



## Portfolio applications in electricity markets review: Private investor and manager perspective trends



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### ABSTRACT

The private sector plays a major role in the expansion and operation of power systems in most countries, especially those running liberalized electricity markets. Policymakers have the task of inducing private agents, through their regulatory designs, to make decisions that point toward social welfare maximization. Conversely, it is a task of private agents to protect themselves against the risks of the sector, including regulatory risks, international fuel price uncertainty, climate change policies, natural resource availability, electricity demand uncertainty, CO<sub>2</sub> clearance prices, etc. Instead of hiding all of these risks within the total project costs and losing competitiveness, private agents can use diversification as a strategy to deal with them. This paper presents a review of the main applications, voids and challenges of portfolio optimization for two key agents of the private sector: investors and managers. The problem of the investor is to design a technology portfolio to invest in that maximizes its expected returns and limits risks, while the manager has to design a portfolio of financial/physical instruments (long-term contracts, futures, etc.) to sell/buy electricity and hedge against price risks. We have found two fundamental issues in the literature; the first and most important is excessive confidence in historical data and statistical analysis for predicting future price behavior for a changing future in detriment of more structural analysis. Structural analysis can include particularities of modern power systems such as future transmission changes, congestion, operational constraints (ramps), new entrants, new technologies, and new demand grow patterns that cannot be taken into account by simply analyzing price historical values. The second is the omission of renewable complementarities, which is a proven characteristic of dispersed renewable plants that may have important risk-mitigation effects, although it has largely been ignored in portfolio analysis due to insufficient data, modeling limitations, and computational complexity.

### 1. Introduction: portfolio optimization opportunities in the private sector

New problems arising in the modern era such as global warming produced by anthropogenic greenhouse gas emissions on one side, and our dependence on electricity on the other, point toward the integration of new and clean technologies into the grid [1]. The concerns about the environment have not only pushed technological development, but also new regulations seeking to limit local and global emissions. New technologies dependent on natural resources such as solar and wind farms, new, more stringent local and global environmental regulations, and the new market arrangements that are necessary to accommodate such changes are added to a global context where **uncertainty** is the common denominator [2,3]. The feasibility of big investments, such as new large power plants and new, high-capacity transmission corridors, hinges on the risk perceptions of market agents on a series of

uncertainties at the operational, commercial, planning, and regulatory levels. The electricity system is now flooded with these uncertainties in multiple time scales, increasing the difficulty of decision making and pushing for the development of new risk management tools, which are fundamental for developing energy projects with limited levels of risks [4,5].

There are three key agents in the electricity sector who are constantly in need of risk management tools: private investors, managers commercializing energy (for large energy holdings, industrial consumers, or load serving entities) and planners, which are often specialized units of the regulator seeking social welfare over both the long and short run. The three interact with each other under the same platform, the energy markets. However, they face entirely different problems with respect to risk management.

The risk management problem for planners, for instance, often consists of long-term planning for the generation mix and transmission

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updates that maximize social welfare along with the policy design to achieve that plan. There are multiple sources of uncertainty including fossil fuel prices, renewable resource availability, technology development, social opposition, and global and local emissions limits, among many other factors that matter in these long time scales. The multiple sources of uncertainty notwithstanding, the vast majority of the literature over the last two decades has focused solely on fossil fuel price uncertainty [6–18]. Thus, the literature is paying limited or no attention to the other sources of uncertainties.

While market participants are key players in today's electricity sector, their risk management problems are less developed compared to the planner problem. However, after a decade of portfolio application for private agents, a systematic literature review is well justified by a number of important articles addressing diversification opportunities and efficient risk taking by trading in multiple markets in different time frames, investing in multiple technologies, and exploiting distant resources with non-coincident production connected to the transmission grid (temporal and geographical complementarity), etc. In addition, there are a number of new concepts, tools, and methodologies available in the literature that have not been fully integrated into private portfolio analysis such as complementarity assessment for multiple renewable sources, structural modeling of the power system physics, and the integration of real option analysis and portfolio optimization. This literature is reviewed in the following sections, highlighting research trends, opportunities, and challenges. Most of the key concepts found in the literature reviewed in this paper are summarized in Fig. 1. The key concepts appearing around the figure of the investor are option value, return and risk measures. Around the figure of the portfolio manager we found trading mechanisms, dynamic and multi-stage, static models, etc. We also found some key concepts around the literature dealing with both market agents, referred to here as cross-cutting issues, among these we are highlighting statistical price modeling, structural modeling, and renewable modeling. All of these concepts are briefly explained and referenced in this review.

