

The 8<sup>th</sup> International Conference on Applied Energy – ICAE2016

## Electricity Market Nash-Cournot Equilibrium Analysis with High Proportion of Gas-Fired Generators

Gang Chen<sup>a</sup>, En Lu<sup>a</sup>, Kaiwen Zeng<sup>a</sup>, Chao Dong<sup>a</sup>, Shizhao Hu<sup>a</sup>,  
Hongye Guo<sup>b</sup>, Qixin Chen<sup>b,\*</sup>

<sup>a</sup> Electric Power Dispatching Center of Guangdong Province, Guangzhou, 510600, China

<sup>b</sup> Department of Electrical Engineering, Tsinghua university, Beijing, 100084, China

### Abstract

With the increasing penetration of renewable energy in the power system and stricter restrictions on the air pollutants, more gas-fired generators have been accessed into the power system. To analyze the impacts of high proportion of gas-fired units on the power system, a liberalized electricity market Nash-Cournot equilibrium model has been proposed in this paper. Then a numerical example is given to present the influence of high proportion of gas-fired units on the energy price, demand and the accommodation of renewable energy in the market equilibrium. Comparisons between high and low proportion of gas-fired units in the power systems are also presented.

*Keywords:* Gas-fired units; electricity market; Nash-Cournot equilibrium

### Nomenclature

ESS	Energy storage system.
MCP	Marginal clearing price.
NCP	Nonlinear complementarity programming.
KKT	Karush-Kuhn-Tucker.
MIQCP	Mixed integer quadratic constrained programming.
LPGF	Low proportion of gas-fired units.
HPGF	High proportion of gas-fired units.
$i$	Unit index.
$t$	Time index.
$T$	Set of time.

\* Corresponding author. Tel. +86-10-62794361; fax:+86-10-62788116.

E-mail address: [qxchen@tsinghua.edu.cn](mailto:qxchen@tsinghua.edu.cn), [hy-guo15@mails.tsinghua.edu.cn](mailto:hy-guo15@mails.tsinghua.edu.cn)

C	Subscript for coal-fired units and set of coal-fired units.
G	Subscript for gas-fired units and set of gas-fired units.
R	Subscript for renewable units and set of renewable units.
D	Subscript for demand.
RMPD	Superscript for ramp down limits of units.
RMPU	Superscript for ramp up limits of units.
$a_{(i)}, b_{(i)}$	Coefficient of the total generation cost.
$\alpha(t), \beta(t)$	Coefficient of the inverse demand function in period $t$ .
$\pi_{(i)}(t)$	Profit of the unit in period $t$ .
$\lambda(t)$	Marginal clearing price of the energy market in period $t$ .
$Q_{(i)}(t)$	the generation output of the unit in the period $t$ .

## 1. Introduction

Facing stricter restrictions on emissions of air pollutants during energy generation and the increasing demand of clean energy, the penetrations of renewable energy are getting higher in many areas. Renewable energy, such as wind power and solar energy, is eco-friendly and low-carbon, but with the intermittent and stochastic generating natures [1, 2], which put plenty of pressure on the operations of power systems and electricity markets at the supply side. Moreover, more and more high power electric equipment, such as electrical vehicles, have got access to power systems, bringing more uncertainty at the demand side of power systems and electricity markets.

To accommodate the trend of high penetration of renewable energy and stochastic power generation and demand, energy generation mix has changed in many areas, where the conventional fossil energy is undergoing a decreasing share in the primary inputs to power generation. At the meantime, the energy storage systems (ESSs) and gas-fired units have grabbed more attention. Due to the ability in charging and discharging within a short time, ESSs have great potential to provide flexibility to the power systems [3]. Since the prices of ESSs are relatively expensive, it is rarely a practical and economic method to build a number of ESSs in the power systems. In the meantime, gas-fired units, which have faster ramping rates, less air pollutants output and more economical cost, have become appropriate alternatives of obsolete coal-fired units [4].

Recently, Chinese government has issued a series of policies, aiming at promoting the accommodation of renewable energy, increasing the proportion of gas-fired units and reducing the proportion of coal-fired units [5, 6]. There have been many studies on the electricity market equilibrium with multiple generations. Reference [4] analyzed the competitiveness of gas-fired units under different environmental policies. But it didn't analyze the impacts of high proportion of gas-fired units on the renewable energy accommodation and the electricity price is given by historical data. Reference [7] used a stochastic Cournot model to represent the strategic behavior of the wind generators. With the bidding strategy proposed in the paper, the profit for wind firms might be increased. Reference [8] proposed a Cournot model solved by potential function to evaluate the contribution of energy storage to support large-scale renewable generation in joint energy and ancillary service markets. Reference [9] presented a multi-nodal intertemporal Cournot gaming model to simulate capacity and energy-only markets under high renewable penetration.

