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Determining the contribution of different effective factors to individual voltage unbalance emission in n-bus radial power systems

Mahyar Abasi *, S. Ghodratollah Seifossadat, Morteza Razaz, S. Sajad Moosapour

Department of Electrical Engineering, Faculty of Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran

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ABSTRACT

In this paper, a new method is presented for determining the contribution of different effective factors such as lateral loads, lines between buses, and voltage supply source to voltage unbalance in an n-bus radial distribution system. For this purpose, the factors affecting voltage unbalance at the place of load were analyzed based on the effects of source, line, and load. Furthermore, novel techniques are suggested to model and analyze passive, active (induction motor), and combined loads. The proposed method is then tested on a real distribution feeder with 9 jumper loads (20 kV/0.4 kV distribution transformer) in Iran. The results show the high accuracy of the proposed method.

1. Introduction

In recent decades, power quality (PQ) has gained more scholarly attention than before due to an increase of sensitive loads to voltage and current changes, power quality issue has been more notable than before. Damages resulting from voltage flicker, voltage unbalance, harmonics and voltage sag have threatened the power industry. On the other hand, the competitive power market has made generation companies and consumers attend more to PQ issues. Mutual effects of the system and the loads from main PQ factors have attracted the attention of researchers and power industry experts. Nowadays, regional power companies enforce big reactive power consumers to implement capacitor allocation, and if they violate reactive power consumption rules they are penalized and must pay for it.

Another issue under study is to determine the necessary standards for PQ factors such as voltage flicker, voltage unbalance, harmonics, and voltage sag.

Assessment of the voltage unbalance emission and determining the contribution of different effective factors to the resulting voltage unbalance in load side is one of the important aspects in the voltage unbalance management in power systems. To assess the voltage unbalance level and establish a basis for comparing the unbalanced of effects different points available in the network, a percentage of negative sequence voltage is considered as a criterion for voltage unbalance. In this paper, the studies published on voltage unbalance are divided into four groups whit the first group studying the effects of DG, the second group studying the effects of induction machines, the third studying the analytical and computational methods of determining the contribution of different factors to voltage unbalance emission in power systems, and finally the fourth group focusing on heuristic and intelligent algorithms.

In [1–7], DG units were used to reduce voltage unbalances in power systems. Authors in [1] focused on the assessment of emission levels in large photovoltaic (PV) units connected to LV and MV networks to limit unbalance emission. Ref. [2] presented a technique to mitigate the voltage unbalance in Low-Voltage Distribution Networks (LVDNs) with a high penetration level of PV system using a controllable Energy Storage Unit (ESU). In [3], the effects of connecting DG units by means of a three-phase connection (instead of a single-phase connection) on voltage unbalance were studied. In [4], two different improvement methods based on the application of series (DVR) and parallel distribution static compensator (DSTATCOM) custom power devices were investigated to improve the voltage unbalance problem. Studying and assessing of voltage unbalance effects in radial distribution networks equipped with single-phase photovoltaic systems were carried out in [5]. To analyze voltage unbalance sensitivity for different maximum rates of a single-phase PV system with several PV penetration levels in a typical radial distribution system in Spain, a new method is presented in this paper. In Ref. [6] describes the efficiency of negative sequence voltage reduction in an MV distribution system with balanced phase currents in the presence of MV static DGs. A distributed cooperative control design to

* Corresponding author.
E-mail address: mahyarabasi1368@yahoo.com (M. Abasi).

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Nomenclature

$V_i^{(k)}$	ith sequence voltage of kth bus
$V_i^{(k-1,k)}$	ith Sequence voltage drop of the line between kth and (k − 1)th buses
$Vuf^{(k)}$	voltage unbalance factor of kth bus
$Vuf^{Line(k-1,k)}$	voltage unbalance factor of the line between kth and (k − 1)th buses
Vuf^{Line}_{total}	sum of voltage unbalance factors of all lines
Vuf^{Load}_{total}	sum of voltage unbalance factors of all loads
$Vuf^{Load(k)}$	voltage unbalance factor of the load in kth bus
Vuf^{Source}	voltage unbalance factor of the source
Vuf^{Cal}	computed voltage unbalance factor through the presented formula at each bus j
$Vuf^{measure}$	computed real voltage unbalance factor at each bus using DIgSILENT Power Factory software
$V_{reg(k,k+1)}$	regulating line voltage between kth and (k − 1)th buses
$I_i^{(k)}$	ith sequence current, load at kth bus
$I_i^{(k-1,k)}$	ith Sequence current, the line between kth and (k − 1)th buses
$I_{i,PL}^{(k)}$	ith sequence current, passive load at kth bus
$I_{i,IM}^{(k)}$	ith sequence current, motor load at kth bus
$Cuf^{(k)}$	current unbalance factor of the load of the kth bus
$Cuf_{PL}^{(k)}$	current unbalance factor of passive load kth bus
$Cuf_{IM}^{(k)}$	current unbalance factor of motor load kth bus
$Cuf_{mix,Load}^{(k)}$	current unbalance factor of combined load kth bus
$S_i^{*(k)}$	complex conjugate power of sequence ith, load kth bus
$Z_{ii}^{(k)}$	self-impedance of sequence ith, load kth bus
$Z_{ij}^{(k)}$	mutual impedance between sequence ith and sequence jth, load *k*th bus
$Z_{ii,PL}^{(k)}$	self -impedance of sequence ith, passive load kth bus
$Z_{ij,PL}^{(k)}$	mutual- impedance between sequence ith and sequence jth, passive load kth bus
$Z_{ii,IM}^{(k)}$	self-impedance of sequence ith, motor load kth bus
$Z_{ii,mix,Load}^{(k)}$	self-impedance of sequence ith, combined load kth bus
$Z_{ii}^{(k-1,k)}$	self-impedance of sequence ith, line between kth and (k − 1)th buses
$Z_{ij}^{(k-1,k)}$	mutual-impedance between sequence ith and sequence jth, line between kth and (k − 1)th buses

compensate voltage unbalance level in an islanded micro grid is presented in [7]. The proposed control design in this paper is able to compensate for voltage unbalances in a natural load and simultaneously divides the compensator dynamically in a dispersed manner.

Induction machines were used in [8–11] to minimize voltage unbalance. Ref. [8] investigated an improved control and operation of a Doubly-Fed Induction Generator (DFIG) system during the network voltage unbalance conditions. Ref. [9] dealt with load-carrying capacity of an induction cage machine under voltage unbalance combined with over- or under-voltage conditions. Ref. [10] aimed at complementing the studies concerning the influence of voltage unbalance on the performance of induction motors using sequence-equivalent circuits to determine the loss increase in the induction motor. Ref. [11] analyzed the unbalance problem of a stand-alone DFIG under unbalanced loads and proposed a compensatory method to balance the stator output voltage.

Analytical methods in [12–25] were employed to study voltage unbalance. The primary objective of Ref. [12] was to develop statistical approaches for compliance assessment using outcomes of deterministic methodologies on VU emission assessment, thus refining the existing general summation law. In order to determine the influence of the network's parameters on voltage unbalances, with the aim of providing exact solutions to reduce or even eliminate voltage unbalances, the present study develops and presents two methods. Ref. [13] developed and presented two methods. The first method involves using sensitivity analysis to determine the influence on the voltage unbalance of each parameter of the system (magnitudes and angles of the three phases); in the second method, attempts are made to change the unbalances based on analytical solutions. The main contribution of Ref. [14] was the investigation of possible discrepancies when comparing values of Voltage Unbalance Factor (VUF) resulting from the calculations, where it employs different aggregation time intervals, as well as the causes for such discrepancies. Using a static load transfer switch, Ref. [15] presents a method to reduce voltage unbalance and power loss resulting from neutral current flow in a bipolar DC distribution system. Surveying the effects of voltage and current unbalances of a three-phase four-wire system on line and transformer loss is performed in [16], This study also investigated the over-capacity of distribution transformers. Authors of [17] have proposed an approximation algorithm for fast calculation of voltage unbalances in three-phase power systems. In the proposed method, root square operations are turned into simple trigonometric equations based on geometric form of the transform, and then these trigonometric equations are approximated using algebraic operations. This technique has reduced the calculation burden. In Ref. [18], a method to determine voltage unbalance emission level was proposed in radial and interconnected systems. In this paper, the radial system is modeled as a power source (upstream network), a transmission line and a load, and the emitted unbalance is found only for the far end bus of the system. To estimate the contribution of single-phase PV inverters to voltage unbalance of distribution systems a stochastic method was used in [19]. In this method, place and phase uncertainties are considered in some stochastic indices. In Ref. [20], the effects of the influencing factors on voltage unbalance coefficients of buses in a radial distribution system have not been completely specified, and since all the obtained relations in the paper characterize a very specific situation, the presented relations cannot be generalized. Definition and analysis of voltage unbalance emission concept in power systems are given in [21]. This paper also illustrates that the changes occurring in voltage unbalance in each system bus due to adding a load, are not essentially because of the considered load. Based on behavior analysis of each sideway transmission line and the connected loads to them in an interconnected power system and in terms of a new concept, Ref. [22] presents a method called voltage unbalance emission vector. Ref. [23] mostly deals experimentally with the analysis and study of voltage unbalance emission assessment in interconnected systems. Authors of [24] have presented a novel and algebraic method to assess voltage unbalance emission in interconnected systems. They were have been able to provide a comprehensive insight towards the realization of IEC – related issues in the fields of voltage unbalance management. A new formulation to determine load, power transmission line and source contributions for the generated voltage unbalance in load consumption area without considering the total effect of loads and system topology was proposed in [25], Of course the formulation is for a far end bus in a radial distribution system.

