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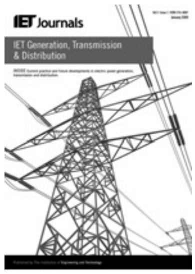

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Weighted least error squares based variable window phasor estimator for distance relaying application

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Abstract: This study presents an adaptive data window algorithm for fast and accurate signal phasor estimation. The proposed approach is based on weighted least error squares algorithm as well as variable data window estimation technique. An adaptive model is used for the input signal during the two stages. The first stage is considered to accelerate the relay first zone operation. This is done through applying a forgetting factor in the cost function computation, by which the effect of older samples on the estimation is reduced to decrease the initial inertia of the phasor estimator. The second stage starts one cycle after occurrence of the fault. This algorithm is used to improve the estimation accuracy and helps to obtain better estimation for the fault location. To verify the performance of the proposed algorithm, various simulation studies are carried out. The algorithm is also evaluated using some field data recorded from a high-voltage transmission system. The obtained results confirm accurate, fast and reliable response of the proposed algorithm. It not only accelerates the relay operation but also improves the digital filters' capability of reducing the estimation error because of power system frequency deviation, harmonics, inter-harmonics and decaying dc component.

1 Introduction

Transmission line protection performs a vital role in power systems to maintain system stability and to minimise equipment damage. In the present days, with the worldwide restructuring of the electrical utility industry, the power systems are utilised at smaller safety margins and are more prone to instability. Therefore improvement in speed and accuracy of protection schemes is one of the first steps to enhance the power system security against instability [1].

In phasor-based digital relays, estimation of the fundamental phasor of the voltage and current signals has an essential role on overall performance of the protection schemes. During a system fault, the voltage and current signals might get severely distorted. Therefore fast and accurate estimation of the signals' fundamental components becomes difficult. This is because of the undesired harmonic and inter-harmonic components, decaying dc component, instrument transformers' error and power system frequency deviation [2–9].

The full-cycle discrete Fourier transform (FCDFT) is one of the most common algorithms for the estimation of signal fundamental component in the relaying applications [10]. Some methods have been proposed to improve the performance of this algorithm by removing the decaying dc component [3–5] and reducing leakage error because of power system frequency deviation [6–8]. A full-cycle algorithm requires one period of the fundamental frequency to completely enter to the post-fault samples window. As a result, its output would not converge to the final value before one period.

It is well known that the speed and accuracy of the phasor estimation algorithms are in compromise. The filter ability to suppress undesired harmonics is relative to its data window length. A long data window provides more accurate results but the response will be delayed. Whereas, a short data window provides faster response but its output might become oscillatory in the case of distorted input signals [11].

In order to reduce the initial inertia of the phasor estimation algorithms, variable-length data window algorithms have been proposed. These algorithms initiate with a short length data window and progressively increase the window length up to a full-cycle window. The noise immunity is gradually improved as the data window length increases. This approach overcomes the speed and accuracy compromise problem, which cannot be solved by using the fixed data window algorithms [11, 12].

This paper presents an adaptive data window algorithm for fast and accurate estimation fault signals' fundamental phasor. The proposed technique employs the weighted least error squares (LES) algorithm to obtain the digital filter coefficients. The weight function is adjusted progressively in two stages to achieve the optimal performance of the phasor estimator for the distance relaying application. Moreover, an adaptive model is proposed for the input signal, in which the components of the signal model are completed progressively. The proposed phasor estimation algorithm has the following advantages:

1. During the first stage, a forgetting factor is proposed to weight the effect of older signal samples with smaller values in the cost function computation. Thereby, initial inertia of

the obtained digital filters is reduced, which helps to achieve faster dynamic response in comparison with the response of the conventional Fourier technique. As the older samples are not completely removed from the data window, the proposed technique provides more stable response compared to the other variable-length data window algorithms, which initiate with a short length window.
2. Unlike the other variable-length data window algorithms, the proposed filters are not fixed at the full-cycle length when the number of post-fault samples exceeds one cycle. In order to improve the estimation accuracy, the final length of the data window in the second stage of the proposed algorithm covers two cycles of the fundamental frequency. This way, the decaying dc component effect is effectively diminished.
3. In the second stage, a Gaussian weight function is proposed to enhance the long-window filters' ability in removing harmonic and inter-harmonic components. The proposed weighting matrix also reduces the estimation error because of the power system frequency deviation. This technique is efficient for accurate identification of the faults occurring at the first zone boundary limit. In addition, it can be employed to accelerate the relay operation for the faults which occur at the end part of the transmission line protected in the second zone of the distance relay.

Extensive simulation studies are carried out to evaluate the performance of the proposed algorithm. Obtained results are compared with those of the full-cycle discrete Fourier transform algorithm and the variable-length data window algorithm proposed in [11]. The proposed method is evaluated further using some field data recorded from a high-voltage transmission system [13]. The results confirm accurate, fast and reliable response of the proposed algorithm.

2 Proposed adaptive algorithm

For ease of illustration, the fundamental frequency used throughout this paper is assumed to be 60 Hz and the signals are sampled at the rate of 12 samples per cycle. Nevertheless, the proposed method can be easily adopted for higher sampling rates as well.

