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Distributionally robust optimization for energy and reserve toward a low-carbon electricity market

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ABSTRACT

This paper proposes a two-stage distributionally robust model for the optimization of energy and reserve under uncertain wind power. The first-stage model considers a day-ahead market that determines the nominal generation and reserves before the realization of wind power uncertainty. The second-stage decisions are made in a realtime market, after the observation of uncertainty, so that the expected emission factor is constrained below a target level. Case studies are conducted to demonstrate that the proposed method is capable of effectively capturing the ambiguous distribution of wind power generation, and can be tractably solved. The influence of different emission constraints is also discussed, showing the trade-off between lowering the total operating cost and reducing the long-term impact of carbon emissions.

1. Nomenclature

We use bold letters to denote vectors or matrices. Entries of vectors or matrices are regular letters with the corresponding subscripts. For example, b_i is the ith entry of vector $\boldsymbol{b}$, and A_{mn} is the entry of matrix $\boldsymbol{A}$ in the mth row and the nth column. The nth column of $\boldsymbol{A}$ is denoted by $\boldsymbol{A}_n$. $|\mathcal{S}|$ is the number of elements in set $\mathcal{S}$, and $\|\cdot\|$ is the 2-norm of a vector. The currency unit used in this paper is US dollars ($). Other notations are listed below.

1.1. Indices and sets

$a/\mathcal{A}$	Indices/set of conventional thermal units
$b/\mathcal{B}$	Indices/set of load buses
$h/\mathcal{H}$	Indices/set of energy storage systems
$i/\mathcal{I}$	Indices/set of random variables
$j/\mathcal{J}$	Indices/set of auxiliary variables
$k/\mathcal{K}$	Indices/set of constraints in the extended support set
$l/\mathcal{L}$	Indices/set of transmission lines
$m/\mathcal{M}$	Indices/set of uncertain constraints
$\mathcal{P}_0(\cdot)$	Set of distributions of given random variables
$s/\mathcal{S}$	Indices/set of wind power sources
$t/\mathcal{T}$	Indices/set of time steps
$\mathcal{V}$	Feasible set of unit commitment decisions
τ	Indices of time steps

1.2. Uncertainty model

$\mathbb{F}$	Ambiguity set of random variables
$\mathbb{G}$	Extended ambiguity set
$\mathbb{P}$	Distribution of random variables $\tilde{\boldsymbol{z}}$
$\mathbb{Q}$	Joint distribution of vectors $\tilde{\boldsymbol{z}}$ and $\tilde{\boldsymbol{u}}$
$\tilde{\boldsymbol{u}}$	Auxiliary variables introduced into the extended sets
$\hat{w}_{st}$	Expected wind power from source s at time t (MW)
$\tilde{z}_{st}$	Forecast error of wind power source s at time t (MW)
$\mathcal{Z}/\hat{\mathcal{Z}}$	Support/extended support set of random variables

1.3. Constants and functions

C_a^d/C_a^u	Cost of downward/upward reserves of unit a ($/MW)
C_a^s	Cost of starting up unit a ($)
E_h	Energy rating capacity of storage system h (MWh)
F^e	Target of the expected emission factor (kg/MWh)
$F_l(\cdot)$	Function of the DC power flow for the lth line
L_{bt}	Load at bus b, during time step t (MW)
M_0	Number of rows in matrix $\boldsymbol{G}$
M_k	Number of rows in matrix $\boldsymbol{A}_k$
N_1	Number of decisions for the day-ahead market
$\bar{P}_a$	The maximum capacity of thermal unit a (MW)
$\underline{P}_a$	The minimum capacity of thermal unit a (MW)

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Q_h Power rating capacity of storage system h (MW)
R_a^d Ramp-down rate limitation of thermal unit a (MW/h)
R_a^u Ramp-up rate limitation of thermal unit a (MW/h)
T_l Transmission capacity of the lth line (MW)
$\bar{Z}_{st}$ Upper bound of forecast error $\tilde{z}_{st}$ (MW)
$\underline{Z}_{st}$ Lower bound of forecast error $\tilde{z}_{st}$ (MW)
α_a Squared term of the cost function of unit a (\$/MWh^2)
β_a Linear term of the cost function of unit a (\$/MWh)
γ_a Constant term of the cost function of unit a (\$)
δ_h^c Charge efficiency coefficient of storage system h
δ_h^d Discharge efficiency coefficient of storage system h
ϵ_a Carbon emission rate of unit a (kg/MWh)
θ_t Constant indicating the skewness of wind power distribution at time t
ϕ_{1t} The mean absolute deviation of the forecast error of total wind power at time t (MW)
ϕ_{2t} The standard deviation of the forecast error of total wind power at time t (MW)

1.4. Daily-ahead market decisions

d_{ht}^0 Nominal discharge of energy storage h at time t (MW)
o_{at} Start-up cost of thermal unit a at time t (\$)
p_{at}^0 Nominal output of thermal unit a at time t (MW)
q_{ht}^0 Nominal charge of energy storage h at time t (MW)
r_{at}^d Downward reserve of thermal unit a at time t (MW)
r_{at}^d Upward reserve of thermal unit a at time t (MW)
v_{at} Unit commitment decision of unit a at time t
w_{st}^0 Nominal output of wind power source s at time t (MW)
$\boldsymbol{x}$ Vector of all daily-ahead market decision variables

1.5. Realtime market decisions and decision rule

$d_{ht}(\boldsymbol{z})$ Discharge of storage h at time t, under uncertainty realization $\boldsymbol{z}$ (MW)
$p_{at}(\boldsymbol{z})$ Output of thermal unit a at time t, under uncertainty realization $\boldsymbol{z}$ (MW)
$q_{ht}(\boldsymbol{z})$ Charge of storage h at time t, under uncertainty realization $\boldsymbol{z}$ (MW)
$w_{st}(\boldsymbol{z})$ Output of wind farm s at time t, under uncertainty realization $\boldsymbol{z}$ (MW)
$\boldsymbol{y}(\boldsymbol{z})$ Vector of all realtime market decisions, under uncertainty realization $\boldsymbol{z}$
$\bar{\boldsymbol{y}}(\boldsymbol{z}, \boldsymbol{u})$ Decision rule as affine functions of random variables $\boldsymbol{z}$ and auxiliary variables $\boldsymbol{u}$

2. Introduction

Global warming has become a serious issue in the 21st century [1–3], and there is a pressing need to reduce carbon emissions in power industry. In an effort to achieve low-carbon electricity markets, clean energy technologies are applied in fast-growing scales in modern power systems. Typical clean energy sources, such as wind and photovoltaic power, are known to be highly uncertain and difficult to dispatch. This is why various optimization approaches are studied these years to model highly volatile uncertain energy sources in joint energy and reserve optimization.

For example, stochastic programming is widely used to achieve the optimal expected performance [4–8], where the uncertainty of renewables is represented by a number of scenarios. Such a scenario-representation, however, requires detailed information on the exact probability distribution of random variables [9], which may be difficult to be accurately identified. Even if the detailed distribution information is available, the number of scenarios might grow exponentially with the increase of random parameters [10]. Though various decomposition algorithms [11–13] are developed to alleviate the computational burden, the stochastic programming problems remain very challenging to solve. Besides stochastic programming, chance-constrained programming [14] also greatly replies on the precise information on probability distributions. Chance constraints are generally non-convex and intractable to solve, except for special cases like having Gaussian distributed random variables [15].

Robust optimization seeks optimal solutions that are robust against the worst-case realizations over a deterministic uncertainty set [16], so it can be applied to energy and reserve optimization problems [17–20] without assuming the actual probability distribution of uncertain parameters. Similar to robust models, the interval optimization is frequently used to protect the system against the worst-case scenarios defined by the boundaries of uncertain renewable generation [21–23]. However, both robust and interval optimization approaches are unable to directly model expected terms, because limited distribution information can be explicitly incorporated into the uncertainty set or the uncertainty boundaries.

In order to address these difficulties, a new method called distributionally robust optimization [24–26] has been introduced to power system optimization [27–29]. This method characterizes the system uncertainties by some descriptive statistics rather than detailed distribution information, so that the worst-case expectation expressions can be formulated in the objective function or in constraints without enumerating scenarios.

