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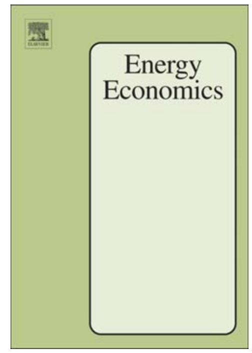

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3/11/2016

Abstract

This paper studies the effect of natural-gas fuel cost uncertainty on capacity investment and price in a competitive electricity market. Our model has a two-stage decision process. In the first stage, an independent power producer (IPP) builds its optimal capacity, conditional on its perceived uncertainties in fuel cost and electricity demand. In the second stage, equilibrium prices and quantities are determined by IPPs competing in a Cournot market. Under the empirically reasonable assumption that per MWh fuel costs are log-normally distributed, we find that a profit-maximizing IPP increases its capacity in response to rising fuel cost volatility. Consequently, the expected profit of the IPP and expected consumer surplus increase with volatility, rejecting the hypothesis that rising fuel cost uncertainty tends to adversely affect producers and consumers. Expected consumer surplus further increases if the IPP hedges the fuel cost risk. However, the IPP's optimal strategy is not to do so. The policy implication of these results is that the government should not intervene to reduce the price volatility of a well-functioning spot market for natural gas, chiefly because such intervention can have the unintended consequence of discouraging generation investment, raising electricity prices, and harming consumers.

Key words: Competitive electricity market, Natural gas price volatility, Generation investment under uncertainty, Electricity price

JEL codes: D24, D43, L11, L94, L95

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1. Introduction

Over the past three decades, the electricity sector in many countries has transitioned from an integrated monopoly to one with a deregulated generation market in which electricity prices and capacity investments reflect the decentralized decision making of independent power producers (IPPs) (Newbery, 1995, 2002, 2005; Joskow, 2006, 2008; Shively and Ferrare, 2010). An IPP's capacity investment is based on an assessment of expected future profits. Thus, the introduction of market competition exposes the IPP to risks previously borne by retail end-users under a regulated monopoly's cost of service ratemaking.

Large-scale natural gas developments (e.g., shale gas in the USA) have caused a price decline that encourages the use of natural gas in electricity generation. Relative to coal-fired generation, natural-gas-fired generation has less emissions and shorter construction periods (MIT, 2011). It is dispatchable in real time, offering operational flexibility for reliable grid integration of intermittent renewable resources. As a result, nearly all new plants in the USA are fueled by natural gas (DECC, 2012; EIA, 2013).

Natural-gas-fired generation faces large fuel cost risks because: (a) natural gas constitutes about 80% of its variable costs, and (b) natural gas has large price volatility, substantially more than those of coal and oil.[1] Indeed, the annualized price volatility of natural gas in 2014 is 96%, far above the 17% and 8% for Brent oil and Australian coal, respectively (Mastrangelo, 2007; Geman and Ohana, 2009; Roesser, 2009; Graves and Levine, 2010; Smead, 2010; BPC, 2011; Whitman and Bradley, 2011; Alterman, 2012); see Figure 1 below. Various consumer organizations have expressed concerns that the IPPs' fuel cost risk exposure and the natural-gas price volatility may impede investments in natural-gas-fired generation plants.[2]

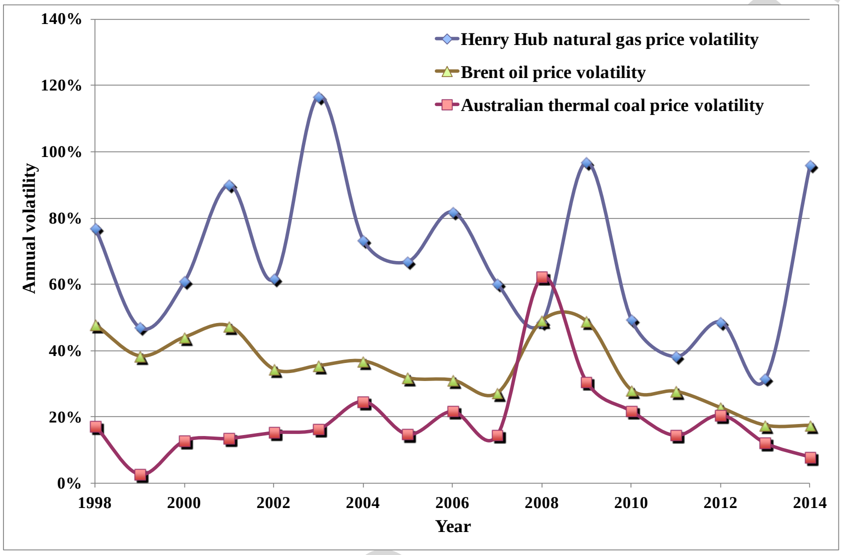
Despite its real-world relevance and importance, the effect of natural-gas fuel cost risk on market price and capacity investment in a liberalized electricity market has attracted little academic attention. To be fair, the seminal work of Dixit and Pindyck (1994) presents a framework for analyzing capacity investment under uncertainty, but

[1] Price volatility is commonly based on the daily percentage price changes over a pre-specified period (Roesser, 2009). The measure of volatility is taken from Eydeland and Wolyniec (2003). The analytical definition of volatility and its characteristics are detailed in the next section.

[2] See, for example, Whitman and Bradley (2011).

that framework was applied to a highly competitive market where a single firm's decision does not affect the market price.[3]

Figure 1: **Volatility of Henry Hub natural gas price, Brent oil price and Australian thermal coal price**



Prices were obtained from IMF Primary Commodity Prices (2014). Computation of the annualized volatility was based on the number of trading days in a year (Eydeland and Wolyniec, 2003).

This paper extends Tishler et al. (2008) to study the effect of uncertain natural gas prices and therefore fuel costs on capacity investment and prices in a competitive electricity market. Our main research hypothesis is based on a common belief that rising fuel cost uncertainty (volatility) likely impedes generation capacity investment and reduces the expected profits of IPPs and consumer surplus. Our key findings, however, qualitatively and numerically, reject this hypothesis.

Our findings come from a two-stage model of a liberalized market of electricity generation. In the first stage, an IPP builds its optimal capacity, conditional on its perceived uncertainties of fuel cost and electricity demand. In the second stage, equilibrium prices and quantities are determined by IPPs competing in a Cournot market environment. We show that a profit-maximizing IPP increases its capacity in

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[3] Applications of real option theory to oligopolistic framework, such as in Bouis et al. (2009), focus on investment timing, but not optimal capacity expansion, which is the main interest of our paper.

response to rising fuel cost volatility, so do the expected consumer surplus and expected producer profit. Expected consumer surplus further increases if the IPP hedges the fuel cost risk. The IPP's optimal strategy, however, is not to do so.

This paper makes two main contributions. First, it offers an analytical framework for assessing the effect of uncertain fuel costs on capacity investment in competitive electricity markets. The framework equally applies to other sectors and industries (e.g., petrochemical and aviation, where capacity investment is highly dependent on fuel cost volatility). Second, it demonstrates that government should not intervene to reduce the price volatility of a well-functioning natural gas spot market because such intervention can have the unintended consequence of discouraging generation investment, raising electricity prices, and reducing consumer surplus.

The paper proceeds as follows. To provide a contextual background, Section 2 discusses natural-gas price volatility. Section 3 is a literature review of electricity capacity investment under uncertainty. Section 4 develops a two-stage model that determines equilibrium capacity and electricity prices in a competitive electricity market under fuel cost uncertainty. Section 5 analyzes a three-stage model in which the fuel cost risk can be hedged via call options. Section 6 illustrates our model's empirics with a simplified electricity market based on the data of California, the eighth largest economy of the world.[4] Section 7 concludes.

2. Natural-gas price volatility

Annual price volatility of a commodity is commonly based on its daily percentage price changes over a 1-year period (Roesser, 2009).[5] Figure 1 shows the US daily natural gas price data's high volatility since the 1978 deregulation of the US natural gas market.

[4] http://www.latimes.com/business/la-fi-california-world-economy-20150702-story.html.

[5] Eydeland and Wolyniec (2003) define the annualized volatility, σ, as follows (page 82, eq. (3.9)):

$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}} - \frac{1}{n}\sum_{i=1}^{n}\frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}}\right)^2}$, where $\{P_i\}$ denotes the time series of historical prices observed at times t_i, $i = 0,\dots,n$, and $t_i - t_{i-1}$ are year fractions. A year fraction equals the length of the interval, in days, between two observations, divided by 365 or by 250 (when only trading days are accounted for).

Natural-gas price volatility is mainly caused by transportation constraints and storage limitations (Eydeland and Wolyniec, 2003). The transportation of natural gas is limited by pipeline capacity and/or liquefied natural gas (LNG) capacity. Natural gas storage is limited to depleted reservoirs, salt formations or LNG tanks. Production disruptions and demand spikes trigger large natural-gas price increases (Alterman, 2012), further magnified by low inventory (Geman and Ohana, 2009).

High natural gas price volatility implies that a natural-gas-fired generation plant's cash flow in a competitive electricity market is highly uncertain and this uncertainty may be further exacerbated by electricity demand uncertainty.[6] The plant's fuel cost risk hurts its owner's project financing (Stern, 1998), thus discouraging capacity investment that in turn causes electricity price spikes on days of high demand.

While an IPP may use call options to hedge against the fuel cost risk and stabilize its cash flows, the risk management strategy is not costless because its expected profit is reduced by the cost of procuring the call options. Hence, a risk-neutral IPP would not hedge *sans* the need for cash flow stability, as verified by our analysis of a three-stage model in Section 5.

