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Reliability analysis of shuffle-exchange network systems

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Abstract

Shuffle-exchange networks (SENs) have been widely considered as practical interconnection systems due to their size of its switching elements (SEs) and uncomplicated configuration. SEN is a network among a large class of topologically equivalent multistage interconnection networks (MINs) that includes omega, indirect binary *n*-cube, baseline, and generalized cube. In this paper, SEN with additional stages that provide more redundant paths are analyzed. A common network topology with a 2×2 basic building block in a SEN and its variants in terms of extra-stages is investigated. As an illustration, three types of SENs are compared: SEN, SEN with an additional stage (SEN+), and SEN with two additional stages (SEN+2). Finally, three measures of reliability: terminal, broadcast, and network reliability for the three SEN systems are analyzed.

Keywords: Interconnection; Multistage; Networks; Reliability; Fault tolerant; Shuffle exchange; Stages; Switching elements

1. Introduction

Interconnection networks play a major role in the performance of modern parallel computers. These networks can provide the communication in a parallel processing system consisting of a large number of processors that are working together to perform a single-overall task. Multistage interconnection networks (MINs) are designed to provide fast and efficient communication at a reasonable cost.

Many different interconnection networks between the extremes of the single bus and the completely connected scheme have been proposed in the literature [1–4]. In general, MINs consist of layers of switching elements (SEs) with a specific topological pattern. These networks provide interconnection between the set of processors (inputs) and the set of memory modules (outputs). They fall within the category of indirect network as they rely on intermediate elements to provide the interconnection between the input and output elements. It has been extensively used in both circuit switching and packet switching networks with the introduction of buffered switches. These include multi-processor and communication network environments such as Ultracomputer [5], IBM RP3, ATM switches [6], and optical network [7]. The number of stages, interconnection topology, and the type of SEs used in the network configuration differentiate each MIN fault tolerant. Examples of the widely used MINs include: shuffle-exchange network (SEN) [8–10], gamma network [11], extra-stage gamma network [12], delta network, Tandem–Banyan network [6] and multilayer MIN [13]. Due to the size of its SEs and uncomplicated configuration of SEN as shown in Fig. 1, it is one of the most commonly used MINs.

In this paper, MINs with additional stages that provide more redundant paths are analyzed. A common network topology with a 2×2 basic building block such as SEN and its variants in terms of extra-stages is presented first. Three types of SENs are studied: SEN, SEN with an additional stage (SEN+), and SEN with two additional stages (SEN+2). Three measures of network system reliability: terminal, broadcast, and network reliability are developed to evaluate the performance of these networks and to assess the effect of additional stages on SEN reliability.

A SEN is a unique path MIN [9,10,14,15]. Therefore, there is only a single path between a particular input and a particular output. In this type of network, all SEs are critical and assumed as series connection. The SE can

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either transmit the inputs straight through itself or has cross connections. The number of switches per stage, the number of links and the connection between stages are consistent. An eight-input/eight-output SEN with three stages, 12 switches (SEs), and 32 links are shown in Fig. 1.

A SEN+ is an $N \times N$ SEN with an additional stage. The SEN+ system has N inputs and N outputs, with two paths between each source–destination pair. It has $n = (\log_2 N) + 1$ stages and each stage has N/2 SEs. In general, the switch complexity for the $N \times N$ SEN+ is N/2 $((\log_2 N) + 1)$. Thus, the additional cost of the SEN+ is $N/2$ switches or a fractional increase of $1/\log_2 N$, which is small for a large N [8]. An example of the 8×8 SEN+ is demonstrated by Fig. 2. The addition of an extra-stage to the SEN allows two paths for communication between each source and destination. While the paths in the first and the last stages of the SEN+ are not disjoint, the paths in the intermediate stages do disjoint links traverse. So the path redundancy in the SEN+ is achieved at the expense of an additional stage to the SEN.