This paper proposes a distributionally robust model for the day-ahead scheduling of energy and reserve considering uncertain wind power generation. Constraints on the expected emission factor of all generators, which cannot be directly modeled by conventional robust or interval optimization approaches, are imposed to control the long-term impact of carbon emissions. Instead of relying on the knowledge of the exact probability distribution, the proposed method captures the uncertainty of wind power generation by an ambiguity set containing a collection of distributions. Compared with the other distributionally robust models [27,28] that merely depends on the mean values and covariance to define the ambiguity set, our method imposes a finite support set and uses additional distribution information, such as mean absolute deviations and the asymmetry of distribution functions, to better describe the possible pattern of distributions, thus improving the quality of solutions. These statistical measures can be more easily evaluated by using point forecast [30–34] and prediction interval approaches [35–37]. For the rest of the paper, the proposed formulation and details of deriving a tractable robust counterpart are presented in the next section. Case studies are provided in Section 4, and the final section concludes our work.

3. Formulation

3.1. Uncertainty model of wind power

In this paper, the uncertain wind power is expressed as Eq. (1).

$$\tilde{W}_{st} = \hat{w}_{st} + \tilde{z}_{st}, \quad \forall s \in \mathcal{S}, \quad \forall t \in \mathcal{T} \tag{1}$$

where $\hat{w}_{st}$ denotes the expected power of wind energy source s at time step t, determined by any forecast technologies [32–34], and $\tilde{z}_{st}$ is a random variable indicating the corresponding forecast error of wind power.

The proposed model uses an ambiguity set $\mathbb{F}$ expressed as Eq. (2) to define a collection of distributions for random variable $\tilde{\boldsymbol{z}}$.

$$\mathbb{F}=\left\{\mathbb{P}\in\mathcal{P}_0\left(\tilde{\boldsymbol{z}}\in\mathbb{R}^{|\mathcal{S}|\times|\mathcal{T}|}\right):\begin{array}{l}\mathbb{E}_{\mathbb{P}}\left\{\tilde{\boldsymbol{z}}\right\}=\boldsymbol{0}\\ \mathbb{E}_{\mathbb{P}}\left\{\left|\sum_{s\in\mathcal{S}}\tilde{z}_{st}\right|\right\}\le\phi_{1t},\quad\forall t\in\mathcal{T}\\ \mathbb{E}_{\mathbb{P}}\left\{\left(\sum_{s\in\mathcal{S}}\tilde{z}_{st}\right)^2\right\}\le\phi_{2t}^2,\quad\forall t\in\mathcal{T}\\ \mathbb{P}\left\{\tilde{\boldsymbol{z}}\in\mathcal{Z}\right\}=1\end{array}\right\} \tag{2}$$

The first line of Eq. (2) suggests that the expected value of forecast errors $\tilde{\boldsymbol{z}}$ is $\boldsymbol{0}$. The second line and the third line imply that the mean absolute deviation (MAD) and standard deviation (SD) of the forecast error of the aggregated wind power over the entire system are lower than ϕ_{1t} and ϕ_{2t}, respectively. The last line suggests that the support set of random variables is $\mathcal{Z}$, which is defined as below to incorporate the boundaries of forecast errors.

$$\mathcal{Z}=\left\{\boldsymbol{z}\in\mathbb{R}^{|\mathcal{S}|\times|\mathcal{T}|}:\underline{Z}_{st}\le z_{st}\le\bar{Z}_{st},\quad\forall s\in\mathcal{S},\quad\forall t\in\mathcal{T}\right\} \tag{3}$$

In order to derive an enhanced decision rule to approximate adjustable decisions [24,38,39,25], the support set $\mathcal{Z}$ in (3) is extended into the lifted form $\hat{\mathcal{Z}}$ in (4) by introducing auxiliary variables $\boldsymbol{u}$ into the uncertainty model.

$$\hat{\mathcal{Z}}=\left\{\begin{pmatrix}\boldsymbol{z}\in\mathbb{R}^{|\mathcal{S}|\times|\mathcal{T}|}\\ \boldsymbol{u}\in\mathbb{R}_+^{4\times|\mathcal{T}|}\end{pmatrix}:\begin{array}{l}\boldsymbol{z}\in\mathcal{Z}\\ \sum_{s\in\mathcal{S}}z_{st}=u_{1t}-u_{2t},\quad\forall t\in\mathcal{T}\\ u_{1t}^2\le u_{3t}\le\left(\sum_{s\in\mathcal{S}}\bar{Z}_{st}\right)^2,\quad\forall t\in\mathcal{T}\\ u_{2t}^2\le u_{4t}\le\left(\sum_{s\in\mathcal{S}}\underline{Z}_{st}\right)^2,\quad\forall t\in\mathcal{T}\end{array}\right\} \tag{4}$$

The MAD constraints in set $\mathbb{F}$ can be transformed into the lifted form (5) using auxiliary variables u_{1t} and u_{2t}.

$$\mathbb{E}_{\mathbb{P}}\left\{\left|\sum_{s\in\mathcal{S}}z_{st}\right|\right\}=\mathbb{E}_{\mathbb{Q}}\left\{|u_{1t}-u_{2t}|\right\}\le\mathbb{E}_{\mathbb{Q}}\left\{u_{1t}+u_{2t}\right\}\le\phi_{1t},\quad\forall t\in\mathcal{T} \tag{5}$$

The equality is obtained when u_{1t} and u_{2t} are respectively the positive and negative part of the expression $\sum_{s\in\mathcal{S}}z_{st}$. Similarly, the SD constraints can also be written as the lifted form (6) by using auxiliary variables u_{3t} and u_{4t}.

$$\mathbb{E}_{\mathbb{P}}\left\{\left(\sum_{s\in\mathcal{S}}z_{st}\right)^2\right\}=\mathbb{E}_{\mathbb{Q}}\left\{(u_{1t}-u_{2t})^2\right\}$$
$$\le\mathbb{E}_{\mathbb{Q}}\left\{u_{3t}+u_{4t}\right\}\le\phi_{2t}^2,\quad\forall t\in\mathcal{T} \tag{6}$$

The inequality (6) is then modified into the following formulation, so that the skewness of probability distribution can be reflected in the ambiguity set.

$$\begin{cases}\mathbb{E}_{\mathbb{Q}}\left\{u_{3t}\right\}\le\theta_t\phi_{2t}^2,\quad\forall t\in\mathcal{T}\\ \mathbb{E}_{\mathbb{Q}}\left\{u_{4t}\right\}\le(1-\theta_t)\phi_{2t}^2,\quad\forall t\in\mathcal{T}\end{cases} \tag{7}$$

so that we can implicitly enforce $\mathbb{E}_{\mathbb{Q}}\left\{u_{1t}^2\right\}\le\theta_t\phi_{2t}^2$ and $\mathbb{E}_{\mathbb{Q}}\left\{u_{2t}^2\right\}\le(1-\theta_t)\phi_{2t}^2$, where θ_t is a constant between zero and one. The modified constraints (7) not only satisfy the SD constraints (6), but also capture the asymmetric pattern of wind power distributions. Intuitively, if $\theta_t<0.5$, the negative part of random variables should have a longer tail, so the distribution is left-skewed. The constant $\theta_t>0.5$ indicates that the distribution is right-skewed.

With the MAD and SD constraints in (2) replaced by the lifted expressions (5) and (7), auxiliary variables $\boldsymbol{u}$ are used to extend the ambiguity set $\mathbb{F}$ into the lifted form [24]. The extended ambiguity set, denoted by $\mathbb{G}$, is given as Eq. (8).

$$\mathbb{G}=\left\{\mathbb{Q}\in\mathcal{P}_0\begin{pmatrix}\tilde{\boldsymbol{z}}\in\mathbb{R}^{|\mathcal{I}|\times|\mathcal{T}|}\\ \tilde{\boldsymbol{u}}\in\mathbb{R}^{4\times|\mathcal{T}|}\end{pmatrix}:\begin{array}{l}\mathbb{E}_{\mathbb{Q}}\left\{\tilde{\boldsymbol{z}}\right\}=\boldsymbol{0}\\ \mathbb{E}_{\mathbb{Q}}\left\{\tilde{u}_{1t}+\tilde{u}_{2t}\right\}\le\phi_{1t},\quad\forall t\in\mathcal{T}\\ \mathbb{E}_{\mathbb{Q}}\left\{\tilde{u}_{3t}\right\}\le\theta_t\phi_{2t}^2,\quad\forall t\in\mathcal{T}\\ \mathbb{E}_{\mathbb{Q}}\left\{\tilde{u}_{4t}\right\}\le(1-\theta_t)\phi_{2t}^2,\quad\forall t\in\mathcal{T}\\ \mathbb{Q}\left\{(\tilde{\boldsymbol{z}},\tilde{\boldsymbol{u}})\in\hat{\mathcal{Z}}\right\}=1\end{array}\right\} \tag{8}$$