3. Literature review

Prior to the 1980s, the electricity sectors in various regions of the world were vertically integrated monopolies. Generation expansion models were designed to find the minimal present value cost of meeting the projected future demand over a long planning horizon (e.g., 20 years), subject to such constraints as fuel availability, resource adequacy, and emissions limit. Demand growth is the main source of uncertainty in these models. Anderson (1972) reviews several optimization models that determine the least-cost investment in a vertically integrated market. All parameters in the optimization are assumed to be deterministic, despite the stochastic nature of future demands and costs.

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[6] Both price and sales risks can be mitigated by forward contracts that specify the must-take quantity at known prices and tolling agreements that set the capacity lease payment and transfer part or all of the natural gas cost risk from the sellers to buyers. A detailed investigation of forward contracts and tolling agreements, however, is beyond the scope of this paper.

Hartman (1972) studies the effect of uncertainty on the investment decisions of a competitive profit-maximizing firm, demonstrating that rising marginal cost volatility tends to increase capacity investment. Levin et al. (1985) extend the capacity investment model to a monopoly facing uncertain fuel costs, showing that for normally distributed fuel costs, the monopoly's optimal capacity investment is insensitive to fuel cost uncertainty.

Restructuring of the electricity sector in the 1990s to introduce competition in the generation segment requires a new modeling approach. Capacity expansion is no longer the result of total cost minimization, but the interactions among profit-maximizing firms. The competitive market models that accommodate these developments fall into two main classes: *equilibrium models* and *simulation models* (Ventosa et al., 2005). The most common equilibrium model uses Cournot competition, where the strategic variable is the electricity output (Andersson and Bergman, 1995; Borenstein and Bushnell, 1999; Murphy and Smeers, 2005; Tishler et al., 2008). Other models use *Bertrand* competition, where the strategic variable is the electricity price, or a *Stackelberg* game, where a market leader first decides the electricity price or quantity, while accounting for the subsequent reactions of the followers (Spulber, 1981).

Assuming that the fuel cost is deterministic and known, several studies discuss the optimal capacity choice of firms under market competition (von der Fehr and Harbord, 1997; Murphy and Smeers, 2005; Tishler et al., 2008). In contrast, Hobbs (1995) discusses optimization planning methods which incorporate fuel cost uncertainty via Monte Carlo simulations.

Ryu and Kim (2011) study the effect of cost uncertainty on equilibrium in a duopoly market, finding that equilibrium production can increase or decrease, depending on each firm's conjecture about the rival's uncertain cost. Siddiqui and Maribu (2009) study investment in distributed generation by micro-grids under natural-gas price uncertainty. Using real options, they show that a micro-grid prefers a direct investment strategy for low levels of volatility and a sequential one for high levels of volatility. Finally, Tishler et al. (2008) and Milstein and Tishler (2012) study capacity investment under demand uncertainty in a Cournot market setting.

Financial theory offers a different line of inquiry. Pindyck (1991) and Dixit and Pindyck (1994) use the real option approach to value irreversible but deferrable investment opportunities with uncertain cash flow. These authors argue that the net present value (NPV) method in the presence of uncertain cash flow is incorrect because the NPV method ignores the opportunity cost of investing now instead of awaiting more information. An important result is that higher volatility leads to higher option value (Pindyck, 1991). However, the real option theory is limited to a highly competitive market, where a single firm's decision does not affect the equilibrium price.

4. Model of capacity investment under uncertainty of natural gas prices

Building a new power plant is a slow process, requiring a long lead time. However, daily fuel costs and electricity demands are volatile. Installed capacity aims to serve projected daily demands, implying potentially large idle capacity during many hours of the day in a given year. When the daily realized peak demand exceeds the available capacity, the daily electricity price increases to curb the excess demand. Hence, daily hourly spot electricity prices fluctuate substantially within a day and across seasons (Bessembinder and Lemmon, 2002; von der Fehr et al., 2005; Tishler et al., 2008).

To explore the inter-connection among capacity expansion, electricity price and fuel cost volatility, we develop a two-stage closed-loop model to capture the lead time in capacity construction, daily fluctuating fuel costs, and short-term demands met by installed capacity. In the first stage, only the probability distribution function of future daily fuel costs is known. Profit-seeking IPPs maximize their expected profits by determining the amount of capacity to be constructed. In the second stage, once daily fuel cost and electricity demand become known, each IPP selects its daily output up to the capacity built in the first stage via the Cournot conjecture which, in turn, determines the equilibrium market price.[7]

[7] The assumption of Cournot competition is common in the electricity market reform literature (e.g., Wolfram, 1999; Borenstein et al., 2002; Murphy and Smeers, 2005; Tishler et al., 2008).

Solved recursively, our two-stage model assumes an oligopoly market with N identical IPPs (firms) operating daily in a period of T days.[8] We first determine the equilibrium quantities, equilibrium price and profits in stage 2. We then determine each firm's optimal capacity to be built in stage 1.

The market's inverse demand function P_t on day t = 1, …, T is assumed to be linear:

[1] $$P_t = a - bQ_t,$$

where $a > 0$ and $b > 0$, $Q_t = \sum_1^N Q_{it}$ = total electricity generated by N firms, and $Q_{it} =$ electricity generated by firm i = 1, …, N.

For simplicity, variable costs other than those for natural gas are assumed to be zero.[9] Following Newbery (1998), Besanko and Dorazelski (2004), Murphy and Smeers (2005) and Tishler et al. (2008), we assumed a linear amortization of the debt service, and thus the cost function of electricity generation of firm i on day t is:

[2] $$C(Q_{it}, \mathrm{K}_i) = \frac{\Theta}{T} K_i + c_t Q_{it},$$

where Θ = marginal capacity cost (\$/MW-year) applicable to the i-th firm's capacity K_i (MW), and c_t = per MWh fuel cost (\$/MWh) = generation heat rate (MMBtu/MWh) × natural-gas price (\$/MMBtu).[10] We also assume that $c_t < a$ because $Q_t = 0$ when the price level completely choke off market demand at $P_t = a$.

Further, let c_0 = initial per MWh fuel cost at the beginning of t = 1. Following Pilipovic (1997) and Geman (2005), we assume that $\ln\left(\frac{c_t}{c_{t-1}}\right)$ is normally distributed

[8] As was noted by a referee, using a single period in the second (operation) stage of the model is sufficient for the derivation and intuition of most of the results in the paper. However, as was shown by Wogrin et al. (2013), the optimal solution of the model with a single period in the second stage may differ than those with multiple periods in the second stage. In particular, it is straightforward to show that optimal capacity and optimal production are the same when only one period is considered in the second stage (see, for example, Wogrin et al., 2013). Hence, we opted to develop the model with multiple periods in the second stage.

[9] The assumption of zero non-fuel cost is later relaxed in our empirical illustration in Section 6.

[10] The decision on capacity and, hence, capacity total cost, is incurred on stage 1. There are several ways to formulate capacity costs in our model. In reality, the choice of the financing period and the stream of debt payments for the capacity are usually related to the lifetime of the investment. We chose to abstract from the optimal financing decision by each investor and assume a linear amortization of the debt over the lifetime of the project.

with mean μ and variance $\sigma_{c_t}^2$. Thus, $\frac{c_t}{c_0}$ has a log-normal distribution, with parameters μt and $\sigma_{c_t}^2 t$:

$$[3] \qquad \ln\frac{c_t}{c_0} \sim N(\mu t, \sigma_{c_t}^2 t) \quad \leftrightarrow \quad \frac{c_t}{c_0} \sim lnN(\mu t, \sigma_{c_t}^2 t)$$

$$[4] \qquad \tilde{g}\left(\frac{c_t}{c_0} | \mu t, \sigma_{c_t}^2 t\right) = \frac{c_0}{c_t \sigma_{c_t} \sqrt{2\pi t}} e^{-\frac{1}{2}\frac{\left[\ln\left(\frac{c_t}{c_0}\right) - \mu t\right]^2}{\sigma_{c_t}^2 t}}$$

The rest of this section uses σ_{c_t} to measure volatility of per MWh fuel cost in period *t*. [11] For expositional simplicity, we denote $\tilde{g}\left(\frac{c_t}{c_0} | \mu t, \sigma_{c_t}^2 t\right) \equiv g(c_t)$.

Stage 2 Solution

Assuming Cournot competition, each of the *N* firms sets Q_{it} in stage 2 to maximize its profits, π_{it} on day *t*, given its available capacity K_i, c_t and Q_{jt} $(j = 1, \ldots, N; j \neq i)$. Each firm's optimization problem is:

$$[5] \qquad max_{Q_{it}} \pi_{it} = (P_t - c_t) Q_{it}$$

s.t. $Q_{it} \leq K_i$, $Q_{it} \geq 0$.

Inserting the demand function in equation [1] into equation [5] yields:

$$[6] \qquad \max_{Q_{it}} \pi_{it} = (a - b \cdot \textstyle\sum_i Q_{it} - c_t) \cdot Q_{it}$$

s.t. $Q_{it} \leq K_i$, $Q_{it} \geq 0$.

The first-order condition is:

$$[7] \qquad a - b \cdot (2Q_{it} + Q_{-it}) - c_t = 0,$$

where $Q_{-it} \equiv \sum_{j \neq i} Q_{jt}$. The optimal solution is:

$$[8] \qquad Q_{it}^* = \begin{cases} K_i, & if\ 0 \leq c_t < a - 2bK_i - bQ_{-it} \\ \frac{1}{2b}(a - c_t - bQ_{-it}), & if\ a - 2bK_i - bQ_{-it} \leq c_t < a \end{cases}$$

Inserting the above solution into the inverse demand function [1] yields the market equilibrium price, similar to the one in Tishler et al. (2008).