As a comparison to SEN and SEN+, a SEN with two additional stages (SEN+2) is presented, and the reliability of an 8×8 network is evaluated. In general, a SEN+2 consists of N inputs and N outputs, $N/2$ SEs per stage, $(\log_2 N) + 2$ stages, and (N) $((\log_2 N) + 3)$ links. The network complexity is defined as the total number of SEs in the MIN, that is, $(N/2)$ $((\log_2 N) + 2)$ which is 20 SEs for an 8×8 SEN+2.

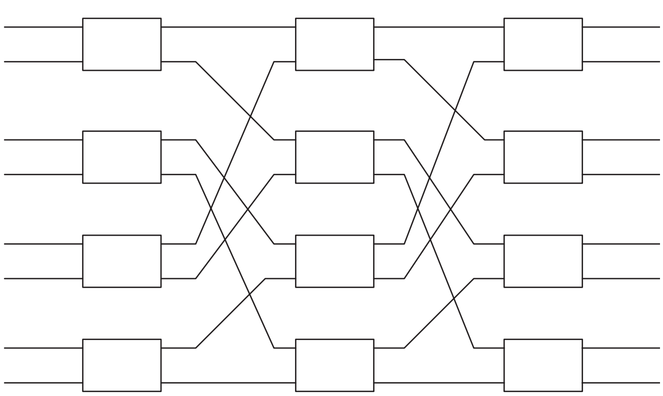

Fig. 1. 8×8 shuffle-exchange network (SEN).

The number of terminal paths between an input and an output switches will be increased to 2^k by adding k extra-stages to the SEN. This is also true for broadcast network. The additional k stages will create 2^k broadcast paths between a particular source and all destinations. Therefore, a SEN is a (2^k-1) fault tolerant. For the 8×8 case, the terminal paths and the broadcast paths of the SEN+ and SEN+2 are 2 and 4, respectively.

In the next sections, three reliability parameters: terminal, broadcast, and network reliability for SEN, SEN+, and SEN+2 will be analyzed.

2. Terminal reliability of SEN, SEN+, and SEN+2

Terminal reliability is defined as the probability of successful communication between an input and an output switches. In this section, terminal reliability of SEN, SEN+, and SEN+2 are compared. As an illustration, the terminal reliability of these three networks is evaluated for the 8×8 network.

The SEN is a unique-path MIN that has N input switches and N output switches and n stages, where $n = \log_2 N$. Each stage consists of $N/2$ interchange boxes, where each box being controlled individually through routing tags. An eight-input/eight-output SEN with three stages, 12 SEs and 32 links is shown in Fig. 1.

Let r be the probability of a switch being operational. Since SEN is a unique-path MIN, the failure of any switch will cause system failure, so from the reliability point of view, there are $\log_2 N$ SEs in series for each terminal path. Hence, the terminal reliability of an $N \times N$ SEN is given by

$$R_t(\text{SEN}) = (r)^{\log_2 N}. \tag{1}$$

It is noted that there is only a single path between a particular input S_i, $i = 1, 2, 3, 4$, and a particular output D_i in the 8×8 SEN so the terminal reliability for $N = 8$ can be

