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Chance-Constrained System of Systems Based Operation of Power Systems

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***Abstract*—In this paper, a chance-constrained system of systems (SoS) based decision-making approach is presented for stochastic scheduling of power systems encompassing active distribution grids. Based on the concept of SoS, the independent system operator (ISO) and distribution companies (DISCOs) are modeled as self-governing systems. These systems collaborate with each other to run the entire power system in a secure and economic manner. Each self-governing system accounts for its local reserve requirements and line flow constraints with respect to the uncertainties of load and renewable energy resources. A set of chance constraints are formulated to model the interactions between the ISO and DISCOs. The proposed model is solved by using analytical target cascading (ATC) method, a distributed optimization algorithm in which only a limited amount of information is exchanged between collaborative ISO and DISCOs. In this paper, a 6-bus and a modified IEEE 118-bus power systems are studied to show the effectiveness of the proposed algorithm.**

***Index Terms*—Active distribution grid, chance-constrained programing, distributed optimization, generation scheduling, stochastic programming, system of systems.**

NOMENCLATURE

i	Index for generating units.
j	Index for active distribution grids.
k	Index for loop iterations.
l	Index for lines.
t	Index for time period.
NA	Number of active distribution grids.
NB	Number of buses.
NG	Number of generating units.
NS	Number of solar power generation unit.
NT	Number of studied period.
NW	Number of wind power generation unit.
P_{it}	Generation of thermal unit i at time t.
R_{it}	Reserve of thermal unit i at time t.
P_{wt}	Power produced by wind power w at time t.
P_{st}	Power produced by solar power s at time t.
D_{bt}	Power demand at bus b at time t.
PL_l, $PL_l^{\max}$	Power flow in line l and its capacity.
I_{it}	Commitment state of unit i at time t.
$F_i(.)$	Generation cost function of thermal unit i.
SUD_{it}	Startup/shutdown costs of unit i at time t.
$E(.)$	Expected value of random variable.
$\Pr\{.\}$	Probability measure.
$LOLP$	Loss of load probability.
$TLOP$	Probability of line overload.
Z_x	$100*(1-x)$th percentile of the standard normal distribution.
σ_x	Standard deviation of parameter x at time t.
$PD_{o,jt}$	Power generated by ISO at period t and supplied to DISCO j in ISO's optimization problem.
$PG_{D,jt}$	Power demanded by DISCO j at time t and supplied by ISO in the $\mathrm{DISCO_j}$'s optimization problem.
SF	Shift factor.
X^o	Variables/parameters defining by the ISO.
X^d	Variables/parameters defining by the DISCO.
α, β	Multipliers of penalty function.

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I. INTRODUCTION

COMPARED with that in conventional power systems, the distribution networks in modern power systems are becoming active grids due to the inclusion of distributed generation (DG) units [1], [2]. These active distribution grids (ADGs) have led to change in the power systems characteristics and make the systems more reliable, efficient, and cost-effective. Without including DGs and ADGs, the independent system operator (ISO) commonly runs a centralized day-ahead generation scheduling problem to find the optimal hourly generation of conventional generating units connected to the transmission network [3], [4]. However, applying this scheduling scheme in modern power systems including DGs and ADGs, there is the

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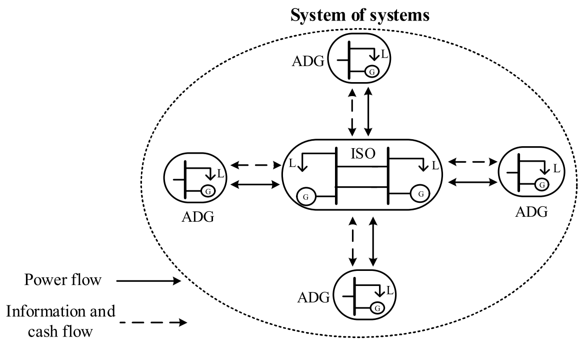

Fig. 1. Power system incorporating active distribution grids.

lack of collaboration and cooperation between transmission and distribution grids; and as a result, it may not obtain the optimal system operating point of the entire power system. Therefore, a new decision-making model is required in order to facilitate the integrated operation of DGs and ADGs, and enhance the security and economics of both transmission and distribution grids [5], [6].

Notice that there are several uncertain phenomena, such as variable demand and volatile renewable energy, which are making the day-ahead generation scheduling in the modern power systems even more sophisticated. Indeed, the value of power consumption at each bus cannot be exactly predicted and the system operator needs to consider load forecast error [7]. In addition, in order to reduce the environmental impacts of power systems and make them sustainable, renewable energy sources such as wind and solar power generating units are being widely employed to produce the electricity for consumers. Although these renewable energy sources produce clean and free energy, they are uncertain generating resources suffering from the variability and limited predictability [8]. Chance-constrained stochastic programming is one of efficient techniques to handle the load, and wind and solar power generation uncertainties in power systems [9], [10]. In this technique, the stochastic constraints can be violated with a pre-specified value of probability which is usually very small [9].

As shown in Fig. 1, in power systems encompassing active distribution grids, the electricity transportation, information and cash flow are bidirectional, from transmission system to ADGs or vice versa, and it complicates system analysis. In such power systems, as the transmission system operator, ISO, and active distribution grid operator, DISCO, are two autonomous entities which can function independently with their own operation and control regulations, the collaboration and cooperation relationship among them can be represented by the concept of system of systems (SoS). A SoS is described as incorporation of task-oriented or dedicated systems in a unique system in which its components: 1) collect their resources and capabilities to construct a more complex system that has more capability and performance than simply the sum of the basic systems, and 2) are able to independently perform valid functions in their own right and continue to work to fulfill those purposes when are separated from the overall system [11], [12]. Formulating the optimization problem, condition assessment and decision making under different circumstances, and modeling the economic issues and collaborative behavior of the independent systems are among the challenges facing the implementation of the SoS framework [11]. In addition, the required data to model the behaviors of the independent systems and the data flow process between the systems should be determined and guaranteed. A centralized stochastic optimization algorithm which needs all the information of the autonomous systems (ISO/DISCO), might not be the appropriate way to find the optimal operating point of such SoS-based power systems since generators, loads and network information, are usually protected by each individual system.

