \Pi^\nu}{\partial p_H^2} = -\frac{2 \mu g_1}{v_H - \delta v_L} \frac{\mu (p_H - p_L)}{(v_H - \delta v_L)^2} g_1' < 0$$ \hspace{1cm} (9)

From $\frac{\partial \Pi^\nu}{\partial p_H} = 0$, we have

$$\frac{p_H - p_L}{v_H - \delta v_L} = \frac{\theta_L}{g_1}.$$ \hspace{1cm} (10)

Substituting Eq.(10) in inequality (9), we get $2g_1' + \theta_H g_1' > 0$.

Substituting Eq.(10) in Eq.(8), we see the following:

$$\frac{\partial \Pi^\nu}{\partial p_L} = -\mu (1 - \delta) \bar{G}_1 + \mu \frac{g_2}{v_L G_2} \left(1 - \frac{g_2}{v_L G_2} (p_L - M)\right).$$ \hspace{1cm} (11)

Where $\bar{G}_1 = 1 - G_1, \bar{G}_2 = 1 - G_2$.

If $G(\cdot)$ is an IGFR distribution, then Eq.(11) is decreasing in $p_L$. Hence,
\[ \frac{\partial^2 \Pi^w}{\partial p^L_h} = -\frac{2\mu \delta}{v_{h} - \delta v_{L}} g_1 - \frac{\mu \delta \left( p_{H} - p_{L} \right)}{\left( v_{H} - \delta v_{L} \right)^2} g'_1 - g'_2 \frac{\mu}{v_{L}} \left( p_{L} - M \right) \frac{\mu.}{v_{L}} \frac{c_{w} q^2}{\Theta f(q)} g_2 + 2 < 0. \]

When \( 2g_2 + \frac{g'_2}{g_2} \left( \partial \theta_{H} + \theta_{L} \right) > 0 \), inequality \( \frac{\partial^2 \Pi^w}{\partial p^L_h} < 0 \) is satisfied in any situation.

We get the Hesse matrices of this problem as follows:

\[
\begin{pmatrix}
-\frac{c_{w}}{\Theta} f\left( \frac{q}{\Theta} \right) & 0 & -\frac{c_{w}}{\Theta} f\left( \frac{q}{\Theta} \right) \frac{q}{v_{L}} g_2 \\
0 & \frac{\partial^2 \Pi^w}{\partial p^H_{n} \partial p^L_{p}} & \frac{\partial^2 \Pi^w}{\partial p^H_{p} \partial p^H_{l}} \\
-\frac{c_{w}}{\Theta} f\left( \frac{q}{\Theta} \right) \frac{q}{v_{L}} g_2 & \frac{\partial^2 \Pi^w}{\partial p^H_{n} \partial p^H_{l}} & \frac{\partial^2 \Pi^w}{\partial p^H_{p} \partial p^H_{l}}
\end{pmatrix}
\]

where \( \frac{\partial^2 \Pi^w}{\partial p^H_{n} \partial p^H_{l}} = \frac{\mu (\delta + 1)}{v_{H} - \delta v_{L}} g_1 + \frac{\mu \delta \left( p_{H} - p_{L} \right)}{\left( v_{H} - \delta v_{L} \right)^2} g'_1. \)

Therefore, the optimizing condition for the profit of the firm is

\[ \left( -\frac{c_{w}}{\Theta} f\left( \frac{q}{\Theta} \right) \right) \left( \frac{\partial^2 \Pi^w}{\partial p^H_{n}} \right) > 0 \]  \hspace{1cm} (12)

and

\[ -\frac{c_{w}}{\Theta} f\left( \frac{q}{\Theta} \right) \left( \frac{\partial^2 \Pi^w}{\partial p^H_{n} \partial p^H_{l}} \right) - \left( -\frac{c_{w}}{\Theta} f\left( \frac{q}{\Theta} \right) \frac{q}{v_{L}} g_2 \right) \left( \frac{\partial^2 \Pi^w}{\partial p^H_{p} \partial p^H_{l}} \right) < 0. \]  \hspace{1cm} (13)

Inequality (12) is always satisfied.

From inequality (13), we obtain inequality (5). The right-hand side of inequality (5) is positive, hence if \( \delta = 1 \), then inequality (5) always holds. There exists \( \delta < 1 \), when \( 1 \geq \delta > \delta' \), inequality (5) is always satisfied and if \( \delta < 0 \), then for any \( 0 \leq \delta \leq 1 \) maximum value point always exists.