Existing articles are mostly focused on portfolio applications from the planner perspective. This is the traditional planning problem, where systems costs are minimized. Here, portfolio theory allows including the risks over such social solution, without specific attention to market details or market agents.

Given the current trends in power systems is every day more relevant considering the private agents' perspective. The private sector has a growing role in power systems, especially in renewable energy

development. This paper is focused on the perspective of private agents and its contributions can be summarized as follows:

- To the best of our knowledge this is the first review on portfolio applications focused on private agents (both investors and managers). This perspective is of growing interest due to the current trend of implementation of electricity markets across the world and increasing the deployment of renewable energy technologies.
- The paper presents an overview of different portfolio tools for the decision making process of private agents in power systems with high penetration of renewable energies.
- In addition to the review of the existing literature, this paper discusses cross-cutting issues emerging from the growing interaction of a new technological paradigm: markets and uncertainties sources driven by renewable energy development and technology evolution.

This paper is organized as follows: Section 2 provides an overview of the applications, problems, and challenges of portfolio optimization for **private investors**. Specifically, Section 2.1 presents the different measures of return/cost and risk typically covered in the literature, Section 2.2 highlights the lack of appropriate modeling of uncertainty factors that are usually ignored even when they play an important role for investors, and finally, Section 2.3 addresses the importance of considering the value of waiting in the investment decision problem and how to address it in a portfolio analysis. Section 3 discusses the main applications of portfolio optimization from the **manager's perspective** and presents two families of approaches: static and dynamic models. Section 3.1 presents static models that assume that all decisions must be made “here and now,” and Section 3.2 presents dynamic models that are much more computationally demanding but they are able to separate “here and now” decisions and “wait and see” decisions, and finally, Section 3.3 presents alternative markets, such as capacity markets, demand response markets and others, to diversify services and mitigate risks. Section 4 presents cross-cutting issues, voids and challenges from both perspectives (investors and managers), Section 4.1 provides an overview of the most used modeling approaches to simulate price evolution, and Section 4.2 focuses on renewable profile complementarities and how they have been ignored by portfolio literature, even when there is literature available that provides estimations and measurements of high complementarity between geographically dispersed renewable resources. Finally,

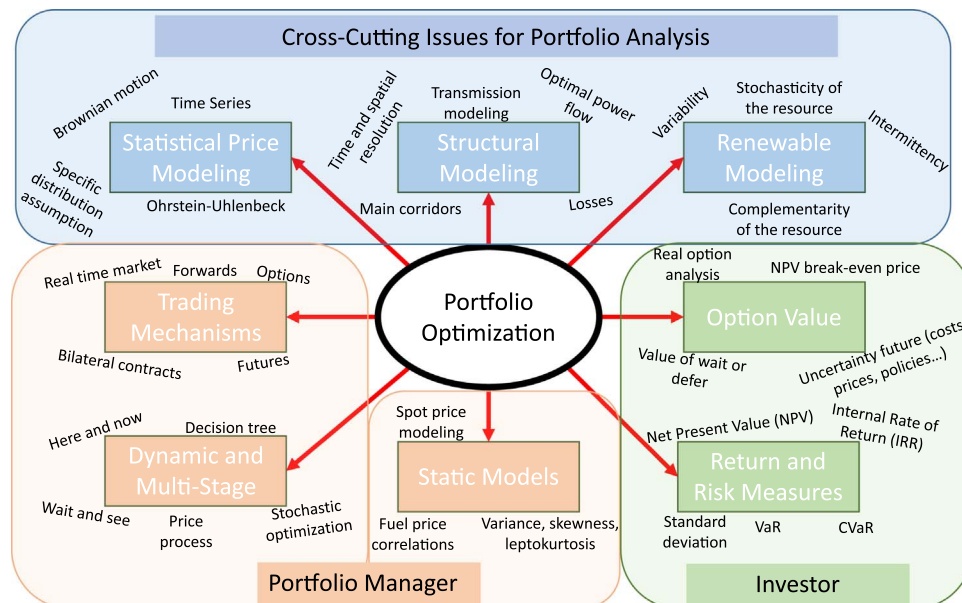


Fig. 1. Most important concepts reviewed in this paper.