This paper presents a chance-constrained system of systems-based decision-making algorithm for stochastic day-ahead generation scheduling of power systems encompassing active distribution grids. The contributions of this paper are summarized as follows:

- The entire interconnected power system is modeled as a system of systems where the ISO and DISCOs are defined as self-governing systems. These systems autonomously utilize their generation resources to optimize their own operation, while collaborating with each other to accomplish the entire power system's interest, such as a secure and economic operation.
- Taking into consideration the demand and renewable generation uncertainties in both ISO and DISCOs, the stochastic interactions between the ISO and DISCOs are modeled by using chance-constrained stochastic programing. The proposed model can be used to study the impact of uncertainties on the tie-line connecting the ISO with other independent DISCOs.
- A distributed optimization algorithm, which is based on analytical target cascading (ATC) method for the optimization of hierarchical and multilevel systems, is used to solve the proposed SoS-based power system operation. In the proposed algorithm, an independent system does not need to share all of its own information (which might be usually commercially sensitive) with others. Only a limited information which is related to crossing borders between the systems is shared among them. This shared information is defined as exchanged power between ISO and DISCOs.

The rest of this paper is organized as follows. The modern power system is defined as a system of systems in Section II. The proposed collaborative stochastic decision-making algorithm and solution process for the SoS-based day-ahead generation scheduling are carried out in Section III. Numerical testing results are presented in Section IV. The concluding remarks are provided in Section V.

II. SoS-Based Stochastic Power System Operation

In the proposed study, the ISO is an autonomous system being responsible for the transmission system operation, and the DISCOs are other independent systems utilizing the active distribution grids. The ISO and each DISCO individually formulate and solve their own generation scheduling problems according to their own available generation sources, network topologies and forecasted loads.

A. Stochastic Generation Scheduling for Autonomous Systems

In this section, an individual stochastic optimization problem is formulated for transmission system/active distribution grid considering the uncertainty brought by variable demand, and volatile wind and solar power generations. Suppose there is no tie-line (connection) between the transmission network and the active distribution grid. It means that these two systems are separated and the operating point of one does not impact the operating point of another. The ISO minimizes (1) as the objective function of its own generation scheduling problem.

$$Min \sum_{t=1}^{NT} \sum_{i=1}^{NGo} (F_i(P_{it}^o) I_{it} + SUD_{it}^o). \tag{1}$$

This objective function includes operating and startup/shutdown costs of the thermal generating units over the scheduling horizon (e.g., 24 hours). In general, the equality and inequality constraints consist of unit commitment constraints such as power balance, system spinning reserve requirements, generating unit capacity, and units' ramping up/down and minimum on/off time limits, as well as network security constraints in which the shift factor technique is used to formulate line power flows. Because of the load and renewable generation uncertainties, some of the constraints such as power balance, spinning reserve requirements, and network security constraints, have stochastic characteristic. Using the expected values of the random variables and applying the chance constraint technique, the corresponding stochastic constraints are formulated by

$$\sum_{i=1}^{NGo} P_{it}^o + \sum_{w=1}^{NWo} E(P_{wt}^o) + \sum_{s=1}^{NSo} E(P_{st}^o) = \sum_{b=1}^{NBo} E(D_{bt}^o) \quad \forall t \tag{2}$$

$$\Pr\left\{ \sum_{i=1}^{NGo} (P_{it}^o + R_{it}^o) + \sum_{w=1}^{NWo} P_{wt}^o + \sum_{s=1}^{NSo} P_{st}^o \geq \sum_{b=1}^{NBo} D_{bt}^o \right\} \geq 1 - LOLP_t^o \quad \forall t \tag{3}$$

$$\Pr\left\{ \begin{array}{l} -PL_l^{\max} \leq \sum\limits_{i=1}^{NGo} SF_{li}^o P_{it}^o + \sum\limits_{w=1}^{NWo} SF_{lw}^o P_{wt}^o \\ \qquad + \sum\limits_{s=1}^{NSo} SF_{ls}^o P_{st}^o - \sum\limits_{b=1}^{NBo} SF_{lb}^o D_{bt}^o \\ \qquad \leq PL_l^{\max} \end{array} \right\} \geq 1 - TLOP_{lt}^o \quad \forall l, \forall t. \tag{4}$$

Constraint (2) indicates the power balance constraints in which the expected values of the uncertain load and renewable generations (wind and solar) are considered. Thermal generating units require to provide adequate reserve to accommodate renewable generation and demand uncertainties with a prescribed probability. This constraint is modeled by (3) in which the ISO needs to set a proper value for loss of load probability at time $t(LOLP_t^o)$ to guarantee the availability of the adequate spinning reserve during the real-time dispatch [10]. Note that $LOLP_t^o$ should be appropriately defined by the ISO through a long-term study in order to compromise between economics and reliability of the system operation. In order to ensure the network security, stochastic constraint (4) requires being satisfied. This constraint guarantees that the stochastic line flow will stay within the capacity limit of the line with an acceptable probability. In (4), $TLOP_{lt}^o$ (probability of line overload) has the same meaning as $LOLP_t^o$ in (3), and should be also properly selected to make a trade-off between economics and reliability [9].

Here, we use superscript (o) in $LOLP_t^o$ and $TLOP_{lt}^o$ denoting that these values are selected by the ISO to be use in its own optimization problem. We will further explain this issue in the next subsection.

B. Deterministic Model of Chance Constraints

Different probability distribution functions (PDFs) have been proposed in the literature to model load, wind and solar power generation. In general, the distribution function is obtained based on using the historical data and statistical techniques. Here, the wind and solar power generations are represented by the normal distribution function, and the load is presented as a truncated normal distribution [1] as shown in (5)–(7) [13], [14].

$$P_{wt}^o \sim E(P_{wt}^o) + PDF(\mu_{wt}^o, (\sigma_{wt}^o)^2) \quad \forall w, \forall t \tag{5}$$

$$P_{st}^o \sim E(P_{st}^o) + PDF(\mu_{st}^o, (\sigma_{st}^o)^2) \quad \forall s, \forall t \tag{6}$$

$$D_{bt}^o \sim E(D_{bt}^o) + PDF(\mu_{bt}^o, (\sigma_{bt}^o)^2) \quad \forall b, \forall t \tag{7}$$

where $E(.)$ is the expected value of the random variable, and μ and σ are the mean value and standard deviation of the distribution function, respectively.