2.1 Proposed variable-window phasor estimation algorithm

To estimate the signal fundamental phasor, the conventional LES algorithm uses a limited rectangle data window. It is also called the windowed-LES algorithm. The data window length determines the number of samples incorporated in the computations. It is well known that for a full-cycle data window, the digital filters for the extraction of fundamental phasor, obtained by the LES algorithm, are identical to the full-cycle Fourier filters [14]. The conventional LES algorithm considers the same weights for all of the elements of the error vector in the cost function computation [15, 16]. A long data window results in an accurate and smooth response but its dynamic response is slow because of the natural inertia of the algorithm. On the other hand, when the data window length is reduced, faster dynamic response would be obtained. Nevertheless, distorted fault signals may result in unstable outputs when short-window algorithms are used.

Weighted-LES method permits to consider different weights in the cost function computation. The cost function and estimation of the unknown parameters using the weighted-LES method are given by [17]

$$V = \underline{\boldsymbol{E}}^{\mathrm{T}} \underline{\boldsymbol{W}}\, \underline{\boldsymbol{E}} \tag{1}$$

$$\hat{\underline{\boldsymbol{\theta}}} = (\underline{\boldsymbol{\phi}}^{\mathrm{T}}\, \underline{\boldsymbol{W}}\, \underline{\boldsymbol{\phi}})^{-1}\, \underline{\boldsymbol{W}}\, \underline{\boldsymbol{\phi}}^{\mathrm{T}} \underline{Y} \tag{2}$$

in which $\boldsymbol{E}$ denotes the estimation error vector, $\boldsymbol{\phi}$ is the regressor matrix comprising vectors of definite variables and $\boldsymbol{W}_{d\times d}$ is the weighting matrix. The proposed phasor estimation algorithm employs the weighted-LES method to achieve the optimal performance of the phasor estimator for the distance relaying application. In the proposed algorithm, the variable data window is progressively adjusted in two stages.

2.1.1 Stage I: The first stage is considered for fast and reliable first zone operation of the distance relay. During this stage, the original data window length (d) is considered to be one cycle of the fundamental frequency and the diagonal elements of the weighting matrix are defined as follows

$$w_n = \lambda^{(N-n)^2}, \quad 1 \le n \le N \tag{3}$$

where N denotes the full-cycle data window length, w_n is the nth diagonal element of W and λ denotes the forgetting factor. The non-diagonal elements of the weighting matrix are considered to be zero.

The forgetting factor value is smaller than one and it is within the range of [0–1]. This way, the older samples are weighted with smaller values in the cost function computation and hence the samples corresponding to the more recent instants would have more effect on the parameter estimation technique. Thereby, inertia of the algorithm would be reduced, which provides faster dynamic response.

The forgetting factor has a determinative role on the effective data window length. A smaller forgetting factor implies a shorter length data window and a forgetting factor equal to one means a rectangular data window, whose length is equal to d. The older samples are used even for the short data windows, albeit with smaller effect. The advantage of the proposed method is that the older samples are not completely removed from the data window, thus resulting in a more stable response compared to the response of the conventional short window algorithms.

Fig. 1*a* illustrates adaptive adjustment of the weighting coefficients during the first stage. In the steady-state condition, a full-cycle data window is used and all of the weights are set at one. The fault is assumed to occur at the instant of $k = 0$. Once a fault occurs, the forgetting factor is reset to an initial value. Using the exponential equation in (3), the corresponding weight for each sample is calculated. The present sample is weighted with one and the older samples are weighted with smaller values. It should be noted that the data windows shown in Fig. 1*a* are not directly applied to the sampled signal, but are applied as the weights for the respective estimation errors in the cost function computation given in (1).

The forgetting factor is progressively increased in fixed steps. The maximum value of the forgetting factor is equal to one. Minimum size of the incremental steps should be chosen such that the forgetting factor value becomes one in a full-cycle interval. One cycle after the fault instant, all of the samples are weighted with the same value and the frequency responses of the obtained digital filters become

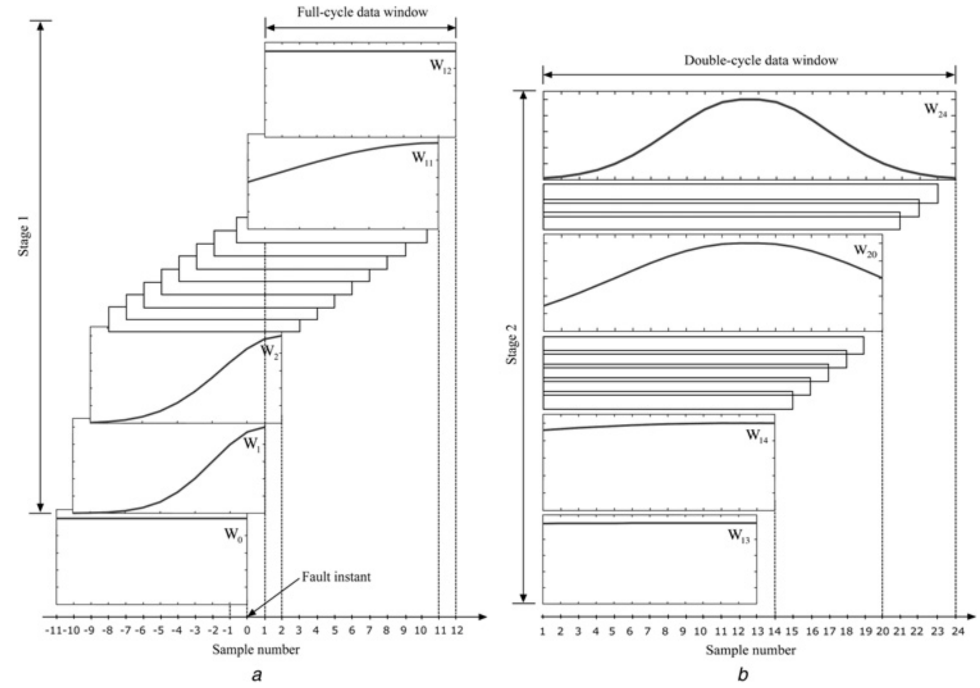

Fig. 1 *Proposed adaptive window algorithm*

a First stage
b Second stage

similar to those of the full-cycle Fourier filters. Thereafter, the data window only contains the post-fault samples.