Section 5 concludes with the main voids in the literature and challenges to be addressed in the future research.

## 2. Portfolio optimization as tool for investors to allocate capital in different generation projects

The problem that investors face is quite different from the problem planners face. Investors aim to define an efficient technological-locational mix by maximizing their return on the investment. Planners, on the other hand, aim to minimize costs. Investors may focus solely on some places and some technologies according to their preferences and possibilities, while planners may focus on the whole arrangement of places and technologies. Additionally, investors have the flexibility of waiting to invest in a project. However, when the investment is done, they have a high degree of inflexibility due to the high sunk costs involved. Planners, on the other hand, have to plan to meet the expected demand, but they also have the possibility of changing the long-term plans. Finally, investors are usually witnesses of policy changes, transmission expansions, new entrants, and environmental standards, while planners have a key decision-making role in these areas. Thus, the investor's portfolio problem of involves a very large amount of capital and high levels of uncertainty in the returns on the investment, so diversification among technologies, resources, and places is a common strategy for hedging risk. Portfolio optimization is a tool used to deal with these risks through diversification.

The generation sector has historically faced high and volatile electricity spot prices caused by the variability of demand and the impact of physical constraints such as generation and transmission limitations. Such volatility has increased in recent years due to the integration of volatile renewable resources including wind and solar. The fast progress and aggressive entry to the market of these technologies (see the examples of penetrations of these technologies in Chile in Fig. 2) has produced a decrease in the levels of spot prices as well as an increase in their variance [19–21]. In addition, the intermittency of renewables requires that high transmission capacities be available at all times to move its energy in the system. However, the time required to develop new transmission is much longer than the time to develop renewable projects, so it is not infrequent to see congestion on transmission lines near a set of renewable projects. This also dramatically impacts spot prices, either by marginal losses or simple by a decoupling of markets caused by congestion. For example, this is exactly the situation produced in the north of Chile where solar PV plants and coal-fired plants are subjected to long hours of zero marginal costs due to transmission congestion [22]. Unlike planners, who usually plan in the long term and therefore they assume that transmission systems will adapt and therefore congestion can be avoided in the portfolio analysis, investors do not have that possibility. If the analysis is done in the long term, investors have to include the transmission system and its future possible congestions in the financial modeling of their portfolio of projects, since electricity prices and energy production may change dramatically by a change in the

transmission structure. Transmission equalizes spot prices over the space through the marginal loss and marginal congestion component of prices and is a key locational signal for generation siting.

Investors' capital allocation in the electricity sector is a particular case of the project portfolio selection problem (PPSP) that studies how to distribute capital among different projects such that the expected return is maximized for a given level of risk [23]. Despite that there are different investment situations,<sup>1</sup> all investors seek the same goal: to maximize their return and limits their risks, so in all of these situations a measure of profitability has to be estimated using the projects' projected cash flows (see Fig. 3). This means that for every year of a generation project's service life, the estimation of its income and its costs is required. At the same time, income and costs essentially depend on uncertain factors like electricity spot prices, project expected generation, fuel prices, and capital costs, among others. Cash flow calculations are then random variables that depend on the realization of different sources of uncertainty as illustrated in Fig. 3. Return and risk measures arising from these cash flows feed into portfolio optimization models to guide investors in the design of efficient return-risk portfolios.

According to the investors' level of risk aversion and their currently set of generation facilities, different portfolios of projects can be selected by buying or developing new projects, or alternatively, the investment could be delayed if the uncertainty is too great. Note that the option of deferral is an important difference compared with the problem of planners, who often have to plan to satisfy the expected demand without the ability to defer generation over time. This additional flexibility afforded to investors and the corresponding modeling approaches are explored in Section 2.3.

