

# Reactive power market management considering voltage control area reserve and system security

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## ABSTRACT

This paper presents a new algorithm to optimize reactive power procurement through commercial transactions considering system voltage security. The proposed algorithm minimizes reactive power provision and transmission loss costs in addition to maximizing system voltage security margin through a multi-objective function. In order to maintain the voltage profile of power system during severe contingencies or due to load uncertainty, all voltage control areas (VCA) of the system are detected and then optimal reactive power reserve is provided for each VCA during the market settlement. A four-stage multiobjective mathematical programming method is proposed to settle the reactive power market. The proposed algorithm has been applied on IEEE-RTS test system. The simulation results show the effectiveness of the proposed algorithm for reactive power market management.

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## 1. Introduction

In restructured power systems, ancillary services are among significant issues which have essential role in reliable operation of electricity markets [1]. In most competitive electricity markets, one basic responsibility of Independent System Operator (ISO) is to provide ancillary services through commercial contracts with market participants [1,2]. Reactive power provision is one of the most important ancillary services in electricity markets. There are several issues which should be regarded by the system operator in the time of reactive power market settlement [1].

The system operator should compensate market participants for reactive power provision. Therefore, economic issues in the reactive power market are very important. Several methods have been proposed for reactive power payment [3–9]. In [3] a piece-wise linear cost curve has been developed for reactive power pricing in accordance with generator's energy bid curve and its reactive power capability curve. In [4–7] a three-region reactive power cost curve for a synchronous generator has been presented. Ref. [8] proposes a value-based method to allocate the cost of reactive power provision by the generators. In [9] a quadratic polynomial has been fitted to calculate the cost of reactive power provided by a generator. In comparison with other proposed reactive power cost curves, the quadratic polynomial is much simpler to be implemented and analyzed in optimization problems without losing

the accuracy. Some references deem it is necessary to compensate not only synchronous generators but also other reactive power sources such as synchronous condensers, capacitor banks and FACTS devices [10].

Because of the important role of reactive power in network operation and security, technical issues have been thought out as well as economic issues in many researches. Different objective functions such as minimization of reactive power cost, transmission loss minimization, and maximization of system loadability, have been used in reactive power market settlement [5,11–14].

Scheduling the reactive power market ignoring total energy loss may lead to increase the network loss and hence market operation cost. Therefore, transmission loss should be considered in the time of reactive power market clearing [3–5].

Reactive power has an essential role in power system voltage security [15,16]. Inadequate reactive power has been known to be one of the most important reasons for some major blackouts in the world [16]. Thus, system voltage security is another important issue which should be regarded in reactive power scheduling. In [17] a cost-based reactive power pricing has been proposed, which integrates reactive power cost minimization and voltage security problem into the optimal power flow problem. Ref. [7] proposes a two-level framework for scheduling reactive power market taking into account system voltage security aspects. In the first level, reactive power procurement has been performed on a seasonal basis while, in the second level, close to real-time reactive power dispatch has been carried out. Because of some difficulties in seasonal reactive power procurement, especially in forecasting the need for reactive power, Ref. [18] proposes a day-ahead reactive power market scheduling.

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As it is shown in Section 3, considering only the voltage security margin does not guarantee that the voltage profile will be kept in acceptable range after any disturbance. Therefore, similar to conventional power systems, in deregulated electricity markets, sufficient reactive power reserve is necessary to prevent unacceptable voltage deviation after any system disturbance or due to load uncertainty. Ref. [19] proposes to provide enough reactive power reserve to improve system voltage security and profile in power systems. Because the voltage and reactive power problems are local issues, it is shown in Section 3 that local reactive power reserve allocation is more efficient in the system. Hence, this paper proposes to provide local reactive power reserve in the stage of reactive power market settlement to achieve a more efficient ancillary market. In [20] an algorithm has been proposed to recognize all voltage control areas (VCA) of the network which is used in this paper to find VCAs in the time of allocating local reactive reserve.

In this paper a new algorithm is proposed for optimal reactive power provision in a day-ahead ancillary market. A full AC constrained OPF problem is developed to find the best schedule of reactive power produced by all synchronous generators and condensers. The proposed algorithm minimizes reactive power provision cost and transmission energy loss payment. Simultaneously, power system voltage security margin and reactive power reserve are maximized through a multiobjective function.

In the proposed algorithm, all voltage control areas of the system (VCA) are detected and optimal reactive power reserve is provided for each VCA separately during the market settlement. A four-stage multiobjective mathematical programming method is used to clear the reactive power market. The proposed algorithm has been applied to IEEE-RTS test system. The simulation results show the effectiveness of the proposed algorithm for reactive power market management.

The rest of the paper is organized as follows: Section 2 describes economical aspects of reactive power market design. In Section 3, voltage security and reactive power reserve have been discussed as two important technical issues of reactive power market clearing. Section 4 presents the proposed multiobjective reactive power market model. Sections 5 and 6 include numerical results and conclusion, respectively.

## 2. Economic aspects of the reactive power market

In deregulated electricity markets, the system operator should provide reactive power in an optimal manner considering both economic and technical issues. Owing to importance of economic aspects in any activity in the market, the economic issues are reviewed in this section and technical issues are the subject of the next section.

The system operator should compensate market participants for providing reactive power. If the operator seeks to minimize only cost of reactive power provision, it will contract with providers which offer the minimum prices. But, it may result in increasing transmission energy loss and consequently increasing total system payment. Therefore, cost of both reactive power provision and transmission energy loss should be considered in the time of reactive power market clearing.