Stage 1 Solution: optimal capacity

[11] Assuming risk neutral investment, $\mu t = rt - \frac{\sigma_{c_t}^2 t}{2}$, where *r* = riskless interest rate (Lee et al., 2010).

In stage 1, each firm determines its optimal capacity, given the distribution function $g(c_t)$. The optimal capacity maximizes the sum of the expected daily profits over the operation period minus the capacity's construction cost:

[9] $$max_{K_i}\left[\sum_{t=1}^{T} e^{-rt}\left(E(\pi_{it}|K_i) - \frac{\Theta}{T}K_i\right)\right].$$

Expected variable profit of the *i*-th firm on day *t* is:

[10] $$E(\pi_{it}|K_i) = \int_0^a g(c_t)\cdot(P_t - c_t)\cdot Q_{it}dc_t.$$

Inserting equation [8] into equation [10] yields:

[11] $$E(\pi_{it}|K_i) = \int_0^{a-2bK_i-bQ_{-it}} g(c_t)K_i\cdot(a - bK_i - bQ_{-it} - c_t)\,dc_t + \int_{a-2bK_i-bQ_{-it}}^{a} g(c_t)\frac{1}{4b}(a - bQ_{-it} - c_t)^2dc_t.$$

The first-order condition for optimal capacity is:

[12] $$0 = \frac{\partial}{\partial K_i}\sum_{t=1}^{T} e^{-rt}[\int_0^{a-2bK_i-bQ_{-it}} g(c_t)K_i\cdot(a - bK_i - bQ_{-it} - - c_t)dc_t + \int_{a-2bK_i-bQ_{-it}}^{a} g(c_t)\frac{1}{4b}(a - bQ_{-it} - c_t)^2dc_t].$$

Applying the Leibniz rule to compute the derivative in equation [12] and assuming symmetry of the *N* firms (i.e. $K_1^* = \cdots = K_N^* = K^*$) yield the optimal capacity condition:[12]

[13] $$0 = \sum_{t=1}^{T} e^{-rt}[\int_0^{c_{K^*}}(c_{K^*} - c_t)\cdot g(c_t)dc_t - \frac{\Theta}{T}],$$

where $c_{K^*} \equiv a - K^*\cdot b\cdot(N+1)$, and K^* is the firm's optimal capacity.

The first-order condition given by [13] implies that the firm should add capacity, so long as the new capacity's present value of expected daily operating profits exceeds the capacity cost.

Since c_t is uncertain, building capacity K^* is equivalent to buying a put option with a strike price c_{K^*}. When $c_t > c_{K^*}$, equilibrium generation in stage 2 is below the available capacity because the option is "out of the money". If $c_t < c_{K^*}$, the firm's

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[12] The optimal capacity satisfies the second-order condition for profit maximization.

equilibrium generation increases to full capacity because the option is "in the money", with a per MWh profit of $(c_{K^*} - c_t) > 0$.

For readability, the proofs of all propositions are in Appendix. Propositions 1 and 2 below characterize the optimal first-stage solution.

Proposition 1: The marginal profit and optimal capacity of an expected profit-maximizing firm in oligopolistic competition *increase* with the volatility of c_t.

Proposition 2: An increase in the volatility of c_t causes:

a. An increase in the expected consumer surplus and expected profit at the optimal first-stage solution on day *t*.
b. A decline in the expected electricity price at the optimal first-stage solution on day *t*.

To better understand Propositions 1 and 2, consider a log-normal distribution which implies the probability of low c_t rises when σ_{c_t} increases, chiefly because the log-normal distribution is skewed to the left, with most of its mass occurring at low c_t values. Consequently, rising fuel cost volatility induces firms to increase their installed capacity, so as to improve their chances of benefiting from low natural gas prices.[13] The installed capacity's increase reduces electricity prices, thus enhancing consumer surplus.

[13] It is straightforward to show that when demand and fuel costs are independent, the positive effect of the per MWh fuel cost's volatility on the optimal level of capacity investment holds in the presence of demand uncertainty. The independence assumption is empirically reasonable, except for a region where natural gas and electric heating are similarly prevalent. When this region experiences cold weather, both natural gas demand and demand for electricity may rise in tandem, thus resulting in a positive correlation between the demand for electricity and the natural gas price. The solution of the model of Tishler et al. (2008), which includes demand uncertainty and non-random fuel price, is similar to the solution in this paper when the demand for electricity follows either normal or uniform distribution. Hence, in the rest of the paper we can reasonably assume no demand uncertainty.

5. The effect of natural gas price hedging on capacity investment

In this section, we evaluate the effects of a possible use of call options to hedge the natural gas price risk. To this end, we add a hedge stage to the two-stage model of Section 4, between generation capacity planning and electricity production. While capacity planning remains stage 1, electricity production becomes stage 3. We denote the hedge stage as stage 2, where the firms can buy call options to hedge their fuel costs for part or all of their generation capacity.[14]

This 3-stage model is solved backwards. We assume that preceding the third stage, the *i*-th firm built capacity, K_i, and purchased R_i fuel cost options at strike price *s*. If the maximal generation (= installed capacity) is hedged, then 100% of the fuel cost will be capped by the strike price. If only part of the generation capacity is hedged, the fuel cost of that part is capped, while the remainder is incurred at the spot natural gas price. Thus, the hedged fuel cost is:

[14] $$c_t^h = \begin{cases} c_t \text{ for any quantity of units of fuel}, & if\ c_t < s \\ s \text{ for the initial } R_i \text{ units; } c_t \text{ for all other } (Q_{it} - R_i) \text{ units}, & if\ c_t \geq s \end{cases}$$

The equilibrium generation is a straightforward extension of equation [8]:

[15] $$Q_{it}^* = \begin{cases} K_i, & if\ 0 \leq c_t < a - 2bK_i - bQ_{-it} \\ \frac{1}{2b}(a - c_t - bQ_{-it}), & if\ a - 2bK_i - bQ_{-it} \leq c_t < a - 2bR_i - bQ_{-it} \\ R_i, & if\ a - 2bR_i - bQ_{-it} \leq c_t < a \end{cases}$$

Equilibrium price is obtained by inserting equation [15] into equation [1]:

[16] $$P_t^* = \begin{cases} a - bK_i - bQ_{-it}, & if\ 0 \leq c_t < a - 2bK_i - bQ_{-it} \\ \frac{1}{2}(a - bQ_{-it} + c_t), & if\ a - 2bK_i - bQ_{-it} \leq c_t < a - 2bR_i - bQ_{-it} \\ a - bR_i - bQ_{-it}, & if\ a - 2bR_i - bQ_{-it} \leq c_t < a \end{cases}$$

If c_t is relatively low (i.e., $c_t < a - 2bR_i - bQ_{-it}$), hedging does not alter the equilibrium quantity. The options' purchase alters the equilibrium quantity only when c_t is relatively high (i.e., $c_t > a - 2bR_i - bQ_{-it}$). When full capacity is hedged

[14] Murphy and Smeers (2010) analyze a similar three-stage model. However, the second stage in their model focuses on a forward contract for electricity sale, while the second stage in our model uses call options for hedging the fuel cost.

with $R_i = K_i$, the equilibrium generation equals $Q_{it}^* = K_i$ for any value of c_t. In that case, the price of electricity is capped at $P_t^* = a - 2bK_i - bQ_{-it}$. If only part of the capacity is hedged with $R_i < K_i$, the price of electricity will be capped at a higher level so that $P_t^* = a - 2bR_i - bQ_{-it}$.

Stage 2 solution: the firm's optimal hedging strategy

In stage 2, each firm sets the number of options to maximize the expected value of the profits, given the capacity constructed in stage 1 and the distribution function $g(c_t)$. For expositional simplicity, we start by stating the *i*-th firm's optimization problem in stage 2 for a particular value of the strike price. The proof of Proposition 4 in the Appendix shows that the results of this section hold for any strike price level.

Suppose $s = a - 2bK_i - bQ_{-it}$. The expected operating profit of firm *i* on day *t* in stage 2 as a function of the number of options purchased is:

[17] $$E(\pi_{it}|R_i) = \int_0^{a-2bK_i-bQ_{-it}} g(c_t)K_i \cdot (a - bK_i - bQ_{-it} - c_t)\,dc_t + \int_{a-2bK_i-bQ_{-it}}^{a-2bR_i-bQ_{-it}} g(c_t)\left[\frac{1}{4b}(a - bQ_{-it} - c_t)^2 + R_i \cdot (c_t - a + 2bK_i + bQ_{-it})\right] dc_t + \int_{a-2bR_i-bQ_{-it}}^{a} g(c_t)R_i \cdot [(a - bR_i - bQ_{-it}) - (a - 2bK_i - bQ_{-it})]\,dc_t$$

The cost of buying a call option is $\int_s^a g(c_t) \cdot (c_t - s)dc_t$ (Lee et al., 2010). Thus, the *i*-th firm's optimization problem is:

[18] $$\max_{R_i}\left(\sum_{t=1}^{T} e^{-rt}\left[E[\pi_{it}|R_i] - R_i \cdot \int_{a-2bK_i-bQ_{-it}}^{a} g(c_t) \cdot (c_t - a + 2bK_i + bQ_{-it})dc_t\right]\right)$$

$s.t. \quad R_i \leq K_i,\ R_i \geq 0$

The first term in equation [18] depicts the expected profit given R_i options and the second the cost of buying R_i options, leading to the following propositions:

<u>Proposition 3</u>: Expected consumer surplus increases with the number of options purchased by the *i*-th firm.