Theorem 1 provides a sufficient condition for the existence of the firm’s maximum profit. From Theorem 1, if customers have enough patience to wait for markdown or more strategies (i.e. \( \delta \to 1 \)), then the two-period pricing policy can bring more benefits to the firm, otherwise if customers’ patience is \( \delta < \delta' \), then the two-period pricing cannot provide more profit.

**Corollary 1.** In a quick-response regime, the equilibrium price \( p^*_L \) in period \( L \), \( p^*_H \) in period \( H \) and optimal quantity \( q^* \) at maximum value point exist and are characterized by the following:
\[
F\left(\frac{q}{\Theta}\right) = 1 - \frac{c}{c_y}
\]  
(14)

\[
g_i = \frac{v_H - \delta v_L}{p_H - p_L} \tilde{G}_i
\]  
(15)

\[
\frac{\delta(p_H - p_L)}{v_H - \delta v_L} g_i + \theta c + \frac{g_2}{v_L} (M - p_L) = 0
\]  
(16)

Proof. Eq.(14), (15) and (16) can be obtained from \(\frac{\partial \Pi^v}{\partial q} = 0\), \(\frac{\partial \Pi^v}{\partial p_H} = 0\) and \(\frac{\partial \Pi^v}{\partial p_L} = 0\).

Let \(x = \frac{p_H - \delta p_L}{v_H - \delta v_L}\), \(y = \frac{p_L}{v_L}\), from Eq. (15) and (16), we have the following:

\[
y = \left(x - \frac{\tilde{G}_i}{g_i} \right) \frac{v_H - \delta v_L}{v_L} (1 - \delta)
\]  
(17)

\[
\tilde{G}_i = \frac{1}{1 - \delta} \left( \tilde{G}_2 - g_2 \left( \frac{y}{v_L} - \frac{M}{v_L} \right) \right)
\]  
(18)

Differentiating \(y\) with respect to \(x\), we obtain:

\[
\left(\frac{\partial y}{\partial x}\right)_1 = \frac{(v_H - v_L) \delta}{v_L (1 - \delta)} \left( \frac{\tilde{G}_2}{g_2} \left( \frac{g_1}{\tilde{G}_1} \right)' + 1 \right) > 0
\]

\[
\left(\frac{\partial y}{\partial x}\right)_2 = \frac{g_1 (1 - \delta)}{g_i (1 - \delta)} - \frac{g_2}{G_2} \left( \tilde{G}_2 - \tilde{G}_1 (1 - \delta) \right) + g_2 \left( \tilde{G}_2 (1 - \delta) + 1 \right) > 0
\]

In Eq. (17), when \(y = 0\), \(0 < x < 1\); when \(y = 1\), \(0 < x < 1\).

In Eq. (18), when \(x = 0\), \(0 < y < 1\); when \(x = 1\), \(0 < y < 1\).

Therefore, the optimal solution \((x^*, y^*)\) always exists, i.e. \((p_H^*, p_L^*, q^*)\) always exists.

4.2 Special case when \(G(*)\) is a Uniform distribution

Without loss of generality, many researchers assume the customer’s taste for value \(\theta\) follows a uniform distribution on \([0, 1]\) (Jerath et al. 2010, García and Tugores 2006, Dong 2012). In the next section, we discuss this situation.

Corollary 2. In a quick response regime, an equilibrium always exists if the customer’s taste for value \(\theta\) follows a uniform distribution.

Proof: When \(\theta\) follows a uniform distribution on \([0, 1]\), inequality (5) change to

\[
\left(\frac{c_y}{\Theta} \int f\left(\frac{q}{\Theta}\right) \left(\delta + 1\right) - \frac{4v_H}{v_L} \right) \left(\frac{\mu^2}{(v_H - \delta v_L)^2}\right) < 0.
\]
This inequality is always satisfied when \( v_H > v_L \) and \( 0 < \delta < 1 \). Therefore \( \Pi^v \) has maximum value when customer’s taste follows a uniform distribution. This completes the proof.