In order to better present this approach, such an extended ambiguity set is rewritten as the compact matrix form as below.

$$\mathbb{G}=\left\{\mathbb{Q}\in\mathcal{P}_0\begin{pmatrix}\tilde{\boldsymbol{z}}\in\mathbb{R}^{|\mathcal{I}|}\\ \tilde{\boldsymbol{u}}\in\mathbb{R}^{|\mathcal{J}|}\end{pmatrix}:\begin{array}{l}\mathbb{E}_{\mathbb{Q}}\left\{\tilde{\boldsymbol{z}}\right\}=\boldsymbol{0}\\ \mathbb{E}_{\mathbb{Q}}\left\{\boldsymbol{G}\tilde{\boldsymbol{u}}\right\}\le\boldsymbol{\sigma}\\ \mathbb{E}_{\mathbb{Q}}\left\{(\tilde{\boldsymbol{z}},\tilde{\boldsymbol{u}})\in\hat{\mathcal{Z}}\right\}=1\end{array}\right\} \tag{9}$$

with $\boldsymbol{G}\in\mathbb{R}^{M_0\times|\mathcal{J}|}$ and $\boldsymbol{\sigma}\in\mathbb{R}^{M_0}$, where $\mathcal{I}=\mathcal{S}\times\mathcal{T}$ is the set of all random variables, $\mathcal{J}=\{1,2,3,4\}\times\mathcal{T}$ is the set of all auxiliary variables, and M_0 is the row number of matrix $\boldsymbol{G}$. Constraints in the extended support set (4) are transformed into the equivalent second-order cone constraints [40], then the set $\hat{\mathcal{Z}}$ can also be expressed as the matrix form below.

$$\hat{\mathcal{Z}}=\left\{\begin{pmatrix}\boldsymbol{z}\in\mathbb{R}^{|\mathcal{I}|}\\ \boldsymbol{u}\in\mathbb{R}^{|\mathcal{J}|}\end{pmatrix}:\begin{array}{l}\|\boldsymbol{A}_k\boldsymbol{z}+\boldsymbol{B}_k\boldsymbol{u}+\boldsymbol{c}_k\|\\ \le\boldsymbol{f}_k^T\boldsymbol{z}+\boldsymbol{g}_k^T\boldsymbol{u}+e_k,\quad\forall k\in\mathcal{K}\end{array}\right\} \tag{10}$$

with $\boldsymbol{A}_k\in\mathbb{R}^{M_k\times|\mathcal{I}|}$, $\boldsymbol{B}_k\in\mathbb{R}^{M_k\times|\mathcal{J}|}$, $\boldsymbol{c}_k\in\mathbb{R}^{M_k}$, $\boldsymbol{f}_k\in\mathbb{R}^{|\mathcal{I}|}$, and $\boldsymbol{g}_k\in\mathbb{R}^{|\mathcal{J}|}$, where M_k is the row number of matrices $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$, and $\mathcal{K}$ is the set of all constraints defining $\hat{\mathcal{Z}}$. Based on the extended sets, the energy and reserve optimization model can be built and solved by applying decision rule approximations. Further details are provided in the next subsection.

3.2. Energy and reserve optimization

The proposed optimization model is formulated as the following two stage problem.

$$\min\ \sum_{t\in\mathcal{T}}\sum_{a\in\mathcal{A}}\left\{\alpha_n\left(p_{at}^0\right)^2+\beta_a p_{at}^0+\gamma_a v_{at}+C_a^u r_{at}^u+C_a^d r_{at}^d+o_{at}\right\} \tag{11}$$

$$\text{s.t.}\quad \boldsymbol{v}\in\mathcal{V} \tag{12}$$

$$o_{at}\ge\max\left\{C_a^s\left(v_{at}-v_{a(t-1)}\right),0\right\} \tag{13}$$

$$\sum_{a\in\mathcal{A}}p_{at}^0+\sum_{h\in\mathcal{H}}\left(d_{ht}^0-q_{ht}^0\right)+\sum_{s\in\mathcal{S}}w_{st}^0=\sum_{b\in\mathcal{B}}L_{bt},\quad\forall t\in\mathcal{T} \tag{14}$$

$$-T_l\le F_l\left(\boldsymbol{p}_t^0,\boldsymbol{w}_t^0,\boldsymbol{d}_t^0,\boldsymbol{q}_t^0,\boldsymbol{L}_t\right)\le T_l,\quad\forall l\in\mathcal{L},\quad\forall t\in\mathcal{T} \tag{15}$$

$$0\le w_{st}^0\le\hat{w}_{st},\quad\forall s\in\mathcal{S},\quad\forall t\in\mathcal{T} \tag{16}$$

$$p_{at}^0+r_{at}^u\le\bar{P}_a v_{at},\quad\forall a\in\mathcal{A},\quad\forall t\in\mathcal{T} \tag{17}$$

$$p_{at}^0-r_{at}^d\ge\underline{P}_a v_{at},\quad\forall a\in\mathcal{A},\forall t\in\mathcal{T} \tag{18}$$

$$r_{at}^u\ge0,\quad r_{at}^d\ge0,\quad\forall a\in\mathcal{A},\quad\forall t\in\mathcal{T} \tag{19}$$

$$0\le q_{ht}^0\le Q_h,\quad 0\le d_{ht}^0\le Q_h,\quad\forall h\in\mathcal{H},\quad\forall t\in\mathcal{T} \tag{20}$$

$$0 \le \sum_{\tau=1}^{t}\left(\delta_h^q q_{h\tau}^0 - \frac{1}{\delta_h^d} d_{h\tau}^0\right) \le E_h, \quad \forall h \in \mathcal{H}, \quad \forall t \in \mathcal{T} \tag{21}$$

$$p_{at}^0 + r_{at}^u - p_{a(t-1)}^0 + r_{a(t-1)}^d \le R_a^u + \bar{P}_a\left(1 - v_{a(t-1)}\right), \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{22}$$

$$p_{a(t-1)}^0 + r_{a(t-1)}^u - p_{at}^0 + r_{at}^d \le R_a^d + \underline{P}_a(1 - v_{at}), \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{23}$$

$$w_{st}(\boldsymbol{z}) \le \hat{w}_{st} + z_{st}, \quad \forall \boldsymbol{z} \in \mathcal{Z}, \quad \forall s \in \mathcal{S}, \quad \forall t \in \mathcal{T} \tag{24}$$

$$p_{at}(\boldsymbol{z}) \le p_{at}^0 + r_{at}^u, \quad \forall \boldsymbol{z} \in \mathcal{Z}, \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{25}$$

$$p_{at}(\boldsymbol{z}) \ge p_{at}^0 - r_{at}^d, \quad \forall \boldsymbol{z} \in \mathcal{Z}, \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{26}$$

$$\sum_{a\in\mathcal{A}} p_{at}(\boldsymbol{z}) + \sum_{h\in\mathcal{H}} (d_{ht}(\boldsymbol{z}) - q_{ht}(\boldsymbol{z})) + \sum_{s\in\mathcal{S}} w_{st}(\boldsymbol{z}) = \sum_{b\in\mathcal{B}} L_{bt}, \quad \forall \boldsymbol{z} \in \mathcal{Z}, \forall t \in \mathcal{T} \tag{27}$$

$$-T_l \le F_l(\boldsymbol{p}_t(\boldsymbol{z}), \boldsymbol{w}_t(\boldsymbol{z}), \boldsymbol{d}_t(\boldsymbol{z}), \boldsymbol{q}_t(\boldsymbol{z}), \boldsymbol{L}_t) \le T_l \ \forall \boldsymbol{z} \in \mathcal{Z}, \quad \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{28}$$

$$0 \le q_{ht}(\boldsymbol{z}) \le Q_h, \quad 0 \le d_{ht}(\boldsymbol{z}) \le Q_h, \quad \forall \boldsymbol{z} \in \mathcal{Z}, \quad \forall h \in \mathcal{H}, \quad \forall t \in \mathcal{T} \tag{29}$$

$$0 \le \sum_{\tau=1}^{t}\left(\delta_h^c q_{h\tau}(\boldsymbol{z}) - \frac{1}{\delta_h^d} d_{h\tau}(\boldsymbol{z})\right) \le E_h, \quad \forall \boldsymbol{z} \in \mathcal{Z}, \quad \forall h \in \mathcal{H}, \quad \forall t \in \mathcal{T} \tag{30}$$

$$\sup_{\mathbb{P}\in\mathbb{F}} \mathbb{E}_{\mathbb{P}}\left\{\sum_{t\in\mathcal{T}}\sum_{a\in\mathcal{A}} \epsilon_a p_{at}(\tilde{\boldsymbol{z}})\right\} \le F^e \sum_{t\in\mathcal{T}}\sum_{b\in\mathcal{B}} L_{bt} \tag{31}$$