2.1.2 Stage II: The second stage is used to increase phasor estimation accuracy by improving the digital filters characteristic against undesired harmonic and inter-harmonic components. It helps to calculate the fault location more accurately and improves identification of the faults occurring at the transmission line boundary limits. The proposed second stage starts one cycle after the fault occurrence. During this stage, the original data window length is increased progressively, as a new sample becomes available. This procedure would continue until the data window length reaches two cycles of the fundamental frequency. Thereafter, the data window length remains fixed. In order to improve the filters' ability to eliminate the undesired harmonic and inter-harmonic components, a Gaussian weight function is used in the second stage. The weighting matrix is adaptively adjusted to gradually form the Gaussian weight-function when the number of post-fault samples exceed one cycle.

Fig. 1*b* illustrates adaptive adjustment of the data window length and the corresponding weight function during the second stage. One cycle after the fault occurrence, the corresponding weighting coefficients for all of the samples present in the data window are equal to one. As a new sample becomes available, the data window length is increased and the forgetting factor is reduced by $-\Delta\lambda$. During the second stage, the decreasing steps are adjusted in a logarithmic manner. For a given data window length $M \leq 2N$, the diagonal elements of the weighing matrix are defined as follows

$$w_n = \begin{cases} \lambda^{(N-n)^2}, & 1 \leq n \leq N \\ w_{n-2(n-N)+1}, & N \leq n \leq M \end{cases} \quad (4)$$

As shown in Fig. 1*b*, when the data window length reaches two cycles, the weighting coefficients form the desired Gaussian weight function. Fig. 2 depicts the frequency responses of the two-cycle filters obtained by the proposed weighted-LES algorithm and those of the conventional one-cycle and double-cycle Fourier algorithms. As can be seen,

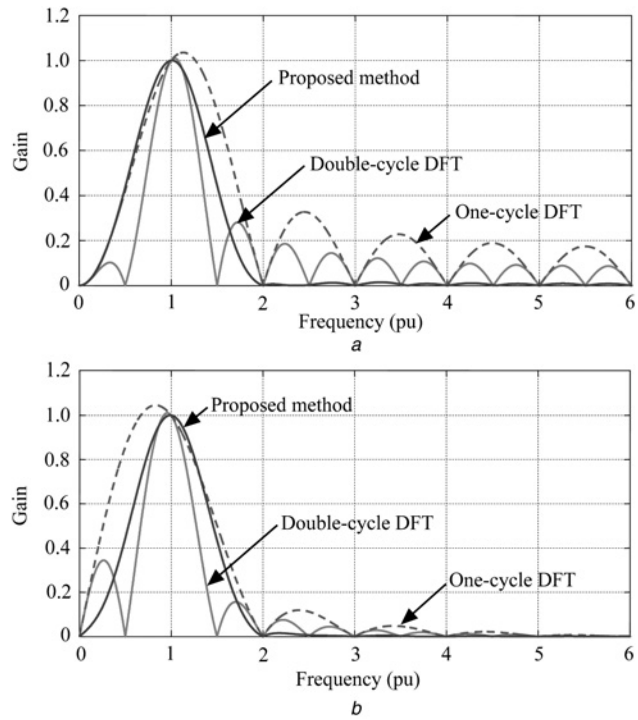

Fig. 2 *Frequency responses of the proposed digital filters, one-cycle and double-cycle Fourier filters*

a Sine filter
b Cosine filter

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the proposed digital filters provide a suitable characteristic to diminish the undesired harmonic and inter-harmonic components. In addition, the ability of the proposed filters for the elimination of integer harmonics does not depend on the nominal frequency.

Frequency responses of the conventional orthogonal double-cycle Fourier filters around the nominal frequency are not similar. Therefore any drift in system frequency results in oscillation of the estimated amplitude. Another advantage of the proposed digital filters is their similar characteristic nearby the fundamental frequency. As such, for small drifts in the power system frequency, output of both orthogonal filters shown in Figs. 2*a* and *b* would have the same deviations. It yields an oscillation free response. In the other words, the proposed method implicitly obviates the problem of spectral leakage of the DFT algorithm because of the power system frequency deviation.

Referring to Fig. 2, for changes of the signal fundamental frequency within the range of acceptable frequency drift in power systems (i.e. 58–62 Hz), the gains of the proposed orthogonal filters are almost constant. Assuming a pure fundamental frequency sinusoidal waveform, even if the signal frequency changes to 55 Hz, the corresponding error is only about 0.02 pu. However, the more important advantage of the proposed algorithm compared to the conventional algorithm is its capability to highly attenuate non-fundamental harmonics even under frequency drift from the nominal frequency. As can be seen in Fig. 2, the gains of the proposed orthogonal filters for integer harmonics are almost zero independent of the fundamental frequency drift.