**Corollary 3.** In a quick-response regime, the equilibrium price \( p_L^* \) in period \( L \), \( p_H^* \) in period \( H \) and optimal quantity \( q^* \) are unique and characterized by the following:

\[
p_L^* = \frac{v_H - \delta v_L}{2v_H - v_L(1+\delta)} \left[ \frac{(\delta+1)v_L}{2} + M \right]
\]

(19)

\[
p_H^* = \frac{(1+\delta)p_L^* + v_H - \delta v_L}{2}
\]

(20)

\[
q^* = (1 - \frac{p_L^*}{v_L})F^{-1}(\frac{c_w - c}{c_w})
\]

(21)

where \( M = \frac{c_w}{\mu} \left[ \mu + \int_0^{F^{-1}(\frac{c_w - c}{c_w})} f(x)dx - (\frac{c_w - c}{c_w})F^{-1}(\frac{c_w - c}{c_w}) \right] \).

### 4.3 Impact of discount factor

**Corollary 4.** The optimal price in period \( L \) \( (p_L^*) \) is strictly increasing in the discount factor \( \delta \).

Proof. Differentiating \( p_L^* \) with respect to \( \delta \), we see

\[
\frac{\partial p_L^*}{\partial \delta} = \frac{A_l}{\left(4v_H - v_L(1+\delta)^2\right)^2},
\]

where

\[
A_l = v_L \left(4v_H^2 + v_L^2(1+\delta)^2 - 2M(-1+\delta)(-2v_H + v_L + \delta)\right) + v_H v_L \left(-3 - 6\delta + 3\delta^2\right).
\]

Differentiating \( A_l \) with respect to \( \delta \), we have the following:

\[
\frac{\partial A_l}{\partial \delta} = -2v_L \left(2M(v_H - v_L) - v_L \left(3v_H - v_L - (v_H + v_L)\delta\right)\right).
\]

Moreover, \( \frac{\partial^2 A_l}{\partial \delta^2} = 2v_L^2 \left(v_H + v_L - 2M\right) > 0 \), because \( M < c_w < v_L < v_H \).

Therefore, \( \frac{\partial A_l}{\partial \delta} \big|_{\delta=0} = 2v_L \left(v_L(3v_H - v_L) - 2Mv_H\right) > 0 \).

Thus, \( \frac{\partial p_L^*}{\partial \delta} > 0 \), i.e. the optimal price in period \( L \) is strictly increasing in the discount factor \( \delta \). □

Corollary 4 shows that when customers have more patience to wait for the lower price, the price in period \( L \) is higher.

**Corollary 5.** The optimal price in period \( H \) \( (p_H^*) \) is initially decreasing, then increasing in the discount factor \( \delta \). When \( \delta = \delta_i \), the optimal price in period \( H \) \( (p_H^*) \) is lowest.

\[
\delta_i = \frac{2v_H v_L - M(3v_H - v_L)v_L - 2(v_H - v_L)\sqrt{v_Hv_L(v_Hv_L - M^2)}}{v_L(2v_H v_L - M(v_H + v_L))}.
\]

Where
Proof. Differentiating $p_H^*$ with respect to $\delta$, we see
\[ \frac{\partial p_H^*}{\partial \delta} = A_2 / \left(4v_H - v_L(1+\delta)^2\right), \]
where $A_2 = -2v_Hv_L(1+\delta)(-2v_H + v_L + \delta v_L) + M \left(4v_H^2 + v_L^2(1+\delta)^2 + v_Hv_L(-3-6\delta+\delta^2)\right)$.

Differentiating $A_2$ with respect to $\delta$, we have the following:
\[ \frac{\partial A_2}{\partial \delta} = 2v_L \left(2v_H(v_H - v_L) - M \left(3v_H - v_L - (v_H + v_L)\delta\right)\right). \]

Moreover,
\[ \frac{\partial^2 A_2}{\partial \delta^2} = -2v_L \left(2v_Hv_L - M \left(v_H + v_L\right)\right) < 0, \]
because $M < c_p < v_L < v_H$.

Thus,
\[ \frac{\partial A_2}{\partial \delta} \bigg|_{\delta=\delta_0} = 4v_L \left(v_H - M\right)(v_H - v_L) > 0. \]

Therefore,
\[ A_{2,\text{max}} = A_2 \bigg|_{\delta=\delta_0} = 4M \left(v_H^2 + v_L^2 - 2v_Hv_L\right) > 0, \]
\[ A_{2,\text{min}} = A_2 \bigg|_{\delta=\delta_0} = M \left(4v_H^2 + v_L^2 - 3v_Hv_L\right) - 2v_Hv_L \left(2v_H - v_L\right) < 0. \]

Hence, the value of $\frac{\partial p_H^*}{\partial \delta}$ is changed from negative to positive with the increase in $\delta$.