Finally, in [26–29] heuristic and intelligent algorithms were suggested to study voltage unbalance. Ref. [26], first analyzed voltage unbalance definitions, and extracted differences between various definitions of voltage unbalance using a genetic algorithm (GA). Ref. [27] presented a new control algorithm for a series active filter that optimized the power factor, balanced the voltages, limited the total harmonic distortion of voltage, and ensured a high-quality voltage supply to sensitive loads. Ref. [28] described a new methodology for a three-phase power system optimization, in which the phase unbalances are taken into account. Ref. [29] addressed the problem of estimating the characteristics of three-phase unbalanced voltage dips from measured phase voltages.

The main novelty and objective of this paper is the way it proposes to determine the effects of various factors on voltage unbalance emission in every single bus of flexible radial distribution systems for different (n-bus) systems. In this paper, using the phasor analysis and the ruling mathematical equations in a radial distribution system in positive, negative and zero sequences, the researchers managed to extract a general formula for the impacts of all effective factors on the generation of voltage unbalance in each bus. The obtained equations can be generalized to be used in any radial system. To the best our knowledge, this subject has never been discussed in any of the papers in this area. To investigate this issue, the generated voltage unbalance in a general bus (j) is first written in terms of the line impedances, the load currents, considering all kinds of their models, and the source voltage unbalance effect. Then, similar to the superposition rule, the impacts of all loads, lines and the source can be determined separately. Finally, to verify the presented paper, all the obtained relationships are implemented on a 20 kV radial distribution system, and the contribution of all effective factors on the generated voltage unbalance in each bus in the system is specified.

In a general view, this paper includes four sections and two appendices. Section 2, after studying the effects of power source, load and line, analyzes the contribution of each of these factors. Then, according to the models of existing loads in the distribution systems and computational requirements, three load models are proposed in the last part of this section. Simulation results of the study are given in Section 3, and conclusions are presented in Section 4. Finally, all required data of the test system are presented in Appendix A.

2. Basic principle and the proposed method

Based on the radial distribution system shown in Fig. 1 it can be seen that KVL equation for positive and negative sequences ruling on the system is written according to the voltage drop between the source bus and the end bus. To realize this, it can be said that the voltage drop between any two buses is written in terms of the product of the current flowing through the line between those two buses and the sequence impedance of that bus. In this case, the whole current flowing through the line between buses k and k − 1 are the sum of the current of bus k loads to the end bus of the system. Voltage equation for positive and negative sequence between the source bus and the end bus are extracted from Eqs. (1) and (2).

$$\begin{aligned} V_1^{(0)} = {} & \left[Z_{11}^{(0,1)}\sum_{k=1}^{n} I_1^{(k)} + Z_{11}^{(1,2)}\sum_{k=2}^{n} I_1^{(k)} + \cdots + Z_{11}^{(j-1,j)}\sum_{k=j}^{n} I_1^{(k)} + \cdots + Z_{11}^{(n-1,n)}\sum_{k=n}^{n} I_1^{(k)}\right] \\ & + \left[Z_{12}^{(0,1)}\sum_{k=1}^{n} I_2^{(k)} + Z_{12}^{(1,2)}\sum_{k=2}^{n} I_2^{(k)} + \cdots + Z_{12}^{(j-1,j)}\sum_{k=j}^{n} I_1^{(k)} + \cdots + Z_{12}^{(n-1,n)}\sum_{k=n}^{n} I_1^{(k)}\right] + V_1^{(n)} \end{aligned} \tag{1}$$

$$\begin{aligned} V_2^{(0)} = {} & \left[Z_{21}^{(0,1)}\sum_{k=1}^{n} I_1^{(k)} + Z_{21}^{(1,2)}\sum_{k=2}^{n} I_1^{(k)} + \cdots + Z_{21}^{(j-1,j)}\sum_{k=j}^{n} I_1^{(k)} + \cdots + Z_{21}^{(n-1,n)}\sum_{k=n}^{n} I_1^{(k)}\right] \\ & + \left[Z_{22}^{(0,1)}\sum_{k=1}^{n} I_2^{(k)} + Z_{22}^{(1,2)}\sum_{k=2}^{n} I_2^{(k)} + \cdots + Z_{22}^{(j-1,j)}\sum_{k=j}^{n} I_2^{(k)} + \cdots + Z_{22}^{(n-1,n)}\sum_{k=n}^{n} I_2^{(k)}\right] + V_2^{(n)} \end{aligned} \tag{2}$$

Due to the need for negative sequence voltage, Eq. (2) is analyzed, expanded and restated as Eq. (3).

$$V_2^{(0)} = \sum_{k=1}^{n} Z_{21}^{(0,k)} I_1^{(k)} + Z_{22}^{(0,k)} I_2^{(k)} + V_2^{(n)} \tag{3}$$

Through Eq. (3) and some other parameters, the voltage unbalance factor in the nth bus is extracted. In order to define the voltage unbalance factor, the current unbalance factor and the line voltage regulation are used as follows:

$$V_1^{(k)} = V_1^{(k,k+1)} + V_1^{(k+1)} \tag{4}$$

$$V_{reg(k,k+1)} = \frac{V_1^{(k,k+1)}}{V_1^{(k+1)}} \tag{5}$$

$$\frac{V_1^{(k)}}{V_1^{(k+1)}} = 1 + V_{reg(k,k+1)} \tag{6}$$

Since the system under study is an n-bus system, $V_{reg}^{(n,n+1)}$ is considered to be zero, where $V_1^{(k,k+1)}$ is the positive sequence voltage along the line between the kth and the (k + 1)th buses, and $V_{reg}^{(k,k+1)}$ is the voltage regulation of the line between the (k − 1)th and the kth buses. Using the parameters above, the voltage unbalance factor at the nth bus is stated as follows:

$$\begin{aligned} VUF^{(n)} = {} & VUF^{(source)}\prod_{i=0}^{n}\left(1 + V_{reg(k,k+1)}\right) \\ & - \sum_{k=1}^{n}\left[\frac{Z_{21}^{(0,k)}}{Z_{11}^{(k)}} + \frac{Z_{22}^{(0,k)}}{Z_{11}^{(k)}} CUF^{(k)}\right]\prod_{i=k}^{n}\left(1 + V_{reg(i,i+1)}\right) \end{aligned} \tag{7}$$

where $VUF^{(n)}$ is the voltage unbalance factor at the nth bus, $VUF^{(source)}$ is the voltage unbalance factor of the source, and $Z_{11}^{(k)}$ is the positive-sequence impedance for the load of the kth bus. In order to determine the level unbalance in the jth bus, the system is divided into two segments, with one segment including the system state before the jth bus and the other segment extending from jth bus. Therefore, the negative sequence voltage equation for the jth bus towards the nth bus is stated as in Eq. (8).

$$\begin{aligned} V_2^{(j)} = {} & \left[Z_{21}^{(j,j+1)}\sum_{k=j+1}^{n} I_1^{(k)} + Z_{21}^{(j,j+2)}\sum_{k=j+2}^{n} I_1^{(k)} + \cdots + Z_{21}^{(n-1,n)}\sum_{k=n}^{n} I_1^{(k)}\right] \\ & + \left[Z_{22}^{(j,j+1)}\sum_{k=j+1}^{n} I_2^{(k)} + Z_{22}^{(j+1,j+2)}\sum_{k=j+2}^{n} I_2^{(k)} + \cdots + Z_{22}^{(n-1,n)}\sum_{k=n}^{n} I_2^{(k)}\right] + V_2^{(n)} \end{aligned} \tag{8}$$

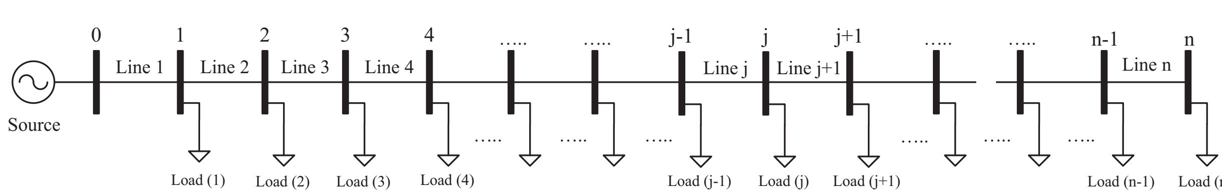

Fig. 1. Single-linear radial distribution feeder.