The first-stage problem (11)–(23) considers a day-ahead market that determines the nominal generation schedule and the upward and downward reserves before the realization of system uncertainties. The objective function attempts to minimize the total operating cost, which (11) is the summation of the nominal power production cost, start-up cost, and the cost of upward and downward reserves. The set $\mathcal{V}$ in (12) is the feasible set of unit commitment decisions $\boldsymbol{v}$ defined by minimum up/down time limitations of generators [41]. Eq. (14) and inequalities (15) enforce the power balance and transmission capacity constraints for nominal generation schedule, respectively. The day-ahead generation and reserves schedule are also constrained by available wind power (16) and generation capacities of thermal units (17) and (18). Inequalities (19) suggest that the upward and downward reserves only take nonnegative values. The power rating capacities of energy storage systems are imposed by constraints (20), and the energy capacities are enforced by inequalities (21). Constraints (22) and (23) define the ramping rate limitations of thermal units.

Constraints (24)–(31) represent the realtime market, where $\boldsymbol{p}_t(\boldsymbol{z})$, $\boldsymbol{w}_t(\boldsymbol{z})$, $\boldsymbol{c}_t(\boldsymbol{z})$, and $\boldsymbol{d}_t(\boldsymbol{z})$ are corresponding second-stage decisions determined under wind power realization $\boldsymbol{z}$. Specifically, the wind energy production in the realtime market is constrained by the realization of uncertain wind power (24), and expressions (25) and (26) imply that the power dispatch $p_{at}(\boldsymbol{z})$ should be within the generation capacity interval specified by the upward and downward reserves. Power balance Eq. (27), transmission limitation constraints (28), power and energy capacities of storage systems (29) and (30) are also enforced in the realtime market. The last distributionally robust constraint (31) imposes an upper bound of the expected emission factor. Note that the expected emission is derived from all possible distributions defined by the ambiguity set $\mathbb{F}$. It is considered more robust against the inexact distribution data than considering a single underlying probability distribution.

Let $\boldsymbol{y}(\boldsymbol{z}) = (\boldsymbol{p}(\boldsymbol{z}), \boldsymbol{w}(\boldsymbol{z}), \boldsymbol{c}(\boldsymbol{z}), \boldsymbol{d}(\boldsymbol{z}))$ be the vector of all second-stage decisions, determined after the realization of uncertain variables $\boldsymbol{z}$. It has been shown in references [10,42,43] that directly solving adjustable decisions $\boldsymbol{y}(\boldsymbol{z})$ under potentially infinite numbers of uncertainty outcomes can be computationally intractable. This is why various decision rule approximations are utilized in adjustable optimization problems to achieve tractable solutions [42,43,39,25,38]. This paper applies an enhanced decision rule that restricts the recourse decisions to be affinely dependent on some random variables $\boldsymbol{z}$ and auxiliary variables $\boldsymbol{u}$. Let $\mathcal{N}$ be the set of all adjustable decisions, the nth decision rule, denoted by $\bar{y}_n(\boldsymbol{z}, \boldsymbol{u})$, can be expressed as Eq. (32).

$$\bar{y}_n(\boldsymbol{z}, \boldsymbol{u}) = y_n^0 + \sum_{i\in\mathcal{I}_n} z_i y_{ni}^z + \sum_{j\in\mathcal{J}_n} u_j y_{nj}^u, \quad \forall n \in \mathcal{N} \tag{32}$$

In this function, coefficient y_n^0 is the nominal term of $\bar{y}_n(\boldsymbol{z}, \boldsymbol{u})$, coefficients y_{ni}^z and y_{nj}^u indicate how the decision rule $\bar{y}_n(\boldsymbol{z}, \boldsymbol{u})$ adjusts to the realization of random variable $\boldsymbol{z}$ and auxiliary variables $\boldsymbol{u}$. Sets $\mathcal{I}_n$ and $\mathcal{J}_n$ stand for the subsets of random variables and auxiliary variables, respectively, that the nth decision rule $\bar{y}_n(\boldsymbol{z}, \boldsymbol{u})$ depends on. In this paper, it is assumed that the adjustable decisions only depend on the random and auxiliary variables at the current time step. All decision rule coefficients are determined by solving a distributionally robust counterpart presented in the next subsection.

After replacing adjustable decisions $\boldsymbol{y}(\boldsymbol{z})$ in (11)–(31) by the decision rule $\bar{\boldsymbol{y}}(\boldsymbol{z}, \boldsymbol{u})$, a tractable formulation (33)–(48) can be derived. In this model, there is no need to consider the nominal DC power flow (14) and (15), wind power capacities (16), or power and energy ratings constraints of storage units (20) and (21) in the day-ahead market, since these nominal decisions in the day-ahead market are $\boldsymbol{y}^0 = \bar{\boldsymbol{y}}(\boldsymbol{0}, \boldsymbol{0})$, and these constraints are already implicitly enforced by expressions (41)–(47), as random variables $\boldsymbol{z} = \boldsymbol{0}$ and auxiliary variables $\boldsymbol{u} = \boldsymbol{0}$.

$$\min \sum_{t\in\mathcal{T}}\sum_{a\in\mathcal{A}} \left\{\alpha_n \left(p_{at}^0\right)^2 + \beta_a p_{at}^0 + \gamma_a v_{at} + C_a^u r_{at}^u + C_a^d r_{at}^d + o_{at}\right\} \tag{33}$$

$$\text{s.t.} \quad \boldsymbol{v} \in \mathcal{V} \tag{34}$$

$$o_{at} \ge \max\left\{C_a^s\left(v_{at} - v_{a(t-1)}\right), 0\right\} \tag{35}$$

$$p_{at}^0 + r_{at}^u \le \bar{P}_a v_{at}, \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{36}$$

$$p_{at}^0 - r_{at}^d \ge \underline{P}_a v_{at}, \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{37}$$

$$r_{at}^u \ge 0, \quad r_{at}^d \ge 0, \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{38}$$

$$p_{at}^0 + r_{at}^u - p_{a(t-1)}^0 + r_{a(t-1)}^d \le R_a^u + \bar{P}_a\left(1 - v_{a(t-1)}\right), \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{39}$$

$$p_{a(t-1)}^0 + r_{a(t-1)}^u - p_{at}^0 + r_{at}^d \le R_a^d + \underline{P}_a(1 - v_{at}), \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{40}$$

$$\bar{w}_{st}(\boldsymbol{z}, \boldsymbol{u}) \le \hat{w}_{st} + z_{st}, \forall (\boldsymbol{z}, \boldsymbol{u}) \in \hat{\mathcal{Z}}, \quad \forall s \in \mathcal{S}, \quad \forall t \in \mathcal{T} \tag{41}$$

$$\bar{p}_{at}(\boldsymbol{z}, \boldsymbol{u}) \le p_{at}^0 + r_{at}^u, \quad \forall (\boldsymbol{z}, \boldsymbol{u}) \in \hat{\mathcal{Z}}, \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{42}$$

$$\bar{p}_{at}(\boldsymbol{z}, \boldsymbol{u}) \ge p_{at}^0 - r_{at}^d, \quad \forall (\boldsymbol{z}, \boldsymbol{u}) \in \hat{\mathcal{Z}}, \quad \forall a \in \mathcal{A}, \quad \forall t \in \mathcal{T} \tag{43}$$

$$\sum_{a\in\mathcal{A}} \bar{p}_{at}(\boldsymbol{z}, \boldsymbol{u}) + \sum_{h\in\mathcal{H}} \left(\bar{d}_{ht}(\boldsymbol{z}, \boldsymbol{u}) - \bar{q}_{ht}(\boldsymbol{z}, \boldsymbol{u})\right) + \sum_{s\in\mathcal{S}} \bar{w}_{st}(\boldsymbol{z}, \boldsymbol{u}) = \sum_{b\in\mathcal{B}} L_{bt}, \quad \forall (\boldsymbol{z}, \boldsymbol{u}) \in \hat{\mathcal{Z}}, \quad \forall t \in \mathcal{T} \tag{44}$$