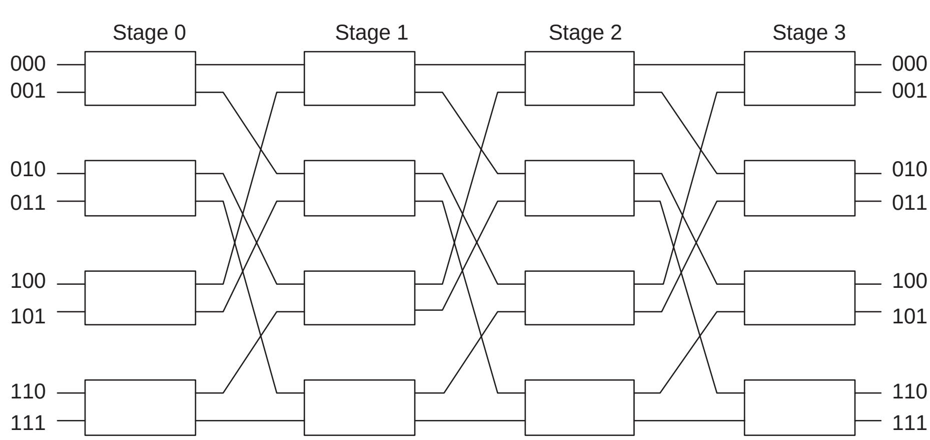

Fig. 2. 8×8 SEN with an extra-stage (SEN+).

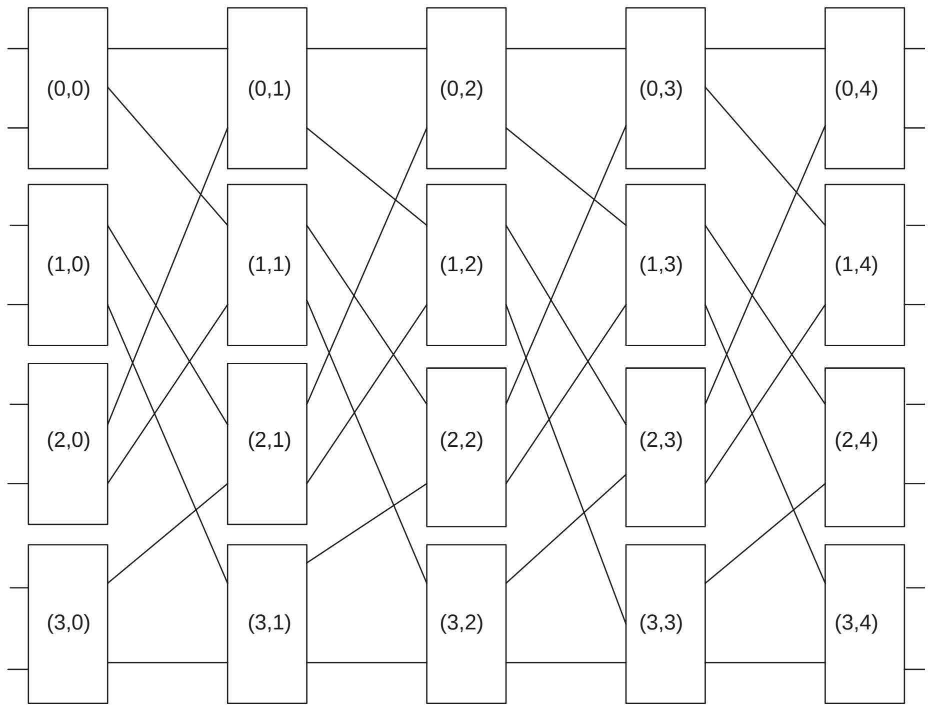

Fig. 3. 8×8 shuffle-exchange network with two additional stages (SEN + 2).

calculated as

$$R_t(\text{SEN}) = (r)^3. \quad (2)$$

SEN+ is a two-path MIN derived from the SEN by adding an extra-stage. Fig. 2 shows an eight-input/eight-output SEN+ with four stages consisting of 16 SEs and 40 links. Since the SEN+ is a two-path MIN, there are two-connection paths between a particular input and output. From the reliability point of view, this system can be represented as a parallel system path, consisting of $(\log_2 N)-1$ SEs each. Where, each path is connecting the input and output SE in series. Hence, the terminal reliability of an $N\times N$ SEN+ is given by

$$R_t(\text{SEN+}) = (r)^2(1-(1-r^{(\log_2 N)-1})^2). \quad (3)$$

By adding an extra-stage to the 8×8 SEN, the number of connecting paths between any input and output switches will increase to two. Therefore, the terminal reliability of the 8×8 SEN+ is higher than that of the 8×8 SEN. From equations (3), the terminal reliability of the 8×8 SEN+ for $N=8$ is given by

$$R_t(\text{SEN+}) = (r)^2(1-(1-r^2)^2) = 2(r)^4-(r)^6. \quad (4)$$

An 8×8 SEN + 2 consists of eight inputs and eight outputs, four SEs per stage, five stages, and 48 links as demonstrated in Fig. 3. It is observed that there are four terminal paths between any pair of input S_i ($i=1$, 2, 3, and 4) and output D_i.