From $A_2 = 0$, we have
\[ \delta = \frac{2v_H^2 - M \left(3v_H - v_L\right)v_L - 2(v_H - v_L)\sqrt{v_Hv_L(v_H - M^2)}}{v_L \left(2v_Hv_L - M \left(v_H + v_L\right)\right)} = \delta_1. \]

It means that the optimal price in period $H$ ($p_H^*$) is decreasing first, then increasing with the increase of $\delta$ and when $\delta = \delta_1$, the optimal price in period $H$ ($p_H^*$) is lowest.

Generally, if customers have more patience to wait for lower prices, then the firm should set a lower price in period $H$ (higher than price in period $L$) to have more customers purchase in high prices. However, when the degree of patience for lower price or the level of strategic behavior is high enough, the equilibrium price in period $H$ will increase with $\delta$. Although the number of customers who purchase in higher price decreases, the firm can obtain more profit in period $L$.

When $v_H \to v_L$, i.e. the value of product is stability, we obtain $\delta_1 \to 1$. It means that the optimal price in period $H$ ($p_H^*$) is always decreasing in the discount factor $\delta$ in this situation.

Figure 2 demonstrates the optimal price in period $H$ ($p_H^*$) for various value of discount factor. In Figure 2 and all other graphical examples, $v_H = 40$, $v_L = 20$, $c = 3$ and $c_p = 6$, $N$ is the normal distribution with mean of 30 and standard deviation of 10.
Corollary 6. The optimal quantity $q^*$ is strictly decreasing in the discount factor $\delta$.

Proof. Differentiating $q^*$ with respect to $\delta$, we see the following:

$$\frac{\partial q^*}{\partial \delta} = -\frac{1}{v_L} \frac{\partial p^*_L}{\partial \delta} e^{-\delta} \left( \frac{c_v c_p}{c_p^*} - 1 \right) < 0.$$  

It means that the optimal quantity $q^*$ is strictly decreasing in the discount factor $\delta$.

Corollary 6 shows that when customers have more patience to wait for lower price, the equilibrium quantity is less.

Corollary 7. The firm’s optimal expected profit $\Pi^{\omega^*}$ is initially decreasing, and then increasing in the discount factor $\delta$. When $\delta = \delta_2$, the optimal profit is lowest.

Proof. Substituting $p^*_L$ (Eq. (19)), $p^*_H$ (Eq. (20)) and $q^*$ (Eq. (21)) in Eq. (3), we can obtain:

$$\Pi^{\omega^*} = \mu \left( \frac{p^*_L \theta_H (1+\delta) + 2p^*_L \theta_H v_H - \theta_H v_L}{2} - M (1-p^*_L) \right).$$

Then, differentiating $\Pi^{\omega^*}$ with respect to $\delta$, we have the following:

$$\frac{\partial \Pi^{\omega^*}}{\partial \delta} = \mu \left( 4v_H - v_L (1+\delta) \right) A_1.$$

Where

$$A_1 = -M^2 (1-\delta) (2v_H - v_L - \delta v_L) + M \left( 4v_H^2 + v_H^2 (1+\delta)^2 + v_H v_L \left( -3 - 6\delta + \delta^2 \right) \right) - v_H v_L \left( -1+\delta \right) \left( -2v_H + v_L + v_\delta \right)$$

Differentiating $A_1$ with respect to $\delta$, we see the following:

$$\frac{\partial A_1}{\partial \delta} = 2 \left( M^2 \left( v_H - v_L \delta \right) + v_H v_L \left( v_H - \delta v_L \right) + M v_L \left( v_H \left( -3+\delta \right) + v_L \left( 1+\delta \right) \right) \right).$$

Moreover,
\[ \frac{\partial^2 A_1}{\partial \delta^2} = -2v_H (v_H - M)(v_L - M) < 0 \]

Therefore,

\[ \frac{\partial A_1}{\partial \delta} \bigg|_{\delta = \delta_1} = 2 \left( M^2 - 2Mv_L + v_H v_L \right) \left( v_H - v_L \right) > 0. \]