Different with the articles listed above, this paper establishes a Nash-Cournot equilibrium model for electricity market with high proportion of gas-fired units, to accurately study the impacts of the alternative of large amount of gas-fired units on the integration of renewable energy and electricity prices.

The main contributions of this paper are given here.

- 1) Proposed a Nash-Cournot market equilibrium model with high proportion of gas-fired units;
- 2) Analyzed the impacts of the high proportion of gas-fired units on the electricity market equilibrium;
- 3) Analyzed the influence of the high proportion of gas-fired units on the renewable energy accommodation.

## 2. Modeling of the Nash-Cournot Equilibrium in the Electricity Market

In the liberalized electricity market consisting of various participants, the Nash-Cournot equilibrium is usually formulated as a bi-level optimization model, in which the upper-level model indicates each participant's profit-maximization objective, whereas the lower-level model represents the clearing conditions of the electricity market. To concentrate on the key research point, only energy market is considered in this paper. Next, by substituting the common lower-level electricity market model into each upper-level model, we can obtain a multi-individual optimization problem. Then, the solution method of this problem is given in the section 3.

As the classic Nash-Cournot model, there is no objective function in the lower-level model. But the optimization problem proposed in this paper have the similar mathematical structure as bi-level models, it should be still called the bi-level optimization model [10]. Then, profit maximization models for each market participant and the lower-level model of the market equilibrium are listed below.

### 2.1. Profit Maximization Model for Coal-Fired Units

The objective function of the individual profit-maximization model of coal-fired unit is formulated as the following:

$$\max \sum_{t=1}^T \pi_{Ci}(t) = \max \sum_{t=1}^T \{ \lambda(t) Q_{Ci}(t) - [a_{Ci} Q_{Ci}^2(t) + b_{Ci} Q_{Ci}(t)] \} \quad (1)$$

where  $\pi_{Ci}$  indicates the profit of the coal-fired unit,  $\lambda(t)$  is the marginal clearing price (MCP) of the energy market in period  $t$ ,  $Q_{Ci}(t)$  indicates the generation output of the coal-fired unit  $Ci$  in the period  $t$ ,  $a_{Ci}$  and  $b_{Ci}$  represent the coefficients of the total generation cost.

The capacity constraints as well as ramp up and down rate limits of the coal-fired units are:  $\forall t \in T$ ,

$$\begin{cases} Q_{Ci \min} \leq Q_{Ci}(t) \leq Q_{Ci \max} \\ Q_{Ci}^{RMPD} \leq Q_{Ci}(t+1) - Q_{Ci}(t) \leq Q_{Ci}^{RMPU} \end{cases} \quad (2)$$

where  $Q_{Ci \min}$  and  $Q_{Ci \max}$  are the minimum and maximum generation output limits of the coal-fired unit  $Ci$ ,  $Q_{Ci}^{RMPD}$  and  $Q_{Ci}^{RMPU}$  are the ramp down and up limits of the coal-fired unit  $Ci$ .

### 2.2. Profit Maximization Model for Gas-Fired Units

The objective function of the individual profit-maximization model of gas-fired unit is in the same formulation as coal-fired units:

$$\max \sum_{t=1}^T \pi_{Gi}(t) = \max \sum_{t=1}^T \{ \lambda(t) Q_{Gi}(t) - [a_{Gi} Q_{Gi}^2(t) + b_{Gi} Q_{Gi}(t)] \} \quad (3)$$

where  $\pi_{Gi}$  indicates the profit of the gas-fired unit,  $Q_{Gi}(t)$  indicates the generation output of the gas-fired unit  $Gi$  in the period  $t$ ,  $a_{Gi}$  and  $b_{Gi}$  represent the coefficients of the total generation cost.