Proposition 4: The optimal strategy of a profit-maximizing firm, for any strike price *s*, is *not* to hedge against natural gas price spikes.

Proposition 3 states that consumers gain from fuel cost hedging, which leads to lower electricity prices and higher generation.

The intuition behind Proposition 4 is as follows. The reduction of profits due to a higher per MWh cost is divided between consumers and producers because part of this reduction is mitigated by an increase of the electricity price. However, the call option premium paid by an IPP for hedging against high natural gas prices accounts for all of the expected difference between the actual gas price and the strike price (for any gas price larger than the strike price). Therefore, the IPP's hedging cost exceeds the expected profit lost by not hedging. Hence, the optimal strategy of a profit-maximizing firm is *not* to hedge the natural gas fuel cost.[15]

6. An illustration of the model using California's market data

We illustrate a simplified competitive market using California's market data.[16] In 2015, natural-gas-fired generation accounted for 56% of California's instate electricity supply, with the remainder coming from hydro, nuclear, and renewable

[15] We chose to model risk neutral firms in order to derive some analytical results and avoid the complexity involved in modelling risk-averse firms. Chin and Siddiqui (2014) show how to model risk-averse electricity producers. They employ only numerical computations in assessing (among other elements) the effect of forward contracts as a suitable hedging tool for hedging against risk under market power. Hedging by risk neutral firms may be due to the following. First, financial institutions may require hedging as a pre-condition to granting a loan to construct new capacity. The mere existence of hedging may result in lower interest rate on such a loan. Second, although the average (over the operation periods, say) natural-gas fuel cost in our model will always be below the average (over the same periods) electricity price, there may be a period in the planning horizon of a particular producer in which the profit margin (inclusive of the periodical payment covering the loan taken for capacity construction) is negative. As a result, a cash-strapped IPP may be prone to bankruptcy in this case. Hedging helps stabilize this IPP's cash flows, thus mitigating the risk of bankruptcy.

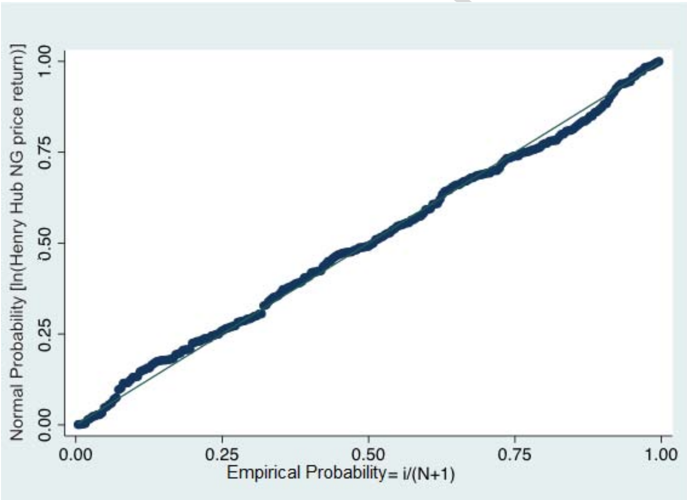
[16] Because of its simplicity, our model does not portray how the Californian electricity market works. Rather, this section provides an illustrative example employing a simplified competitive market that is similar in structure to the California electricity market. We use the California data for the sole purpose of numerically demonstrating how fuel cost volatility variations may affect prices, quantities, profits and consumer surplus.

resources.[17] The average hourly consumption during 2011 was 26,300 MWh (California ISO, 2013) at an average electricity price of $138/MWh (EIA, 2012).

6.1. Daily per MWh fuel cost's distribution

Figure 2 depicts the fit of $\ln\frac{c_t^G}{c_{t-1}^G}$ to a normal distribution, where c_t^G = daily per MWh fuel cost during January 2004 to December 2014 = daily Henry Hub price $\times$constant heat rate of 6.44 MMBtu per MWh, and c_{t-1}^G = per MWh fuel cost on the previous day. It shows that $\ln\frac{c_t^G}{c_{t-1}^G}$ is normally distributed, corroborated by the Jarque-Bera normality test and documented by Pilipovic (1997) and Geman (2005).

Figure 2: Fit of $\ln\frac{c_t^G}{c_{t-1}^G}$ to a normal distribution (2004-2014)



The annualized volatility of the natural gas price during 2004-2014 ranges from 30% to 120% (with a value of about 100% in 2014). The implication of a 100% annualized volatility is that over a period of one year there is a 67% probability that the price of natural gas will change by 100%.[18]

6.2. Optimal Capacity

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[17] California Energy Commission, Energy Almanac – Electric Generation Capacity & Energy: http://energyalmanac.ca.gov/electricity/electric_generation_capacity.html.

[18] An annualized volatility of 100%, implies a daily standard deviation of $100\%/\sqrt{365}$=5.2%. That is, a probability of 67% that over one day the price will change by 5.2%.

Following Milstein and Tishler (2015), we assume that the price elasticity of demand is -0.25, implying that the inverse demand function in California is $P_t = 690 - 0.021Q_t$. Next, we assume that the capacity cost is $\theta/T =$ \$285.12/MW-day, based on a \$1023/kW total installed cost, a 6% WACC, 20 years depreciation and an annual fixed cost of \$15.37/kW (EIA 2013, Table 1). The initial per MWh fuel cost is $c_0 =$\$29.5/MWh, based on an average Henry Hub natural-gas price of \$4/MMBtu in 2013-2014, a constant heat rate of 6.44 MMBtu/MWh, a variable cost of \$3.2/MWh (EIA 2013, Table 1), and a riskless interest rate of 4%.

Figure 3 depicts the marginal capacity unit's annual profit as a function of the fuel cost volatility (calculated for an electricity market with five identical firms) over 365 days. It shows that the marginal profit increases with volatility. As a result, a firm's optimal capacity increases with per MWh fuel cost volatility. The optimal capacity for an annualized volatility of 40% is 5152 MW, less than the 5193 MW for an annualized volatility of 100%. This finding's importance is not so much the small increase in a firm's optimal capacity, but it is rather the absence of a capacity decrease in response to rising fuel cost volatility.

Based on the equilibrium solution in equation [13], Figure 4 presents the optimal generation capacity for different numbers of firms in the market as a function of the annualized volatility of the per MWh fuel cost. As expected, higher volatility implies higher optimal capacity, and an increase in the number of firms in the market increases the market's total capacity. Moreover, an increase in σ_{c_t} from 20% to 120% increases the optimal generation capacity by about the same 1.2% for N = 3, 5, 10, 20.

Figure 3: Value and cost of marginal capacity unit as a function of the annualized volatility of the per MWh fuel cost (N=5)

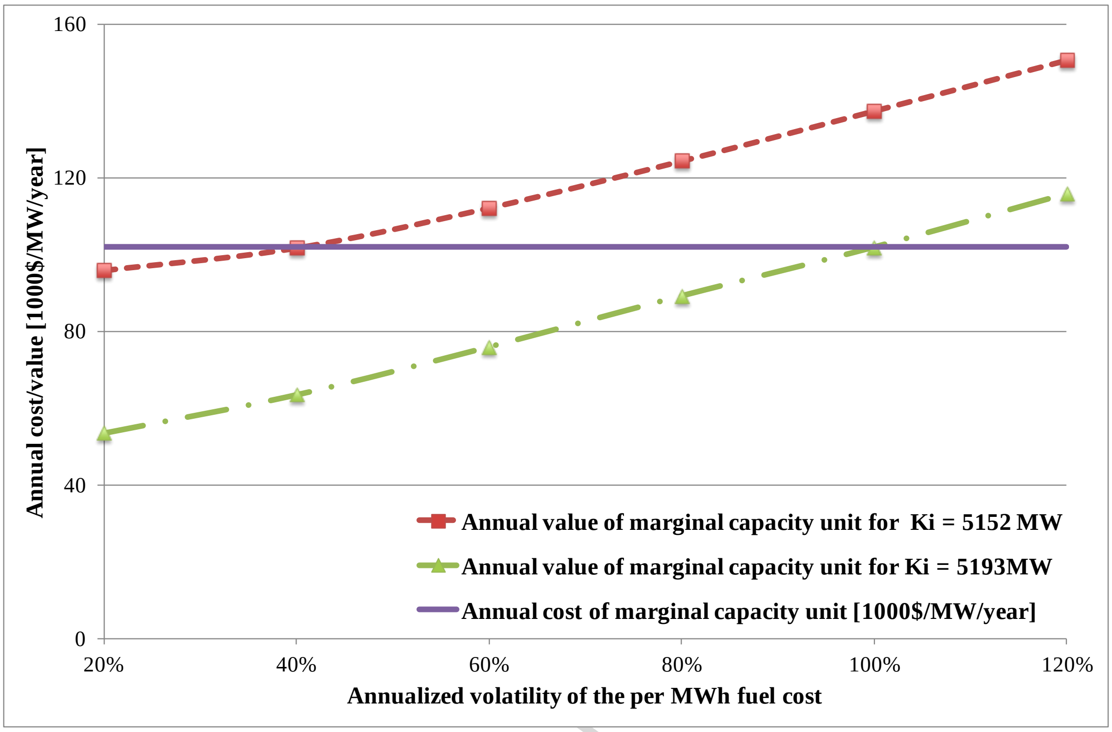
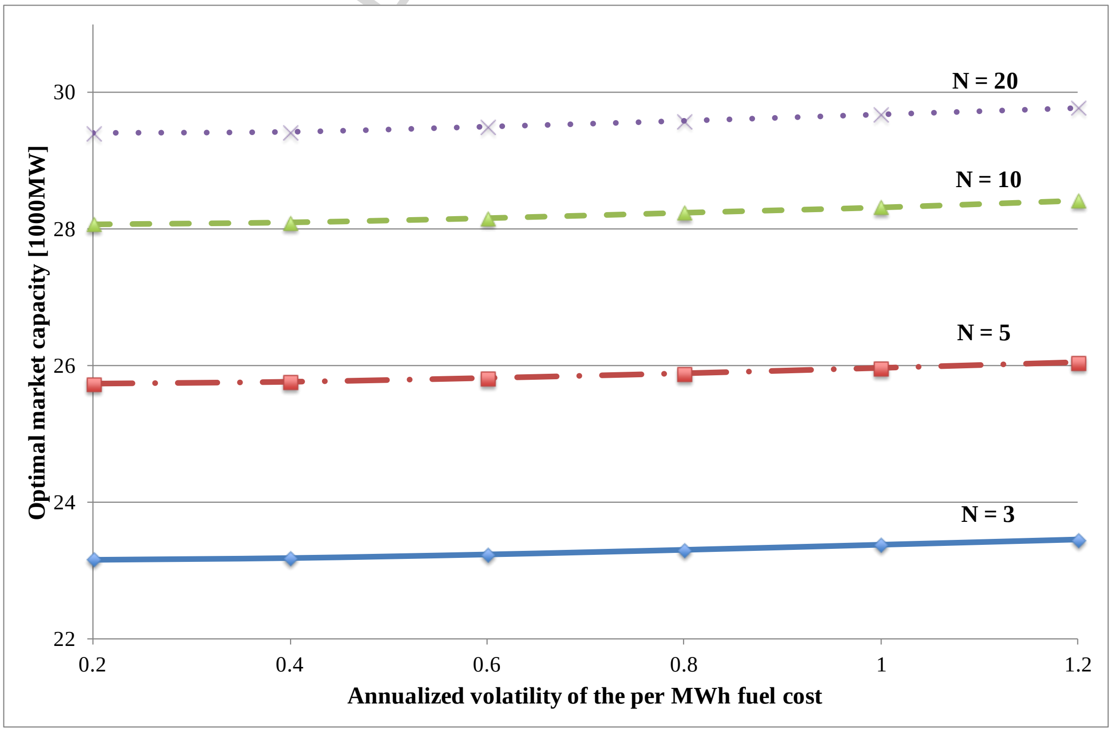