Hence,

\[ A_{\text{max}} = A_1 \bigg|_{\delta = \delta_1} = 4M \left( v_H^2 + v_L^2 - 2v_H v_L \right) > 0 \]

\[ A_{\text{min}} = A_1 \bigg|_{\delta = \delta_0} = -M^2 \left( 2v_H - v_L \right) + M \left( 4v_H^2 + v_L^2 - 3v_H v_L \right) - v_H v_L \left( 2v_H - v_L \right) < 0 \]

Therefore the value of \( \frac{\partial \Pi^\star}{\partial \delta} \) is changed from negative to positive with the increase of \( \delta \).

From \( A_{\text{min}} = 0 \), we have

\[ \delta = \frac{M v_L + v_H v_L - 2M v_H}{v_H v_L - M v_L} = \delta_1. \]

Such outcome means that the optimal profit \( \Pi^\star \) is initially decreasing, and then increasing with the increasing of \( \delta \) and when \( \delta = \delta_1 \), the optimal profit \( \Pi^\star \) is lowest.

Corollary 7 shows that the firm’s maximum profit may be obtained when customers have enough patience or have no patience to wait for lower price. From \( \Pi^\star \bigg|_{\delta = \delta_1} > \Pi^\star \bigg|_{\delta = \delta_0} \), we have \( M > 2v_H - v_L - \sqrt{5v_H v_L + v_L^2} = M_1 \). Therefore, if \( M > M_1 \), then the firm obtains maximum profit when customers have enough patience (\( \delta = 1 \)). Otherwise the firm obtains maximum profit when customers have no patience to wait for lower price (\( \delta = 0 \)). Some papers believe that the strategic consumers hurt revenues (e.g. Anderson and Wilson 2003, Levin et al. 2009). However, in our model, we found that in some scenarios, the strategic consumers may yield more revenues.

Furthermore, if the firm committed to sell products in price \( p_H \) during the whole selling season, then the strategic consumers will not wait for lower price. It is a special case of our model when \( p_H = p_L \). From our model, the total profit during such situation is not higher than the optimal solution of the model. However, commitment to a non-decreasing pricing scheme will lead more customers to purchase in the price \( p_H \). Therefore, a few firms may commit to sell products in price \( p_H \) throughout, but will markdown the prices in the second period in order to gain more profit through increasing the number of customers who will purchase in the price \( p_H \). If they proceed as described, then firms will obtain a profit equal with the condition \( \delta = 0 \) in our model. From Corollary 7, we found that the firm could not always obtain more profit through a breach of the price commitment if customers have
enough patience to wait for lower price apart from reputation loss.

Figure 3 demonstrates the optimal profit for various value of the discount factor.

![Figure 3. The optimal profit $\prod^{\pi^*}$ for various value of discount factor](image)

**Corollary 8.** When $v_H = v_L$, if $\delta \to 1$, $p_H \to p_L$.

Proof. When $v_H = v_L$, $p_L^* = \frac{2}{\delta + 3} \left[ (\delta + 1) v_L + M \right]$, $p_H = \frac{(1 + \delta) p_L^* + v_H - \delta v_L}{2}$.

If $\delta \to 1$, then we have $p_L^* \to \frac{1}{2} [v_L + M]$, $p_H \to p_L^*$. □

When the value of product doesn’t change during the selling season, the firm can gain more profit through price differentiation in two periods if $\delta < 1$. In addition, if customers have no discount for future purchases, then the price in two periods is equal. Figure 4 demonstrates the optimal profit for various value of discount factor when $v_H = v_L$.

![Figure 4. The optimal profit for various value of discount factor when $v_H = v_L$](image)

5. **Extended analysis – the case with Beta distribution**

In the previous section, we discussed the condition when the customer’s taste for value $\theta$ follows a uniform distribution on $[0, 1]$. Here, we will analyze the situation when $\theta$ follows a
more general \([0, 1]\) distribution function (i.e. Beta distribution, \(\theta \sim \text{Be}(\alpha, \beta)\)) through numerical example. Beta distribution is a family of continuous probability distributions defined on the interval \([0, 1]\), parameterized by two positive shape parameters, denoted by \(\alpha\) and \(\beta\).

When \(\alpha = 1\) and \(\beta = 1\), Beta distribution turns into uniform distribution on \([0, 1]\).