Eq. (8) can be expanded and stated in a simpler way, as in (9):

$$V_2^{(j)} = \sum_{k=j+1}^{n} Z_{21}^{(j,k)} I_1^{(k)} + Z_{22}^{(j,k)} I_2^{(k)} + V_2^{(n)} \tag{9}$$

Finally, Eq. (10) states the voltage unbalance factor at the nth bus.

$$\begin{aligned} VUF^{(n)} &= VUF^{(j)} \prod_{i=j}^{n} (1 + V_{reg(i,i+1)}) \\ &\quad - \sum_{k=j+1}^{n} \left[\frac{Z_{21}^{(j,k)}}{Z_{11}^{(k)}} + \frac{Z_{22}^{(j,k)}}{Z_{11}^{(k)}} CUF^{(k)} \right] \prod_{i=k}^{n} (1 + V_{reg(i,i+1)}) \end{aligned} \tag{10}$$

where $VUF^{(j)}$ is the voltage unbalance factor at the jth bus. With respect to the computed $VUF^{(n)}$ in Eqs. (7) and (10), and by setting these two equations equal, the voltage unbalance factor at the jth bus is calculated using Eq. (11).

$$\begin{aligned} VUF^{(j)} &= VUF^{(source)} \prod_{i=0}^{j-1} (1 + V_{reg(i,i+1)}) - \sum_{k=1}^{n} \frac{f(k)}{Z_{11}^{(k)}} CUF^{(k)} Z_{L22}^{(0,k)} \\ &\quad - \sum_{k=1}^{j} \sum_{m=k}^{n} \frac{f(m)}{Z_{11}^{(m)}} Z_{21}^{(k-1,k)} \end{aligned} \tag{11}$$

where, $f(m) = f(k)$ and $Z_{L22}^{(0,k)}$ are calculated using Eqs. (12) and (13), respectively.

$$f(k) = \frac{\prod_{i=0}^{j-1} 1 + (V_{reg(i,i+1)})}{\prod_{i=0}^{k-1} 1 + (V_{reg(i,i+1)})} \tag{12}$$

$$Z_{L22}^{(0,k)} = \left\{ \begin{array}{ll} Z_{22}^{(0,k)} & k < j \\ Z_{22}^{(0,j)} & k \geqslant j \end{array} \right\} \tag{13}$$

As it can finally be observed, the bus j voltage unbalance factor equation in term of the effective factors of Eq. (11) is extracted separately. It is seen that in this equation the effects of three important factors, i.e. load, line, and source are separated, which will be analyzed in the following.

2.1. Analyzing the effect of source

A distribution substation feeding by several transmission lines has one or more output feeders that are in a radial form. In case the receiving voltage by the feeding from the transformer located within inside the substation is totally balanced i.e. when the negative voltage of the feeding feeder is zero, then there will not be any effects by the source on the voltage unbalance of each buses, in other words the system is fed by a balanced source.

However, if the distribution substation transformer, which works as the feeding source for the feeder, envisages voltage unbalance due to any external or internal factor, then the system will be fed by an unbalanced source and will have a considerable impact on voltage unbalanced of each bus. Based on Eq. (11) it is obvious that the contribution of the source to voltage unbalance of bus j is obtained according to the voltage unbalance factors of the source as in Eq. (14) at $VUF^{(j)}$:

$$VUF^{(S)} = VUF^{(source)} \prod_{i=0}^{j-1} (1 + V_{reg(i,i+1)}) \tag{14}$$

Eq. (14) shows the effect of source on voltage unbalance factor at the jth bus. As shown, this equation is composed of multiplication of voltage unbalance of source by regulating voltage lines to the jth bus. According to line voltage regulation, this value exists in balance and unbalance voltages. A non-zero value for this parameter may not be interpreted as dealing with an unbalanced system. In fact, $VUF^{(source)}$ is created because of the source unbalance, so that a non-zero value for this parameter is equivalent to having an unbalanced source. Therefore, if the source is unbalanced, $VUF^{(source)}$ is equal to zero and one may see no source unbalance effect on the jth bus.

2.2. Analyzing the effect of line

Summation of voltage unbalance factors over all the lines is formulated as follows:

$$VUF_{total}^{line} = -\sum_{k=1}^{j} \sum_{m=k}^{n} \frac{f(m)}{Z_{11}^{(m)}} Z_{21}^{(k-1,k)} \tag{15}$$

Eq. (15) gives summation of the line effects between the source bus and the jth bus in voltage unbalance factor of bus jth. Eq. (16) indicates the line effect between the (k − 1)th and the kth buses in voltage unbalance factor of the jth bus.

$$VUF^{line(k-1,k)} = -\sum_{m=k}^{n} \frac{f(m)}{Z_{11}^{(m)}} Z_{21}^{(k-1,k)} \tag{16}$$

Since this amount is composed of coupling impedance between the negative-positive sequence of line and $f(m)/z_{11}^{(m)}$, the impedance between the negative-positive sequence of line is because of non-transposing between the (k − 1)th and the kth buses, and $f(m)/z_{11}^{(m)}$ exists when the system is either balanced or unbalanced (source, load, line) whit its values always being non-zero.

Therefore, Eq. (16) shows the line effect asymmetry between the (k − 1)th and kth buses and Eq. (15) shows sum of line effects between the source bus to the jth bus in voltage unbalance of the jth bus. Considering the mentioned equation, symmetry or asymmetry of lines between the jth bus to the nth bus does not have any influence on voltage unbalance factor of bus jth.

2.3. Analyzing the effect of load

Eq. (17) shows summation of the effects of all existing loads in the system on voltage unbalance factor of the jth bus.

$$VUF_{total}^{load} = -\sum_{k=1}^{n} \frac{f(k)}{Z_{11}^{(k)}} CUF^{(k)} Z_{L22}^{(0,k)} \tag{17}$$

Furthermore, Eq. (18) indicates the effect of the connected load to the kth bus.

$$VUF^{load(k)} = -\frac{f(k)}{Z_{11}^{(k)}} CUF^{(k)} Z_{L22}^{(0,k)} \tag{18}$$

where $VUF^{load(k)}$ is the voltage unbalance factor of the kth bus load.

Eq. (18) consists of two parts: $CUF^{(k)}$ and $-(f(m)/z_{11}^{(m)})\, Z_{22}^{(0,k)}$. As shown, $-(f(m)/z_{11}^{(m)})\, Z_{22}^{(0,k)}$ exists in any state (balanced or unbalanced load, source and line) and is non-zero, which does not mean that the system is unbalanced. However, $CUF^{(k)}$ is directly related to the load. If the system load is passive, $CUF^{(k)}$ will be zero provided the load is balanced. Otherwise, it will have a non-zero value. If the motor is of on induction type (since these kinds of motors are naturally symmetric and their asymmetric performance is due to system unbalance), the value of this term in the motor load state will be zero given the system is zero. Otherwise, it will have a non-zero value.

2.3.1. Passive load

2.3.1.1. Passive load with specific impedance. This model is used when details of impedance load are specified. Regarding KVL equation governing passive load impedance at the kth bus, we will have the following equation:

$$\begin{aligned} V_1^{(k)} &= Z_{11,PL}^{(k)} I_{1,PL}^{(k)} + Z_{12,PL}^{(k)} I_{2,PL}^{(k)} \\ V_2^{(k)} &= Z_{21,PL}^{(k)} I_{1,PL}^{(k)} + Z_{22,PL}^{(k)} I_{2,PL}^{(k)} \end{aligned} \tag{19}$$

where $Z_{ii},_{PL}^{(k)}$ is the impedance of the ith sequence, passive load at the kth bus; $Z_{ij},pl^{(k)}$ is the coupling impedance between the ith and jth sequences, passive load at kth bus; and $I_{i,PL}^{(k)}$ is the ith sequence current, passive load at the kth bus. Studies on load distribution show that the contribution of voltage drop $Z_{12,PL}^{(k)}$ $I_{2,PL}^{(k)}$ on $V_1^{(k)}$ is very insignificant [25]. Therefore, voltage drop of $Z_{12,PL}^{(k)}$ $I_{2,PL}^{(k)}$ is ignored and Eq. (19) will turn into Eq. (20).

$$\begin{aligned} V_1^{(k)} &= Z_{11,PL}^{(k)} I_{1,PL}^{(k)} \\ V_2^{(k)} &= Z_{21,PL}^{(k)} I_{1,PL}^{(k)} + Z_{22,PL}^{(k)} I_{2,PL}^{(k)} \end{aligned} \tag{20}$$