Fig. 3 compares estimated amplitude of the fundamental phasor using the proposed two-cycle filters and the conventional double-cycle DFT algorithm. The input signal comprises of the fundamental and integer harmonic components up to the Nyquist frequency. The frequency drift is considered to be $\Delta f = -1$ Hz. The amplitude of all harmonic components is considered as 0.5 pu, whereas the initial amplitude of the fundamental component is 1 pu. As can be seen, the proposed method works very well despite the presence of harmonics and also fundamental frequency drift, and can provide an accurate and stable response.

2.2 Elimination of the decaying dc component

When the fault signals only contain the fundamental frequency and integer harmonic components, the full-cycle DFT algorithm is able to yield the accurate fundamental phasor after one cycle interval. However, in most cases, the fault current

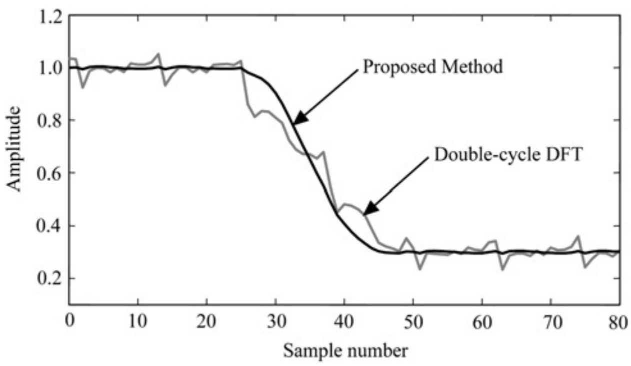

Fig. 3 *Estimated phasor amplitudes using the proposed and the conventional double-cycle fourier algorithms for a frequency drift case*

signals contain a decaying dc component as well. Since the frequency spectrum of the decaying dc component includes non-integer harmonic components, the DFT algorithm is not able to filter out the decaying dc component [4].

Several approaches have been proposed to avoid the estimation error because of the decaying dc component. Some proposed methods first remove the decaying dc component from the input signal and then calculate the exact fundamental phasor. The decaying dc component can be estimated from the input signal directly [18] or using some successive estimated phasor [5]. Further, it is also possible to calculate the error caused by the decaying dc component on the estimated phasor. Then, the fundamental phasor could be obtained by subtracting the calculated error from the estimated value [4, 19, 20]. Most of these methods require at least one cycle of the fundamental frequency plus some extra samples to extinguish the decaying dc component error. Meanwhile, they mostly do not provide a smooth response during the transient interval when variable data window algorithms are used.

The other approach is employing the LES algorithm considering the decaying dc component in the model of the input signal [1, 12]. Thereby, the obtained digital filters would be able to simultaneously remove the decaying dc component effect, and no further computation is required. For this purpose, the model components of the input signal are as follows

$$x(t) = A_0\,\mathrm{e}^{-t/\tau} + \sum_{n=1}^{N/2} A_n \cos(n\omega_0 t + \theta_n) \qquad (5)$$

Exponentially decaying component can be estimated by two terms of its Taylor expansion as

$$A_0\,\mathrm{e}^{-t/\tau} = A_0\left(1 - \frac{t}{\tau}\right) \qquad (6)$$

This method is effective for long-window algorithms. However, long-window algorithms cannot provide the fast response required for the relaying application. On the other hand, in the case of short window algorithms, if the decaying dc component is included in the signal model, the frequency response of the obtained digital filters in elimination of harmonic and inter-harmonic components would deteriorate [19].

By virtue of the variable-window estimation technique, one can achieve both the required speed and accuracy for the elimination of decaying dc component simultaneously. For this purpose, an adaptive model is proposed for the input signal. During the first stage, it considers only the dc offset (the first term of (6)) as one of the signal model components. It provides the required accuracy for the first zone operation of the relay. By increasing the data window length during the second stage, decaying dc component is also considered in the signal model. This way, the decaying dc component would be effectively eliminated. It yields a more accurate result for the identification of faults occurring at the first zone boundary limits and those at the end part of the transmission line protected in the second zone of the distance relay.

2.3 Proposed adaptive signal model

Table 1 illustrates the proposed adaptive model in which the components of the signal model are completed progressively. During the first stage of the algorithm that is the first cycle where $k \leq 12$, the dc offset component is considered in the

Table 1 Proposed adaptive signal model

Samples number	Harmonics				dc offset	Decay-dc
	2nd	3rd	4th	5th		
$1 \le k < 6$		×		×	×	
$6 \le k < 12$	↑	×	×	×	×	
$12 \le k < 16$	×	×	×	×	×	
$k \ge 16$	×	×	×	×	×	×

↑ denotes that the ability of the filters to eliminate this harmonic progressively increases as the forgetting factor approaches to one

signal model. This is done to achieve fast dynamic response as well as eliminating the dc component influence on the estimations. The ability of the proposed filters for eliminating even harmonics increases progressively during the first stage. After one cycle, all of the integer harmonics up to the Nyquist frequency are modelled and would be completely eliminated. It should be noted that the input signal is passed through an anti-aliasing filter before the analog-to-digital conversion. It limits the bandwidth of the input signal to the Nyquist frequency, which is equal to half of the sampling frequency. One cycle after the fault occurrence (12 samples), the signal model consists of all integer harmonics as well as the dc offset component. The decaying dc component is considered during the second stage when the data window length exceeds 16 samples. Thereby, the desired accuracy besides fast dynamic response would be achieved.