In order to handle the chance constraints in the scheduling problem, we convert them to the equivalent deterministic constraints[2]. Note that, however, the power balance constraint (2) is a stochastic constraint, it uses the expected values of the random variables and can be directly considered in the scheduling problem in the present form without any need to be converted to another model. Considering the PDF of each random variable in (3) and (4), and using its mean value and standard deviation, the chance constraints can be converted into the linear constraints by performing some manipulations. For more details, please see [15] and [16]. Thus, constraints (8) and (9) are equivalent linear inequalities which respectively replace chance constraints (3) and (4) in the system's generation scheduling problem [9].

$$\sum_{i=1}^{NGo} (P_{it}^o + R_{it}^o) + \sum_{w=1}^{NWo} E(P_{wt}^o) + \sum_{s=1}^{NSo} E(P_{st}^o) \geq \sum_{b=1}^{NBo} E(D_{bt}^o) + Z_{LOLP_t^o} \left[\sum_{w=1}^{NWo} (\sigma_{wt}^o)^2 \right.$$

[1]Normal distribution of load and renewable generation is widely adopted in the renewable energy integration studies [13], [14]. This paper follows this assumption and transforms the proposed chance constraints into closed-form deterministic equivalents for better utilization of the problem characteristics.

[2]Note that the chance-constraint programming can be solved by the Monte-Carlo scenario based approach. [9] provided a comparison between the deterministic method that is used in this paper and the scenario-based method, where the chance constraints are imposed by the indicator functions, which measure the number of occurrences when the chance constraints (3) and (4) are violated in a given scenario. It was found that the deterministic method is more sensitive than the scenario-based method to load and renewable generation forecast errors, and the deterministic method provides near-optimal solutions with a much faster CPU time compared to the scenario-based method.

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$$+ \sum_{s=1}^{NSo} (\sigma_{st}^{o})^2 + \sum_{b=1}^{NBo} (\sigma_{bt}^{o})^2 \Bigg]^{1/2} \quad \forall t \tag{8}$$

$$\Bigg| \sum_{i=1}^{NGo} SF_{li}^{o} P_{it}^{o} + \sum_{w=1}^{NWo} SF_{lw}^{o} E(P_{wt}^{o}) + \sum_{s=1}^{NSo} SF_{ls}^{o} E(P_{st}^{o}) - \sum_{b=1}^{NBo} SF_{lb}^{o} E(D_{bt}^{o}) + Z_{TLOP_{lt}^{o}} \Bigg[\sum_{w=1}^{NWo} (SF_{lw}^{o} \sigma_{wt}^{o})^2 + \sum_{s=1}^{NSo} (SF_{ls}^{o} \sigma_{st}^{o})^2 + \sum_{b=1}^{NBo} (SF_{lb}^{o} \sigma_{bt}^{o})^2 \Bigg]^{1/2} \Bigg| \le PL_{lt}^{\max} \quad \forall l, \forall t. \tag{9}$$

Similarly, the chance-constrained stochastic generation scheduling problem can be also formulated for the ADG. Notice that since in the SoS-based power systems, the ISO and the DISCO are independent systems, the security level (*LOLP* and *TLOP*) selected by them to be used in their own generation scheduling problem could be different. For example, the ISO may prefer to have a high security level and select a small value of *LOLP* and *TLOP* of its own transmission lines, however, this issue restricted the constraints and may increase the operation cost of the ISO. Likewise, as an independent system, the DISCO may prefer to reduce its own operating cost rather than increasing the security level of the distribution system, and so it may select larger values for *LOLP* and *TLOP* compared with the ISO. Hence, in the DISCO's problem formulation, superscript (d), which shows the variables/parameters are exclusively for DISCO, should be used instead of (o) denoting the variables/parameters of the ISO.

III. Collaborative Stochastic Decision-Making Solution

According to the above assumption that the transmission network is not connected to the active distribution grid, both ISO and DISCO can separately solve their own local chance-constrained-based optimization problem to find the hourly generation schedule of their own generating units. However, when the transmission network and ADGs are indeed linked together, the optimal operating point of one of them impacts the operating point and security margin of others. In this section, to model the interaction between the systems in the SoS-based stochastic generation scheduling problem, and to find the optimal operating point of the ISO and DISCOs, a pseudo generation/load model is presented for the shared variables between the systems. Then, a collaborative stochastic decision-making solution is presented considering uncertainties brought by both demand and renewable power supplies.

A. Shared Variables Modeling

Assume the ISO and ADG are connected together as shown in Fig. 2(a). The power exchange between the ISO and DISCO (power flow in the tie-line connecting the transmission and distribution systems together) is the shared variable between these two independent systems. Considering a hypothetical power flow direction in the tie-line, the shared variable can be modeled using a pseudo generation source and a pseudo load. Suppose that the power is transferred from the transmission system toward $\mathrm{ADG_j}$. In order to separate the independent systems, as shown in Fig. 2(b), from the DISCO's perspective, the line flow is modeled as a pseudo generator supplying to DISCO; and from the ISO's perspective, the line flow is modeled as a pseudo load supplied by ISO. Thus, PG_D is the power demanded by DISCO and supplied by ISO in the DISCO's optimization problem; and PD_O is the power generated by ISO and supplied to DISCO in ISO's optimization problem.