Suppose that the position of a SEs i in stage j is represented by $SE_{i,j}$. Since there are 20 SEs in the 8×8 SEN + 2 and five stages (0, 1, 2, 3, and 4), the SEs are numbered from $SE_{0,0}$, $SE_{1,0}$, …, $SE_{2,4}$, $SE_{3,4}$. As an example, the terminal reliability between $SE_{0,0}$ and $SE_{0,4}$ in Fig. 3 are examined. There are four terminal paths between those SEs that involve the $SE_{0,1}$, $SE_{1,1}$, $SE_{0,2}$, $SE_{1,2}$, $SE_{2,2}$, $SE_{3,2}$, $SE_{0,3}$, and $SE_{2,3}$ in the intermediate stages.

Table 1
Terminal reliability comparison of the 8×8 SEN, SEN+, and SEN + 2

Switching reliability	Terminal reliability of the SEN	Terminal reliability of the SEN+	Terminal reliability of the SEN + 2
0.99	0.970299	0.979712	0.924345
0.98	0.941192	0.958894	0.856787
0.96	0.884736	0.915935	0.742687
0.95	0.857375	0.893921	0.694677
0.94	0.830584	0.871628	0.651818
0.92	0.778688	0.826431	0.579273
0.90	0.729000	0.780759	0.520995

The terminal reliability of the 8×8 SEN with two additional stages for $N=8$ can be computed as follows:

$$R_t(\text{SEN}+2) = r^{10}+2r^9(1-r)+8r^8(1-r)^2+8r^7(1-r)^3 + 2r^7(1-r)^2+4r^6(1-r)^3+4r^6(1-r)^2+4r^5(1-r)^2. \quad (5)$$

The comparison of the SEN, SEN+, and SEN + 2 for the 8×8 networks is presented in Table 1. It shows that the terminal reliability of the SEN+ is the highest and the terminal reliability of SEN + 2 is the lowest among the three networks. Therefore, it is noted that there is not a direct relation between additional paths and the terminal reliability increment in SEN because the additional paths

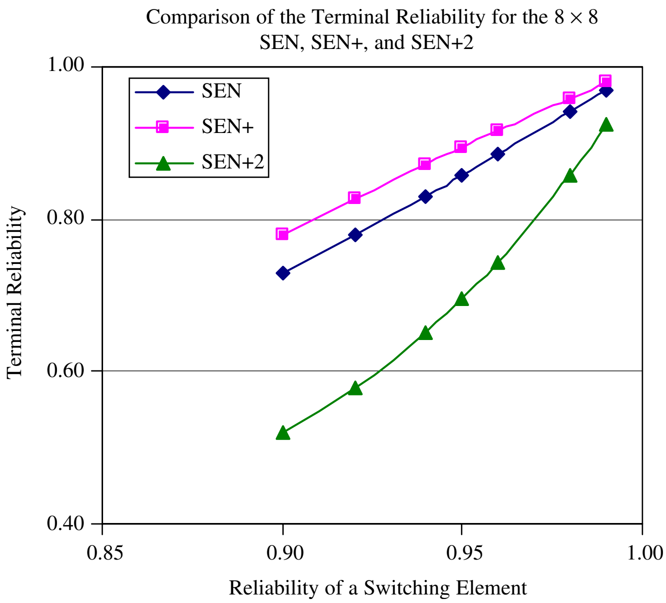

Fig. 4. Terminal reliability graph of the 8 × 8 SEN, SEN+, and SEN+2.

may increase the links complexity of the network, leading to a higher failure. Hence, it can be concluded that adding one additional stage to the SEN is more efficient way to improve terminal reliability than an addition of two stages to the SEN. The comparison of the terminal reliability graph for these three networks is illustrated in Fig. 4.