According to NERC (North American Electric Reliability Council) Operation Policy 10, only reactive power produced by synchronous generators has been considered as ancillary service and is eligible for financial compensation [7].

In this paper, it is assumed that both synchronous generators and synchronous condensers should be compensated due to their contribution in reactive power control. The reactive power pricing algorithms are presented in the following subsections.

### 2.1. Cost of generator's reactive power

Different reactive power payment structures can be used for synchronous generators [7–9]. In [9] a quadratic reactive power cost curve for a typical synchronous generator has been proposed. This cost curve accurately models the investment cost, operational cost and also lost opportunity cost of a synchronous generator. It is defined as follows:

$$\text{Cost}(Q_{gi}) = a_{q,i}Q_{gi}^2 + b_{q,i}Q_{gi}c_{q,i} \quad (1)$$

where  $Q_{gi}$  is reactive power output of  $i$ th generator and equation coefficients,  $a_q$ ,  $b_q$  and  $c_q$  can be calculated based on active power cost curve [9]. This equation can provide accurate results in reactive power market while it is very simple to be implemented.

### 2.2. Reactive power cost of condensers

Synchronous condenser is a synchronous machine without any prime mover which can provide only reactive power. The reactive power cost curve of a condenser consists of the investment and operating costs. The operating cost contains the cost of energy consumed to overcome the mechanical friction and electrical loss, and the maintenance cost. Consequently, the reactive power cost curve of a synchronous condenser can be formulated by (2):

$$\text{Cost}(Q_{ci}) = (\beta_{ci} + \sigma_{ci})Q_{ci} \quad (2)$$

where  $Q_{ci}$  is the reactive power output of condenser,  $\sigma$  (\$/Mvar-h) is the operating cost of condenser and  $\beta_{ci}$  (\$/Mvar-h) which is formulated by (3) models the investment cost [21].

$$\beta_{ci} = \frac{\text{capital investment cost}}{8760 \times \text{lifespan} \times \text{average usage rate}} \quad (\$/\text{Mvar-h}) \quad (3)$$

## 3. Technical aspects of the reactive power market

There are some important technical issues such as voltage security, reactive power reserve, bus voltage profiles, generators' capability limits and transmission lines' limits which should be regarded in the time of reactive power market management [5,11–14]. All of the mentioned subjects should be thought out for both normal and post contingency conditions.

After a contingency in the power system, the voltage profile of each bus may become unacceptable and also the system goes toward the voltage instability point. If the system operator thinks only about the system voltage security margin in the time of reactive power market clearing, the system will be strengthened against the voltage instability even after a severe contingency. But, it does not guarantee that the voltage profile will be kept in the acceptable range. Vice versa, if the operator pays only attention to the voltage profile of buses, the system may have good voltage profile but it does not guarantee that the system have adequate voltage security margin. In Fig. 1 two different  $p-v$  curves are demonstrated to explain the above discussion. If the voltage magnitude is equal to  $V_1$ , which is considered to be acceptable voltage magnitude, the system with  $p-v$  curve 1 does not have adequate voltage security margin, while in the case of curve 2, the voltage security margin is sufficient.

Consequently, in this paper, system voltage security margin and reactive power reserve are considered as two important technical problems during reactive power planning.

### 3.1. Voltage security assessment

The voltage security is defined as an important subject in power systems and should be taken into account in many programming

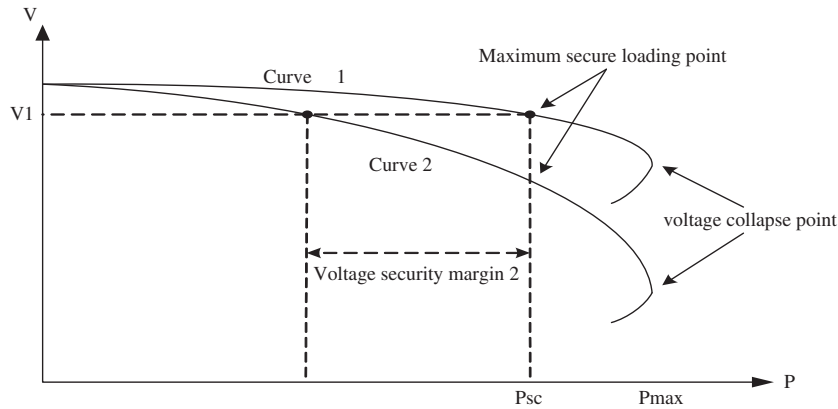


Fig. 1. Different voltage security margins for two p–v curves.

and planning problems [15–18,22]. To ensure reliable operation of the power system, it is necessary to maintain adequate voltage security margin in both normal condition and under contingency cases. Therefore, in this paper, maximization of voltage security margin is considered as one objective of the multiobjective optimal reactive power scheduling.

There are several approaches to estimate voltage security margin in a typical power system. Using different voltage security indices is a useful and conventional method for voltage security margin assessment [23–26]. Various indices for determination of the distance to the voltage collapse or instability has been proposed; active power margin based on p–v curves, reactive power margin based on q–v curves and  $L_{mn}$ -index are some good examples of these indices [16,23–26].

After a comprehensive studies and researches in order to find a suitable index for normal systems, the authors decided to use line stability index ( $L_{mn}$ ) to assess system voltage security in this work. Among different indices, the  $L_{mn}$ -index is one of the most accurate ones that its calculation does not involve computational complexities. Therefore,  $L_{mn}$ -index can be easily implemented in order to have a fast and accurate estimation of system voltage stability margin. Moreover, using this index, the system operator can monitor the status of all connected lines of the network and can identify the lines which are in stressed condition and also would be able to locate the exact location of voltage collapse [23,24].