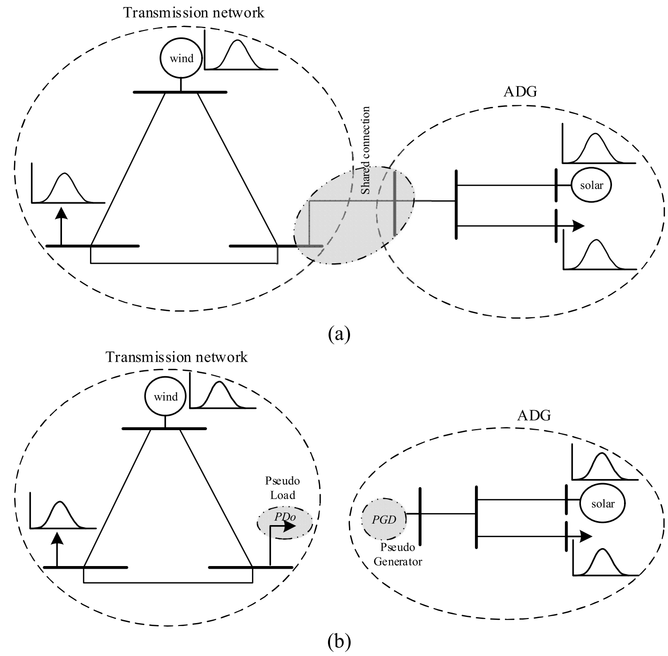

Fig. 2. Modeling shared variables between an ADG and ISO.

Note that we model the power flow in the tie-line by two pseudo variables and both variables PG_D and PD_O should be between minimum/maximum capacity of the line connecting the transmission network to $\mathrm{ADG_j}$. However, as the load, wind and solar power uncertainties influence the power flow in the tie-line, we cannot only restrict pseudo variables PG_D and PD_O between the maximum capacity of the tie-line using the deterministic constraints. Therefore, we formulate a chance constraint for pseudo generation in the DISCO's optimization problem, and similarly a chance constraint for pseudo load in the ISO's scheduling problem. These constraints model the influence of the uncertainties on the power flow in the tie-line and guarantee that the power flowing does not exceed the maximum capacity of the tie-line with a specified probability (level of security). The corresponding chance constraints are presented in the next subsection to model the uncertainties on the shared variables.

B. Coupling Constraints Handling

According to the above modeling, the generation scheduling problems of ISO and DISCOs get separated from each other using pseudo generation and load. The ISO and DISCOs are located in two different levels in the power system (ISO is in the upper level, and DISCOs are in the lower level). Thus, according to the analytical target cascading (ATC) method, a hierarchical two-level optimization technique can be implemented to perform the collaborative generation scheduling problem for the entire SoS-based power system [17]–[19].

Assume the optimization problem of the $\mathrm{DISCO_j}$, located in the lower level. A set of augmented Lagrangian penalty functions are added to the objective function of the DISCOj's problem each of which represents impact of a pseudo generation (a shared variable with ISO) at a specific time period. Using these penalty function as well as penalty multipliers, $\mathrm{DISCO_j}$ can individually solves its own local SCUC problem receiving a prescheduled value for the pseudo load from ISO.

$$Min \sum_{t=1}^{NT}\sum_{i=1}^{NGd_\mathrm{j}} \left(F_i(P_{it}^d)I_{it} + SUD_{it}^d\right) + \sum_{t=1}^{NT}(\alpha_{jt}(PD_{O,jt}^{*} - PG_{D,jt}) + \left\|\beta_{jt}\circ(PD_{O,jt}^{*} - PG_{D,jt})\right\|_2^2) \tag{10}$$

where the symbol $\circ$ represents the Hadamard product. The first term of (10) is summation of operating and startup/shutdown costs of the dispatchable generating units located in $\mathrm{DISCO_j}$, and the second term is the quadratic penalty function related to the shared variables with ISO over the time horizon. Notice that in the penalty function, variables $PG_{D,jt}$ need to be determined, but the values of $PD_{O,jt}^{*}$ are pre-scheduled received from the ISO. Meanwhile, the regular SCUC constraints as well as (8) and (9) should be satisfied. Moreover, in order to model the stochastic characteristics of the demand and renewable power generations on the shared variables between $\mathrm{DISCO_j}$ and the ISO, a new chance constraint (11) needs to be satisfied in the $\mathrm{DISCO_j}$'s problem.

$$\Pr\left\{PG_{D,jt} \le PL_{lt}^{\max}\right\} \ge 1 - TLOP_{lt} \quad \forall t, l \in tie - line. \tag{11}$$

Note that as we mentioned before, the $\mathrm{DISCO_j}$ and ISO may select different values for loss of load probability and probability of line overload to model the uncertainty in their own optimization problem, but they need to work together to set the same $TLOP_{lt}$ for the chance constraints modeling the shared variables between them.

In order to handle the chance constraint (11) in the DISCO's optimization problem, we convert it into an equivalent deterministic constraint (12). This constraint makes sure that the power exchange between the $\mathrm{DISCO_j}$ and ISO is within the acceptable range with a pre-specified probability.

$$PG_{D,jt} \le PL_{lt}^{\max} - Z_{TLOP_{lt}}\left[\sum_{w=1}^{NWd}\left(SF_{lw}^d\sigma_{wt}^d\right)^2 + \sum_{s=1}^{NSd}\left(SF_{ls}^d\sigma_{st}^d\right)^2 + \sum_{b=1}^{NBd}\left(SF_{lb}^d\sigma_{bt}^d\right)^2\right]^{1/2} \tag{12}$$

where SF_{lw}^d, SF_{ls}^d and SF_{lb}^d are local shift factors of the ADG representing the impact of uncertain demand, and wind and solar power generations on the power flow in the tie-line connecting $\mathrm{ADG_j}$ to the ISO; and σ_{lw}^d, σ_{ls}^d and σ_{lb}^d are standard deviations of the corresponding random variables located in $\mathrm{ADG_j}$. Notice that only the random variables of the $\mathrm{DISCO_j}$'s appear in (12).