3. Broadcast reliability of SEN, SEN+, and SEN+2

Broadcast reliability is defined as the probability that a single-input switch is able to broadcast data or connected to all the output switches. As an illustration, the broadcast reliability of SEN, SEN+, and SEN+2 for the 8 × 8 case are examined.

Since SEN is a unique-path MIN that has N inputs and N outputs, there is only one broadcast path in this network. The broadcast reliability of the SEN can be calculated by assuming that the SE in the input stage and the SEs in the output stage have to work in order the network to be operational. Then by conditioning on the first stage and listing all possible combinations of paths between an input and all outputs, the broadcast reliability can be computed.

The failure of any switch in a broadcast path will cause the system failure, so from the reliability point of view; all SEs in a broadcast path are critical and can be assumed as in series. Since a broadcast path in the 8 × 8 SEN consists of seven SEs, the broadcast reliability as a function of the reliability of a SE for $N = 8$ can be calculated as follows:

$$R_b(\text{SEN}) = (r)^7. \tag{6}$$

Since the SEN+ is a two-path MIN, there are two broadcast paths from every input to all outputs. It can be observed that by having two broadcast paths, the SEN+ is much more reliable than the SEN that has only one broadcast path. The broadcast reliability of the 8 × 8 SEN+ for $N = 8$ can be calculated by

$$R_b(\text{SEN+}) = 2(r)^8 + 2(r)^9 - 4(r)^{10} + (r)^{11}. \tag{7}$$

In this section, the broadcast reliability for the 8 × 8 SEN+2 is evaluated. Since two extra-stages are added to the SEN, there are four broadcast paths in the SEN+2. The broadcast reliability of the 8 × 8 SEN with two additional stages for $N = 8$ can be calculated as follows:

$$\begin{aligned} R_b(\text{SEN}+2) = {} & r^{15} + 4r^{14}(1-r) + 20r^{13}(1-r)^2 \\ & + 32r^{12}(1-r)^3 + 16r^{11}(1-r)^4 + 10r^{11}(1-r)^2 \\ & + 12r^{10}(1-r)^3 + 4r^9(1-r)^2. \end{aligned} \tag{8}$$

Table 2
Broadcast reliability comparison of the 8 × 8 SEN, SEN+, and SEN+2

SE reliability	Broadcast reliability of the SEN	Broadcast reliability of the SEN+	Broadcast reliability of the SEN+2
0.99	0.932065	0.950334	0.897863
0.98	0.868126	0.901462	0.809831
0.96	0.751447	0.806756	0.667692
0.95	0.698337	0.761192	0.610132
0.94	0.648478	0.716965	0.559696
0.92	0.557847	0.632844	0.475794
0.90	0.478297	0.554872	0.408837

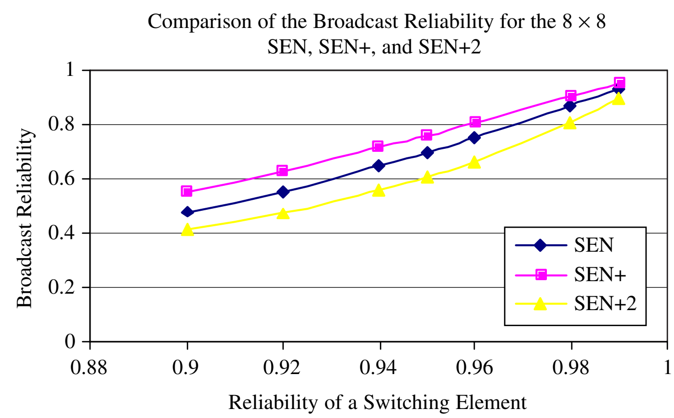

Fig. 5. Broadcast reliability graph of the 8 × 8 SEN, SEN+, and SEN+2.