Since $Z_{12,PL}^{(k)}$ is identical to $Z_{22,PL}^{(k)}$ in the passive load, the current unbalance factor of the kth bus (load connected to kth bus) is calculated as follows,

$$CUF_{PL}^{(k)} = VUF^{(k)} - \frac{Z_{21,PL}^{(k)}}{Z_{11,PL}^{(k)}} \tag{21}$$

where $CUF_{pl}^{(k)}$ is the unbalance factor of current of passive load in kth bus. In this model, the load with impedance is given. Finally, a combination of Eqs. (17) and (21) will result in Eq. (22):

$$VUF^{load(k)} = -\frac{f(k)}{Z_{11,PL}^{(k)}} Z_{L22}^{(0,k)} \left(VUF^{(k)} - \frac{Z_{21,PL}^{(k)}}{Z_{11,PL}^{(k)}} \right) \tag{22}$$

2.3.1.2. Passive load with specific power. This model is used when details of the impedance load are not clearly specified, but the power load is known. Since power information, unlike the impedance information, is known in this model, all load impedances mentioned in the previous equations should be changed to corresponding load power terms. Regarding Eq. (16) and positive sequence impedance in terms of voltage and power load, $Z_{22,PL}^{(k)} = |v_1^{(k)}|^2/S_1^{*(k)}$, the contribution of voltage line unbalance factor between the (k – 1)th and the kth buses is indicated as in Eq. (23) if the passive load with specific power is determined at the mth bus.

$$VUF^{line(k-1,k)} = -f^{/}(m) S_1^{*(m)} Z_{21}^{(k-1,k)} \tag{23}$$

Also the effect of the connected load to the kth bus is stated as in Eq. (24):

$$VUF^{load(k)} = -\frac{f^{/}(k) S_2^{*(k)} Z_{L22}^{(0,k)}}{VUF^{(k)^*}} \tag{24}$$

where $S_i^{*(m)}$ is the complex conjugate power of the ith sequence, the mth bus load and $f^{/}(m) = f^{/}(k)$ is computed as follows:

$$f^{/}(k) = \frac{\left[\prod_{i=0}^{j-1} 1 + (V_{reg(i,i+1)})\right] \left[\prod_{i=0}^{k-1} 1 + (V_{reg(i,i+1)})\right]^*}{|V_1^{(0)}|^2} \tag{25}$$

As it is evident, there is no effect of load impedance in Eqs. (17) and (24). Effects of voltage line unbalance factor, voltage line regulating factor, and line impedances that are determined from known information under "passive load with specific power" state can be calculated.

2.3.1.3. Passive load with specific current. When the load model is of a determined-current type, regardless of balanced or unbalanced source, the current drawn by the load does not have the same range. However, the negative sequence current related to the load may be important due to unequal change it results in the phase angle concerning the voltage drop which means that asymmetric load behavior with a specific current can be determined to be the same as the load with power .

2.3.2. Active load (induction motor)

Regarding the IEEE model for induction motor in Fig. 2, negative-positive sequence impedance is calculated as in Eq. (26) [30]. where $s_i = s$ in is the CCW mode and $s_i = s$ in is the CC mode. $s_i = s$

$$\begin{aligned} Z_{11,M} &= (R_1 + X_1 j) + X_m j || (X_2 j + \tfrac{R_2}{S}) \\ Z_{22,M} &= (R_1 + X_1 j) + X_m j || (X_2 j + \tfrac{R_2}{2-S}) \end{aligned} \tag{26}$$

Due to the negative-positive sequence impedance of the motor and the KVL equation governing the induction motor, the $CUF^{(k)}$ of the motor in terms of impedance is calculated as in Eq. (28).

$$\begin{aligned} V_1^{(k)} &= Z_{11,IM}^{(k)} I_{1,IM}^{(k)} \\ V_2^{(k)} &= Z_{22,IM}^{(k)} I_{2,IM}^{(k)} \end{aligned} \tag{27}$$

$$CUF_{IM}^{(k)} = VUF^{(k)} \frac{Z_{11,IM}^{(k)}}{Z_{22,IM}^{(k)}} \tag{28}$$

where $Z_{ii,IM}^{(k)}$ is the impedance of the ith sequence in the motor load of the kth bus, $I_{i,IM}^{(k)}$ is the ith sequence current in motor load of the kth bus, and $CUF_{i,IM}^{(k)}$ is current unbalance factor of motor load at the kth bus. Therefore, regarding Eqs. (28), (16) and (18), the effect between the (k – 1)th and the kth buses and the load effect of the mth bus are extracted in Eqs. (29) and (30) as below. voltage unbalance factor of the line between the (k – 1)th and the kth buses when the motor load exists in the *m*th bus is given by

$$VUF^{line,(k-1,k)} = \frac{f(m)}{Z_{11,IM}^{(m)}} Z_{21}^{(k-1,k)} \tag{29}$$

The contribution of voltage unbalance of motor load in the mth bus on voltage unbalance of the kth bus is expressed as

$$VUF^{load(k)} = -\frac{f(m)}{Z_{22,IM}^{(m)}} VUF^{(k)} Z_{L22}^{(0,k)} \tag{30}$$

2.3.3. Combined load

If the load connected to the kth bus is a combination of the motor load and the passive load, the combined load model is used. In order to analyze this state considering the value of $CUF_{mixload}^{(k)}$, which is calculated by Eq. (31), the contribution of voltage unbalance to the effect of the combined load is computed by Eq. (32).

$$CUF_{mixload}^{(k)} = Z_{11mix}^{(k)} \left[\frac{CUF_{PL}^{(k)}}{Z_{11,PL}^{(k)}} + \frac{CUF_{IM}^{(k)}}{Z_{11,IM}^{(k)}} \right] \tag{31}$$

where $CUF_{mixload}^{(k)}$ is the current unbalance factor of the combined load at the kth bus and $Z_{11,mix}^{(k)}$ is the positive sequence impedance in the combined load at the kth bus.

$$VUF^{load(k)} = -\frac{f(k)}{Z_{11,PL}^{(k)}} CUF_{PL}^{(k)} Z_{L22}^{(0,k)} - \frac{f(k)}{Z_{11,IM}^{(k)}} CUF_{IM}^{(k)} Z_{L22}^{(0,k)} \tag{32}$$

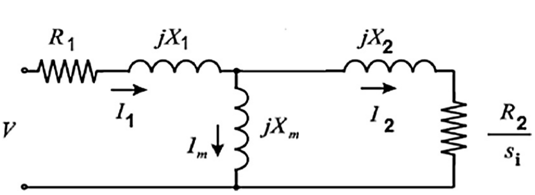

Fig. 2. IEEE model of induction motor.

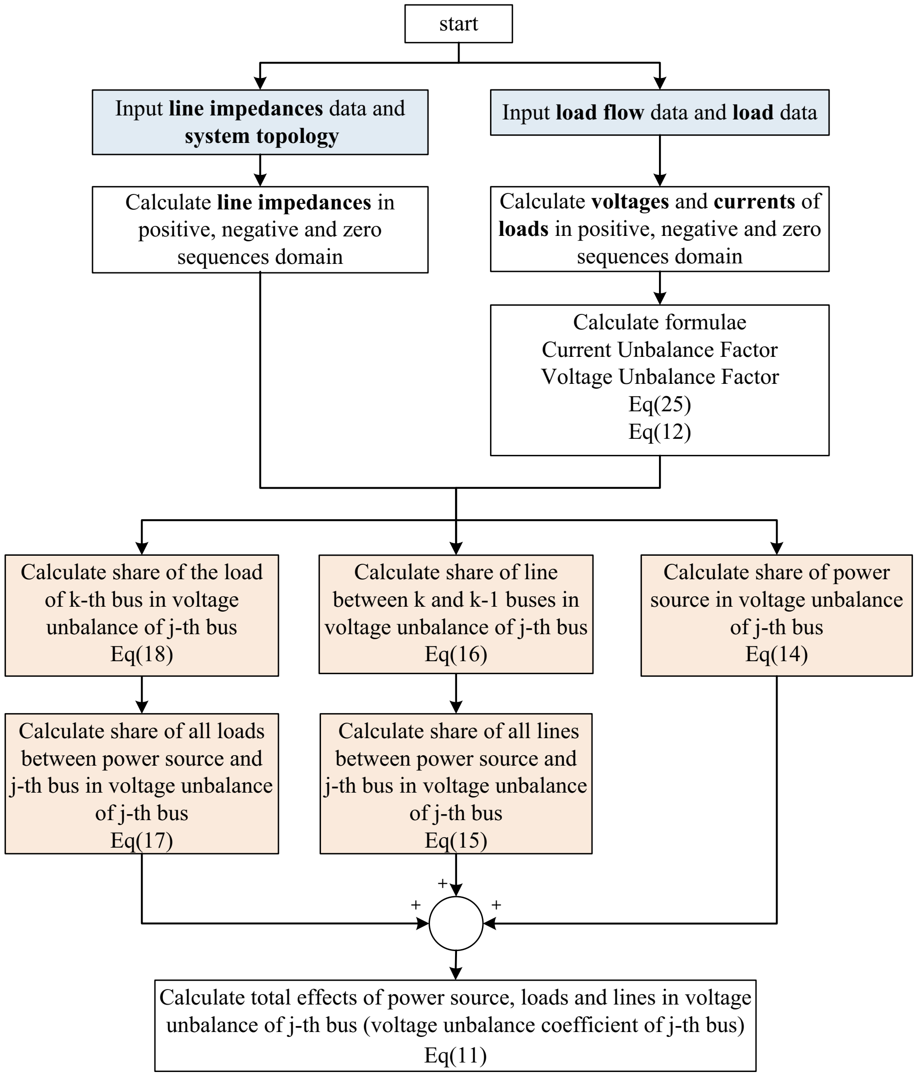

Fig. 3. Overall flowchart of determining the contribution of different effective factors to voltage unbalance coefficient generated on bus *j* in a radial power system.