The optimization problem (13) is for the ISO in which the objective is the summation of operating and startup/shutdown costs of the dispatchable generating units of the ISO as well as the augmented Lagrangian penalty function associated with the shared variables. The prescheduled values for the pseudo generation received from the DISCOs are used to build the penalty function. Thus, in this problem, $PD_{O,jt}$ is treated as the vector of decision variables while $PG_{D,jt}^{*}$ is treated as a constant term.

$$Min \sum_{t=1}^{NT}\sum_{i=1}^{NGo}(F_i(P_{it}^o)I_{it} + SUD_{it}^o) + \sum_{t=1}^{NT}\sum_{\mathrm{j}=1}^{NA}(\alpha_{jt}(PD_{O,jt} - PG_{D,jt}^{*}) + \left\|\beta_{jt}\circ(PD_{O,jt} - PG_{D,jt}^{*})\right\|_2^2). \tag{13}$$

The regular SCUC constraints as well as (8) and (9) should be satisfied. Similar to the DISCO's optimization problem, the following chance constraint for the tie-line should be satisfied in the ISO's problem.

$$\Pr\left\{PD_{O,jt} \le PL_{lt}^{\max}\right\} \ge 1 - TLOP_{lt} \quad \forall t, l \in tie - line \tag{14}$$

which can be converted to a deterministic constraint as (15):

$$PD_{O,jt} \le PL_{lt}^{\max} - Z_{TLOP_{lt}}\left[\sum_{w=1}^{NWo}(SF_{lw}^o\sigma_{wt}^o)^2 + \sum_{s=1}^{NSo}(SF_{ls}^o\sigma_{st}^o)^2 + \sum_{b=1}^{NBo}(SF_{lb}^o\sigma_{bt}^o)^2\right]^{1/2} \tag{15}$$

where SF_{lw}^o, SF_{ls}^o and SF_{lb}^o are local shift factors of the ISO representing the impact of uncertain wind power, solar power, and load on the power flow in the tie-line connecting ISO to the $\mathrm{ADG_j}$; and σ_{lw}^o, σ_{ls}^o and σ_{lb}^o are standard deviations of the random variables located in transmission network. Similar to the DISCO's problem, only the random variables of the ISO's are used in (15). Assume that $\mathrm{ADG_j}$ is radially connected to the transmission network through the tie-line. So, the shift factors associated with all buses of the distribution system on this tie-line is minus one (as the default positive direction of the shift factors is from transmission network toward ADGs), which means any change in the power injected on buses of the ADG will result in the same but opposite change in the power flow through the tie-line. Therefore, the power flow limit of the tie-line (12), from the $\mathrm{DISCO_j}$'s perspective, $PG_{D,jt}$, can be rewritten as (16).

$$PG_{D,jt} \le PL_{lt}^{\max} - Z_{TLOP_{lt}}\left[\sum_{w=1}^{NWd}\left(-\sigma_{wt}^d\right)^2 + \sum_{s=1}^{NSd}\left(-\sigma_{st}^d\right)^2 + \sum_{b=1}^{NBd}\left(-\sigma_{bt}^d\right)^2\right]^{1/2}. \tag{16}$$

Under similar assumption that the $\mathrm{ADG_j}$ is radially connected to the transmission network through one tie-line, the shift factors associated with all buses of the transmission network on the tie-line connecting ISO to $\mathrm{ADG_j}$ are zero (as the reference bus of the entire system is located within the ISO).

It means that any change in the power injected to the buses of the transmission network has no influence on this tie-line. Thus, the power flow limit of the tie-line (15), from the ISO's perspective, $PD_{O,jt}$, is written as follows:

$$PD_{O,jt} \leq PL_{lt}^{\max}. \tag{17}$$

C. Solution Procedure

An iterative solution procedure is presented to solve this chance-constrained SoS-based generation scheduling problem and determine the hourly unit commitment and generation dispatch for the ISO and DISCOs. This algorithm is explained as follows.

Step 1: Set the iteration index $k = 1$, and choose initial values for $PD_{O,jt}^{*k-1}$, α_{jt}^{k} and β_{jt}^{k}.

Step 2: Solve the generation scheduling problem for each DISCO with the variable $PG_{D,jt}^{k}$ and the values of $PD_{O,jt}^{*k-1}$ from the previous iteration.

Step 3: Solve the generation scheduling problem for ISO with the variable $PD_{O,jt}^{k}$ and the values of $PG_{D,jt}^{*k}$ obtained in Step 2.

Step 4: Check the following necessary-consistency (18) and sufficient (19) stopping criteria. If they are not satisfied, go to Step 5; otherwise, the converged optimal result is obtained and the solution procedure stops.

Necessary-consistency condition:

$$PD_{O,jt}^{k} - PG_{D,jt}^{k} \leq \varepsilon_1 \quad \forall j,\ \forall t. \tag{18}$$

Sufficiency condition:

$$\left| \frac{f_s(\mathbf{x}^{(k)}) - f_s(\mathbf{x}^{(k-1)})}{f_s(\mathbf{x}^{(k)})} \right| \leq \varepsilon_2 \tag{19}$$

where f_s is objective function of the independent system S. Note that the smaller stopping criteria are, the more accurate result will be obtained; however, the algorithm needs more iterations and computation time. Thus, a tradeoff between the computation time and accuracy should be considered.

Step 5: Set $k = k + 1$ and update the values of multipliers α_{jt}^{k} and β_{jt}^{k} using (20) and (21), and return to Step 2

$$\alpha_{jt}^{k} = \alpha_{jt}^{(k-1)} + 2(\beta_{jt}^{(k-1)})^2 (PD_{o,jt}^{k-1} - PG_{D,jt}^{k-1}) \tag{20}$$

$$\beta_{jt}^{k} = \lambda \beta_{jt}^{(k-1)} \tag{21}$$

where the coefficient λ is necessary to be equal or larger than one in order to get the converged optimal results. The success of the augmented Lagrangian relaxation depends on the ability of the algorithm to drive Lagrangian multipliers to the value of multipliers associated with consistency constraints at the optimal solution. Updating schemes (20) and (21) for computing new values of Lagrangian multipliers are presented in [17]. In fact, applying the combination of augmented Lagrangian Relaxation method and (20) and (21) is known as the method of multipliers. The method of multipliers is proven to converge to the optimal results [17], [20]. In this method, if the difference between the shared variables gets a negative value, the penalty multiplier α, (20), also gets a negative value and therefore we have a positive value for the first term of the penalty function in (10) and (13). Thus, the algorithm tries to minimize the difference between the shared variables.