$$-T_l \le F_l\left(\bar{\boldsymbol{p}}_t(\boldsymbol{z},\boldsymbol{u}), \bar{\boldsymbol{w}}_t(\boldsymbol{z},\boldsymbol{u}), \bar{\boldsymbol{d}}_t(\boldsymbol{z},\boldsymbol{u}), \bar{\boldsymbol{q}}_t(\boldsymbol{z},\boldsymbol{u}), \boldsymbol{L}_t\right) \le T_l, \quad \forall(\boldsymbol{z},\boldsymbol{u}) \in \hat{\mathcal{Z}}, \quad \forall t \in \mathcal{T} \tag{45}$$

$$0 \le \bar{q}_{ht}(\boldsymbol{z},\boldsymbol{u}) \le Q_h, \quad 0 \le \bar{d}_{ht}(\boldsymbol{z},\boldsymbol{u}) \le Q_h, \ \forall(\boldsymbol{z},\boldsymbol{u}) \in \hat{\mathcal{Z}}, \forall h \in \mathcal{H}, \forall t \in \mathcal{T} \tag{46}$$

$$0 \le \sum_{\tau=1}^{t}\left(\delta_h^c \bar{q}_{a\tau}(\boldsymbol{z},\boldsymbol{u}) - \frac{1}{\delta_h^d}\bar{d}_{h\tau}(\boldsymbol{z},\boldsymbol{u})\right) \le E_h, \quad \forall(\boldsymbol{z},\boldsymbol{u}) \in \hat{\mathcal{Z}}, \quad \forall h \in \mathcal{H}, \quad \forall t \in \mathcal{T} \tag{47}$$

$$\sup_{\mathbb{Q}\in\mathbb{G}} \mathbb{E}_{\mathbb{Q}}\left\{\sum_{t\in\mathcal{T}}\sum_{a\in\mathcal{A}} \epsilon_a \bar{p}_{at}(\tilde{\boldsymbol{z}},\tilde{\boldsymbol{u}})\right\} \le F^e \sum_{t\in\mathcal{T}}\sum_{b\in\mathcal{B}} L_{bt} \tag{48}$$

The formulation above is equivalent to a tractable robust counterpart. Details of deriving the robust counterpart are provided in the next subsection. Let $\boldsymbol{x}$ be the vector of all day-ahead market decisions, including binary unit commitment decisions $\boldsymbol{v}$, nominal generation $\boldsymbol{p}^0$ and $\boldsymbol{w}^0$, upward reserve $\boldsymbol{r}^u$ and downward reserve $\boldsymbol{r}^d$, the problem (33)–(48) can also be written as the compact matrix formulation (49)–(51).

$$\min \quad \boldsymbol{x}^T\boldsymbol{C}\boldsymbol{x} + \boldsymbol{b}^T\boldsymbol{x} \tag{49}$$

$$\text{s.t.} \quad \boldsymbol{T}\boldsymbol{x} + \boldsymbol{W}\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u}) \le \boldsymbol{h}(\boldsymbol{z}), \quad \forall(\boldsymbol{z},\boldsymbol{u}) \in \hat{\mathcal{Z}} \tag{50}$$

$$\sup_{\mathbb{Q}\in\mathbb{G}} \mathbb{E}_{\mathbb{Q}}\left\{\boldsymbol{\lambda}^T\bar{\boldsymbol{y}}(\tilde{\boldsymbol{z}},\tilde{\boldsymbol{u}})\right\} \le \Lambda \tag{51}$$

with matrices $\boldsymbol{C} \in \mathbb{R}^{N_1\times N_1}$, $\boldsymbol{T} \in \mathbb{R}^{|\mathcal{M}|\times N_1}$, $\boldsymbol{W} \in \mathbb{R}^{|\mathcal{M}|\times|\mathcal{M}|}$, and vectors $\boldsymbol{b} \in \mathbb{R}^{N_1}$, $\boldsymbol{\lambda} \in \mathbb{R}^{|\mathcal{M}|}$, where N_1 is the number of first-stage decisions, $\mathcal{M}$ is the set of constraints (34)–(47). Note that random variables $\boldsymbol{z}$ only appear in the right-hand-side vector of constraints (41) so the vector $\boldsymbol{h}(\boldsymbol{z})$ is generalized into an affine function (52) of uncertainty $\boldsymbol{z}$.

$$\boldsymbol{h}(\boldsymbol{z}) = \boldsymbol{h}^0 + \boldsymbol{H}\boldsymbol{z} \tag{52}$$

where vector $\boldsymbol{h}^0 \in \mathbb{R}^{|\mathcal{M}|}$ denotes constant term coefficients, and matrix $\boldsymbol{H} \in \mathbb{R}^{|\mathcal{M}|\times|\mathcal{I}|}$ indicates linear term coefficients. The extended ambiguity set $\mathbb{Q}$ is expressed in the matrix form (9). The quadratic objective function (33) is represented by function (49). Constraints (34)–(47) are generalized into the matrix inequalities (50), and the expected emission factor constraint is expressed as (51). The adjustable decisions $\boldsymbol{y}(\boldsymbol{z})$ are approximated by the enhanced linear decision rule $\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u})$, which is also expressed in the following matrix form.

$$\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u}) = \boldsymbol{y}^0 + \boldsymbol{Y}^z\boldsymbol{z} + \boldsymbol{Y}^u\boldsymbol{u} \tag{53}$$

where $\boldsymbol{y}^0 \in \mathbb{R}^{|\mathcal{M}|}$ represents constant term coefficients, and entries of matrices $\boldsymbol{Y}^z \in \mathbb{R}^{|\mathcal{M}|\times|\mathcal{I}|}$ and $\boldsymbol{Y}^u \in \mathbb{R}^{|\mathcal{M}|\times|\mathcal{J}|}$, specified by Eqs. (54) and (55), are linear term coefficients.

$$Y_{ni}^z = \begin{cases} y_{ni}^z, & \text{if } i \in \mathcal{I}_n, \quad \forall n \in \mathcal{N} \\ 0, & \text{if } i \in \mathcal{I}\backslash\mathcal{I}_n, \quad \forall n \in \mathcal{N} \end{cases} \tag{54}$$

$$Y_{nj}^u = \begin{cases} y_{nj}^u, & \text{if } j \in \mathcal{J}_n, \quad \forall n \in \mathcal{N} \\ 0, & \text{if } j \in \mathcal{J}\backslash\mathcal{J}_n, \quad \forall n \in \mathcal{N} \end{cases} \tag{55}$$

3.3. Derivation of the robust counterpart

Let $f(\boldsymbol{z},\boldsymbol{u})$ be the continuous probability measure of random vectors $(\boldsymbol{z},\boldsymbol{u})$, then the worst-case expected term in constraint (51) can be expressed as the following semi-infinite optimization problem.

$$\sup_{\mathbb{Q}\in\mathbb{G}} \mathbb{E}_{\mathbb{Q}}\left\{\boldsymbol{\lambda}^T\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u})\right\} = \sup \int_{\hat{\mathcal{Z}}} \boldsymbol{\lambda}^T\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u})\, df(\boldsymbol{v},\boldsymbol{u}) \tag{56}$$

$$\text{s.t.} \quad \int_{\hat{\mathcal{Z}}} z_i df(\boldsymbol{z},\boldsymbol{u}) = 0, \quad \forall i \in \mathcal{I} \tag{57}$$

$$\int_{\hat{\mathcal{Z}}} (\boldsymbol{G}\boldsymbol{u})_m df(\boldsymbol{z},\boldsymbol{u}) \le \sigma_m, \quad \forall m = 1, 2, \ldots, M_0 \tag{58}$$

$$\int_{\hat{\mathcal{Z}}} df(\boldsymbol{z},\boldsymbol{u}) = 1 \tag{59}$$

where expressions (57)–(59) represent the three constraints that define the extended ambiguity set $\mathbb{G}$ in Eq. (9). Let $\boldsymbol{\rho} \in \mathbb{R}^{|\mathcal{I}|}$, $\boldsymbol{\eta} \in \mathbb{R}^{M_0}$, and $\kappa \in \mathbb{R}$ denote the dual variables associated with these three constraints, respectively, the distributionally robust optimization model (49)–(51) can be transformed into the robust optimization problem (60)–(64) by taking the dual of the supremum expectation (56)–(59).