Consider a one-line transmission line of an interconnected network as shown in Fig. 2. This single line is connected with other line and form the power network. Using the power flow results, the  $L_{mn}$ -index of the line between bus  $m$  (sending end) and  $n$  (receiving end) is expressed by the following equation [23]:

$$L_{mn} = \frac{4XQ_r}{[V_m \sin(\theta - \delta_m + \delta_n)]^2} \quad (4)$$

where  $V_m$  is the voltage magnitudes of  $m$ th bus,  $\delta_m$  and  $\delta_n$  are the voltage phase angles of  $m$ th and  $n$ th bus,  $\theta$  is line impedance angle,  $X$  is line reactance and  $Q_r$  is the reactive power at the receiving end. This index should be calculated for all transmission lines. This index is a quantitative measure to estimate the distance between actual state of the system and the stability limit. It varies from 0

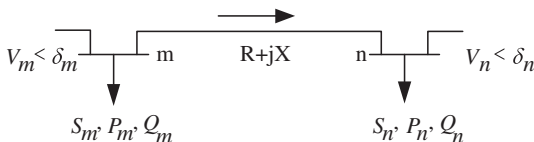


Fig. 2. Typical one-line diagram of a transmission line.

(no load condition) to 1 (voltage collapse). The more the index is far from 1 the better voltage security margin is achieved.

Contingencies such as transmission line or generator outages can result in voltage instability in power systems. The system is secure if no contingency can cause the voltage collapse in the system. The maximum  $L_{mn}$ -index of the system after a contingency gives a measure of the contingency severity.

Based on the concept of voltage security margin, the system operating condition can be classified into different classes. In this paper four different security levels including normal, alert, emergency and acute emergency are defined to classify system status. According to the calculated  $L_{mn}$ -index, the voltage security status can be categorized as described in Table 1 [27].

### 3.2. Reactive power reserve

Reactive power reserve is an important issue which has a significant effect on reliable operation of the power system. Some contingencies may result in voltage drop in the network and therefore the reactive power generating by shunt capacitors and line charging will reduce, thenceforth the reactive power losses will also increase [19]. It can be said that adequate reactive reserve should be available in the system to prevent voltage drop after any contingency.

The amount of reactive power that a synchronous machine can provide depends on the armature and field current ratings, the system operating condition and location of the machine in the network [16]. Based on the armature capability curve, the generator's maximum reactive power output can be expressed by the following equation:

$$Q_{g,max-armorure} = \sqrt{V_g^2 \cdot I_{g,max}^2 - P_g^2} \quad (5)$$

where  $V_g$  is the terminal voltage,  $P_g$  is the real power output and  $I_{g,max}$  is the maximum armature current of the generator. Eq. (6) can be used to determine maximum reactive power output due to field current limitation:

$$Q_{g,max-field} = -\left(\frac{V_g^2}{X_d}\right) + \sqrt{\left(\left(\frac{V_g^2 \cdot E_{max}^2}{X_d^2}\right) - P_g^2\right)} \quad (6)$$

where  $X_d$  is direct axis reactance and  $E_{max}$  is internal maximum voltage corresponding to the maximum field current of generator.

Table 1  
Voltage security classification levels.

$0 \leq L_{mn} < v_1$	$v_1 \leq L_{mn} < v_2$	$v_2 \leq L_{mn} < v_3$	$v_3 \leq L_{mn} \leq v_4$
Normal	Alert	Emergency	Acute emergency

Generator's maximum reactive power output is the minimum value of (5) and (6).

$$Q_{g,\max} = \min(Q_{g,\max-\text{armature}}, Q_{g,\max-\text{field}}) \quad (7)$$

Therefore, the maximum reactive power reserve of  $g$ th generator is:

$$Q_{g,\max,\text{res}} = Q_{g,\max} - Q_g \quad (8)$$

### 3.3. Voltage control area

In a typical power system, if the voltage of a bus varies, it significantly results in increasing or decreasing the voltage of some other buses and the voltage of many buses of the system will not considerably change due to this variation. This fact develops the idea that the buses of a power system can be separated into some different groups.

One idea which has come up in some references is that a given power system can be classified to different voltage control areas (VCAs) [6,20]. A voltage control area consists of some buses which are electrically close to each other and sufficiently far from other areas. Any variation in reactive power absorption or generation of a bus in a VCA has a significant impact on voltage of other buses in that area. Therefore, the reactive power sources in a VCA can be utilized to control the voltage profile of that area. These sources have less influence on the voltage in other VCAs [20].

Based on the method reported in [20] for determining VCAs in French power system, in this paper, the following two steps are used to identify all VCAs of a typical power system.