3 Test results and discussion

3.1 Test signal cases

In order to evaluate the performance of the proposed algorithm, initially different simulation studies are carried out using some test signals. The fundamental frequency is 60 Hz and the signals are sampled at the rate of 12 samples per cycle. In the performed simulations, the fault detection delay is considered 1.4 ms (1/60/12 s). To demonstrate the capabilities of the proposed algorithm, obtained results are compared with those of the full-cycle DFT algorithm and the variable-length data window algorithm proposed in [11].

In the variable-length data window algorithm, the filter starts with an initial half-cycle data window length. This window is used until the pre-fault samples are completely removed from the data window. Thereafter, the phasor is computed using a variable-length data window technique. After each estimation process, the filter progressively increases its data window length. The data window length will be kept fixed when the data window covers one cycle of the fundamental frequency.

3.1.1 Case I: At first, the proposed algorithm response to a pure sinusoidal signal is investigated. The input signal is as follows

$$x(t) = 1.0\sin(\omega_0 t) \tag{7}$$

Fig. 4 depicts the estimated phasor amplitude using the proposed method as well as the FCDFT and the variable-length data window algorithms. As can be seen, the proposed algorithm provides a faster dynamic response compared to the response of the conventional FCDFT algorithm. Meanwhile, since the input signal contains no harmonic component, the variable-length data window algorithm converges to the final value only after a half-cycle interval.

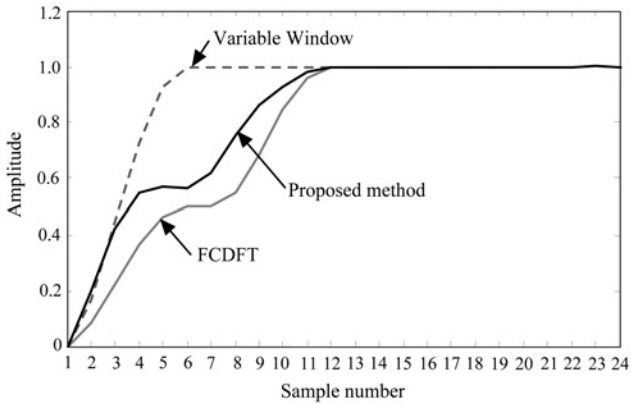

Fig. 4 *Estimated phasor amplitude using the proposed, FCDFT and variable-window algorithms for a pure sinusoidal signal, case I*

3.1.2 Case II: In this case, the input signal comprises of the fundamental component as well as a decaying dc component as follows

$$x(t) = 1.0\,e^{-t/0.0265} + 1.0\sin(\omega_0 t + \pi/2) \tag{8}$$

The time constant of the decaying dc component is considered to be 26.5 ms, which corresponds to the transmission line reactance-to-resistance ratio of $X/R = 10$. Fig. 5a compares

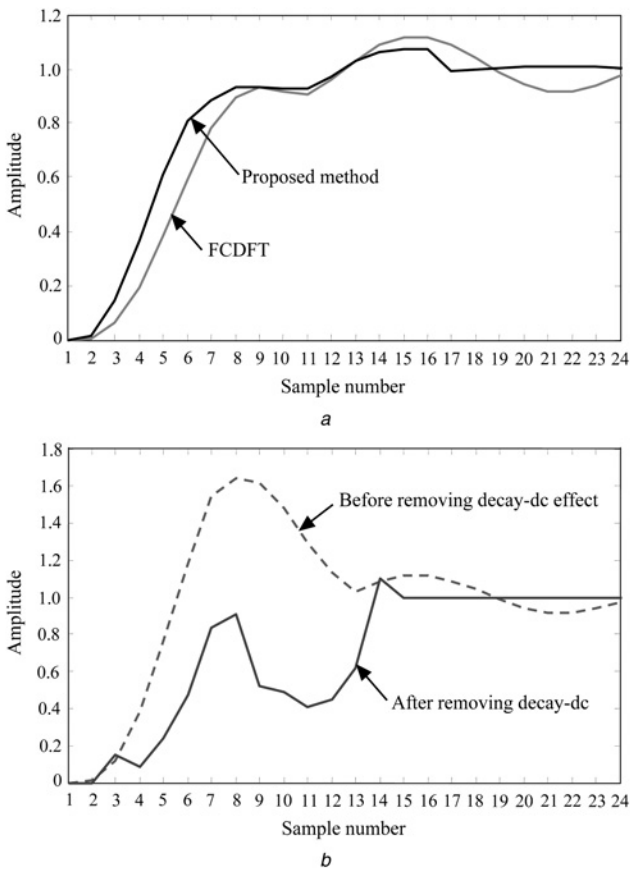

Fig. 5 *Estimated phasor amplitudes for case II*

a Using the proposed and FCDFT algorithms
b Using the variable data window algorithm before and after removing the decaying dc error

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the estimated phasor amplitude using the proposed method and the conventional FCDFT algorithm. As can be seen, the response of the FCDFT algorithm oscillates around the final value, which is due to the effect of the decaying dc component, whereas the proposed algorithm is able to effectively reduce this effect in addition to providing faster dynamic response. This would improve identification of the faults which occur at the end part of the transmission line.