It has been proven that when the problem is a convex optimization, the ATC method used in this paper can converge to a feasible result that is indeed optimal result of the entire optimization problem [17], [19]. Although there is a convergence proof of the ATC algorithm for convex optimization problems, there is no direct proof for non-convex MIP problems which is studied in this paper. However, for a nonconvex optimization problem, the non-convexity can be mitigated by the augmented Lagrangian method in which quadratic penalty terms as a local convexifier [17] are added to the objective function to improve the convexity of the problem. In addition, according to our experiments/testing experiences, we would like to mention that the effectiveness of the ATC algorithm on the proposed non-convex collaborative power system operation problem is satisfactory and acceptable, which can be supported by the following case studies.

In the proposed distributed decision-making framework, all DISCOs collaborate and communicate with one ISO. Thus, the ISO can be committed as the entity that is in charge of updating the penalty multipliers and sending them to the DISCOs. However, an active distribution system can have its own right to refuse the penalty multipliers and work in islanded mode. In this condition, there is no need to consider this system in the distributed optimization process.

IV. Case Studies

The proposed chance-constrained SoS-based optimization algorithm is applied on a six-bus and the IEEE 118-bus test systems and the results are discussed. For a simulation purpose, in our paper, we solve the independent DISCOs' problems in a serial manner on a single PC. All cases utilize ILOG CPLEX 12.4's MIQP solver on a 3.4 GHZ personal computer.

A. Six-Bus Test System

The system topology and the characteristics of generating units, network information, and the hourly load distribution over 24-h horizon are given in [5]. The six-bus test system has 3 thermal generating units, 7 branches, and 3 demand sides in the transmission system. A wind farm is located at bus 3. Two active distribution grids are connected to the transmission system through buses 3 and 4, and one passive distribution grid is connected to bus 5. ADG1 includes 9 buses, 8 distribution lines, 5 load points, 2 dispatchable DG units, and 1 solar power generation located at bus 4; and ADG2 has 7 buses, 6 distribution lines, 4 loads, 2 dispatchable DG units, and 1 solar power generation located at bus 5.

In order to analyze the effectiveness of the proposed algorithm, we consider the following three case studies. In all cases, let us assume that the ISO and DISCOs select the same value for loss of load probability (*LOLP*) and probability of line overload (*TLOP*).

TABLE I
GENERATION DISPATCH (MW) IN CASE 1

Hour	ISO			DISCO1		DISCO2	
	Unit1	Unit2	Unit3	DG1	DG2	DG1	DG2
1	112	0	0	0	18	25	0
2	106	0	0	0	18	25	0
3	103	0	0	0	18	25	0
4	93	0	0	0	18	25	0
5	93	0	0	0	18	25	0
6	103.4	0	0	0	18	25	0
7	112.3	0	0	0	18	25	0
8	115.1	0	0	0	18	25	0
9	101.8	0	25	0	18	25	0
10	119.7	0	25	0	18	25	0
11	128	0	25	15	18	25	3.3
12	132.7	0	25	15	18	25	6.4
13	137.2	0	25	15	18	25	8.7
14	138.3	0	25	15	18	25	9.3
15	142.3	0	25	15	18	25	11.2
16	146.5	0	25	15	18	25	14.1
17	144.5	0	25	15	18	25	14
18	137.2	0	25	15	18	25	10.6
19	135.7	0	25	15	18	25	10.4
20	129.5	0	25	15	18	25	7
21	138.7	0	25	15	18	25	7.3
22	126.8	0	25	15	18	25	6.2
23	123	0	25	0	18	25	0
24	123	0	25	0	18	25	0

Case 1: The generation scheduling with $\sigma_D = \sigma_w = \sigma_{s1} = \sigma_{s2} = 0\%$

Case 2: The generation scheduling with $\sigma_D = 1\%$, $\sigma_w = 20\%$, $\sigma_{s1} = \sigma_{s2} = 25\%$, and $LOLP_t = TLOP_{lt} = 25\%$.

Case 3: The generation scheduling with $\sigma_D = 1\%$, $\sigma_w = 20\%$, $\sigma_{s1} = \sigma_{s2} = 25\%$, and $LOLP_t = TLOP_{lt} = 5\%$.

1) Case 1: Based on the SoS concept, in this case, ISO and both ADGs are regarded as three independent systems with their own operation rules and information privacy. The standard deviation of hourly load forecast error (σ_D), generation forecast error of wind (σ_w), and solar panels of ADG1 (σ_{s1}) and ADG2 (σ_{s2}) is 0%. Hence, there are no stochastic constraints, and we have deterministic optimization problems for each independent system. The constraint related to the power transferred between the ISO and ADGS (shared variables) are also deterministic. The initial value for $\boldsymbol{\alpha}_{jt}^0 = \boldsymbol{\beta}_{jt}^0 = 1$ (j = 1, 2, and t = 1 : 24); and convergence thresholds ε_1 and ε_2 are set to 0.01 p.u. and 0.001, respectively. The proposed algorithm is applied to find the optimal operating point of the transmission system and ADGs. The algorithm converges after 5 iterations within 2 seconds.

The generation dispatch of the generating units is listed in Table I. Unit 2 of the ISO is a very expensive unit and is scheduled to be OFF all over the scheduling horizon; and DG1 of ADG1 and DG2 of ADG2 are (expensive) committed to be ON when the load is near the peak hours. During off-peak hours, unit 1 of the ISO, DG2 of ADG1 and DG1 of ADG2 are ON to supply the power consumed by the loads and spinning reserve. When the load is near the pick hours, unit 3 of the ISO provides power for the loads, however, it does not provide spinning reserve, and the required reserve is provided by unit 1 of the ISO and DG2 of ADG2. The operating cost of the independent systems is shown in Table II, and the total operating cost of the SoS-based power system is $55,360.