Figure 4: Optimal market capacity as a function of the annualized volatility of the per MWh fuel cost and of the number of firms in the market



6.3. Equilibrium generation and equilibrium price

Using the solution for equilibrium quantity in equation [8] and volatility values of 80%, 100% and 120%, we compute the distributions of equilibrium generation and equilibrium electricity prices as a function of σ_{c_t}. Figure 5 depicts the distribution function of equilibrium generation, showing that higher volatility implies more installed capacity, which tends to increase equilibrium quantities.

Figure 5: Distribution function of equilibrium generation (*N*=5)

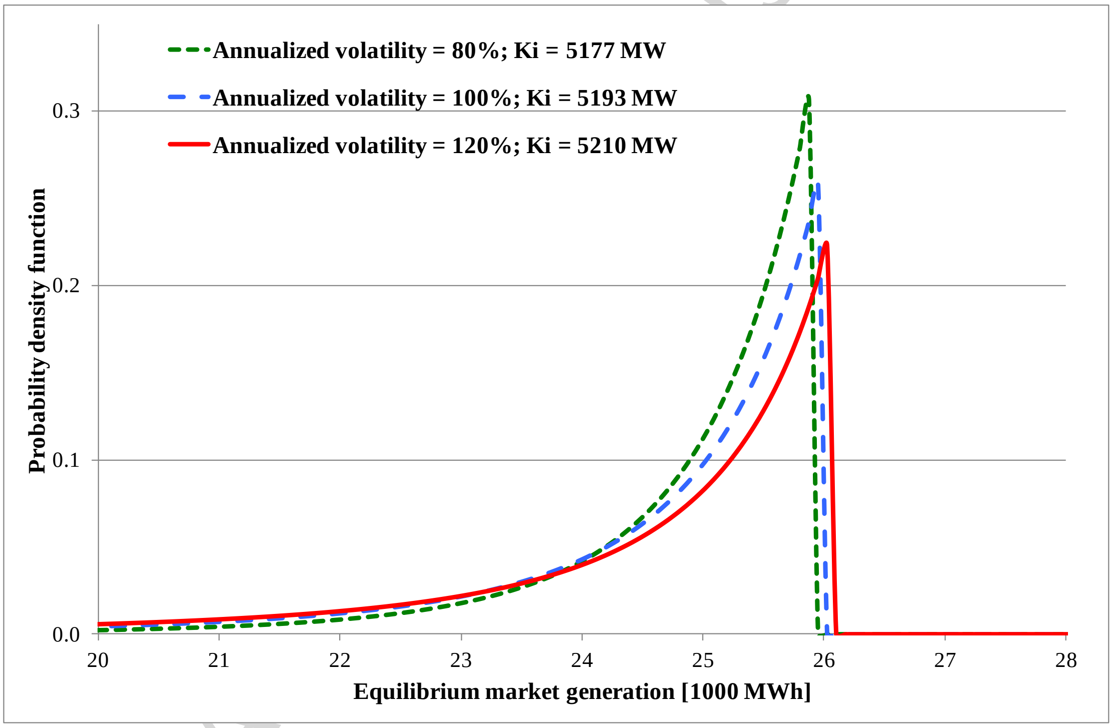
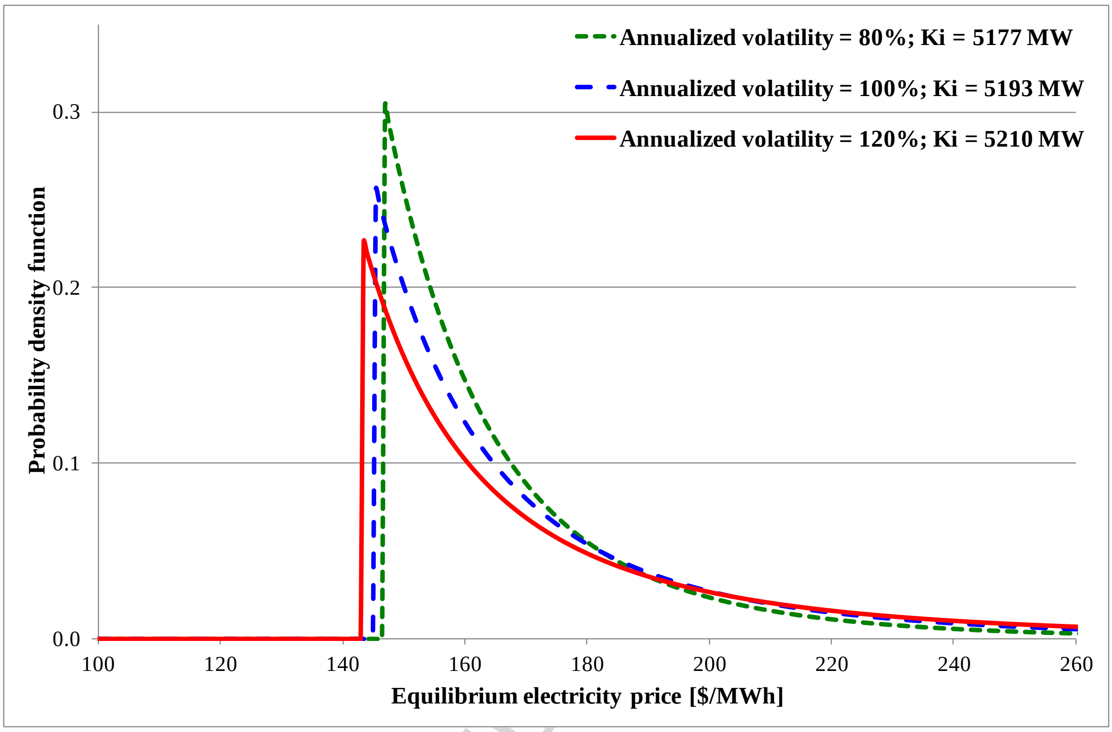


Figure 6 presents the distributions of the equilibrium electricity price. As expected, higher capacity investment, due to higher volatility, reduces the minimal electricity price (as shown in Figure 6), and the average price (as shown in Proposition 2b). The probability of price spikes,[19] however, is higher since the higher per MWh fuel cost volatility leads to higher probability that production will be at less than full capacity.

Figure 6: Distribution function of equilibrium electricity price (*N*=5)



6.4. Producers' profits and consumers' surplus

Figure 7 depicts the distribution of the producers' profits as a function of σ_{c_t}. The increase in per MWh fuel cost volatility affects producer's profit in two ways: (a) lower electricity prices reduce profits; and (b) lower per MWh fuel costs improve profits. The overall effect is that rising per MWh fuel cost volatility increases expected profits, as proved in Proposition 2.

Finally, Figure 8 depicts the distribution function of consumer surplus as a function of σ_{c_t}. It shows that consumer surplus is capped from above (for a given volatility, σ_{c_t}). This cap corresponds to the price floor given by the minimal electricity price at full capacity (see Figure 6). This cap increases with volatility, because higher volatility yields higher capacity, which leads to a lower price floor. Note also that higher

[19] Note that price spikes occur when per MWh fuel cost (which is a result of high natural gas price) is high and, therefore, the profit-maximizing IPP prefers to produce at less than full capacity. Furthermore, the higher the natural gas price the more significant is the electricity price spike.

volatility tends to increase the probability of higher consumer surplus because an increase in volatility increases the probability of low per MWh fuel cost, which leads to higher generation and lower price.