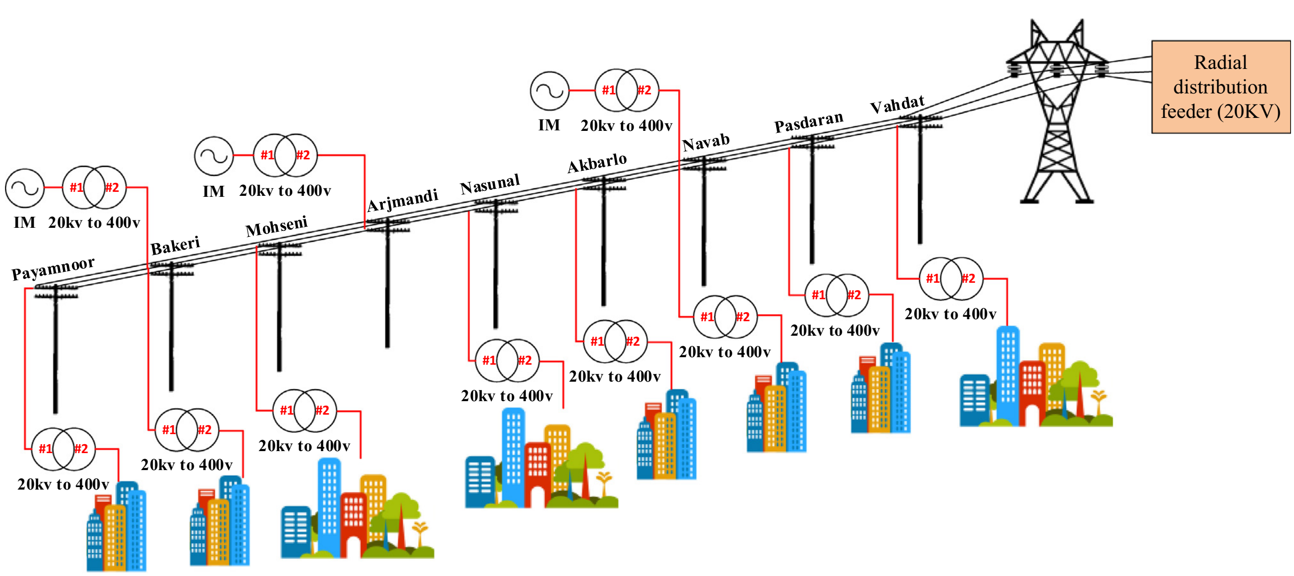

Fig. 4. 20 kV distribution feeder studied in the present research.

Table 1
Definition of test system parameters.

Parameters	Definition of test system parameters
Vuf(vahdat)%	Voltage unbalance factor created in the **vahdat** bus
Vuf(pasdaran)%	Voltage unbalance factor created in the **pasdaran** bus
Vuf(navab)%	Voltage unbalance factor created in the **navab** bus
Vuf(akbarlo)%	Voltage unbalance factor created in the **akbarlo** bus
Vuf(nasiunal)%	Voltage unbalance factor created in the **nasiunal** bus
Vuf(arjmandi)%	Voltage unbalance factor created in the **arjmand** bus
Vuf(mohseni)%	Voltage unbalance factor created in the **mohseni** bus
Vuf(bakeri)%	Voltage unbalance factor created in the **bakeri** bus
Vuf(payamenoor)%	Voltage unbalance factor created in the **payamenoor** bus
Vuf%Load(1)-vahdat	Contribution of passive load connected to the **Vahdat** bus, in voltage unbalance created at the test system buses
Vuf%$_{Load(2)-pasdaran}$	Contribution of passive load connected to the **pasdaran** bus, in voltage unbalance created at the test system buses
Vuf%$_{Load(3)-navab}$	Contribution of passive load connected to the **navab** bus, in voltage unbalance created at the test system buses
Vuf%$_{Load(4)-akbarlo}$	Contribution of passive load connected to the **akbarlo** bus., in voltage unbalance created at the test system buses
Vuf%$_{Load(5)-nasunal}$	Contribution of passive load connected to the **nasiunal** bus, in voltage unbalance created at the test system buses
Vuf%$_{Load(6)-mohseni}$	Contribution of passive load connected to the **mohseni** bus**,** in voltage unbalance created at the test system buses
Vuf%Load(7)-bakeri	Contribution of passive load connected to the **bakeri** bus, in voltage unbalance created at the test system buses
Vuf%Load(8)-payamnoor	Contribution of passive load connected to the **payamenoor** bus, in voltage unbalance created at the test system buses
Vuf%motor(1)-navab	Contribution of Active load connected to the **navab** bus, in voltage unbalance created at the test system buses
Vuf%motor(2)-arjmand	Contribution of Active load connected to the **arjmand** bus, in voltage unbalance created at the test system buses
Vuf%motor(3)-bakeri	Contribution of Active load connected to the **bakeri** bus, in voltage unbalance created at the test system buses
Vuf%Line(1):	Contribution of lines Between **source** and **Vahdat** buses**,** in voltage unbalance created at the test system
Vuf%Line(2)	Contribution of lines Between **Vahdat** and **pasdaran** buses, in voltage unbalance created at the test system
Vuf%Line(3)	Contribution of lines Between **pasdaran** and **navab** buses, in voltage unbalance created at the test system
Vuf%Line(4)	Contribution of lines Between **navab** and **akbarlo** buses, in voltage unbalance created at the test system
Vuf%Line(5)	Contribution of lines Between **akbarlo** and **nasiunal** buses, in voltage unbalance created at the test system
Vuf%Line(6)	Contribution of lines Between **nasiunal** and $_{\mathbf{arjmand}}$ buses, in voltage unbalance created at the test system
Vuf%Line(7)	Contribution of lines Between **arjmand** and **mohseni** buses, in voltage unbalance created at the test system
Vuf%Line(8)	Contribution of lines Between **mohseni** and **bakeri** buses, in voltage unbalance created at the test system
Vuf%Line(9)	Contribution of lines Between **bakeri** and **payamenoor** buses, in voltage unbalance created at the test system

where the first and second terms refer to passive load and motor load effects, respectively. In this case, analysis of passive and motor loads follows the same approach similar to that of passive and motor load model mentioned in the previous section. Line voltage unbalance factor between the (k − 1)th and the kth buses when combined load exists in the mth bus is expressed as:

$$VUF^{line(k-1,k)} = -\frac{f(m)}{Z^{(m)}_{11,PL}} Z^{(k-1,k)}_{21} - \frac{f(m)}{Z^{(m)}_{11,IM}} Z^{(k-1,k)}_{21} \tag{33}$$

Table 2
Results of determining contribution of effective factors using presented formulations and Vuf $^{(Source)}$%=1.5∡26.87.