TABLE II
OPERATING COST OF THE INDEPENDENT SYSTEMS IN CASE 1

Independent system	Operating cost($)
ISO	39,321
DISCO1	5,765
DISCO2	10,274

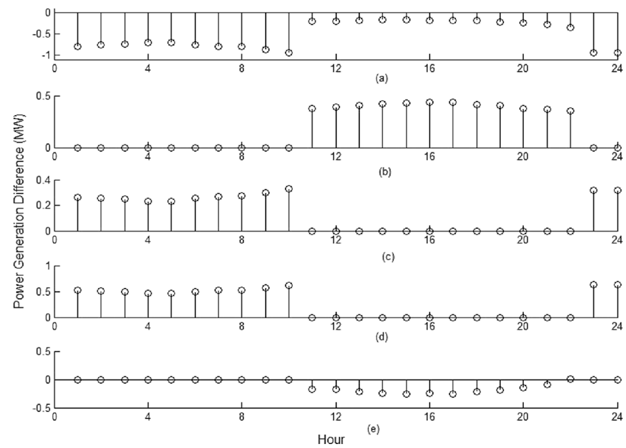

Fig. 3. Difference between power generated by (a) unit 1 of ISO, (b) DG1 of ADG1, (c) DG2 of ADG1, (d) DG1 of ADG2, and (e) DG2 of ADG2 in Cases 1 and 2.

To validate the above result obtained from the proposed distributed algorithm, we assume the entire grid as a single system and solve a conventional centralized optimization problem. The total operating cost obtained by the centralized optimization method is $55,360 which is the same as the cost obtained by the proposed distributed algorithm.

2) Case 2: The standard deviations of hourly load forecast error (σ_D), generation forecast error of wind (σ_w), and solar panels of ADG1 (σ_{s1}) and ADG2 (σ_{s2}) are 1%, 20%, 25%, and 25%, respectively; and $LOLP_t$ and $TLOP_{lt}$ are 25%. Compared with Case 1, in this case, there are several stochastic constraints, and we need to apply the chance-constrained optimization technique to this SOS-based power system. The proposed decision-making algorithm takes 2 seconds to converge to an optimal solution after 5 iterations. In this case, in order to ensure that the stochastic constraints are within the acceptable range with pre-specified probability, the hourly power dispatch of unit 1 of the ISO, and DGs of both ADGs 1 and 2 have changed compared with Case 1. These changes are depicted in Fig. 3. For example, in hour 24, the power generated by unit 1 of the ISO has decreased almost 1 MW. The operating cost of the ISO, DISCO1 and DISCO2 is shown in Table III. The total operating cost of the SoS-based power system is $55,510 which is $150 larger than that in Case 1. These differences between Cases 1 and 2 are because of considering demand and renewable generation uncertainties in optimization problems of the independent systems in Case 2, and taking into account the chance-constraints to satisfy the risk level of the systems in accordance with the value of $LOLP_t$ and $TLOP_{lt}$ indices.

TABLE III
OPERATING COST OF THE INDEPENDENT SYSTEMS IN CASE 2

Independent system	Operating cost($)
ISO	39,493
DISCO1	5,715
DISCO2	10,302

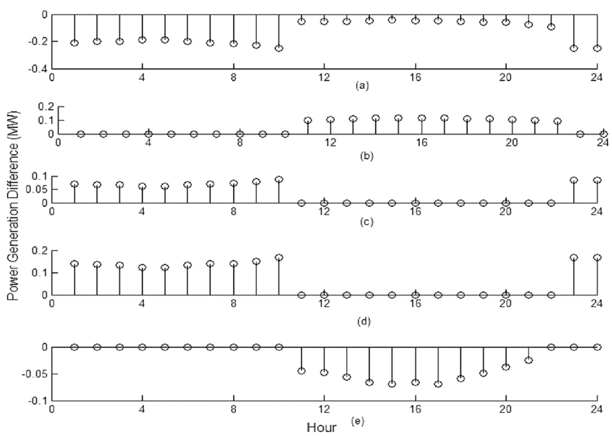

Fig. 4. Difference between power generated by (a) unit 1 of ISO, (b) DG1 of ADG1, (c) DG2 of ADG1, (d) DG1 of ADG2, and (e) DG2 of ADG2 in Cases 2 and 3.

TABLE IV
OPERATING COST OF THE INDEPENDENT SYSTEMS IN CASE 3

Independent system	Operating cost($)
ISO	39,560
DISCO1	5,700
DISCO2	10,320

Similarly, we solve Case 2 using a conventional centralized optimization method and get a same result as using the proposed distributed algorithm.

3) Case 3: Using the same standard deviations as Case2, but $LOLP_t$ and $TLOP_{lt}$ are set to 5%. It reduces loss of load and transmission congestion expectation, and makes more limitation for the chance-constrained stochastic generation scheduling. The proposed algorithm converges after 5 iterations within 2 seconds. Changes in $LOLP_t$ and $TLOP_{lt}$ from 25% to 5% impact the power generations by dispatchable generating units. Fig. 4 shows these changes compared with Case 2. For example, the power output of unit 1 of the ISO at hour 16 increases by 0.32 MW; and DG2 of ADG2 requires to provide 0.44 MW more power at hour 17 compared with that in Case 2. As shown in Table IV, the operating costs of ISO, DISCO1 and DISCO2 are \$39,560, \$5,700 and \$10,320, respectively. The total operating cost of the SoS-based power system is \$55,580 which is \$220 more than that in Case 1. Also, this distributed result is the same as the centralized one.

B. Sensitivity Analysis on the Tie-Lines Chance Constraints

In order to analyze the impact of chance constraints of the tie-lines connecting the ISO to the DISCOs on the SoS-based distributed scheduling algorithm, a sensitivity analysis is performed. The $TLOP_{lt}$ and $LOLP_t$ of all independent systems are 25%. Setting different values for the $TLOP_{lt}$ of the tie-lines (100% to 10%), the proposed algorithm is implemented. In fact, we vary the chance that the power flow in each tie-line is less than its maximum capacity (the security level of each tie-line). Moreover, different values are considered for the standard deviation of the PDFs of the wind and solar power generations, which means we have different accuracy levels for wind and solar power prediction. The standard deviation of the wind power varies from 15% to 65% of the wind power output; and it varies from 20% to 70% of solar power production. The total operating point of the SoS-based power system is shown in Fig. 5. The operating cost increases by decreasing the $LOLP_t$ (increasing the security level of the tie-lines). Also, the results show that better prediction for the wind and solar power generation causes reduction in the total operating cost of the SoS-based power system.