Figure 7: Distribution function of firm profits (*N*=5)

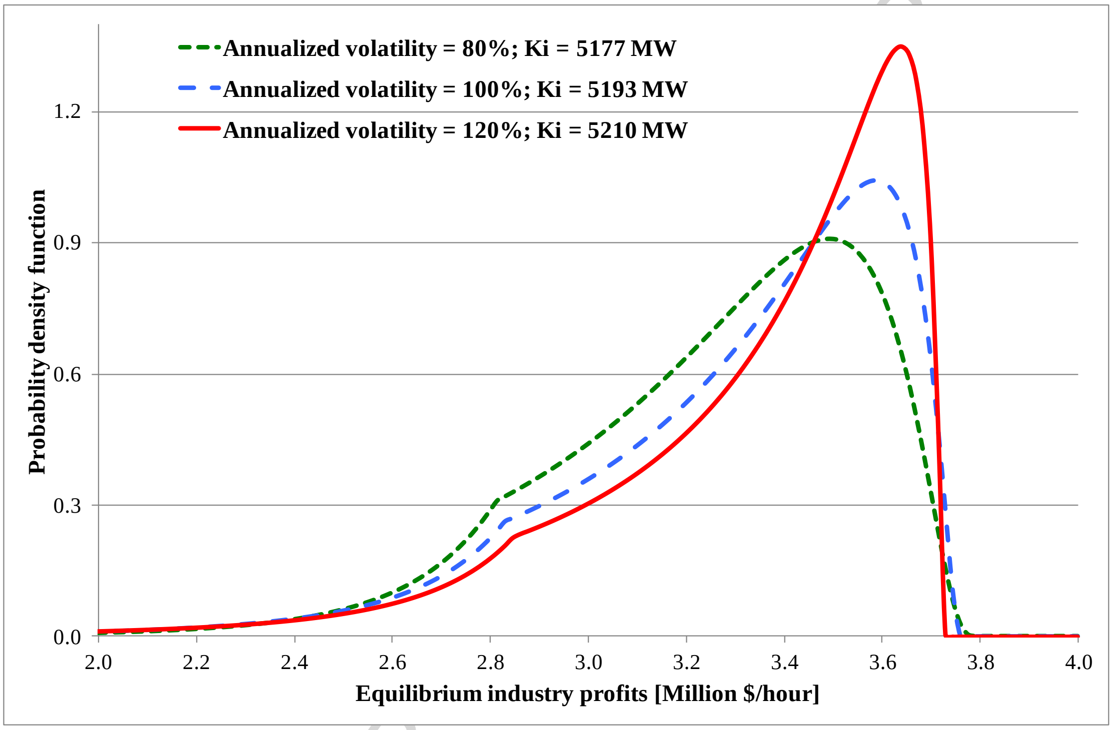
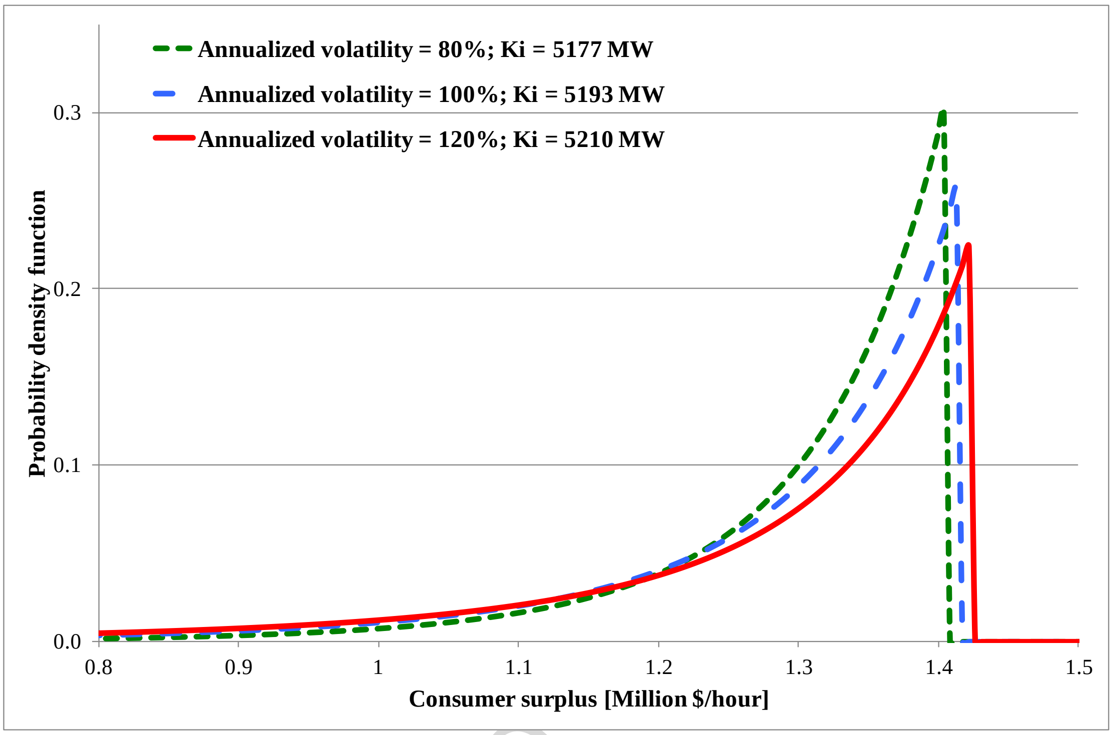


Figure 8: Distribution function of consumer surplus (*N*=5)



In summary, Figures 3 - 8 and extensive simulations show that the effect of a change in σ_{c_t} on generation capacity and, hence, on the average electricity price and quantity and on average profits and consumer surplus, is fairly small. Nevertheless, the response of the market electricity price and quantity on a particular day to a change in σ_{c_t} may be very large.[20]

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[20] We calculated per MWh fuel costs for each day in a year for five different values of annualized volatility (40%, 60%, 80%, 100% and 120%). We repeated this process 100 times. Then, for a given optimal capacity, we calculated electricity prices, industry production, profits and consumer surplus for each day in the year. The results of these simulations show, as is stated in Proposition 2, that average electricity price moderately decreases when fuel cost volatility increases, from $150/MWh for $\sigma_{c_t} = 40\%$ to $148/MWh for $\sigma_{c_t} = 120\%$. Consequently, industry production increases by 0.3%, industry profits by 0.8% and consumer surplus by 0.7%. These changes, however, can be substantial in extreme cases. For example, when annualized volatility is 120%, the per MWh fuel cost can reach a maximum (minimum) value of $240/MWh ($29/MWh) in one simulation, leading to an average electricity price of $214/MWh and a maximum price of $316/MWh. In another simulation, we find the maximum

7. Summary and conclusions

Natural gas prices in a competitive wholesale market are volatile, more so than the prices of other fossil fuels. Hence, large natural-gas price spikes occasionally occur, exposing IPPs to high natural gas fuel costs and low profits. Thus, the introduction of competition into the electricity market exposes IPPs to risks previously borne by end-users.

This paper extends Tishler et al. (2008) to study the effect of natural-gas cost risk on capacity investment in a competitive electricity market. Under the empirically reasonable assumption that per MWh fuel costs are log-normally distributed, we find that optimal capacity investment increases with fuel cost volatility. This is because rising volatility increases the probability of low fuel costs, which in turn improves the IPP's expected profit. The resulting increase in installed capacity lowers electricity prices and increases generation output, thus enhancing consumer surplus.

We also demonstrate that the use of call options to hedge natural gas fuel cost increases consumer surplus by mitigating electricity price spikes. However, profit-maximizing electricity producers are unwilling to hedge against fuel cost spikes. This latter finding highlights the gap between the producers' and consumers' propensity for risk taking.[21] The policy implication is clear: the government should not intervene to reduce the price volatility of a well-functioning competitive natural gas spot market because such an action can have the unintended consequence of discouraging generation investment, raising electricity prices, and reducing consumer surplus.

(minimum) fuel cost to be \$30/MWh (\$1.3/MWh), with the average and maximum electricity prices equal to \$143/MWh.

[21] Note that Proposition 4, proving that firms choose not to hedge against natural gas price rise, holds for a market with any number of IPPs. It is straightforward to model (almost) perfect competition by letting the number of IPPs in our model, *N*, be very large. A larger number of IPPs (i.e., more competition) will reduce the average price of electricity and each firm's profit margin. For example, when volatility = 100% and *N* = 100 (1000), the average electricity price in equilibrium is \$46.5/MWh (\$40.8/MWh) at the average fuel cost of \$26.6/MWh and fixed capacity cost of \$11.9/MW-hour (=\$285.12/MW-day ÷ 24 hours/day). In short, market power and profit margin diminish in response to rising competition.

We would be remiss had we ignored two important limitations to our model presented herein. First, our model only characterizes a simplified electricity market with one generation technology and symmetric firms, thus abstaining from the reality of heterogeneous power plants with differing heat rates (e.g., high, medium and low) and fuel types (e.g., coal, natural gas and oil). Second, fuel cost hedging in our model is limited to the use of call options.