		Vuf$^{(vahdat)}$%	Vuf$^{(pasdaran)}$%	Vuf$^{(navab)}$%	Vuf$^{(akbarlo)}$%	Vuf$^{(nasiunal)}$%	Vuf$^{(arjmandi)}$%	Vuf$^{(mohseni)}$%	Vuf$^{(bakeri)}$%	Vuf$^{(payamenoor)}$%
1	**Vuf%$_{Load(1)-vahdat}$**	0.00084∡156.43	0.00084∡156.48	0.00084∡156.51	0.00085∡156.54	0.00084∡156.55	0.00085∡156.55	0.00084∡156.56	0.00084∡156.56	0.00084∡156.56
2	**Vuf%$_{Load(2)-pasdaran}$**	0.00017∡−103.49	0.00038∡−103.44	0.00038∡−103.37	0.00038∡−103.66	0.00038∡−103.49	0.00038∡−103.3	0.00038∡−103.3	0.00038∡−103.3	0.00038∡−103.32
3	**Vuf%$_{Load(3)-navab}$**	0.00045∡−78.25	0.00098∡−78.2	0.00132∡−78.1	0.00133∡−78	0.00133∡−78.13	0.00133∡−78.11	0.00132∡−78.08	0.00132∡−78.7	0.00132∡−78.11
4	**Vuf%$_{motor(1)-navab}$**	0.00055∡−177.46	0.0011∡−177.4	0.00161∡−177.3	0.00161∡−177.3	0.00161∡−177.3	0.00162∡−177.3	0.001616∡−177.32	0.001617∡−177.3	0.001617∡−177.3
5	**Vuf%$_{Load(4)-akbarlo}$**	0.0047∡105.3	0.01∡105.3	0.0139∡105.39	0.0178∡105.4	0.0179∡105.43	0.01791∡105.44	0.01791∡105.44	0.01792∡105.44	0.01792∡105.45
6	**Vuf%$_{Load(5)-nasunal}$**	0.0035∡128.04	0.0077∡128.09	0.01053∡128.12	0.0134∡128.151	0.0168∡125.03	0.0168∡125.01	0.01683∡125.04	0.01683∡125.01	0.01684∡125.01
7	**Vuf%$_{motor(2)-arjmand}$**	0.0010∡−175.86	0.0022∡−175.81	0.00308∡−175.7	0.0039∡−175.7	0.00492∡−178.87	0.0057∡179.34	0.0057∡179.34	0.00573∡179.34	0.00573∡179.35
8	**Vuf%$_{Load(6)-mohseni}$**	0.00041∡−110.6	0.0009∡−110.5	0.00122∡−110.52	0.00156∡−110.49	0.00195∡−113.61	0.00226∡−115.39	0.0025∡−116.6	0.00257∡−116.68	0.00257∡−116.67
9	**Vuf%$_{Load(7)-bakeri}$**	0.0025∡53.72	0.0055∡53.77	0.00746∡53.8	0.00953∡53.83	0.0119∡50.71	0.01385∡48.75	0.0157∡47.64	0.0189∡43.94	0.0189∡43.95
10	**Vuf%$_{motor(3)-bakeri}$**	0.0009∡−168.9	0.0019∡ − 168.9	0.0026∡−168.91	0.0033∡−168.8	0.00419∡−171.96	0.0048∡−173.74	0.0055∡−177.02	0.0066∡−178.7	0.0066∡−178.72
11	**Vuf%$_{Load(8)-payam}$**	0.00051∡−174.6	0.00112∡−174.6	0.00154∡−174.77	0.00196∡−174.5	0.00245∡−177.7	0.00285∡−179.4	0.0032∡179.23	0.0039∡175.5	0.0048∡172.15
12	**Vuf%$_{Line(1)}$**	0.0124∡−171.4	0.0127∡−171.41	0.0127∡−171.3	0.0128∡−171.3	0.0128∡−171.34	0.01288∡−171.33	0.01282∡−171.3	0.0128∡−171.33	0.0128∡−171.33
13	**Vuf%$_{Line(2)}$**	0	0.0136∡−171.4	0.0136∡−171.4	0.0136∡−171.34	0.0136∡−171.49	0.0136∡−171.32	0.01369∡−171.3	0.01366∡−171.32	0.0136∡−171.32
14	**Vuf%$_{Line(3)}$**	0	0	0.0079∡−171.5	0.0079∡−170.5	0.0079∡−171.52	0.0079∡−171.51	0.0079∡−171.5	0.00794∡−171.51	0.0079∡−171.51
15	**Vuf%$_{Line(4)}$**	0	0	0	0.0076∡−171.7	0.0076∡−170.7	0.0076∡−171.51	0.0076∡−170.7	0.0076∡−170.7	0.0076∡−170.72
16	**Vuf%$_{Line(5)}$**	0	0	0	0	0.0050∡−173.1	0.00508∡−173.118	0.005∡−173.11	0.005∡−173.11	0.005∡−173.11
17	**Vuf%$_{Line(6)}$**	0	0	0	0	0	0.00275∡−174.82	0.0027∡−174.8	0.00274∡−174.84	0.00273∡−174.82
18	**Vuf%$_{Line(7)}$**	0	0	0	0	0	0	0.0025∡−174.8	0.00259∡−174.85	0.00259∡−174.85
19	**Vuf%$_{Line(8)}$**	0	0	0	0	0	0	0	0.00175∡−175.5	0.0017∡−175.52
20	**Vuf%$_{Line(9)}$**	0	0	0	0	0	0	0	0	0.00037∡−166.7
21	**Vuf ^{s} %**	1.502∡26.92	1.505∡26.97	1.5051∡26.97	1.508∡27.03	1.509∡27.04	1.510∡27.05	1.511∡27.05	1.512∡27.06	1.512∡27.06
22	**Vuf ^{cal} %**	1.488∡27.46	1.477∡28.12	1.471∡28.53	1.465∡29.01	1.463∡29.17	1.462∡29.37	1.461∡29.49	1.462∡29.53	1.46∡29.59
23	**Vuf ^{measur} %**	1.488∡27.49	1.477∡28.15	1.471∡28.57	1.465∡29.02	1.463∡29.13	1.462∡29.40	1.461∡29.48	1.462∡29.55	1.46∡29.59

Finally, the flowchart of analyzing the problem of determining the contribution of different effective factors' contributions on voltage unbalance generated on each bus in the radial system is given in Fig. 3 in the following.

3. Case study and simulation results

In this study, a real 20 kV feeder consisting of 9 jumper loads (20 kV/400 V) located in Iran was analyzed (see Fig. 4). These were fed through 6 passive loads, 1 motor load (induction), and 2 combined loads. All the required information for analyzing the problem were received and recorded simultaneously by data logger and smart meters through logic electricity from distribution and beyond distribution unit instantaneously. System data are presented in Appendix A.

The results obtained from the formulations are presented in Table 2. They can be used to determine the effective factors and the unbalanced power flow via DIgSILENT PowerFactory software. At the investigation site, the earth ground resistance was measured to be 100 Ω m, the network frequency was 50 Hz, and post-output voltage unbalance factor was found to be $1.5\measuredangle 26.87\%$

In rows 1–11 of Table 1, the results indicating the effects of loads are obtained using Eq. (24) (in the passive load mode), Eq. (30) (in the motoring load mode), or Eq. (32) (in the mixed load mode), as the effects of motoring and passive loads are separated in this case study. The results in rows 12–20, representing the effects of lines with regard to the type of system loads, were obtained using from Eq. (23) (in the passive load mode), Eq. (29) (in the motoring load mode), or Eq. (33) (in the mixed load mode), separating the effects of motoring and passive loads. To indicate the effect of source on the considered bus, the results in row 21 were obtained using Eq. (14). The results in row 22 indicate the total calculated voltage unbalance using Eq. (11) regarding the nature and the type of loads. The results given in row 23 are the values of total voltage unbalance at each bus, which were obtained using unbalanced power flow in DIgSILENT PowerFactory software. Comparing the results given in rows 22 and 23, it can be observe that the extracted equations in this paper are completely correct and precise. According to Iran Generation and Transmission Management Organization standard, the allowable voltage unbalance percentage in distribution systems is 2%. The measured values using data loggers are under the control of the Electric Power Distribution Management Company. Based on the obtained results using DIgSILENT in row 23 and the measured results using formulation analysis of the paper in row 22, it is observed that from a voltage unbalance point of view and compared to the defined standards in Iran, this network is in a properly acceptable range [31].

3.1. Unbalanced voltage in passive loads

Among seven passive loads (i.e. Vahdat, Pasdaran, Navab, Akbarlo, Nasunal, Bakeri and Payamnoor), the polar graph of unbalanced voltage in passive loads at Bakeri bus is illustrated in Fig. 5a. Based on Fig. 5a, it can be claimed that the highest value for Vuf is obtained at the Bakeri bus, while the lowest value is seen in Payamnoor bus. This event originates from the distance between the buses. Indeed, Bakeri bus is located at maximum and minimum distances from Vahdat and Payamnoor buses, respectively.

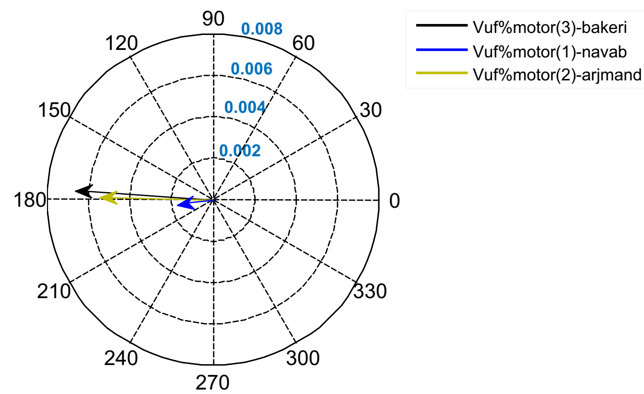

Fig. 5b. Polar graph of (induction motor) at Bakeri bus.

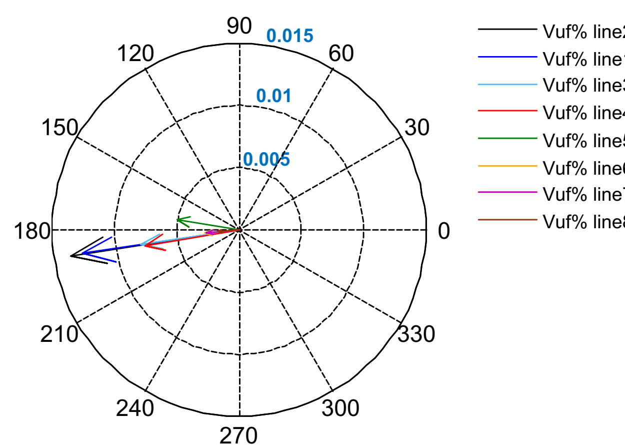

Fig. 5c. Polar graph of unbalanced voltage of lines at Bakeri bus.