With the high flexibility in power output, the ramp up and down limits of gas-fired units are usually omitted. To simulate the gas-fired unit model more accurately, the ramp limits are still considered in this paper, which are:

$$\begin{cases} Q_{Gi \min} \leq Q_{Gi}(t) \leq Q_{Gi \max} \\ Q_{Gi}^{RMPD} \leq Q_{Gi}(t+1) - Q_{Gi}(t) \leq Q_{Gi}^{RMPU} \end{cases} \quad (4)$$

where  $Q_{Gi \min}$  and  $Q_{Gi \max}$  are the minimum and maximum generation output limits of the gas-fired unit  $Gi$ ,  $Q_{Gi}^{RMPD}$  and  $Q_{Gi}^{RMPU}$  are the ramp down and up limits of the gas-fired unit  $Gi$ .

### 2.3. Profit Maximization Model for Renewable Units

The objective function of the individual profit-maximization model of wind farms or solar stations is to maximize the payments from the energy market:

$$\max \sum_{t=1}^T \pi_{Ri}(t) = \max \sum_{t=1}^T \lambda(t) Q_{Ri}(t) \quad (5)$$

where  $Q_{Ri}(t)$  is the generation output decided by the renewables  $Ri$  in period  $t$ .

The capacity constraints of renewable energy are decided by the time-varying weather:  $\forall t \in T$ ,

$$Q_{Ri \min}(t) \leq Q_{Ri}(t) \leq Q_{Ri \max}(t) \quad (6)$$

where  $Q_{Ri \min}(t)$  and  $Q_{Ri \max}(t)$  are the minimum and maximum generation output limits.

### 2.4. Market Clearing Conditions

The Nash-Cournot energy market clearing conditions incorporate the multi-period power balance constraints and the linear inverse demand function. For  $t = 1, 2, \dots, T$ , there are:

$$\begin{cases} Q_D(t) = \sum_{i=1}^N Q_{Ti}(t) = \sum_{i \in C} Q_{Ci}(t) + \sum_{i \in G} Q_{Gi}(t) + \sum_{i \in R} Q_{Ri}(t) \\ \lambda(t) = \alpha(t) - \beta(t) Q_D(t) \end{cases} \quad (7)$$

where  $T = C + G + R$  is the total assembly of various generators participating in the energy market, and the number of  $T$  is  $N$ ,  $Q_{Ti}(t)$  represents total power output of all generation units,  $Q_D(t)$  is the electricity demand in period  $t$ ,  $\alpha(t)$  and  $\beta(t)$  are the coefficients of the inverse demand function in period  $t$ . With the help of the decreasing inverse demand function, the procedure how the generators strategically decide the energy output with the consideration of other rivals' possible reactions to maximize its individual profit could be simulated clearly.

## 3. Solution Method

The liberalized electricity market equilibrium with various units, is a bi-level model, solved by calculating multiple participants' model, which is really difficult to be solved directly. There are, in general, two methods to solve the Nash-Cournot model [11]. The first method is the diagonalization method based on Gauss-Seidel iteration and Jacobi iteration, which calculates each participant's profit-maximum decision, based on other participants' changeless decisions, until the equilibrium is figured out. Based on nonlinear complementarity programming (NCP), the second method deduces the Karush-Kuhn-Tucker (KKT) conditions of each participant's profit-maximum problem. Then all the KKT conditions are combined to be solved together, which gives the equilibrium. Considering the low efficiency, robustness and stability of the first method, in this paper, we used the second method to solve the equilibrium problem.

Specifically, the equation (7) in the lower level model, is substituted into various units’ constraints in the upper level. Then, we can get a “Multi-Individual Problem”, which consists of various units’ profit-maximum model combined with market clearing conditions. Next, the KKT conditions of each unit’s model are calculated and solved together in “Integrated Model”, which belongs to the mixed integer quadratic constrained programming (MIQCP). Moreover, the CPLEX under GAMS is used to solve the MIQCP problem.

#### 4. Numerical Examples

##### 4.1. Basic Data

Numerical examples are implemented on a modified scale power system with various types of generation. The total installed capacity is 2580 MW, including the renewable units. The slope of the inverse demand function  $\beta(t)$ , representing the demand elasticity is constant as 0.01 and the intercept  $\alpha(t)$ , representing the highest payment level consumers can afford, which is shown in the Fig. 1 (a).

Two basic scenario settings are included for comparison in this paper. The low proportion of gas-fired units (LPGF) consists of 5 coal-fired units (40.7% of the total generation mix), 9 gas-fired units (10.4%) and 8 wind turbines (24.8%) and 9 photovoltaic stations (24.1%), representing a case with low proportion of gas-fired units. In the case of the high proportion of gas-fired units (HPGF), all coal-fired units have been replaced by gas units, which means that gas-fired units’ proportion of total generation mix exceeds 50%.