#### 3.3.1. Step 1 (calculating electrical distance)

In fact, electrical distance represents the degree of influence arising from voltage change in a bus on voltage changes on other buses. The concept of electrical distance is based on the sensitivity matrix  $[\partial V/\partial Q]$  that is inverse of  $[\partial Q/\partial V]$  as a part of the Jacobian matrix [20]. The elements of this sensitivity matrix reflect the impact of injected reactive power at a bus on the voltage magnitude of all other buses. We can quantify the voltage coupling magnitude between two buses by the maximum attenuation of voltage variation between these two buses. The following step-by-step approach can be used to calculate electrical distance between two nodes:

- (1) Compute the sub-matrix  $J_4 = [\partial Q/\partial V]$  from Jacobean matrix  $J$ .
- (2) Compute the elements of sensitivity matrix  $B$  which are inverse of  $J_4$  ( $b_{ij} = [\partial V_i/\partial Q_j]$ ).
- (3) Obtain the matrix of attenuation  $\alpha$  between all buses of the system whose terms are written by  $\alpha_{ij} = b_{ij}/b_{ji}$ .
- (4) In order to have a symmetric property in the electrical distance, Eq. (9) can be used to define the electrical distance between nodes  $i$  and  $j$ .

$$D_{ij} = -\log(\alpha_{ij} \cdot \alpha_{ji}) \quad (9)$$

- (5) Normalize all  $D_{ij}$  as below and construct the distance matrix  $D$ .

$$D_{ij} = D_{ij} / \max(D_{i1} \dots D_{in}) \quad (10)$$

#### 3.3.2. Step 2 (determination of all VCAs)

After defining the electric distance between all buses of the system, the boundary of all VCAs can be determined. There are different ways to do it. In this paper, at first, each element of the electrical distance matrix is subdivided to  $n$  levels as bellow to construct the closeness level matrix  $H$ .

If  $R_{k-1} \geq D_{ij} < R_k$  then  $H_{ij} = k \forall i \in SM \quad j \in bus$

where  $SM$  is set of synchronous machines. Also,  $R_0$  is equal to zero.  $H$  is an  $N_{Synch} \times N_{bus}$  matrix where  $N_{Synch}$  is the number of synchronous generators and condensers. Then, according to method described in Ref. [6], those buses which are closer to each other based on the  $H$  matrix can be bring together and define a VCA.

It is notable that  $R_k - R_{k-1}$  and the parameter  $k$  determine the number and distance of each range in classification. The smaller the distance of ranges, the better the grouping will be. But, the distance of each range should not be selected very small to avoid having very small VCAs.

## 4. The proposed reactive power market optimization algorithm

In this paper, a four-stage multiobjective day-ahead reactive power market optimization algorithm is proposed. In the first stage, the total payment for reactive power generation and system energy loss is minimized. The objective of the second stage is to find the best accessible system voltage security margin. In the third stage, all voltage control areas of the system are detected, and then local reactive power reserve is maximized for each VCA individually. Finally, in the last stage, the results of three previous stages are used to formulate a multiobjective mathematical programming for reactive power market settlement. The proposed algorithm is depicted as a flowchart in Fig. 3. Different stages of the algorithm are described in the following subsections.

### 4.1. Total system payment minimization (Stage 1)

The first stage of the algorithm has two steps. In step 1 a quadratic polynomial is fitted to calculate the cost of reactive power provided by each generator. It should be noted that the coefficients of the quadratic functions are determined according to generators' active power cost curve and limitations.

In the second step the objective of the algorithm is minimization of Total System Payment (TSP). Total system payment comprises reactive power payment and energy loss cost. It can be formulated as follows:

$$\text{Minimize } TSP = \sum_{i \in \text{gen}} \text{Cost}(Q_i) + \lambda P_{\text{loss}} \quad (11)$$

Subject to :

$$P_{Gi} - P_{Di} = \sum_{j=1}^n V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \quad \forall i \quad (12)$$

$$Q_{Gi} + Q_{Ci} - Q_{Di} = - \sum_{j=1}^n V_i V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \quad \forall i \quad (13)$$

$$Q_{G\min,i} \leq Q_{Gi} \leq Q_{G\max,i} \quad \forall i \in SM \quad (14)$$

$$Q_{C\min,i} \leq Q_{Ci} \leq Q_{C\max,i} \quad \forall i \in SC \quad (15)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad \forall i \quad (16)$$

$$|S_{ij}(V, \delta)| \leq S_{ij}^{\max} \quad \forall ij \quad (17)$$

$$L_{mn,ij} \leq v \quad \forall ij \quad (18)$$

where  $n$  is number of buses;  $\lambda$  is market energy price;  $SM$  is set of synchronous machines;  $SC$  is set of capacitors;  $P_{Gi}$  is active power generated at  $i$ th bus;  $Q_{Gi}$  is reactive power generated at  $i$ th bus;  $P_{Di}$  is active power demand at  $i$ th bus;  $Q_{Di}$  is reactive power demand at  $i$ th bus;  $Q_{Ci}$  is capacitors generated reactive power at  $i$ th bus;  $V_i$  is voltage magnitude of bus  $i$ ;  $\delta_i$  is voltage angle of bus  $i$ ;  $Y_{ij}$  is The  $ij$ th element of admittance matrix;  $S_{ij}$  is MVA of line between bus  $i$  and  $j$ ;  $L_{mn,ij}$  is The line stability index of line between bus  $i$  and  $j$ .