Fig. 5*b* shows the estimated phasor amplitude using the variable-length data window algorithm. The error corresponding to the decaying dc component is calculated using three successive estimated phasors, as described in [11]. The exact phasor is obtained by subtracting the calculated error from the output of the variable window algorithm. Although this method is able to eliminate the decaying dc error in the steady-state condition, it does not provide an appropriate response during the transient interval.

3.1.3 Case III: In this case, the input signal comprises of the fundamental and decaying dc components as well as all of the integer harmonic components up to the Nyquist frequency as follows

$$x(t) = 1.0\,e^{-t/0.0265} + 1.0\,\sin(\omega_0 t + \pi/2) + 0.2\sum_{h=2}^{5}\sin(h\omega_0 t) \tag{9}$$

As shown in Fig. 6, the proposed algorithm provides faster dynamic response compared to the response of the full-cycle DFT algorithm and the decaying dc effect is effectively diminished using the proposed adaptive model after the 16th sample.

3.1.4. Case IV: In order to investigate the effect of power system frequency deviation on the performance of the proposed phasor estimation algorithm, a frequency drift of $\Delta f = -1$ Hz is considered. The input signal composition is considered the same as the previous case.

Fig. 7 depicts the responses of the proposed adaptive method and the FCDFT algorithm. As can be seen, the proposed method provides an appropriate result, which is superior to the conventional FCDFT result for both of the initial transient and final steady-state intervals. Since the ability of the proposed algorithm in elimination of the integer harmonic components does not depend on the value of the nominal frequency, it can effectively diminish the harmonics despite the nominal frequency drift. Meanwhile, the decaying dc component effect is effectively reduced.

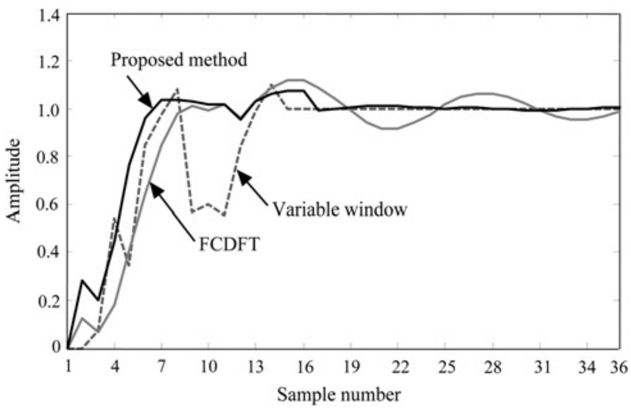

Fig. 6 *Estimated phasor amplitude using the proposed, FCDFT and variable-window algorithms, case III*

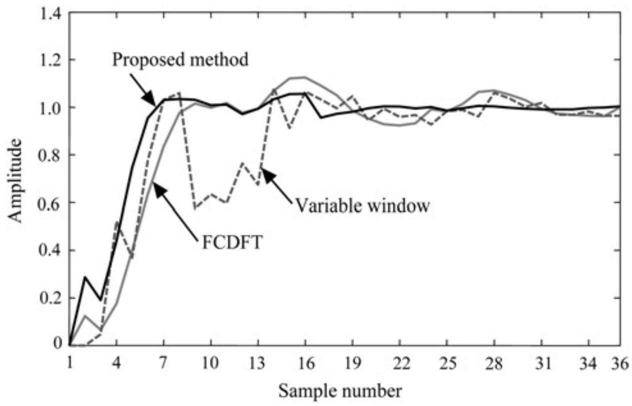

Fig. 7 *Estimated phasor amplitude using the proposed, FCDFT and variable-window algorithms for a power frequency drift, case IV*

3.2 Application to a power system

The performance of the proposed algorithm is evaluated for distance relaying application on a transmission line using the power system simulation software EMTDC/PSCAD. Fig. 8 depicts the one line diagram of the power system under study, which is a part of a real transmission system. In this figure, *R* denotes the distance relay location, and the relay protects the transmission line AB. The parameters of the simulated transmission system are given in the Appendix. The simulation data are used in a MATLAB program in the playback mode to implement the proposed algorithm.

Extensive simulation studies are carried out considering different fault conditions and some of the results are summarised in Table 2. The results are compared to those of the conventional full-cycle DFT algorithm. The distance relay is composed of three phase-to-ground and three phase-to-phase elements, which estimate the positive sequence impedance of the fault loop. An internal fault is concluded if three successive computed fault loop impedances lie inside the first protective zone. The obtained results indicate that the proposed algorithm provides a reliable response and is able to accelerate the relay operation time. On average, it can increase the relay operation speed to about 25% compared to the speed of the FCDFT algorithm.