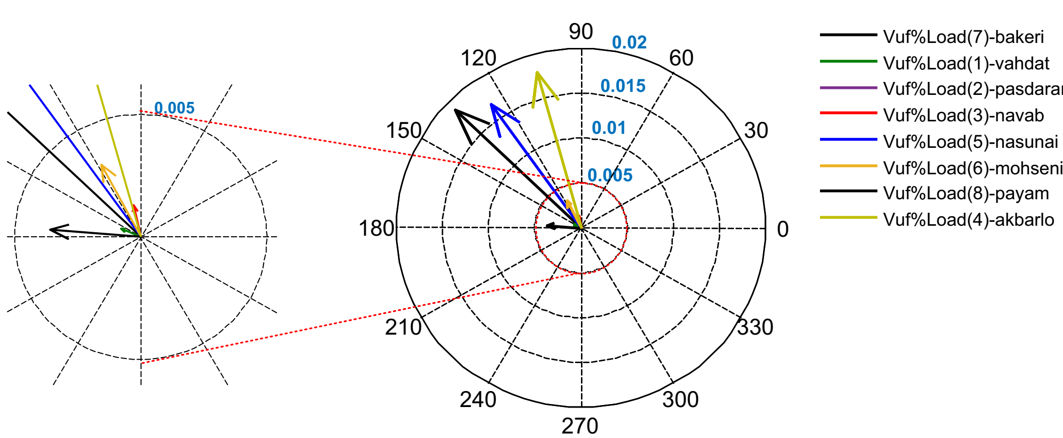

Fig. 5a. Polar graph of unbalanced voltage in passive loads at Bakeri bus.

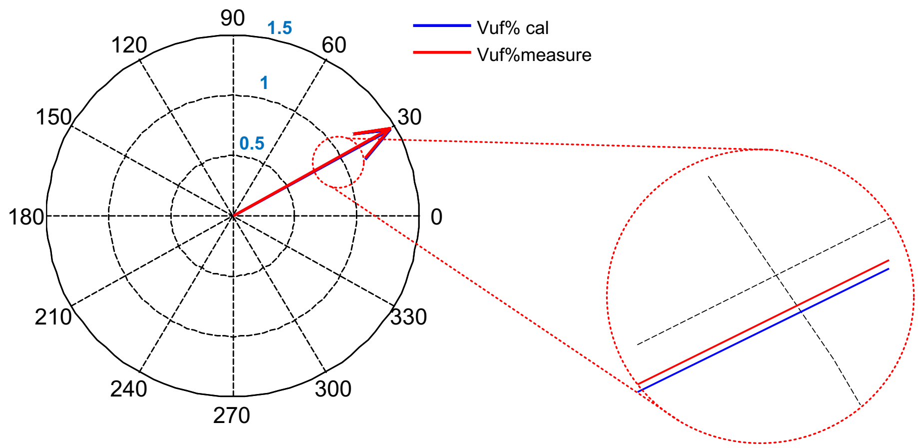

Fig. 5d. Polar graph of the calculated and measured results at Bakeri bus.

3.2. Unbalanced voltage in active load (induction motor)

The test system has three induction motors installed at Bakeri, Navab and Arjmand buses. Fig. 5b shows polar graph of unbalanced voltage of active load (induction motor) at Bakeri bus. Considering Fig. 5b, it is clear that the largest differences occur in Arjmand bus. Navab bus presents the lowest values of unbalanced voltage of active load. Note that no phase difference is observed across the values.

3.3. Unbalanced voltage of the lines

In the studied system, there were nine lines; unbalanced voltage effect at Bakeri bus is illustrated in Fig. 5c. Based on Fig. 5c, Line 1 (connecting Bakeri bus to Payamnoor bus) and Line 7 (connecting Pasdaran bus to Vahdat bus) present the minimum and maximum values, respectively.

3.4. Comparing the calculated and measured results

Finally, polar graph of the effect of the obtained values for unbalanced voltage values via calculation and measurement at Bakeri bus are shown in Fig. 5d. From the figure, it can be said that the difference is very minor and negligible amounts.

4. Conclusion

Managing voltage unbalance in power systems is one of the most fundamental and important issues in power quality management. Therefore, determining the contribution of effective factors on the produced voltage unbalance at the place of load is vital. This paper shows that the produced voltage unbalance at the place of load in a radial distribution system is not only because of the applied load at that bus, but is also due to all the existing loads within the system, the transfer power lines from the source to a given bus, and the voltage supply source. Considering the presented formula in this paper, percentages of effectiveness of all the mentioned factors can precisely be calculated. Then, based on the obtained results, it is possible to control and measure the voltage unbalance at the place of loads in radial distribution systems. As it can be seen in Section2 2 (Basic principle and the proposed method), results of determining the contribution of effective factors using the formulation extracted in Section3 3 (Case study and Simulation results) on 20 kV radial distribution system in Iran are tested and verified, and the results and accuracy of effects of all loads, lines and source on voltage unbalance factor of Bakeri bus are studied case by case, which verifies the accuracy of determining the contribution of all effective factors. Such structures can open new research opportunities in the field of statistical techniques existing in international standards with regard to voltage unbalance management.

Appendix A

Data of the test system

Passive load, induction motor, and line information are presented in Tables A1–A3, respectively.

Employing the date given in Table A3 and Karson's calculation rule for calculating the self-Impedance and mutual–Impedance, the method of calculating the elements of line impedance matrix is described as below. In 1926, John Karson by published a paper proposed and proposed a method for calculating the self-impedance and mutual-impedance of the lines. According to his calculations, the self-impedance and mutual-impedance phases are obtained from Eqs. (A.1) and (A.2) [32].

$$Z_{ii} = r_i + 0.00158836F + j0.00202237F\left[Ln\frac{1}{GMR_i} + 7.6786 + \frac{1}{2}Ln\frac{\rho}{F}\right] \quad (\Omega/\text{mile}) \tag{A.1}$$

$$Z_{ij} = 0.00158836F + j0.00202237F\left[Ln\frac{1}{D_{ij}} + 7.6786 + \frac{1}{2}Ln\frac{\rho}{F}\right] \quad (\Omega/\text{mile}) \tag{A.2}$$

where frequency is in Hz, the earth resistance is in Ω m, geometric average radial and the distribution between the phases in ft, and AC resistance in is Ω/mile

Considering the equations above, impedance matrix of the lines is given as Eq. (A.3).

$$[Z_{abc}]_{3*3} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \tag{A.3}$$

And, finally, the impedance matrix of lines in sequential systems is extracted as in Eq. (A.4).

Table A1
Passive load information.

	Passive load information							Transformator information					
	Name	S_a (KVA)	S_b (KVA)	S_c (KVA)	Cosφ_a	Cosφb	Cosφc	V_1/V_2	S_n (KVA)	Connection	I_{noload}%	U_k%	P_{noload})w(
1	**vahdat**	51.53	59.15	62.73	0.959	0.949	0.948	20/0.4	250	DYn5	2.1	6	610
2	**pasdaran**	75.87	71.9	65.19	0.941	0.981	0.954	20/0.4	315	DYn5	2	6	500
3	**navab**	55.87	53. 21	51.112	0.941	0.971	0.971	20/0.4	250	DYn5	1.6	6	700
4	**akbarlo**	111.21	181.74	117.74	0.961	0.982	0.991	20/0.4	630	DYn5	1	5	900
5	**nasiunal**	101.12	141. 11	134.211	0.971	0.948	0.945	20/0.4	630	DYn5	1	5	900
6	**mohseni**	101.115	102.11	105.21	0.931	0.931	0.945	20/0.4	500	DYn5	1.6	6	900
7	**bakeri**	118.1	125.121	109.213	0.98	0.942	0.951	20/0.4	500	DYn5	1.5	5	870
8	**Payame noor**	20.24	21.31	26.11	0.981	0.981	0.96	20/0.4	100	DYn5	2.6	4	320

Table A2
Induction motor information.

Name	V_n (v)	Induction motor information															Transformator information	
		P (kw)	Cosφ	Efficiency%	F (Hz)	Speed(rpm)	Pole	R_s (Ω)	X_s (Ω)	R_r (Ω)	X_r (Ω)	X_m (Ω)	$V_2/V_{1\ (kv)}$	S_n (KVA)	connection	I_{noload}%	U_k%	P_{noload} (w)
navab	400380/660	75	0.96	0.97	50	1480	4	0.035	0.158	0.11	0.158	5.34	20/0.4	200	DYn5	1.2	6	300
arjmandi	400380/660	75	0.96	0.97	50	1480	4	0.035	0.158	0.11	0.158	5.34	20/0.4	100	DYn5	1.11	4	240
bakeri	400380/660	45	0.92	0.96	50	1480	4	0.066	0.87	0.165	0.069	6.09	20/0.4	100	DYn5	1	3	240

Table A3
Line information.