In the above OPF formulation, (12) and (13) are the nodal active and reactive power flow equations. The constraints of provision of reactive power by generators, condensers and existing capacitors are considered by (14) and (15), respectively. Limits of all bus voltages and transmission line power flows are imposed by (16)



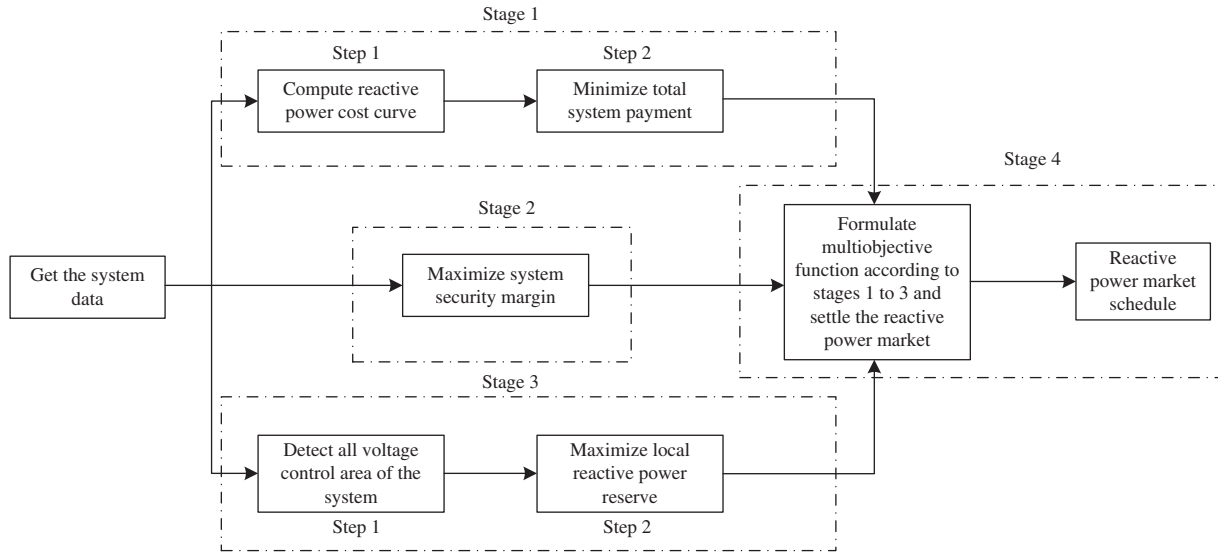


Fig. 3. Flowchart of the proposed algorithm.

and (17). In addition, the voltage security margin is regarded through (18). The parameter  $\nu$  should be sufficiently less than the unity but lessening it more than necessity will increase the operating cost.

#### 4.2. System voltage security maximization (Stage 2)

Voltage instability is an important challenge in the power system operation. In the second stage of optimization, the algorithm seeks to find the best accessible system voltage security margin. Minimization of the largest  $L_{mn}$ -index is the objective function of stage 2. It can be formulated by (19):

$$\text{Min. } WL_{mn} \quad (19)$$

where  $WL_{mn}$  is the largest  $L_{mn}$ -index among all transmission lines (the worst line stability index). Constraints of the optimization can be expressed by (12)–(17).

#### 4.3. Local reactive power reserve provision (Stage 3)

As previously discussed, the reactive power reserve can help to decrease the voltage profile deviation especially after severe contingencies. In the third stage, at first, all voltage control areas are detected and then the optimal Reactive Power Reserve (RPR) is allocated for each area separately. The reactive power reserve provision can be formulated by (20):

$$\text{Max. } RPR_i = \sum_j W_j Q_{res,j} \quad \forall j \in SMA_i \quad \text{and } i = 1, \dots, b \quad (20)$$

where  $Q_{res,j}$  is the reactive power reserve of  $j$ th generator,  $SMA_i$  is set of synchronous machine located in  $i$ th VCA,  $b$  is the number of VCAs and  $W_j$  is the participation factor of  $j$ th generator which can be selected based on the relative participation of each generator to increase reactive power reserve.

This optimization model should be implemented for all VCAs. Constraints of the optimization are the same as the stage 1 and are formulated by (12)–(18).

#### 4.4. Multiobjective model (Stage4)

Finally, in the last stage, a Multiobjective Mathematical Programming (MMP) is formulated to settle the reactive power market.

In [27] an MMP has been presented which can find the best compromise among the different objects. Based on the MMP proposed in [27], Multiobjective Cost Function (MCF) is developed here for reactive power market clearing which is formulated by (21):

$$MCF = \sqrt{\alpha \left( \frac{TSP}{TSP^*} \right)^2 + \beta \left( \frac{WL_{mn}}{WL_{mn}^*} \right)^2 + \sum_{i=1}^b \mu_i \left( \frac{RPR_i^*}{\sum_{gen} Q_{res,gen}} \right)^2} \quad \forall gen \in VCA_i \quad \text{and } i = 1, \dots, b \quad (21)$$

where  $RPR_i^*$  is the maximum reactive power reserve for  $i$ th VCA calculated in the third stage,  $TSP^*$  and  $WL_{mn}^*$  are the minimum values for  $TSP$  and  $WL_{mn}$  which have been calculated in stages 1 and 2, respectively. The parameters  $\alpha$  and  $\beta$  are weighting factors for total system payment and voltage security objective functions, respectively and  $\mu_i$  is weighting factor of reactive reserve of  $i$ th voltage control area. Because these weighting factors determine the relative importance of different terms in the MCF, it is proposed to select  $\beta$  so that the system works in normal or alert voltage security class even after severe contingencies. The system operator can select these weighting factors regarding market conditions. For example, if minimization of total system payment is privileged to voltage security margin and reactive power reserve, the system operator can set small values for  $\beta$  and  $\mu_i$ .

To settle the reactive power market with the optimal manner, the system operator should minimize MCF subject to market constraints as presented by (12)–(18). The proposed algorithm is a multiobjective nonlinear programming and can be solved by conventional nonlinear programming optimization techniques. To solve the proposed optimization problem, the sequential quadratic programming is implemented in Matlab.