$$\min \quad \boldsymbol{x}^T\boldsymbol{C}\boldsymbol{x} + \boldsymbol{b}^T\boldsymbol{x} \tag{60}$$

$$\text{s.t.} \quad \boldsymbol{T}\boldsymbol{x} + \boldsymbol{W}\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u}) \le \boldsymbol{h}(\boldsymbol{z}), \quad \forall(\boldsymbol{z},\boldsymbol{u}) \in \hat{\mathcal{Z}} \tag{61}$$

$$\kappa + \boldsymbol{\sigma}^T\boldsymbol{\eta} \le \Lambda \tag{62}$$

$$\kappa + \boldsymbol{z}^T\boldsymbol{\rho} + \boldsymbol{u}^T\boldsymbol{G}^T\boldsymbol{\eta} \ge \boldsymbol{\lambda}^T\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u}), \quad \forall(\boldsymbol{z},\boldsymbol{u}) \in \hat{\mathcal{Z}} \tag{63}$$

$$\kappa \in \mathbb{R}, \quad \boldsymbol{\rho} \in \mathbb{R}^{|\mathcal{I}|}, \quad \boldsymbol{\eta} \in \mathbb{R}_+^{M_0} \tag{64}$$

Note that the extended support set $\hat{\mathcal{Z}}$ in Eq. (10) is defined by second-order cone (SOC) constraints, the formulation above is equivalent to the following mixed-integer convex optimization problem.

$$\min \quad \boldsymbol{x}^T\boldsymbol{C}\boldsymbol{x} + \boldsymbol{b}^T\boldsymbol{x} \tag{65}$$

$$\text{s.t.} \quad h_m^0 + \sum_{k\in\mathcal{K}}\left(\boldsymbol{c}_k^T\boldsymbol{\pi}_k^m + e_k\mu_k^m\right) \ge (\boldsymbol{T}\boldsymbol{x})_m + \left(\boldsymbol{W}\boldsymbol{y}^0\right)_m, \quad \forall m \in \mathcal{M} \tag{66}$$

$$\sum_{k\in\mathcal{K}}\left(\boldsymbol{A}_k^T\boldsymbol{\pi}_k^m - \boldsymbol{f}_k\mu_k^0\right) = \left(\boldsymbol{H}^T\right)_m - \left[\left(\boldsymbol{W}\boldsymbol{Y}^z\right)^T\right]_m, \quad \forall m \in \mathcal{M} \tag{67}$$

$$\sum_{k\in\mathcal{K}}\left(\boldsymbol{B}_k^T\boldsymbol{\pi}_k^m - \boldsymbol{g}_k\mu_k^0\right) = -\left[\left(\boldsymbol{W}\boldsymbol{Y}^u\right)^T\right]_m, \quad \forall m \in \mathcal{M} \tag{68}$$

$$\|\boldsymbol{\pi}_k^m\| \le \mu_k^m, \quad \forall k \in \mathcal{K}, \quad \forall m \in \mathcal{M} \tag{69}$$

$$\boldsymbol{\pi}_k^m \in \mathbb{R}^{M_k}, \quad \mu_k^m \in \mathbb{R}_+, \quad \forall k \in \mathcal{K}, \quad \forall m \in \mathcal{M} \tag{70}$$

$$\kappa + \boldsymbol{\sigma}^T\boldsymbol{\eta} \le \Lambda \tag{71}$$

$$\kappa - \boldsymbol{\lambda}^T\boldsymbol{y}^0 + \sum_{k\in\mathcal{K}}\left(\boldsymbol{c}_k^T\boldsymbol{\pi}^0 + e_k\mu_k^0\right) \ge 0 \tag{72}$$

$$\sum_{k\in\mathcal{K}}\left(\boldsymbol{A}_k^T\boldsymbol{\pi}_k^0 - \boldsymbol{f}_k\mu_k^0\right) = \boldsymbol{\rho} - \left(\boldsymbol{Y}^z\right)^T\boldsymbol{\lambda} \tag{73}$$

$$\sum_{k\in\mathcal{K}}\left(\boldsymbol{B}_k^T\boldsymbol{\pi}_k^0 - \boldsymbol{g}_k\mu_k^0\right) = \boldsymbol{G}^T\boldsymbol{\eta} - \left(\boldsymbol{Y}^u\right)^T\boldsymbol{\lambda} \tag{74}$$

$$\|\boldsymbol{\pi}_k^0\| \le \mu_k^0, \quad \forall k \in \mathcal{K} \tag{75}$$

$$\kappa \in \mathbb{R}, \quad \boldsymbol{\rho} \in \mathbb{R}^{|\mathcal{I}|}, \quad \boldsymbol{\eta} \in \mathbb{R}_+^{M_0}, \quad \boldsymbol{\pi}_k^0 \in \mathbb{R}^{M_k}, \quad \mu_k^0 \in \mathbb{R}_+, \quad \forall k \in \mathcal{K} \tag{76}$$

In this formulation, uncertain constraints (61) are reformulated into the robust counterpart (66)–(70), with vectors $\boldsymbol{\pi}^m$ and $\boldsymbol{\mu}^m$ indicating the dual variables for the mth constraint. Similarly, the uncertain constraint (63) can also be transform into its robust counterpart (72)–(76), where vectors $\boldsymbol{\pi}^0$ and $\boldsymbol{\mu}^0$ are corresponding dual variables. Details of deriving the robust counterparts are provided in Appendix A. Such a mixed-integer convex program is considered tractable, because it has a dimension proportional to the size of sets $\mathcal{I}$, $\mathcal{J}$, and $\mathcal{K}$. The performance of this distributionally robust optimization model is demonstrated by case studies presented in the next section.

4. Case studies

In this section, a modified IEEE 118-bus test system [44] with 184 transmission lines and 54 generators is studied to illustrate the implementation of the proposed method. Generators with capacities higher than or equal to 300 MW are set to be coal-fired units, with an emission rate of 762 kg/MWh. Generators no larger than 100 MW are natural gas units with carbon capture technology, and the emission rate is 52 kg/MWh. The remaining units are natural gas fired without carbon capture, and the emission rate is 367 kg/MWh. The emission rates are assigned according to Ref. [45]. The price of both upward and downward reserve is set to be 15% of the linear term of generation cost function. It is assumed that there are four wind farms located at bus 17, 49, 59, and 100, and the total load is increased by 15% in order to create a more adverse case for the transmission system. According to the data of hourly aggregated wind output for ERCOT on June the 3rd, 2015, the wind power profile is given as Fig. 1. An energy storage system with a power rating of 145 MW and an energy rating of 1650 MWh is installed at bus 49. The charging and discharging efficiencies are set to be 90% and 85%. The first part of this section demonstrates that the ambiguous distribution information can be incorporated into the proposed model, and the distributionally robust solution can well adapt to different levels of wind variability. The second part focuses on discussing the trade-off between efficient system operation and lowering carbon emissions. The final subsection presents the computational experience.

4.1. The influence of wind power distribution

The proposed approach attempts to model the expected system emission factor without knowing the exact distribution of wind power. It is assumed that the only available information on wind power distribution is in terms of the MAD, the SD, the constant θ_t for skewness, and the boundaries of forecast errors. The following case studies demonstrate that such ambiguous distribution information, especially the MAD and the SD, can be well captured by the distributionally robust model.

In this section, the upper bound $\bar{Z}_{st}$ and lower bound $\underline{Z}_{st}$ of the forecast error $\tilde{z}_{st}$ are both set to be 55% of the expected wind power $\hat{w}_{st}$. Such an interval is considered sufficiently large to cover almost all uncertainty realizations. It has been shown in reference [46] that the distribution of forecast error could be left-skewed or right-skewed, so in this paper, the constant θ_t is assumed to be 0.45, implying that the distribution is slightly left-skewed. The emission factor target F^e is set to be 500 kg/MWh. Fig. 2 shows the total costs in the day-ahead market as decisions are made under various relative mean absolute deviations (RMAD) and the relative standard deviations (RSD). It can be seen that as the RMAD and the RSD increase, suggesting larger wind power variability, the total cost in the day-ahead market also increases, because higher generation flexibility is required to follow the fluctuations of wind power generation. For small values of RMAD or RSD, wind power is unlikely to vary too much from the expected value, especially the chance of having too much wind power is low. In this case, excessive wind energy can be spilled without greatly increasing the total emission. As a result, only small amount of downward reserve is needed, and the total cost is relatively lower. For higher wind power variability, however, the occurrence of having excessive wind energy becomes more frequent. In order to keep the expected emission below the target level, more downward reserve is needed to enhance the flexibility of conventional generation and to promote the usage of emission-free wind energy, thus leading to higher total cost.