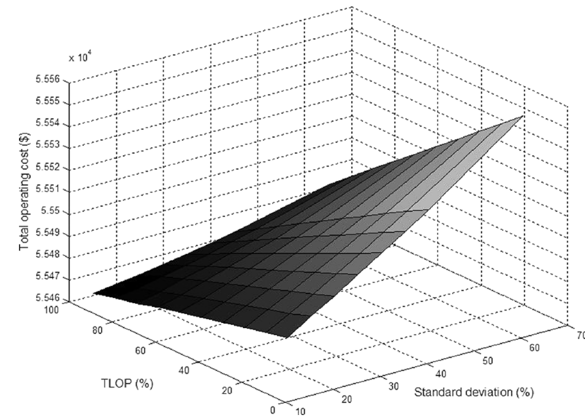

Fig. 5. Total operating cost versus TLOP and standard deviation.

C. The IEEE 118-Bus Test System

The proposed chance-constrained SoS-based generation scheduling has been applied to the modified IEEE 118-bus test system. We have considered 30 active distribution grids connected to the transmission network each of which is operated by an independent DISCO. Therefore, in the SoS-based generation scheduling, there are 31 independent systems (30 DISCOs and one ISO) collaborating together to operate the entire power systems in a secure and economic manner. The ISO has 54 thermal generating units, 61 loads (or inactive distribution grids), 187 branches, and 3 wind farms connected to buses 10, 70 and 110, respectively. There are 30 solar power farms connected to the ADGs. The standard deviations of hourly demand (σ_D), wind generation (σ_w), and solar generation (σ_s) of ADGs are 1%, 20%, 25%, and 25%, respectively; and $LOLP_t$ and $TLOP_{lt}$ are 5%. To find the optimal hourly unit commitment and generation dispatch, the proposed algorithm is implemented setting the initial values $\boldsymbol{\alpha}_{jt}^0 = \boldsymbol{\beta}_{jt}^0 = 1$ (j = 1, 2, and t = 1 : 24), $\varepsilon_1 = 0.01$ p.u., and $\varepsilon_2 = 0.001$. The distributed SoS-based decision-making algorithm takes around 1.5 minutes to converge to an optimal solution after 17 iterations. As an example, Fig. 6 shows the converged amount of power exchange (shared variables) between ISO and DISCO1 at hour 19 in each iteration. Operating costs of individual DISCOs are shown in Fig. 7; and operating cost of the ISO

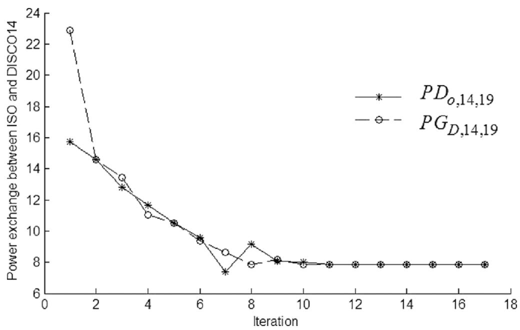

Fig. 6. Power exchange (shared variable) between ISO and ADG14 at hour 19.

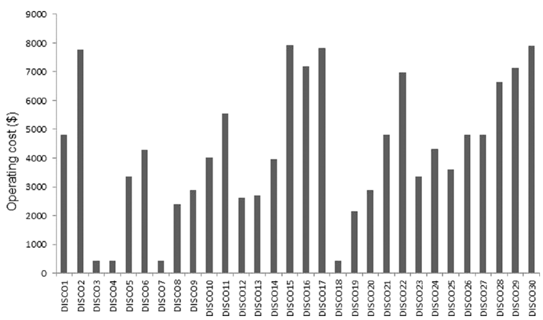

Fig. 7. Total operating cost of the DISCOs.

TABLE V
TOTAL OPERATING COST OF SoS-BASED POWER SYSTEM

Solution Algorithm	Operating cost($)	# of iteration
Centralized	1,184,000	-
Distributed	1,186,200	17

is $1,058,000. As listed in Table V, the total operating cost of the SoS-based power system is $1,186,200. To check the validity of the results, the conventional centralized algorithm is implemented and the total operating cost is $1,184,000. The difference between the operating costs obtained by these two algorithms is 0.18% ($2,200) which is acceptable.

V. CONCLUSION

In modern power systems, incorporation of generation sources of active distribution grids in the power system operation enhances the economic and security of the entire system. Since the transmission and distribution grids are operated by different system operators in the electricity market, making a collaborative and optimal operation among these systems is an important and challenging problem. This becomes more challenging when there are uncertain decision variables in the systems such as load and renewable power generation uncertainties. This paper modeled a power system as a system of systems in which the ISO and each DISCO were autonomous systems, but can cooperate with each other. Considering the uncertainties, the expected stochastic power balance constraint was regarded, and the chance-constrained stochastic programing was applied to model the reserve requirements and line flow limits for each independent system.

The ISO and DISCOs can use the proposed distributed decision-making algorithm for a collaborative day-ahead generation scheduling under uncertainty. The ISO formulates its own SCUC problem taking into account the shared information between the ISO and the DISCOs. Each DISCO also formulates its own local SCUC problem considering the shared information between the distribution system and the ISO. As a result, the proposed distributed algorithm can determine the optimal operating point of the entire system with no need of sharing all the systems' information between the ISO and DISCOs. Thus, the information privacy of each autonomous system can be respected. In addition, the chance-constraints enforced in the local optimization problem of each independent system make sure that transmission/distribution line flows as well as the power flow in the tie-lines are within their limit with pre-specified confidence levels. The chance-constraints also ensure that there is adequate local reserve in each independent system in response to generation/demand uncertainties.

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