## 5. Case study

In order to demonstrate the accuracy and effectiveness of the proposed algorithm, it has been applied to IEEE-RTS test system and the results are discussed [28]. The system consists of ten generators, one synchronous condenser and seventeen load points. Table 2 shows the market participants' parameters and their active power cost curve.

Two different cases are studied and discussed here. The first case compares global and local reactive power reserve provision

**Table 2**  
Reactive power providers' parameters ( $S_{base} = 100\text{MVA}$ ).

Provider	Related bus	$P_{min}$ (pu)	$P_{max}$ (pu)	$Q_{min}$ (pu)	$Q_{max}$ (pu)	$a_p^*$	$b_p^*$	$c_p^*$
G <sub>1</sub>	1	0	2.00	-0.50	1.20	0.0200	2.50	160
G <sub>2</sub>	2	0	1.92	-0.50	1.00	0.0284	2.90	250
G <sub>3</sub>	7	0	3.00	0	1.20	0.0310	2.98	205
G <sub>4</sub>	13	0	5.91	0	2.40	0.0312	3.23	310
G <sub>5</sub>	15	0	2.15	-0.50	1.20	0.0296	2.99	285
G <sub>6</sub>	16	0	1.55	-0.50	0.80	0.0309	3.15	295
G <sub>7</sub>	18	0	4.00	-0.50	2.00	0.0299	3.21	410
G <sub>8</sub>	21	0	4.00	-0.50	2.00	0.0300	3.04	344
G <sub>9</sub>	22	0	3.00	-0.60	0.96	0.0288	2.92	296
G <sub>10</sub>	23	0	6.60	-1.25	3.10	0.0300	2.89	190
Condenser	14	0	0	-0.50	2.00	0	0	0

$$^* \text{Cost}(P_{gi}) = a_{p,i}P_{gi}^2 + b_{p,i}P_{gi} + c_{p,i}.$$

**Table 3**  
Power demand and active power generation for all buses.

Bus	Pd (pu)	Qd (pu)	Pg (pu)
1	1.10	0.25	1.30
2	1.08	0.30	1.40
3	2.03	0.50	0
4	0.84	1.41	0
5	0.81	0.30	0
6	1.46	0.40	0
7	1.37	0.60	2.60
8	1.85	0.40	0
9	1.87	0.46	0
10	2.09	0.44	0
11	0	0	0
12	0	0	0
13	2.75	0.62	4.14
14	2.10	0.45	0
15	3.35	0.70	2.15
16	1.15	0.34	1.55
17	0	0	0
18	3.46	0.75	4.00
19	1.95	0.45	0
20	1.48	0.37	0
21	0	0	4.00
22	0	0	3.00
23	0	0	6.60
24	0	0	0

to demonstrate the importance of local reserve allocation. In case 2 which comprises three sub-cases 2-1 to 2-3, the proposed algorithm is implemented to settle reactive power market to show the aptness of proposed approach. The scheduled values of active power generation for all generators which have been determined in energy market and also the electrical power demand at each bus are declared in Table 3.

In all cases, participation factors of all generators ( $W_j$ ) are assumed to be the unity. The parameters  $v_1$ ,  $v_2$ ,  $v_3$  and  $v_4$  of Table 1 which define the voltage security classification levels are selected to be 0.5, 0.8, 0.9 and 1, respectively. To construct  $H$  matrix from  $D$  matrix,  $D_{ijs}$  are subdivided into four levels and their ranges are selected to be 0.1, 0.2, 0.25 and 0.3.

### 5.1. Comparison between results of global and local reserve provision (case 1)

In Case 1, the proposed optimization problem is reduced to just stage 3 of the algorithm which is reactive power reserve maximization to show the privilege of local reactive power reserve provision in comparison with global provision. In local reactive power provision strategy, all VCAs of the system should be found out by

the method described in Section 3. Table 4 shows the closeness value ( $H_{ij}$ ) of each network bus to each reactive power source. Based on the classified electrical distances, VCAs are recognized. Fig. 4.a shows four different identified VCAs. Since VCA2 includes just one reactive power provider, it is better to merge it to other VCAs if it is possible. Comparing the electrical distance of buses 7 and 8 which belong to VCA2 with other buses, the best new VCA for these buses is VCA1. Fig. 4.b. depicts the new separation of network into 3 VCAs.

In the next step, the system reactive power reserve is determined for both global and local reserve allocation separately. The results of these simulations are presented in Table 5.

As it is shown in Table 5, if the system operator determines the reactive power reserve globally, the reserve of VCA1 is much less than VCAs 2 and 3 and seems to be insufficient. Therefore, in the case of disturbances in VCA1, the voltage magnitude of buses of VCA1 may decrease. While, if the system operator determines the reactive power reserve locally, there are adequate reactive power reserves in all VCAs especially in the case of network disturbances.

In order to show the preference of the local reactive power reserve provision, a single severe contingency is simulated. This contingency is outage of line between buses 4 and 9. In [18] an index has been presented to calculate the Voltage Deviation (VD) of a system. The index is used as follows:

$$VD = \sum_{i \in \text{bus}} \frac{|V_{i,a} - V_{i,b}|}{V_{i,b}} \quad (22)$$

where  $V_{i,b}$  and  $V_{i,a}$  are the voltage magnitude of bus  $i$  before and after the contingency, respectively. The smaller VD index, the system condition is better.

The amounts of VD and the Worst Voltage Magnitude (WVM) indices have been calculated for both global and local reactive power reserve provision. The results are presented in Table 6.

It can be concluded from this study that local reactive power reserve provision gives better VD and WVM indices. Therefore, the system operator should regard reactive power reserve in each voltage control area individually.