This is why the increased flexibility is mainly reflected in the downward reserve, and this point is supported by Fig. 3, which depicts the cost of downward reserve under different levels of wind power uncertainty. As the MAD and SD increases, implying that wind power has a larger variability, the cost of downward reserve is changed from less than two hundred to eight thousand dollars, as a result of assigning more downward reserve to follow the change of wind power, so that the emission factor can be constrained below the given target.

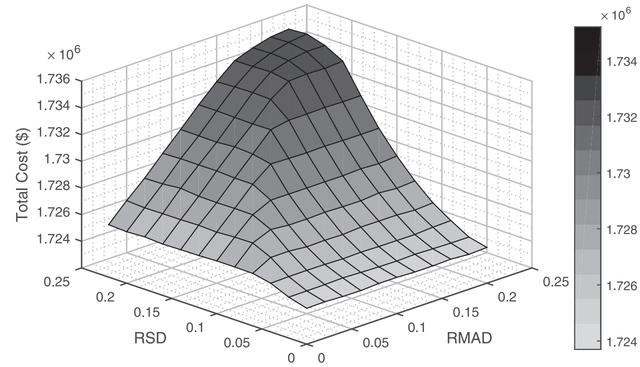

Fig. 2. Total cost in the day-ahead market under different RMADs and RSDs.

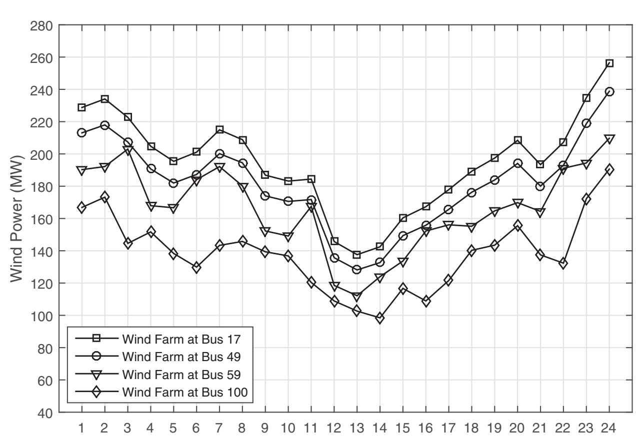

Fig. 1. Wind power profile of wind farms.

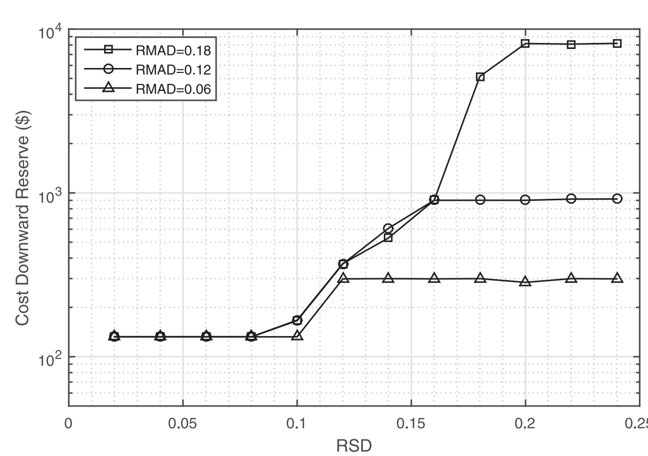

Fig. 3. The total cost of downward reserve under some representative values of RMAD and RSD.

Results above suggest that the solutions of the distributionally robust optimization model greatly depend on the ambiguous distribution information, in terms of MAD and SD. Such distribution information is typically missing from conventional robust optimization approaches, so they are not suitable for modeling the long-term impact of carbon emissions. Compared with stochastic programming methods, the proposed model may be more conservative because decisions are supposed to be robust against the ambiguity of probability distributions. Therefore, our method is probably more practical for complicated systems where the exact distribution of wind power cannot be accurately identified or represented.

4.2. Trade-off between efficient system operation and reducing carbon emission

In this subsection, the MAD and SD are respectively fixed to be 0.12 and 0.16 of the expected value. The target emission factor F^e is changed between 410 kg/MWh and 620 kg/MWh, and the other parameters remain the same. The curve of the total costs in the day-ahead market is shown by Fig. 4.

Apparently, tighter constraints of emission factor lead to higher generation and reserve cost. As the emission factor is decreased from around 600 kg/MWh to 410 kg/MWh, the total cost is increased by approximately 0.3 million dollars. Further reducing the emission factor makes the optimization problem infeasible. This trend can be attributed to the fact that expensive gas units, especially units with carbon capture technology are required to produce more energy, and the cheaper coal-fired units may have smaller output or even be shut down, in order to attain lower carbon emission. More details are provided in Fig. 5, where the energy production from each type of generators is displayed.

Among all generators, gas units with carbon capture technology have the lowest emission rate, but they are most expensive. The coal-fired units are the cheapest, but they produce the highest emission. It is observed that for the tightest emission constraints, the majority of the energy is produced by gas units. As the target emission factor increases, the requirement for clean energy decreases, so the generation from expensive gas units with carbon capture is quickly replaced by the production of cheap coal-fired units. The total operating cost reduces accordingly, as shown by Fig. 4. The generation of gas units without carbon capture only changes slightly, until the generation from carbon-capture units drops to nearly zero as the target emission factor reaches 500 kg/MWh. If the target emission factor further increases, the energy production from gas unit without carbon capture starts to

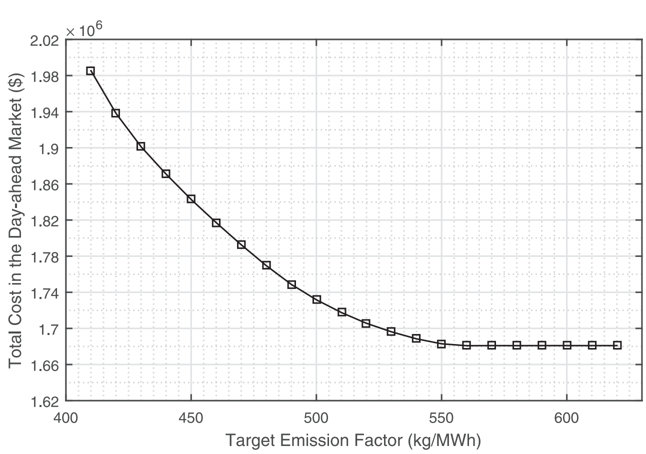

Fig. 4. The total costs in the day-ahead market as various emission factor constraints are enforced.

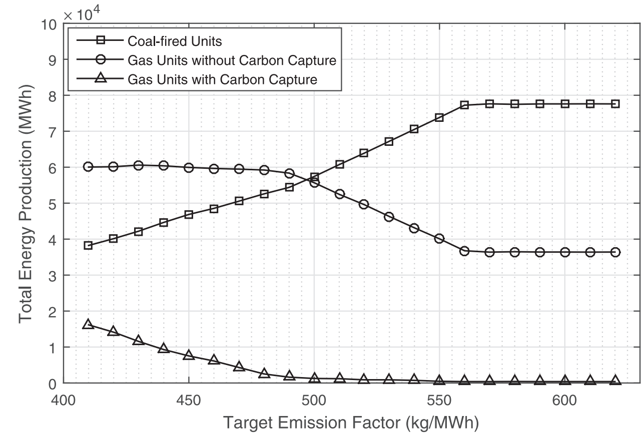

Fig. 5. The total energy production from all three types of units.

drop rapidly, more load is supplied by the cheap coal-fired units in order to lowering the operating cost. As the emission factor level grows larger than 550 kg/MWh, the emission constraint may become inactive, so the generation from each type of generators approaches a steady level, where the majority energy is provided by the cheapest coal-fired units. Results in this subsection suggest that the cheap coal-fired units are more likely to dominate an electricity market without emission constraints. While in a low-carbon market, gas units with low emission rate is preferred and the generation companies may have a strong incentive to invest on this type of generators.

4.3. Computational experience

The proposed model is a large-scale mixed-integer convex optimization problem. This type of problems are still computationally challenging. We therefore use linear piecewise functions to approximation the quadratic objective function (33) and the quadratic constraints of the extended support set $\hat{\mathcal{Z}}$ in Eq. (8). The distributionally robust model is then reduced to a mixed-integer linear programming (MILP) problem. The size of this MILP problem can be further reduced by eliminating transmission capacities constraints that are always inactive. Details of identifying these constraints are provided in Appendix B.