### 2.1. Return and risk measures of investments in energy projects

A measure of profitability must be estimated in order to account for the risk of different projects. The main tool to estimate a project's return is cash flow analysis. Different estimations of profitability can be obtained from a discount cash flow analysis such as the Internal Rate of Return (IRR), the Net Present Value (NPV), or the Present Value Index, among others [24]. In fact, investors will choose the projects with highest NPV. This is the Marshallian approach [25,26] where utility is maximized subject to budget constraints. As an example, Roques et al. [27] used NPV in their portfolio model to design efficient investment combinations among baseload technologies (coal, nuclear, and CCGT plants). They studied how the impact of fuel, electricity, and CO<sub>2</sub> price uncertainties affect optimal portfolios. On the other hand, Muñoz et al. [28] used the internal rate of return (IRR) as a measure of profitability when analyzing renewable project portfolios for investment in the Spanish market. Both publications used the standard deviation of their return variables as a measure of risk. Table 1 presents the return measures and uncertainty factors modeled in some related publications by optimizing a portfolio from the investor perspective.

Although IRR and NPV are both derived from discounted cash flows, they differ from one another. Indeed, when investments are ranked using these two methods, the result is not necessarily the same [29,30]. Tang and Tang [29] go deeply into the difference between these two measures. They argue that IRR gives the private investor's point of view, while NPV gives the society's point of view. The authors explain this view because IRR varies with a change of financial arrangements (e.g., a change of taxation rate or equity-loan ratio), while NPV does not, so they proposed IRR as a financial indicator and NPV as an economic indicator.

<sup>1</sup> For example: individual investors who have the opportunity to invest in any generation technology and their decision variables are continuous (i.e., they can invest part of their budget, from 0% to 100%, in one project or in a group of projects) or big energy companies that normally focus on investing in projects in areas of their technological expertise and their decision variables are more discrete—to invest or not to invest in a certain project, etc.

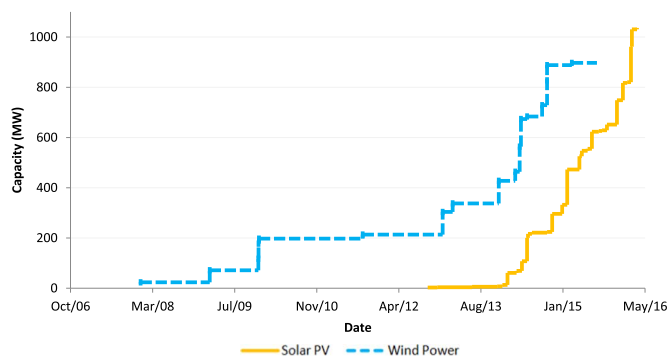


Fig. 2. Accumulated installed capacity of solar PV and wind power plants in Chile.

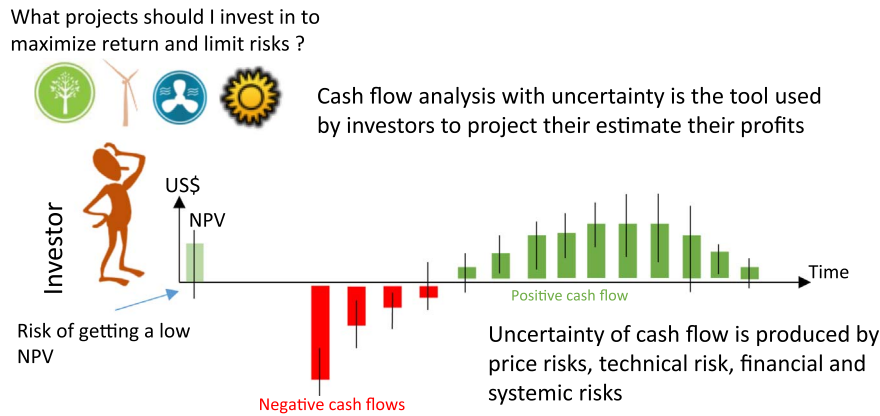


Fig. 3. Portfolio problem of the investor: defining an efficient investment plan to maximize return.

Table 1  
Return measure and uncertainty factors modeled in different papers.