### 5.2. Results of the proposed algorithm (case 2)

The purpose of this case study is to evaluate performance of the proposed algorithm. The schedules of active power generation for all generators and also the electrical power demand at each bus are shown in Table 3. These values are the results of energy market clearing which is known at the time of reactive power provision. The market energy price ( $\lambda$ ) is considered to be 100 \$/MWh which is assumed to be the cost of energy loss.

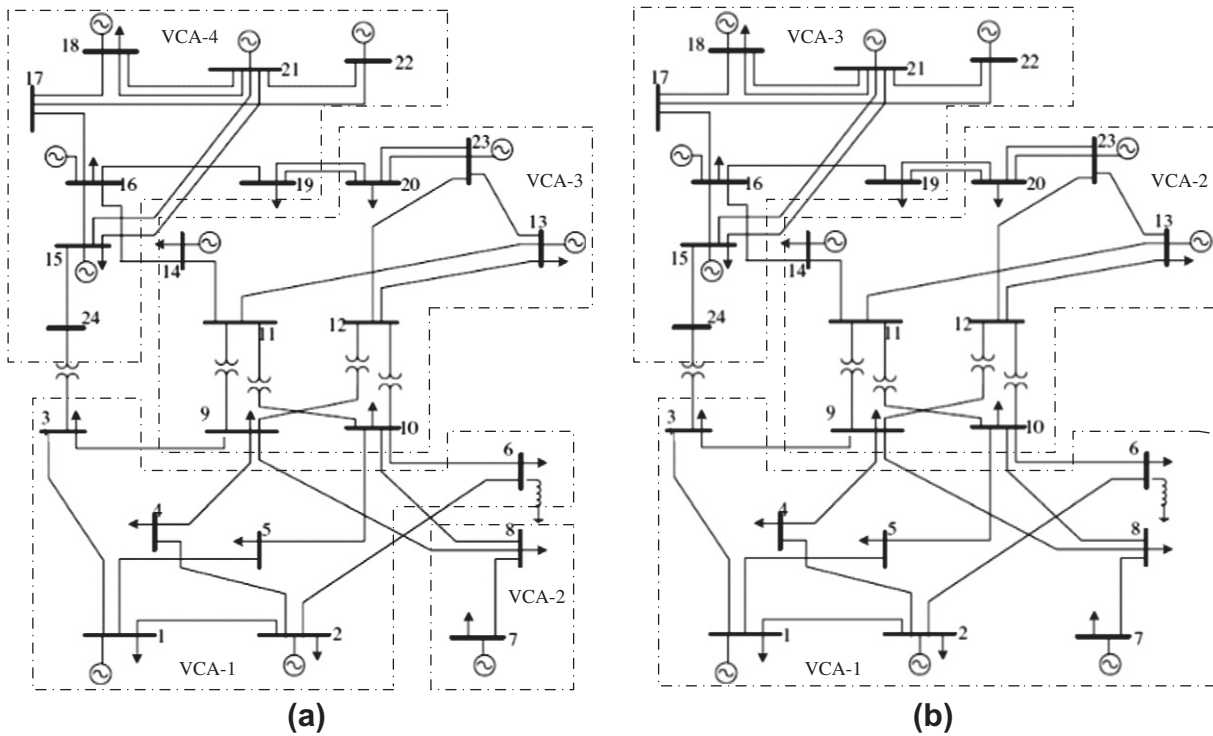
In order to analyze the effect of different objective functions on the performance of the proposed algorithm, three different sub-cases are studied. In case 2-1 the objective function is reduced to minimize only the TSP which is achieved by setting  $\alpha = 1$ ,  $\beta = 0$ ,  $\mu_i = 0$ ,  $\forall i \in \text{VCAs}$  in (21). The objective function of case 2-2 is minimization TSP and local reactive power reserve provision as well ( $\alpha = 1$ ,  $\beta = 0$ ,  $\mu_i = 1$ ,  $\forall i \in \text{VCAs}$ ). In case 2-3, TSP minimization, local reactive power reserve provision and also voltage security margin maximization are considered in the algorithm ( $\alpha = 1$ ,  $\beta = 1$ ,  $\mu_i = 1$ ,  $\forall i \in \text{VCAs}$ ).

The results of these simulations are presented in Table 7. Moreover, to compare the results, TSP index, Voltage Security Classification Level (VSCL) and %  $\Delta\text{TSP}$  have been also demonstrated in the table.

In case 2-1 the reactive power cost will be the minimum possible and operable value, but the system does not have adequate reactive power reserve in VCA1. Another noticeable result

**Table 4**  
Closeness level ( $H_{ij}$ ) of each bus with respect to each reactive provider.

Provider	$D_{ij} \leq 0.1$	$0.1 < D_{ij} \leq 0.2$	$0.2 < D_{ij} \leq 0.25$	$0.25 < D_{ij} \leq 0.3$
	$H_{ij}$ level = 1	$H_{ij}$ level = 2	$H_{ij}$ level = 3	$H_{ij}$ level = 4
G <sub>1</sub>	2	5	10	3,6,11,12,9
G <sub>2</sub>	1		5,10	3,4,6,9,11,12
G <sub>3</sub>			8	
G <sub>4</sub>		9,10,11,12,13,14,23	16,19,20	15
G <sub>5</sub>	16,17,21	18,19,20,24	11,14,23	3,9,10,12,13
G <sub>6</sub>	15,17,19	18,20,21,23	11,12,13,14,24	9,10
G <sub>7</sub>	15,16,17,21	19	20,23	11,12,13,14,22,24
G <sub>8</sub>	15,16,17,18	19	20,22	11,12,13,14,23
G <sub>9</sub>			15,16,17,18,21	
G <sub>10</sub>	20	12,13,16,19	9,10,11,14,15	17,18,21
Condenser		11	9,10,11,12,13,15,16,19,20,23	17,18,21