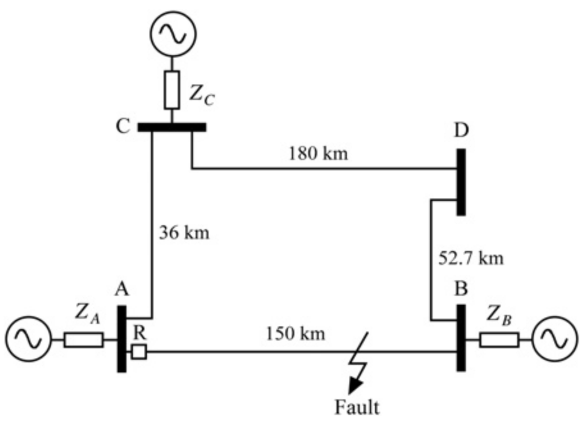

Fig. 8 *Model of the simulated transmission system*

Table 2 Summary of test results for internal fault cases

Fault type	Distance, km	Fault resistance, Ω	Fault angle, deg	Operation time, ms	
				Proposed adaptive method	Full-cycle Fourier algorithm
A-G	20	0.1	5	8.33	11.11
A-G	20	10	5	9.72	12.50
A-G	60	0.1	5	11.11	15.28
A-G	60	0.1	85	13.89	15.28
A-G	60	1.0	85	13.89	18.06
A-G	60	10	85	16.67	18.06
A-G	100	0.1	15	13.89	15.28
A-B	20	0.1	5	9.72	13.89
A-B	20	0.1	85	8.33	11.11
A-B	80	0.1	15	13.89	15.28
A-B-G	50	0.1	45	9.72	12.50
A-B-G	80	0.1	45	11.11	13.89
A-B-C	60	0.1	5	11.11	15.28
A-B-C	100	0.1	85	16.67	19.44
A-B-C-G	60	0.1	85	12.50	16.67

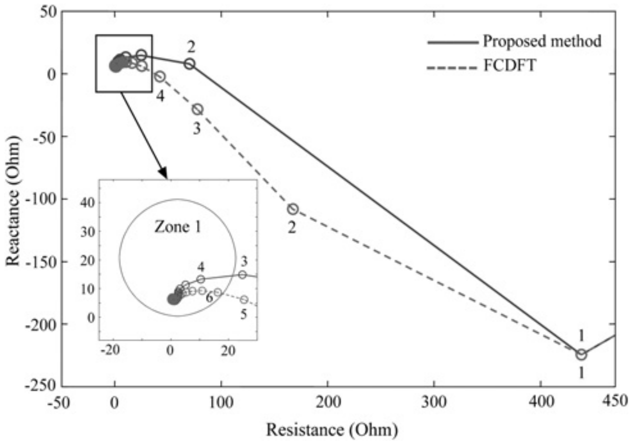

Fig. 9 *Trajectories of the measured impedance for the A-G internal fault*

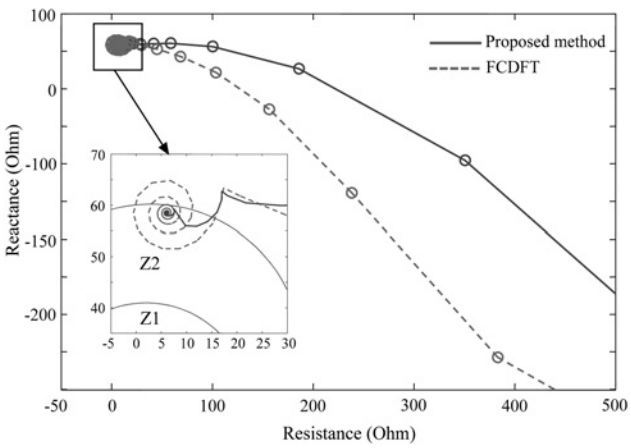

Fig. 10 *Trajectories of the measured impedance for the A-B-G external fault*

As an example, trajectories of the measured impedance for the first case in Table 1 using the proposed and the FCDFT algorithms are depicted in Fig. 9. As can be seen, the measured impedance using the proposed algorithm enters the first zone at the fourth sample, whereas it takes six samples for the FCDFT algorithm to fall into the first zone. Therefore the proposed method detects the fault faster than the FCDFT algorithm.

Fig. 10 depicts the trajectories of the measured impedance for a double-phase-to-ground A-B-G external fault on the