In light of these limitations, our future research plans to analyze two generation technologies, so as to recognize the tradeoffs that may exist when various fuels exhibit different price volatilities. It also intends to introduce dual-fuel plants into our model as means of hedging against fuel cost uncertainty. We also plan to develop a two-stage dynamic model of capacity and operations in the presence of heterogeneous power plants. To be solved by numerical analysis, this model will further assess the fuel cost volatility's impacts on such important issues as risk management, capacity investment, market prices, IPP profits, and consumer surplus.

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Appendix: Proofs

Proof of Proposition 1:

The expected marginal profit of a profit-maximizing firm in oligopolistic competition on day *t* is:

[A.1] $$e^{-rt}\int_0^{c_{K_i}}(c_{K_i}-c_t)\cdot g(c_t)dc_t,$$

where

[A.2] $$c_{K_i}\equiv a-bK_i-bQ_{-it}.$$

We have to show that, for any time *t*:

[A.3] $$\frac{\partial}{\partial\sigma_{c_t}}e^{-rt}\int_0^{c_{K_i}}(c_{K_i}-c_t)\cdot g(c_t)dc_t>0.$$

Using Lee et al. (2010), we have that if $\frac{c_t}{c_0}$ follows a lognormal distribution, and $\mu=r-\frac{\sigma_{c_t}^2}{2}$, then [A.1] can be rewritten as:

[A.4] $$e^{-rt}\int_0^{c_{K_i}}(c_{K_i}-c_t)\cdot g(c_t)dc_t=c_{K_i}e^{-rt}\Phi(-d_2)-c_0\cdot\Phi(-d_1),$$

where $\Phi(\cdot)$ is the standard cumulative normal distribution function, *r* is the riskless interest rate,

[A.5] $$d_1=\frac{ln\left(\frac{c_0}{c_{K_i}}\right)+(r+\frac{1}{2}\sigma_{c_t}^2)t}{\sigma_{c_t}\sqrt{t}},\text{ and }\quad d_2=d_1-\sigma_{c_t}\sqrt{t}=\frac{ln\left(\frac{c_0}{c_{K_i}}\right)+(r-\frac{1}{2}\sigma_{c_t}^2)t}{\sigma_{c_t}\sqrt{t}}.$$

Differentiating the right-hand side of [A.4] with respect to σ_{c_t} yields:

[A.6] $$\frac{\partial}{\partial\sigma_{c_t}}\left[e^{-rt}\int_0^{c_{K_i}}(c_{K_i}-c_t)\cdot g(c_t)dc_t\right]=c_{K_i}e^{-rt}\frac{\partial\Phi(-d_2)}{\partial d_2}\cdot\frac{\partial d_2}{\partial\sigma_{c_t}}-c_0\frac{\partial\Phi(-d_1)}{\partial d_1}\cdot\frac{\partial d_1}{\partial\sigma_{c_t}},$$

where $\frac{\partial\Phi(-d_1)}{\partial d_1}=-\phi(-d_1)$, $\frac{\partial\Phi(-d_2)}{\partial d_2}=-\phi(-d_2)$, and $\phi(\cdot)$ is the normal density function. Therefore,

[A.7] $$\phi(-d_1)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}{d_1}^2},$$

$$\phi(-d_2)=\phi\left(\sigma_{c_t}\sqrt{t}-d_1\right)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\sigma_{c_t}\sqrt{t}-d_1\right)^2}.$$

Thus, we can rewrite [A.6]:

[A.8] $$\frac{\partial}{\partial\sigma_{c_t}}\left[e^{-rt}\int_0^{c_{K_i}}(c_{K_i}-c_t)g(c_t)dc_t\right] = -c_{K_i}e^{-rt}\phi(-d_2)\cdot\frac{\partial d_2}{\partial\sigma_{c_t}} + c_0\phi(-d_1)\cdot\frac{\partial d_1}{\partial\sigma_{c_t}} \quad = -c_{K_i}e^{-rt}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\sigma_c\sqrt{t}-d_1\right)^2}\cdot\frac{\partial d_2}{\partial\sigma_{c_t}} + c_0\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1{}^2}\cdot\frac{\partial d_1}{\partial\sigma_{c_t}}.$$

Using [A.5], c_{K_i} may be written as follows:

[A.9] $$c_{K_i} = c_0 e^{-d_1\sigma_c\sqrt{t}+\left(r+0.5\sigma_c{}^2\right)t}.$$

Inserting [A.9] into [A.8] and rewriting:

[A.10] $$\frac{\partial}{\partial\sigma_{c_t}}\left[e^{-rt}\int_0^{c_{K_i}}(c_{K_i}-c_t)\cdot g(c_t)dc_t\right] =$$

$$= \left[-c_0 e^{-d_1\sigma_{c_t}\sqrt{t}+\left(r+0.5\sigma_c{}^2\right)t}e^{-rt}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\sigma_{c_t}\sqrt{t}-d_1\right)^2}\frac{\partial d_2}{\partial\sigma_{c_t}} + c_0\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1{}^2}\frac{\partial d_1}{\partial\sigma_{c_t}}\right]$$

$$= \left[-c_0\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1{}^2}\frac{\partial d_2}{\partial\sigma_{c_t}} + c_0\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1{}^2}\frac{\partial d_1}{\partial\sigma_{c_t}}\right]$$

$$= c_0\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1{}^2}\left[\frac{\partial d_1}{\partial\sigma_{c_t}} - \frac{\partial d_2}{\partial\sigma_{c_t}}\right].$$

Substituting $d_2 = d_1 - \sigma_{c_t}\sqrt{t}$ in [A.10] yields:

[A.11] $$\frac{\partial}{\partial\sigma_{c_t}}\left[e^{-rt}\int_0^{c_{K_i}}(c_{K_i}-c_t)\cdot g(c_t)dc_t\right] =$$

$$= c_0\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1{}^2}\left[\frac{\partial d_1}{\partial\sigma_c} - \left(\frac{\partial d_1}{\partial\sigma_c} - \sqrt{t}\right)\right] = c_0\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_1{}^2}\sqrt{t} > 0\,.\ \blacksquare$$

Next we show that optimal capacity investment, K^*, also increases with σ_{c_t}. The total derivative of [13] with respect to the natural gas cost volatility, σ_{c_t}, is:

[A.12] $$\frac{d\left(e^{-rt}\int_0^{c_{K^*}}(c_{K^*}-c_t)\cdot g(c_t)dc_t\right)}{d\sigma_{c_t}} = 0 = \frac{\partial\left(e^{-rt}\int_0^{c_{K^*}}(c_{K^*}-c_t)\cdot g(c_t)dc_t\right)}{\partial\sigma_{c_t}} + \frac{\partial c_{K^*}}{\partial K^*}\cdot\frac{\partial K^*}{\partial\sigma_{c_t}}.$$

We proved above that the first element on the RHS of [A.12] is positive. Therefore, the second element on the RHS of [A.12] has to be negative. By definition, $c_{K^*} \equiv a - K^*\cdot b\cdot(N+1)$, $\frac{\partial c_{K^*}}{\partial K^*} = -b(N+1) < 0$. Therefore: $\frac{\partial K^*}{\partial\sigma_{c_t}} > 0$. ■

Proof of Proposition 2:

Part a:

[A.13] $E[CS_t] = \int_0^a g(c_t) \cdot \frac{1}{2} \cdot (a - P_t) \cdot Q_t dc_t.$

Inserting the equilibrium solution, we get:

[A.14] $E[CS_t|K^*] = \int_0^{c_{K^*}} g(c_t) \cdot \frac{1}{2} \cdot b(NK^*)^2 dc_t + \int_{c_{K^*}}^a g(c_t) \cdot \frac{1}{2} \cdot \frac{N^2(a-c_t)^2}{b(N+1)^2} dc_t.$

Applying the chain rule, we get:

[A.15] $\frac{\partial E[CS_t|K^*]}{\partial \sigma_{c_t}} = \frac{\partial E[CS_t|K^*]}{\partial K^*} \cdot \frac{\partial K^*}{\partial \sigma_{c_t}}.$

From Proposition 1, we have $\frac{\partial K^*}{\partial \sigma_{c_t}} > 0$. Differentiation of [A.14] with respect to K^* yields

[A.16] $\frac{\partial E[CS_t|K^*]}{\partial K^*} = \int_0^{c_{K^*}} g(c_t) \cdot b \cdot N^2 K^* \, dc_t > 0.$

Therefore, $\frac{\partial E[CS_t|K^*]}{\partial \sigma_{c_t}} > 0$. ■

Inserting the equilibrium solution into [10], we get:

[A.17] $E[\pi_{it}|K^*] = \int_0^{c_{K^*}} g(c_t) \cdot (a - b \cdot NK^* - c_t) \cdot K^* dc_t + \int_{c_{K^*}}^a g(c_t) \cdot \frac{(a-c_t)^2}{b(N+1)^2} dc_t.$

Applying the chain rule, we get:

[A.18] $\frac{\partial E[\pi_{it}|K^*]}{\partial \sigma_{c_t}} = \frac{\partial E[\pi_{it}|K^*]}{\partial K^*} \cdot \frac{\partial K^*}{\partial \sigma_{c_t}}.$

From Proposition 1, we have $\frac{\partial K^*}{\partial \sigma_{c_t}} > 0$. Differentiation of [A.17] with respect to K^* yields:

[A.19] $\frac{\partial E[\pi_{it}|K^*]}{\partial K^*} = \int_0^{c_{K^*}} g(c_t) \cdot [a - b(N+1)K^* - c_t] \, dc_t = \int_0^{c_{K^*}} g(c_t) \cdot (c_{K^*} - c_t) \, dc_t = \frac{\theta}{T} > 0.$