**Fig. 4.** IEEE-RTS: four voltage control areas (a), reduced to three voltage control areas (b).

**Table 5**  
Results of reactive power market clearing for global and local reserve provision.

VCA	Providers	Global reserve provision		Local reserve provision	
		$Q_G$ (p.u)	Total reactive reserve	$Q_G$ (p.u)	Total reactive reserve
One	G <sub>1</sub>	0.807	0.488	0.227	1.318
	G <sub>2</sub>	1.000		1.000	
	G <sub>3</sub>	1.105		0.855	
Two	G <sub>4</sub>	1.512	3.723	2.400	3.044
	G <sub>10</sub>	0.650		0.482	
	Condenser	1.615		1.574	
Three	G <sub>5</sub>	1.200	4.817	1.200	4.385
	G <sub>6</sub>	0.800		0.800	
	G <sub>7</sub>	0.143		0.575	
	G <sub>8</sub>	0.000		0.000	
	G <sub>9</sub>	0.000		0.000	

in this case is that the VSCL is in alert condition even before any contingency. In case 2–2, there is adequate reserve in each VCA but the system is in alert condition similar to case 2–1. Finally, when the system operator considers the complete objective

function in case 2–3, there is adequate reserve in each VCA and also the system operates in normal condition.

Table 8 presents VSCL condition and VD and WVM indices of the system after outage of line between buses 4 and 9 as a severe

**Table 6**

System indices after contingency in case 1 (outage of line 4–9).

	VD	WVM (pu)
Global reserve provision	0.4520	0.731
Local reserve provision	0.2655	0.795

**Table 7**

Results of reactive power market clearing for case 2.

VCA	Providers	Case 2–1		Case 2–2		Case 2–3	
		$Q_{Gi}$ (p.u.)	Total reactive reserve	$Q_{Gi}$ (p.u.)	Total reactive reserve	$Q_{Gi}$ (p.u.)	Total reactive reserve
One	G <sub>1</sub>	1.200	0.168	0.802	1.075	0.719	1.007
	G <sub>2</sub>	0.998		0.739		0.740	
	G <sub>3</sub>	1.034		0.784		0.934	
Two	G <sub>4</sub>	0.903	3.758	1.851	2.942	1.836	2.903
	G <sub>10</sub>	0.839		0.833		0.861	
	Condenser	2.000		1.874		1.900	
Three	G <sub>5</sub>	0.814	4.765	1.200	4.845	1.200	4.819
	G <sub>6</sub>	0.110		0.258		0.238	
	G <sub>7</sub>	0.593		0.383		0.466	
	G <sub>8</sub>	0.555		0.387		0.362	
	G <sub>9</sub>	0.123		–0.113		–0.125	
	VSCL		Alert		Alert		Normal
TSP (\$)		6153.96		6302.23		6342.58	
% $\Delta$ TSP*		0%		%2.4		%3.1	

\* Relative to case 2–1.

**Table 8**

System indices after contingency in case 2 (outage of line 4–9).

	VD	WVM (pu)	VSCL
Case 2–1	0.8915	0.603	Acute emergency
Case 2–2	0.2844	0.781	Emergency
Case 2–3	0.2962	0.779	Alert

contingency. In case 2–1 the system will shift to the acute emergency condition which is the worst class in the table. Also, the worst values of VD and WVM indices after contingency belong to this case. It can be deduced that it is not suitable to minimize just TSP as the objective function of reactive power provision. The VD and WVM indices of case 2–2 are better in comparison with previous case but the system will be in emergency condition after the contingency which is not desirable. For the next case, the system will shift to alert condition which is the best situation among studied cases. Furthermore, the best values of post contingency VD and WVM indices will be obtained in case 2–3. It is noticeable that this improvement is attained with just 3.1% increment of the operating cost.

The results show the efficiency of proposed objective function and reactive power provision algorithm so that reactive power is provided with the optimum cost and sufficient local reactive power reserve in a secure operating point.

## 6. Conclusion

Reactive power provision is one of the most important ancillary services in deregulated environments and should be regarded in many aspects of market scheduling and operation. While the main concern in electricity markets is financial matters, because of the important role of reactive power in system security, technical is-

suess should be thought out as well as economic issues in market clearing.

In this paper a new multiobjective nonlinear optimization algorithm is proposed for reactive power market settlement. Because of the influence of reactive power plan on both economic and technical aspects of the network operation, the proposed algorithm tries to consider both of these issues in a multiobjective mathematical programming. To fulfill these requirements the proposed algorithm comprises four stages. In the first stage, the algorithm seeks to minimize reactive power provision payment and total transmission loss cost. In addition to market economic issues that are considered in stage one, the technical issues are taken into account in stages 2 and 3. In the second stage, reactive power is provided with the objective of maximizing system voltage security margin. In stage 3, at first, all voltage control areas are recognized based on electrical distance between different buses. Then, an optimization problem is solved to maximizing local reactive power reserve in each voltage control area of the network. Finally, in the last stage, a multiobjective mathematical programming compromises between three previous objective functions using the optimal results of the stages. Case studies on IEEE-24 RTS system prove the suitability of the proposed algorithm.

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