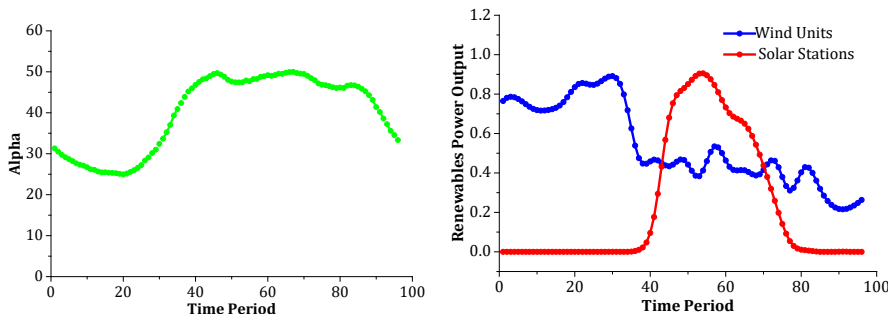


Fig. 1 (a) the curve of  $\alpha(t)$ , (b) the typical power output curves of wind and solar power

The generation costs and the technical parameters of the coal-fired and gas-fired units, are referenced from reference [12]. The wind farms have three kinds of rated capacities: 60 MW, 80 MW, 100 MW and 120 MW and the number of each kind of wind units are 4, 1, 2 and 1, respectively. For solar stations, they have four kinds of rated capacities: 60 MW, 80 MW and 100 MW, with the number of each kind of solar units are 5, 2 and 1.

The power output of wind units and solar stations is based on the typical curves of renewable energy in one day, which is shown in Fig. 1 (b).

##### 4.2. Comparison Analysis between High and Low Proportion of Gas-Fired Units

The market equilibriums in the scenes of HPGF and LPGF are solved, with the energy price and energy demand curves shown in Fig. 2. Due to the relatively lower operation cost of coal-fired units, the energy prices in the scene of LPGF is lower than the energy price of HPGF. Accordingly, the energy demand of HPGF is obviously lower than the demand of LPGF, indicating all high-cost gas-fired units prefer to operate at the low limits.

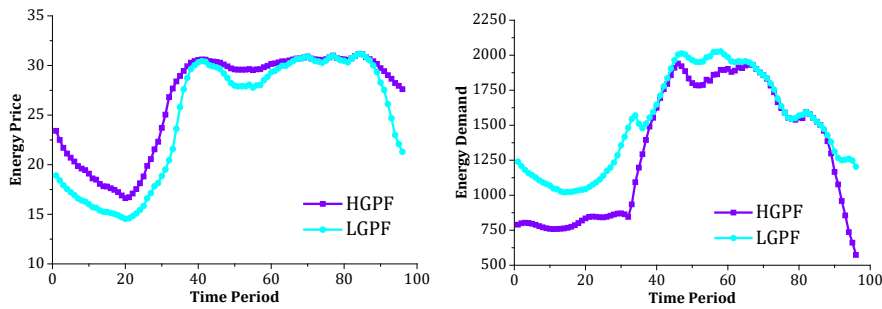


Fig. 2 (a) Energy Price and (b) Energy Demand in the scenes of HGPF and LGPF

As the Fig. 1 (b) shows, the renewables output curves are really smooth when used for day-ahead economic dispatch, which is actually fluctuant and unpredictable, like Fig. 3 (a) shows. Thus, it is necessary to analyze the different scenes' integration ability of renewable energy. It should be noted that the Nash-Cournot equilibrium model cannot simulate the rigidity the load. Thus, the fair way to make comparison is to put the two cases in the same operation status. We use the LGPF operation status as the base status in this numerical examples. The ramping resources needs of renewables are presented in Fig. 3 (b).

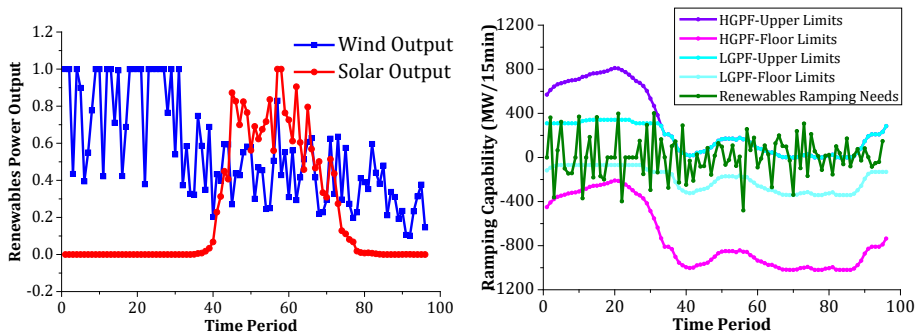


Fig. 3 (a) Actual Renewables Power Output, (b) Renewables Ramping Needs and Ramping Limits of HGPF and LGPF