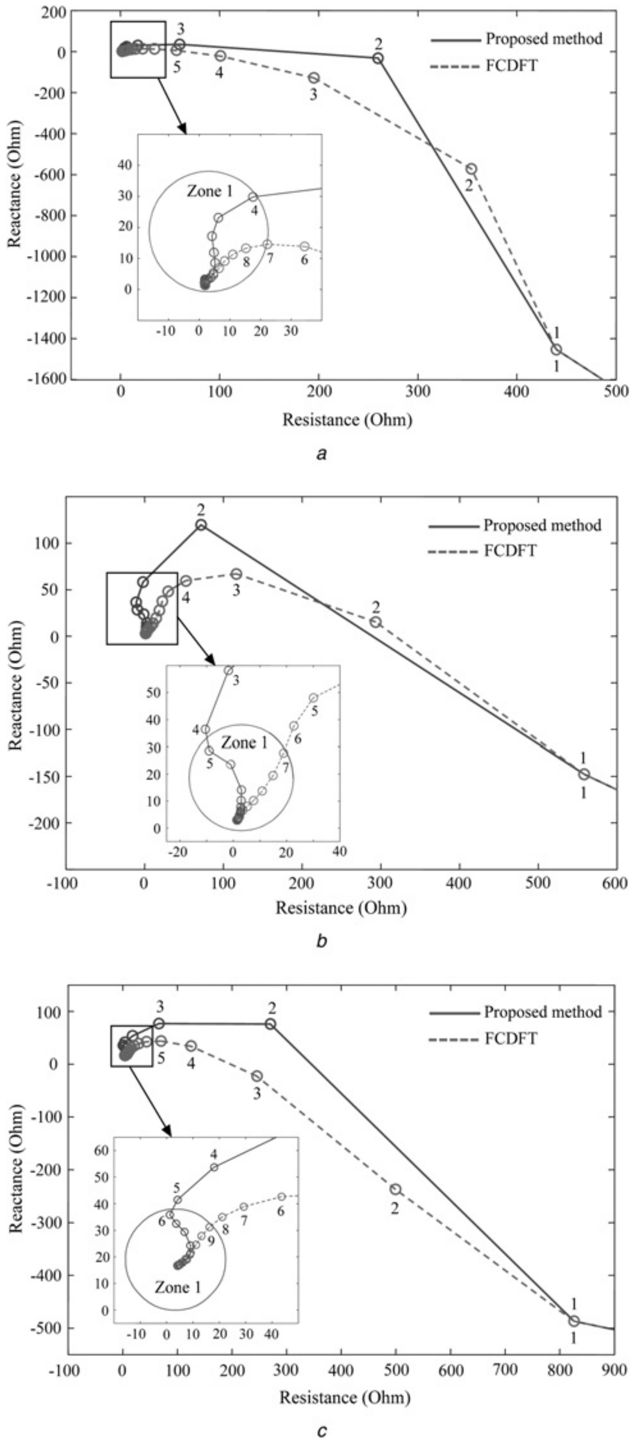

Fig. 11 *Trajectories of the measured impedances for the real data test cases*

a A-B-G fault
b C-G fault
c A-G fault

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transmission line BD. The fault location is within the second protective zone of the distance relay under study. As can be seen, the proposed algorithm provides a more accurate and less oscillatory estimation of the fault loop impedance. This can be employed to enhance reliability and speed of the relay second zone operation.

3.3 Real data test

The performance of the proposed algorithm is further verified using some field data recorded from a 240 kV transmission system [13] and some of the obtained results are described. The voltage and current waveforms are sampled by a digital fault recorder at a high sampling rate. The recorded data are first filtered using a finite impulse response (FIR) filter, and then are down sampled to 720 samples per second. The low-pass FIR filter limits the bandwidth of the input signal to the Nyquist frequency in order to avoid the aliasing error.

Fig. 11*a* demonstrates trajectories of the measured impedance for a double-phase-to-ground A-B-G fault on a 135 km transmission line. The measured impedance using the proposed algorithm enters the first zone at the fourth sample, whereas it takes seven samples for the FCDFT algorithm to fall into the first zone. An internal fault is concluded if three successive computed fault loop impedances lie inside the first protected zone. Therefore the proposed adaptive algorithm and the FCDFT algorithm detect the fault at the sixth and ninth samples, respectively. Considering the sampling frequency of 720 Hz, the operation time of the proposed and FCDFT algorithms are 8.33 and 12.5 ms, respectively.

The estimated impedance for a single-phase-to-ground C-G fault occurring on the 135 km transmission line is depicted in Fig. 11*b*. In this case, the measured impedance using the proposed algorithm enters the first zone at the fifth sample. On the other hand, the estimated impedance using the FCDFT algorithm enters the first zone at the seventh sample. In the next case, the performance of the proposed method is evaluated for identification of another single-phase-to-ground A-G fault on the same transmission line. Fig. 11*c* depicts the trajectories of the measured impedance. For both of these cases the proposed method is faster than the FCDFT algorithm.

4 Conclusions

This paper introduced an adaptive window algorithm for phasor estimation using weighted least error squares method. The proposed algorithm progressively adjusts the weighting coefficients during two stages to provide fast and accurate estimation of the fundamental phasor for relaying applications. In this paper, a low sampling frequency was used for ease of illustration. Nevertheless, higher sampling rates can also be employed to obtain better performance of the protection algorithm.

The simulation studies showed that the proposed algorithm was able to improve the distance relaying performance for fast and reliable fault clearance. It provides more accurate and less oscillatory response, and therefore can enhance the relay operation for the faults occurring at the transmission line boundary limits through improving the estimation accuracy. The performance of the proposed algorithm was also verified using some field data recorded from a high-voltage transmission system. The contributions of this paper are summarised as follows:

- The proposed approach can accelerate the relay first zone operation.
- The influence of the decaying dc component on the phasor estimation is effectively diminished.
- The proposed digital filters provide better characteristics to eliminate undesired harmonic and inter-harmonic components.
- It obviates the problem of spectral leakage because of the power system frequency deviation.
- Both of the proposed orthogonal filters have similar characteristic nearby the fundamental frequency, and hence, a non-oscillatory response would be obtained in the case of power system frequency deviation.

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6 Appendix

The parameters of the 400 kV transmission system used for the simulation studies are as follows

Equivalent source impedances

$$Z_{A1} = Z_{A0} = 0.381 + \mathrm{j}14.42\ (\Omega)$$

$$Z_{B1} = Z_{B0} = 0.183 + \mathrm{j}10.11\ (\Omega)$$

$$Z_{C1} = Z_{C0} = 0.412 + \mathrm{j}17.30\ (\Omega)$$

Transmission line parameters

$$R_1 = 0.0317\ (\Omega/\mathrm{km}), \quad R_0 = 0.3192\ (\Omega/\mathrm{km})$$

$$X_1 = 0.3204\ (\Omega/\mathrm{km}), \quad X_0 = 1.0083\ (\Omega/\mathrm{km})$$

$$C_1 = 10.838\ (\mathrm{nF/km}), \quad C_0 = 7.6493\ (\mathrm{nF/km})$$

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