References	Return measure	Uncertainty factor
Roques et al. [27]	NPV	Fuel, electricity, and CO <sub>2</sub> prices are represented by normally distributed random variables whose cross-correlation and standard deviation are derived from historical time series.
Madlener and Wenk [29]	NPV	Time series of electricity spot price of both base and peak load are used to best fit a distribution (log-normal distribution) Capacity factor: based on historical time series. Hydro capacity factor follows a log-normal. Annual variability for solar PV and wind power is approximated with the data from hydro technologies. Fuel costs: time series. Natural gas follows a Gumbel distribution, while a Gamma distribution is used for uranium.
Muñoz et al. [28]	IRR	Electricity price for the wind, mini-hydro, and solar thermo-electrical modeled with Pearson distribution adjusted from historical values. Electricity price for solar PV is regulated, and the value is pre-established. Other values (investment ratio, service operation life, capacity factor, etc.) of the cash flow are assumed normal with standard deviation depending on different scenarios proposed by the author.
Glensk and Madlener [30]	NPV	Historical series of electricity, fuel, and CO <sub>2</sub> prices used to fit different distributions. Electricity prices were fitted using a beta distribution.
Rohlfis and Madlener [31]	NPV	Future electricity price and future coal, gas, and CO <sub>2</sub> prices modeled assuming Geometric Brownian Motions. Monte Carlo method used to simulated paths of the price development.
Fleten et al. [25]	NPV	Future electricity price modeled assuming Geometric Brownian Motions.

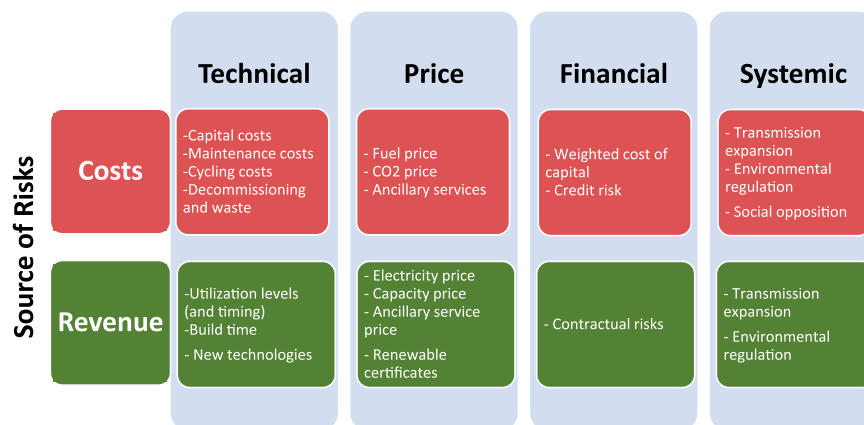


Fig. 4. Risks affecting a firm's cash flow calculation (adapted from [36]).

Organizations may have additional requirements beyond profitability for investing in projects. In the case of power generation investment, for example, renewable generators have benefits that conventional technologies do not, including fewer environmental externalities, flexibility in production, modularity, and reversibility, among others, which rarely are included in the investment decision-making process [31,32]. However, there is research on investment decision-making that considers measures beyond profitability that depend on the strategy of the organization. Davoudpour et al. [33] used an approach based on Analytic Hierarchy Process (AHP) to select renewable projects for an R & D organization by using expert opinion to

find a hierarchy model of a renewable technology portfolio considering market, competitiveness, technical, capability, and learning.

A project may add value in addition to its own return if it helps decrease risks. A new project could be used to enter the market or consolidate a company's position, or it could be develop or acquire to learn about a specific technology or process [34]. Most literature on optimization portfolio does not take these factors into account, although they are already an important part of the literature on project valuation. Therefore, this is a line of research that needs to be exploited in order to better align the literature on portfolio optimization with reality and thus make it useful to investors.

2.2. Others risk sources in addition to electricity prices: technical, financial, systemic

Uncertainty is present in different dimensions and stages of a project development, from technical to systemic risks, including regulatory risks which are commonly accepted as one important risk in the sector [35], causing cost variations on one side and revenues variations on the other, as presented in Fig. 4.

Most technical, financial, and systemic risks are difficult to explicitly include in optimization models, so most works either explicitly or implicitly assume that these factors remain constants and therefore do not affect income or costs or use scenario approaches to quantify them. On the other hand, uncertainty of prices is most treatable in optimization models, both on the income (electricity price) and the cost (fuel prices and CO<sub>2</sub> prices) sides. There are numerous methodologies for electricity spot price forecasting, as reviewed by Weron [37]. However, despite that, most portfolio papers only focus on statistical methods based on past information. This backward-looking strategy has limited value on a system that is evolving to a new carbon-free technological paradigm.