It is obvious that the ramping limits range of HGPF is much wider and the range of LGPF, owing to the high ramping ability of gas-fired units. As for the curve of renewables ramping needs, it often breaks through the boundary of LGPF ramping limits but is usually in the range of HGPF ramping limits. Based on the data, HGPF could integrate 543 MWh renewable energy more than LGPF.

## 5. Conclusions

This paper gives an electricity market Nash-Cournot equilibrium analysis with high proportion of gas-fired generators. The results of numerical examples show that:

- 1) During the valley time, the energy price of HGPF is relatively higher than those of LGPF. During the peak time, the energy price of HGPF is slightly higher than those of LGPF
- 2) During the valley time, the energy demand of HGPF is much lower than those of LGPF. During the peak time, the energy demand of HGPF is slightly lower than those of LGPF.
- 3) In the actual operation, HGPF has more ramping capability than LGPF and is able to accommodate more renewable energy, especially during the valley time.

And it is also worthy of studying the market equilibrium of high gas-fired units by other methods in the future.

## Acknowledgements

This work is supported by Scientific Research Project of Guangdong Province Power Grid Company "Coordinated Dispatch and Control Technology for Combined Natural Gas and Power Grid Systems in Guangdong Province".

## References

- [1] R. Jiang, J. Wang, M. Zhang, and Y. Guan, "Two-Stage Minimax Regret Robust Unit Commitment," *IEEE Transactions on Power Systems*, vol. 28, pp. 2271-2282, 2013.
- [2] C. Woo, J. Zarnikau, J. Kadish, I. Horowitz, J. Wang, and A. Olson, "The Impact of Wind Generation on Wholesale Electricity Prices in the Hydro-Rich Pacific Northwest," *IEEE Transactions on Power Systems*, vol. 28, pp. 4245-4253, 2013.
- [3] G. He, Q. Chen, C. Kang, P. Pinson, and Q. Xia, "Optimal Bidding Strategy of Battery Storage in Power Markets Considering Performance-Based Regulation and Battery Cycle Life," *IEEE Transactions on Smart Grid*, p. 1-1.
- [4] R. Tian, Q. Zhang, G. Wang, H. Li, S. Chen, Y. Li, and Y. Tian, "Study on the promotion of natural gas-fired electricity with energy market reform in China using a dynamic game-theoretic model," *Applied Energy*, 2015.
- [5] EIA, "China-international energy data and analysis," 2015.
- [6] Reuters, "Chinese capital shuts third coal-fired plant in war on smog," 2015.
- [7] K. C. Sharma, R. Bhakar and N. P. Padhy, "Stochastic Cournot Model for Wind Power Trading in Electricity Markets," in IEEE Power and Energy Society General Meeting PESGM NEW YORK: IEEE, 2014.
- [8] P. Zou, Q. Chen, Q. Xia, G. He, and C. Kang, "Evaluating the Contribution of Energy Storages to Support Large-Scale Renewable Generation in Joint Energy and Ancillary Service Markets," *IEEE Transactions on Sustainable Energy*, vol. 7, pp. 808-818, 2016.
- [9] D. Chattopadhyay and T. Alpcan, "Capacity and Energy-Only Markets Under High Renewable Penetration," *IEEE Transactions on Power Systems*, vol. 31, pp. 1692-1702, 2016.
- [10] D. Fudenberg and J. Tirole, *Game Theory*, vol. I. Cambridge: MIT Press, 2013.
- [11] S. A. Gabriel, A. J. Conejo and J. D. Fuller, *Complementarity Modelling in Energy Markets*. New York: Springer, 2013.
- [12] V. Vahidinasab and S. Jadid, "Stochastic multiobjective self-scheduling of a power producer in joint energy and reserves markets," *Electric Power Systems Research*, vol. 80, pp. 760-769, 2010.



## Biography

Qixin Chen received the Ph.D. degree from the Department of Electrical Engineering at Tsinghua University, Beijing, China, in 2010. He is currently an associate professor at Tsinghua University. His research interests include low-carbon electricity, power economics, power markets, and power system planning.