	Conductive wire	Sectional area (mm^2)	Length (km)	Radius (m)	Ac resistance $\frac{\Omega}{km}$	Number of Strings		Rated current (A)	Rated active power (MW)	D_{ab} (m)	D_{ac} (m)	D_{bc} (m)	Arrangement	Start line	End line	Voltage regulation of the line
						Steel	Aluminum									
Line(1)	ACSR (Dog)	120	0.815	0.00412	0.27	1	6	278	8.6	1.15	2.3	1.15	Horizontal	post	vahdat	$0.00187\measuredangle 27.97$
Line(2)	ACSR (Dog)	120	0.95	0.00412	0.27	1	6	278	8.6	1.15	2.3	1.15	Horizontal	vahdat	pasdaran	$0.00194\measuredangle 28.97$
Line(3)	ACSR (Dog)	120	0.62	0.00412	0.27	1	6	278	8.6	1.15	2.3	1.15	Horizontal	pasdaran	navab	$0.00114\measuredangle 28.47$
Line(4)	ACSR (Dog)	120	0.662	0.00412	0.27	1	6	278	8.6	1.15	2.3	1.15	Horizontal	navab	akbarlo	$0.00113\measuredangle 28.00$
Line(5)	ACSR (Mink)	70	0.6	0.00314	0.45	1	6	255	7.9	0.7	1.4	0.7	Horizontal	akbarlo	nasunal	$0.00092\measuredangle 11.87$
Line(6)	ACSR (Mink)	70	0.485	0.00314	0.45	1	6	255	7.9	0.7	1.4	0.7	Horizontal	nasunal	arjmandi	$0.00051\measuredangle 9.45$
Line(7)	ACSR(Mink)	70	0.465	0.00314	0.45	1	6	255	7.9	0.7	1.4	0.7	Horizontal	arjmandi	mohseni	$0.00051\measuredangle 10.79$
Line(8)	ACSR (Fox)	35	0.545	0.00222	0.78	1	6	147	4.5	0.7	1.4	0.7	Horizontal	mohseni	bakeri	$0.0005\measuredangle 0.59$
Line(9)	ACSR (Fox)	35	0.74	0.00222	0.78	1	6	147	4.5	0.7	1.4	0.7	Horizontal	bakeri	payamenoor	$0.0001\measuredangle 48.85$

$$[Z_{012}]_{3*3} = T[Z_{abc}]T^{-1} = \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \quad \text{(A.4)}$$

where T is the Fortesque matrix.

Reference

[1] Hernández JC, Ortega MJ, De la Cruz J, Vera D. Guidelines for the technical assessment of harmonic, flicker and unbalance emission limits for PV-distributed generation. Electr Power Syst Res 2011;81:1247–57.
[2] Huat Chua K, Wong J, Lim YS, Taylor P, Morris E, Morris S. Mitigation of voltage unbalance in low voltage distribution network with high level of photovoltaic system. Energy Proc 2011;12:495–501.
[3] Meersman B, Renders B, Degroote L, Vandoorn T, Vandevelde L. Three-phase inverter-connected DG-units and voltage unbalance. Electr Power Syst Res 2011;81:899–906.
[4] Shahnia F, Ghosh A, Ledwich G, Zare F. Voltage unbalance improvement in low voltage residential feeders with rooftop PVs using custom power devices. Electr Power Energy Syst 2014;55:362–77.
[5] Ruiz-Rodriguez FJ, Hernández JC, Jurado F. Voltage unbalance assessment in secondary radial distribution networks with single-phase photovoltaic systems. Electr Power Energy Syst 2015;64:646–54.
[6] Savio A, Bignucolo F, Caldon R. Contribution of MV static distributed generation to voltage unbalance mitigation. In: AEIT international annual conference; 2016. p. 1–6.
[7] Guo F, Wen C, Mao J, Chen J, Song YD. Distributed cooperative secondary control for voltage unbalance compensation in an islanded micro grid. IEEE Trans Ind Inform 2015;11(5):1078–88.
[8] Hu J, He Y, Xu L. Improved rotor current control of wind turbine driven doubly-fed induction generators during network voltage unbalance. Electr Power Syst Res 2010;80:847–56.
[9] Gnacinski P. Derating of an induction machine under voltage unbalance combined with over or undervoltages. Energy Convers Manage 2009;50:1101–7.
[10] Donolo P, Bossio G, Angelo CD. Analysis of voltage unbalance effects on induction motors with open and closed slots. Energy Convers Manage 2011;52:2024–30.
[11] Phan VT, Logenthiran T, Woo WL, Atkinson D, Pickert V. Analysis and compensation of voltage unbalance of a DFIG using predictive rotor current control. Electr Power Energy Syst 2016;75:8–18.
[12] Jayatunga U, Perera S, Ciufo P, Agalgaonkar AP. A refined general summation law for VU emission assessment in radial networks. Electr Power Energy Syst 2015;73:329–39.
[13] Garcia DC, Filho ALF, Oliveira MAG, Fernandes OA, Nascimento FAD. Voltage unbalance numerical evaluation and minimization. Electr Power Syst Res 2009;79:1441–5.
[14] Oliveira JPJd, Filho ALF, Angarita JAC. The proposal of a new aggregation time interval for calculating the voltage unbalance factor. Electr Power Syst Res 2012;91:71–7.
[15] Gwon GH, Kim CH, Oh YS, Noh CH, Jung TH, Han J. Mitigation of voltage unbalance by using static load transfer switch in bipolar low voltage DC distribution system. Electr Power Energy Syst 2017;90:158–67.
[16] Tavakoli Bina M, Kashefi A. Three-phase unbalance of distribution systems: complementary analysis and experimental case study. Electr Power Energy Syst 2011;33:817–26.
[17] Wen H, Cheng D, Teng Z, Guo S, Li F. Approximate algorithm for fast calculating voltage unbalance factor of three-phase power system. IEEE Trans Ind Inform 2014;10(3):1799–805.
[18] Jayatunga U, Perera S, Ciufo P, Agalgaonkar AP. Deterministic methodologies for the quantification of voltage unbalance propagation in radial and interconnected networks. IET Gener Transm Distrib 2015;9(11):1069–76.
[19] Schwanz D, Möller F, Rönnberg SK, Meyer J, Bollen MHJ. Stochastic assessment of voltage unbalance due to single-phase-connected solar power. IEEE Trans Power Deliv 2017;32(2):852–61.
[20] Seiphetlho TE, Rens APJ. On the assessment of voltage unbalance. In: 2010 14th Int conf harmonics quality power. p. 1–6.
[21] Renner H. Voltage unbalance emission assessment. Electr Power Qual Supply Reliab 2010;16(18):43–8.
[22] Paranavithana P, Perera S. Location of sources of voltage unbalance in an interconnected network. Presented at the IEEE power energy soc gen meeting, Calgary 2009, AB, Canada; 2009.
[23] Paranavithana P, Perera S, Sutanto D, Koch R. A systematic approach towards evaluating VU problem in interconnected sub-transmission networks: separation of contribution by lines, loads and mitigation. In: 2008 13th Int conf. harmonics qual power, Wollongong, Australia; 2008.
[24] Jayatunga U, Perera S, Ciufo P, Agalgaonkar AP. Voltage unbalance emission assessment in interconnected power systems. IEEE Trans Power Deliv 2013;28(4):2383–93.
[25] Jayatunga U, Perera S, Ciufo P. Voltage unbalance emission assessment in radial power systems algorithms for characterizing measured three-phase unbalanced voltage dips. IEEE Trans Power Deliv 2012;27(3):1653–61.
[26] Chen TH, Yang CH, Yang NC. Examination of the definitions of voltage unbalance. Electr Power Energy Syst 2013;49:380–5.

[27] George S, Agarwal V. A DSP-based control algorithm for series active filter for optimized compensation under nonsinusoidal and unbalanced voltage conditions. IEEE Trans Power Deliv 2006;22(1):302–10.
[28] Araujo LR, Penido DRR, Carneiro S, Pereira JLR. A three-phase optimal power-flow algorithm to mitigate voltage unbalance. IEEE Trans Power Deliv 2013;28(4):2394–402.
[29] Bollen MHJ. Algorithms for characterizing measured three-phase unbalanced voltage dips. IEEE Trans Power Deliv 2003;18(3):937–44.
[30] Sandhu KS, Chaudhary V. MATLAB & PSIM based analysis of three-phase induction motor operation with unbalanced supply system. In: 2010 Supply reliability conf, vol. 16(18). p. 43–8.
[31] Iran Electrical Industry Standard. Iran power generation and transmission management organization, part seven: quality of power delivery to different consumers; 2002.
[32] Distribution System Modeling and Analysis, Kersting WH; 2002. p. 85.