The results of the broadcast reliability evaluation are summarized in Table 2. The comparison of broadcast reliability graph among these three networks is illustrated by Fig. 5.

It can be observed that the broadcast reliability of the 8 × 8 network is the highest in SEN+ and the lowest in SEN+2. Although the number of broadcast paths in the SEN+2 is greater than that of the SEN+, the broadcast reliability of the SEN+2 is the lowest among these three networks. Therefore, it can be concluded that based on the broadcast reliability results, SEN+ is the most reliable network among the three networks.

4. Network reliability of SEN, SEN+, and SEN+2

The ability of interconnecting all inputs to all outputs can be demonstrated by the network reliability. Network reliability of the 8×8 SEN, SEN+, and SEN+2 is evaluated in this section.

Since SEN is a single-path MIN, the failure of any switch will cause the system failure. Therefore, from the reliability point of view, there are $(N/2)$ $(\log_2 N)$ SEs in series. The reliability of an $N\times N$ SEN is given by

$$R_n(\text{SEN}) = (r)^{(N/2)(\log_2 N)}. \quad (9)$$

As for 8×8 SEN system, the network reliability for $N=8$ can be calculated as

$$R_n(\text{SEN}) = (r)^{12}. \quad (10)$$

The network reliability of the 8×8 SEN+ (consists of four stages: stage 0, 1, 2, and 3) can be calculated as follows:

1. All SEs in stages 0 and 3 must be working for the system to be operational. For reliability evaluation, it can be assumed that these SEs are in series. Therefore, the reliability for stages 0 and 3 is equal to r^8.
2. In stage 1, consider the state of each switch as conditional and proceed with all possible combinations through stage 2 that make the input–output connections. A k out of n redundancy system is applied where there is at least k out of n components to function for the system to work.
3. Every SE has only two possible states, working or failed.

Hence, the network reliability of the 8×8 SEN+ for $N=8$ can be derived as

$$\begin{aligned} R_n(\text{SEN+}) &= (r^8)\,(\text{the sum of all reliability in the stages 1 and 2}), \\ &= 2r^{12} + 4r^{14} - 8r^{15} + 3r^{16}. \end{aligned} \quad (11)$$

It is noted that the network reliability of the 8×8 SEN+ is much higher than that of the 8×8 SEN for all reliability values of the SEs. By using equivalent approach as described above for the SEN+, the network reliability of the 8×8 SEN+2, where $N=8$ with two additional stages can be derived as

$$\begin{aligned} R_n(\text{SEN}+2) = {} & r^{20} + 4r^{19}(1-r) + 36r^{18}(1-r)^2 \\ & + 120r^{17}(1-r)^3 + 168r^{16}(1-r)^4 \\ & + 2r^{16}(1-r)^2 + 96r^{15}(1-r)^5 \\ & + 20r^{15}(1-r)^3 + 16r^{14}(1-r)^6 \\ & + 14r^{14}(1-r)^4 + 4r^{13}(1-r)^3. \end{aligned} \quad (12)$$

The comparison of the network reliability evaluation of the SEN, SEN+, and SEN+2 for the 8×8 case is summarized in Table 3. The comparison of the network reliability for the three networks is also illustrated in Fig. 6 accordingly. It can be observed from the table that the 8×8 SEN+ has the highest network reliability and the SEN+2 has the lowest network reliability. This result again proves that adding just one additional stage to the SEN, leads to a SEN+ with the highest network reliability for a SEN.