Therefore, $\frac{\partial E[\pi_{it}|K^*]}{\partial \sigma_{c_t}} > 0$. ■

Part b:

[A.20] $E[P_t] = \int_0^a g(c_t) \cdot P_t dc_t.$

Inserting the equilibrium solution, we get:

[A.21] $E[P_t|K^*] = \int_0^{c_{K^*}} g(c_t)(a - bNK^*)dc_t + \int_{c_{K^*}}^a g(c_t) \cdot \frac{(a+Nc_t)}{(N+1)} dc_t.$

Applying the chain rule, we get:

[A.22] $\frac{\partial E[P_t|K^*]}{\partial \sigma_{c_t}} = \frac{\partial E[P_t|K^*]}{\partial K^*} \cdot \frac{\partial K^*}{\partial \sigma_{c_t}}.$

From Proposition 2, we have $\frac{\partial K^*}{\partial \sigma_{c_t}} > 0$. Differentiation of [A.21] with respect to K^* yields:

[A.23] $\frac{\partial E[P_t|K^*]}{\partial K^*} = \int_0^{c_{K^*}} g(c_t) \cdot (-bN)\, dc_t < 0.$

Therefore, $\frac{\partial E[P_t|K^*]}{\partial \sigma_{c_t}} < 0$. ■

Proof of Proposition 3:

Inserting [15] and [16] into [A.13], we get the expected consumer surplus, given the fuel cost distribution, as a function of R_i:

[A.24] $E[CS_t] = \int_0^{c_{K_i}} g(c_t) \cdot \frac{1}{2} \cdot b(K_i + Q_{-it})^2\, dc_t + \int_{c_{K_i}}^{c_{R_i}} g(c_t) \cdot \frac{1}{8b}(a - c_t + bQ_{-it})^2 dc_t + \int_{c_{R_i}}^a g(c_t) \cdot \frac{1}{2} \cdot b(R_i + Q_{-it})^2\, dc_t,$

where $c_{R_i} \equiv a - 2bR_i - b \cdot Q_{it}$ and c_{K_i} is given by [A.2].

Differentiation of [A.24] with respect to R_i yields:

[A.25] $\frac{\partial E[CS_t]}{\partial R_i} = \int_{c_{R_i}}^a g(c_t) \cdot b \cdot (R_i + Q_{-it})\, dc_t \geq 0.$ ■

Proof of Proposition 4:

The proof of Proposition 4 is divided into four parts, depending on the value of the strike price, *s*.

I. Differentiation of the objective function in [18] with respect to R_i:

[A.26] $\frac{\partial E[\pi_{it}|R_i]}{\partial R_i} - \int_{a-2bK_i-bQ_{-it}}^a g(c_t) \cdot (c_t - a + 2bK_i + bQ_{-it})dc_t =$

$$\int_{a-2bK_i-bQ_{-it}}^{a-2bR_i-bQ_{-it}} g(c_t)\,[c_t - a + 2bK_i + bQ_{-it}]dc_t + \int_{a-2bR_i-bQ_{-it}}^{a} g(c_t)[(a-2bR_i-bQ_{-it})-(a-2bK_i - bQ_{-it})]\,dc_t - \int_{a-2bK_i-bQ_{-it}}^{a} g(c_t)\cdot(c_t - a + 2bK_i + bQ_{-it})dc_t = 0.$$

Rearranging [A.26] yields

[A.27] $$\int_{c_{K_i}}^{c_{R_i^*}} g(c_t)\,(c_t - c_{K_i})dc_t + \int_{c_{R_i^*}}^{a} g(c_t)(c_{R_i^*} - c_{K_i})\,dc_t - \int_{c_{K_i}}^{a} g(c_t)\,(c_t - c_{K_i})dc_t = -\int_{c_{R_i^*}}^{a} \mathrm{g}(c_t)\cdot(c_t - c_{R_i^*})dc_t = 0,$$

where $c_{R_i^*} \equiv a - bR_i^* - b\cdot Q_{-it}$ and c_{K_i} is given by [A.2].

[A.27] holds iff $a = c_{R_i^*}$. That is, $R_i^* = 0$ for $i = 1, \dots, N$.

II. Suppose that $a - 2bK_i - bQ_{-it} < s < a - 2bR_i - bQ_{-it}$. Then, the objective function of firm *i* in stage 2 is as follows:

[A.28] $$E(\pi_{it}|R_i) - R_i\cdot\int_s^a g(c_t)\cdot(c_t - s)dc_t = \int_0^{a-2bK_i-bQ_{-it}} g(c_t)K_i\cdot(a - bK_i - bQ_{-it} - c_t)\,dc_t + \int_{a-2bK_i-bQ_{-it}}^{s} g(c_t)\frac{1}{4b}(a - bQ_{-it} - c_t)^2 dc_t + \int_s^{a-2bR_i-bQ_{-it}} g(c_t)\left[\frac{1}{4b}(a - bQ_{-it} - c_t)^2 + R_i\cdot(c_t - s)\right]dc_t + \int_{a-2bR_i-bQ_{-it}}^{a} g(c_t)R_i\cdot(a - bR_i - bQ_{-it} - \mathrm{s})\,dc_t - R_i\cdot\int_s^a g(c_t)\cdot(c_t - s)dc_t$$

Differentiating [A.28] with respect to R_i yields

[A.29] $$\int_s^{c_{R_i^*}} g(c_t)\,(c_t - s)dc_t + \int_{c_{R_i^*}}^{a} g(c_t)(c_{R_i^*} - s)\,dc_t - \int_s^a g(c_t)\,(c_t - s)dc_t = -\int_{c_{R_i^*}}^{a} \mathrm{g}(c_t)\cdot(c_t - c_{R_i^*})dc_t = 0$$

Thus, $R_i^* = 0$ for $i = 1, \dots, N$.

III. Suppose that $s < a - 2bK_i - bQ_{-it}$. Then, the objective function of firm *i* in stage 2 is as follows:

[A.30] $$E(\pi_{it}|R_i) - R_i\cdot\int_s^a g(c_t)\cdot(c_t - s)dc_t = \int_0^s g(c_t)K_i\cdot(a - bK_i - bQ_{-it} - c_t)\,dc_t + \int_{\mathrm{s}}^{a-2bK_i-bQ_{-it}} g(c_t)[R_i\cdot(a - bK_i - bQ_{-it} - \mathrm{s}) + (K_i - R_i)\cdot(a - bK_i - bQ_{-it} - c_t)]\,dc_t + \int_{a-2bK_i-bQ_{-it}}^{a-2bR_i-bQ_{-it}} g(c_t)\left[\frac{1}{4b}(a - bQ_{-it} - c_t)^2 + R_i\cdot(c_t - s)\right]dc_t +$$

$\int_{a-2bR_i-bQ_{-it}}^{a} g(c_t)R_i \cdot (a - bR_i - bQ_{-it} - \mathrm{s})\,dc_t - R_i \cdot \int_s^a g(c_t) \cdot (c_t - s)dc_t$

Differentiating [A.31] with respect to R_i yields

[A.31] $$\int_s^{c_{K_i}} g(c_t)\,(c_t - s)dc_t + \int_{c_{K_i}}^{c_{R_i^*}} g(c_t)\,(c_t - s)dc_t + \int_{c_{R_i^*}}^{a} g(c_t)\big(c_{R_i^*} - s\big)\,dc_t - \int_s^a g(c_t)\,(c_t - s)dc_t = -\int_{c_{R_i^*}}^{a} \mathrm{g}(c_t) \cdot \big(c_t - c_{R_i^*}\big)dc_t = 0$$

Thus, $R_i^* = 0$ for $i = 1, \ldots, N$.

IV. Suppose that $\mathrm{s} = a - 2bR_i - bQ_{-it}$. Then, the objective function of firm *i* in stage 2 is as follows:

[A.32] $$E(\pi_{it}|R_i) - R_i \cdot \int_{a-2bR_i-bQ_{-it}}^{a} g(c_t) \cdot (c_t - s)dc_t = \int_0^{a-2bK_i-bQ_{-it}} g(c_t)K_i \cdot (a - bK_i - bQ_{-it} - c_t)\,dc_t + \int_{a-2bK_i-bQ_{-it}}^{a-2bR_i-bQ_{-it}} g(c_t)\frac{1}{4b}(a - bQ_{-it} - c_t)^2 dc_t + \int_{a-2bR_i-bQ_{-it}}^{a} g(c_t)R_i \cdot (bR_i)\,dc_t - R_i \cdot \int_{a-2bR_i-bQ_{-it}}^{a} g(c_t) \cdot (c_t - a + 2bR_i + bQ_{-it})dc_t$$

Differentiating [A.32] with respect to R_i yields

[A.33] $$\int_{c_{R_i^*}}^{a} g(c_t)2bR_i^*\,dc_t - \int_{c_{R_i^*}}^{a} g(c_t)\left(c_t - c_{R_i^*} + 2bR_i^*\right)dc_t = -\int_{c_{R_i^*}}^{a} \mathrm{g}(c_t) \cdot \big(c_t - c_{R_i^*}\big)dc_t = 0$$

Thus, $R_i^* = 0$ for $i = 1, \ldots, N$. ■

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Research highlights (Fuel Cost Uncertainty, Capacity Investment and Price in a Competitive Electricity Market):

- We assess the effect of fuel cost volatility on electricity capacity and price.
- The price volatility of natural gas price follows a log-normal distribution.
- Electricity capacity increases in response to rising fuel cost volatility.
- Expected consumer surplus increases in response to rising fuel cost volatility.
- Government should not reduce price volatility of the spot market for natural gas.