**Example 1.** \(\alpha = 2, \beta = 2, \delta = 0.5\). In Example 1 and 2: \(v_H = 40, v_L = 20, c = 3\) and \(c_p = 6\). \(N\) is normal distributed with the mean of 30 and standard deviation of 10. Figure 7 illustrates the channel profit for varying \(p_L\) and \(p_H\). In this case, \(p_L^* = 8.55, p_H^* = 18.92, \pi^{\omega*} = 247.76\), i.e. there is a unique optimal solution when \(\theta\) follows a more general \([0, 1]\) distribution function if the inequality (5) in Theorem 1 is satisfied.

![Figure 7. The channel profit for varying \(p_L\) and \(p_H\).](image)

**Example 2.** Figure 8 illustrates the optimal price in period \(H\) (\(p_H^*\)) and in period \(L\) (\(p_L^*\)), and the optimal profit (\(\pi^{\omega*}\)) for various values of discount factor under different values of \(\alpha\) and \(\beta\). In Beta distribution, when parameter \(\alpha\) is smaller than parameter \(\beta\), the number of customers who have high tastes regarding the products’ value is smaller than those with low tastes. Otherwise, the number of customers who have high tastes concerning the product’s value is always greater than those with low tastes. Therefore, in this case if more people prefer the product with high value (\(\alpha > \beta\), Figure 8-C), the optimal prices (\(p_H^*, p_L^*\)) are higher and the supply chain has more profit. Interestingly, if more than enough people prefer the product with low value (Figure 8-A), the highest channel profit can be obtained when \(\delta = 1\). Otherwise, the highest channel profit may be obtained when \(\delta = 0\) (Figure 8-B and Figure 8-C). This is consistent with Corollary 7. Therefore, if more customers prefer low-value products, then the supply chain has a greater chance to obtain the highest profit when customers have enough patience (\(\delta = 1\)).
A. $\alpha = 2, \beta = 8$

B. $\alpha = 2, \beta = 2$

C. $\alpha = 8, \beta = 2$

Figure 8. The optimal price in period $H(p^*_H)$, in period $L(p^*_L)$ and the optimal profit $\prod^{\theta^*}$ for various values of discount factor.

6. Conclusion

If the value of the product is decreasing with time, then the firm can use a dynamic pricing strategy that maximizes its revenue. If all customers are strategic, then this is clearly the best pricing strategy for the firm. However, in the presence of myopic customers, we indicated that dynamic pricing strategies may reduce the firm’s profits. Accordingly, we considered a dynamic pricing and inventory decision with differentiated product value periods. Customers are heterogeneous and have different levels of strategic behavior. With strategic customers, markdown prices during the selling season have both positive and negative influences on the firm’s profit. On the one hand, a lower price can increase the demand and that may increase revenues. Conversely, it leads some strategic customers to delay their purchase in anticipation of a lower price.

First, we showed that to maximize the firm’s profit, an equilibrium exists if and only if the level of customer strategic behavior is sufficiently high. Especially when all customers are strategic, the maximum value must exist. This finding means that if there are enough myopic customers in all customers, then the dynamic pricing strategy may not be a good choice for the firm.
Second, we investigated the impact of strategic consumer behavior on the firm’s two-period pricing and inventory strategies when the customer’s view of product value follows a uniform distribution without any loss of generality. We proved that if a customer has more patience to wait for a lower price, the price in the markdown period will be higher and the equilibrium quantity will be lower. However, the price in a normal selling period and the firm’s optimal expected profit will first decrease, and will eventually lead to an increase in customer strategic behavior. Therefore, we obtained an interesting conclusion that firms may achieve more profit when all customers are either strategic or myopic. This supposition implies that in certain scenarios, the strategic consumers may yield greater revenue.

An extended analysis where customers’ taste for product value following the Beta distribution was also presented. We found that if more people prefer products with low value, the supply chain has a greater chance to obtain the highest profit when all customers are rational. Otherwise, obtaining the highest profit is more likely when all customers are irrational.

Further, future research can extend our analysis in several directions. First, the firm pricing and inventory decisions when the value of product continuously decreases with time can be examined. It would be interesting to investigate an optimal time to mark prices down during a selling season. Second, a firm’s decision, when using a quick response system, should be compared with decisions in a traditional system to gain further insights on the impact of strategic customer behavior.

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