All case studies in this paper are solved by the mixed-integer programming (MIP) solver CPLEX 12.6.2, on a laptop with 2.20 GHz quad-core Intel i7 processors and 16 GB memory. As the relative MIP duality gap set to be 10^{-4}, the average solution time is 239 s, and the median is 235 s. The longest computation time is 402 s, in solving the case with an emission factor of 500 kg/MWh and the RMAD and RSD are set to be 0.10 and 0.22, respectively. In general, these numerical cases can be solved quite efficiently.

5. Conclusion

This paper proposed a distributionally robust model for the joint optimization of energy and reserve in a day-ahead market. The random forecast error of wind power is expressed as ambiguous distribution information, such as the MAD, SD, and the support set of random forecast errors. By incorporating the ambiguous distribution information into the proposed ambiguity set, the long-term impact of carbon emission can be effectively modeled by the worst-case expected emission factor. Case studies in this paper show that the proposed method is capable of capturing the variability of wind power, and can be tractably solved. The trade-off between lowering system operating cost and reducing the expected emission is also discussed. Results show that in a low-carbon electricity market, the low-emission gas units, especially units with carbon capture technology, are favored, while the coal-fired units which produce more emission gases are forced to generate less energy or to be shun down. This might be a strong incentive for generation companies to reduce emission in a low-carbon electricity market.

Appendix A. Robust counterpart of uncertain constraints

In this section, we will only prove that the uncertain constraint (63) is equivalent to deterministic constraints (72)–(76). The robust counterpart (66)–(70) of constraints (61) can be derived by repeating the same procedure.

Proof. Constraint (63) is equivalent to inequality (A.1).

$$\min_{(\boldsymbol{z},\boldsymbol{u})\in\hat{\mathcal{Z}}}\left\{\kappa+\boldsymbol{z}^T\boldsymbol{\rho}+\boldsymbol{u}^T\boldsymbol{G}^T\boldsymbol{\eta}-\boldsymbol{\lambda}^T\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u})\right\}\geq 0 \tag{A.1}$$

Because the extended support set $\hat{\mathcal{Z}}$ is a conic quadratic set, the minimum expression in the constraint above can be written as a SOC program (A.2) and (A.3) of variables $(\boldsymbol{z},\boldsymbol{u})$.

$$\min \quad \kappa+\boldsymbol{z}^T\boldsymbol{\rho}+\boldsymbol{u}^T\boldsymbol{G}^T\boldsymbol{\eta}-\boldsymbol{\lambda}^T\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u}) \tag{A.2}$$

$$\text{s.t.} \quad \|\boldsymbol{A}_k\boldsymbol{z}+\boldsymbol{B}_k\boldsymbol{u}+\boldsymbol{c}_k\|\leq\boldsymbol{f}_k^T\boldsymbol{z}+\boldsymbol{g}_k^T\boldsymbol{u}+e_k,\quad\forall k\in\mathcal{K}, \tag{A.3}$$

where the decision rule $\bar{\boldsymbol{y}}(\boldsymbol{z},\boldsymbol{u})$ is defined as (53). By taking the dual of the SOC program above, the uncertain constraint (63) can be transformed into the following expressions.

$$\kappa-\boldsymbol{\lambda}^T\boldsymbol{y}^0+\sum_{k\in\mathcal{K}}\left(\boldsymbol{c}_k^T\boldsymbol{\pi}^0+e_k\mu_k^0\right)\geq 0 \tag{A.4}$$

$$\sum_{k\in\mathcal{K}}\left(\boldsymbol{A}_k^T\boldsymbol{\pi}_k^0-\boldsymbol{f}_k\mu_k^0\right)=\boldsymbol{\rho}-\left(\boldsymbol{Y}^z\right)^T\boldsymbol{\lambda} \tag{A.5}$$

$$\sum_{k\in\mathcal{K}}\left(\boldsymbol{B}_k^T\boldsymbol{\pi}_k^0-\boldsymbol{g}_k\mu_k^0\right)=\boldsymbol{G}^T\boldsymbol{\eta}-\left(\boldsymbol{Y}^u\right)^T\boldsymbol{\lambda} \tag{A.6}$$

$$\begin{aligned}&\kappa\in\mathbb{R},\quad\boldsymbol{\rho}\in\mathbb{R}^{|\mathcal{I}|},\quad\boldsymbol{\eta}\in\mathbb{R}_+^{M_0},\\&\|\boldsymbol{\pi}_k^0\|\leq\mu_k^0,\quad\boldsymbol{\pi}_k^0\in\mathbb{R}^{M_k},\quad\mu_k^0\in\mathbb{R}_+,\quad\forall k\in\mathcal{K}\end{aligned} \tag{A.7}$$

where the vector $\boldsymbol{\pi}_k^0$ and the scaler μ_k^0 are dual variables associated with the kth constraint in (A.3). Likewise, it can be shown that uncertain constraints (61) are equivalent to the robust counterpart (66)–(70).

Appendix B. Inactive transmission capacity constraints

It is observed that some transmission capacity constraints (45) are always inactive, these constraints are hence removed so that the computational cost of the proposed model is greatly reduced. Notice that the transmission capacity constraints at each time step t are affected by the generation output $\boldsymbol{p}_t$, the wind power generation $\boldsymbol{w}_t$, the discharge $\boldsymbol{d}_t$ and charge $\boldsymbol{q}_t$ of storage systems, and also implicitly affected by the unit commitment $\boldsymbol{v}$. Then we can construct a feasible set of these decision variables by enforcing basic system operation constraints, such as power balance equation, generation capacities of convention units and wind power sources, as well as the power ratings of storage systems. Such a set is denoted by $\mathcal{F}_t$ in Eq. (B.1).

$$\mathcal{F}_t=\left\{\begin{pmatrix}\boldsymbol{v}_t\in\mathbb{B}^{|\mathcal{A}|}\\\boldsymbol{p}_t\in\mathbb{R}^{|\mathcal{A}|}\\\boldsymbol{w}_t\in\mathbb{R}^{|\mathcal{S}|}\\\boldsymbol{d}_t\in\mathbb{R}^{|\mathcal{H}|}\\\boldsymbol{q}_t\in\mathbb{R}^{|\mathcal{H}|}\end{pmatrix}:\begin{array}{l}\sum\limits_{n\in\mathcal{N}}p_{at}+\sum\limits_{h\in\mathcal{H}}(d_{ht}-q_{ht})+\sum\limits_{s\in\mathcal{S}}w_{st}=\sum\limits_{b\in\mathcal{B}}L_{bt}\\\underline{P}_av_{at}\leq p_{at}\leq\bar{P}_av_{at},\forall a\in\mathcal{A}\\0\leq w_{st}\leq\hat{w}_{st}+\bar{Z}_{st},\quad\forall s\in\mathcal{S}\\0\leq d_{ht}\leq Q_h,\quad\forall h\in\mathcal{H}\\0\leq q_{ht}\leq Q_h,\quad\forall h\in\mathcal{H}\end{array}\right\} \tag{B.1}$$

Clearly, any feasible operation decisions should belong to the set $\mathcal{F}_t$, since all constraints listed in $\mathcal{F}_t$ must be strictly satisfied. Therefore we can solve a series of linear programming problems (B.2) and (B.3) for each line $l\in\mathcal{L}$ and time step $t\in\mathcal{T}$, to determine if a transmission constraint is always inactive.

$$\begin{aligned}\zeta_{lt}^+=\max\quad&F_l(\boldsymbol{p}_t,\boldsymbol{w}_t,\boldsymbol{d}_t,\boldsymbol{q}_t)\\\text{s.t.}\quad&(\boldsymbol{p}_t,\boldsymbol{w}_t,\boldsymbol{d}_t,\boldsymbol{q}_t)\in\mathcal{F}_t\end{aligned} \tag{B.2}$$

$$\begin{aligned}\zeta_{lt}^-=\min\quad&F_l(\boldsymbol{p}_t,\boldsymbol{w}_t,\boldsymbol{d}_t,\boldsymbol{q}_t)\\\text{s.t.}\quad&(\boldsymbol{p}_t,\boldsymbol{w}_t,\boldsymbol{d}_t,\boldsymbol{q}_t)\in\mathcal{F}_t\end{aligned} \tag{B.3}$$

If $\zeta_{lt}^+<T_l$ and $\zeta_{lt}^->-T_l$, the corresponding transmission capacity constraint is always inactive for feasible operation decisions, and can be removed from the optimization model.

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