Table 3
Network reliability comparison of the 8×8 SEN, SEN+, and SEN+2

SE reliability	Network reliability of the SEN	Network reliability of the SEN+	Network reliability of the SEN+2
0.99	0.886385	0.921659	0.854252
0.98	0.784717	0.846842	0.733541
0.96	0.612710	0.708630	0.549891
0.95	0.540360	0.645470	0.480112
0.94	0.475920	0.586322	0.421465
0.92	0.367666	0.479906	0.329657
0.90	0.282430	0.388708	0.262133

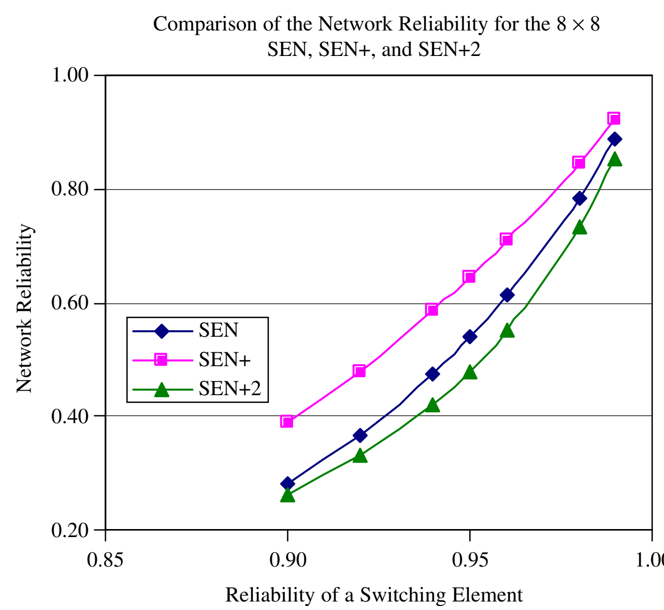

Fig. 6. Network reliability graph of the 8×8 SEN, SEN+, and SEN+2.

5. Conclusions

In this paper, three types of shuffle-exchange network (SEN) systems are compared: SEN, SEN with an additional stage (SEN+), and SEN with two additional stages (SEN+2). As measures of network performance, the terminal, broadcast, and network reliability of these three networks are evaluated.

In general, a SEN+2 consists of N inputs and N outputs, $N/2$ SEs per stage, $(\log_2 N+2)$ stages, and (N) $(\log_2 N+3)$ links. The network complexity, defined as the total number of SEs in the MIN, is $(N/2)$ $(\log_2 N+2)$ which for the 8×8 SEN+2 is 20 SEs, resulting in the lowest reliability among the three networks.

It can be concluded that by adding k extra-stages to the SEN, the number of terminal paths between S and D will be increased to 2^k. This is also true for broadcast network. The additional k stages will create 2^k broadcast paths between a particular source and all destinations. Therefore, a SEN is a (2^k-1) fault tolerant. For the 8×8 case, the terminal paths and the broadcast paths of the SEN+ and SEN+2 are 2 and 4, respectively.

It can be observed from the numerical results that the terminal reliability of the SEN+ is the highest among the three networks. Also, it is noted that the additional terminal paths in the SEN+2 do not increase the terminal reliability of the network since the links complexity leads to higher network system failure.

The broadcast reliability of the 8×8 for the three networks is also analyzed. Although the number of broadcast paths in the SEN+2 is greater than that of the SEN+, the broadcast reliability of the SEN+2 is the lowest among these three networks. From this evaluation, it can be concluded that the SEN+ is the most reliable network in terms of the broadcast reliability.

Similar conclusions are drawn for the network reliability of the three systems. That is, the SEN+ has the highest and SEN+2 have the lowest reliability. This fact again proves that adding an additional stage to the SEN leads to the most reliable SEN since the SEN+ has the highest terminal, broadcast, and network reliability comparing to the original SEN and SEN with two additional stages (SEN+2).

Some future works of the research would include the analysis of the system where switching elements that compose the network are not identical, there are some dependency issues among switching elements, and when the links in the system are less reliable than switching elements.

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