2.3. A dynamic problem and the value of waiting/project deferral

One important feature of project development in a competitive energy industry is that investors can “wait to invest,” for example, to acquire more information about a regulatory reform. Considering the option of waiting before committing resources is very important because it recognizes that the firm has an opportunity cost and the possibility of improving its outcome. This is especially important in the renewable energy field, taking into account the possibility of waiting is very important because renewable projects show a high technological progress rate and require short construction times [1].

Static NPV cannot capture the value of waiting, so Real Option theory is the tool to include this flexibility in the evaluation [38]. Real Option Analysis (ROA) has been applied to the electricity sector for decades to account for the irreversibility of investments. A good **comprehensive review** of ROA is presented by Dixit and Pindyck [26]. In the electricity generation sector, there are several examples of applications of ROA. Indeed, Fernandes et al. [1] present a complete review of applications of ROA applied in the electricity sector. They found that **ROA applications applied to the renewable sector are still limited**. Moreover, the technique is mostly applied to wind and hydropower to the detriment of other newer renewable technologies like photovoltaic. However, recent publications are filling this gap. For example, Zhang et al. [39] present a good review of studies on renewable energy investment using real options method. The authors also propose a real option model for evaluating renewable energy investments by considering uncertain factor such as: CO<sub>2</sub> prices, non-renewable energy costs, investment costs and market prices of electricity. They use their model to evaluate the investment decision of a solar PV power plant in China and its optimal timing.

There are countless works using ROA to analyze investments in conventional technologies and also to evaluate the implementation of policies. For example, Ming Yang et al. [40] use a real option approach for analyzing the effects of government climate change policy in power investments. The authors investigated the flexibility that companies have to optimally time their investments given regulatory uncertainty. Climate change policy uncertainty is represented by means of an uncertain carbon price. Similarly, Sekar [41] uses a real options valuation methodology to evaluate investments in three coal-fired power generation technologies (pulverized coal (PC)), integrated coal gasification combined cycle (IGCC), and IGCC with pre-investments that make future retrofit for CO<sub>2</sub> capture less expensive in an environment of uncertain CO<sub>2</sub> prices. Boomsma et al. [42] analyze investment timing and capacity choices for renewable energy projects under different support schemes, namely, feed-in tariffs and renewable

energy certificates trading. The authors found, through an applied case of study in the Nordic electricity market that feed-in tariffs encourage earning investment in wind power, while certificates trading creates incentives for larger projects. Fleten et al. [25] use ROA to show that investment in a decentralized wind power generator facing uncertainty in electricity prices should be made at a price considerably above the NPV break-even price (electricity price that makes NPV negative) because of price uncertainty.

While optimization methodologies using ROA are usually performed from a power producer perspective to evaluate a single power plant, a large investor would typically prefer to invest in a portfolio of technologies [43]. There are only a few publications that combine ROA and portfolio optimization analysis to find efficient combinations of investments along with its timing. The first research to explicitly combine these methodologies from the perspective of an investor in the electricity sector is, to our knowledge, the research by Fortin et al. [43]. They use ROA to find the optimal timing of investing in carbon capture and storage modules for coal- and biomass-fired power plants and optimal installation time for wind power plants. Using different electricity price evolution paths, the authors derive return distribution for the investment of these technologies. These return distributions (which already include the value of flexibility given by project deferral) are then employed as the input of a CVaR portfolio optimization as presented in Fig. 5.

Other papers expand the work of Fortin et al. [43] by taking into account diversification over time by considering the option of having a different portfolio at a future point. Indeed, Szolgayová et al. [44] find that the possibility of adapting the portfolio actually have a relevant effect on today's portfolio investment decisions. On the other hand, the paper by Fuss et al. [45] further contributes by applying the methodology to different socio-economic scenarios and different targets in greenhouse gasses emissions. Their extension takes into account that investors are completely uncertain about future carbon prices, and therefore it is impossible to assign probabilities to different targets. Thus, investors would seek robust portfolios that perform well even in the worst scenarios. They find that uncertainty associated with CO<sub>2</sub> prices has a profound effect on the optimal composition of technologies portfolios. Moreover, the authors find that uncertainty about stabilization is more important in the energy mix composition than the socio-economic scenario, especially for risk-averse investors.

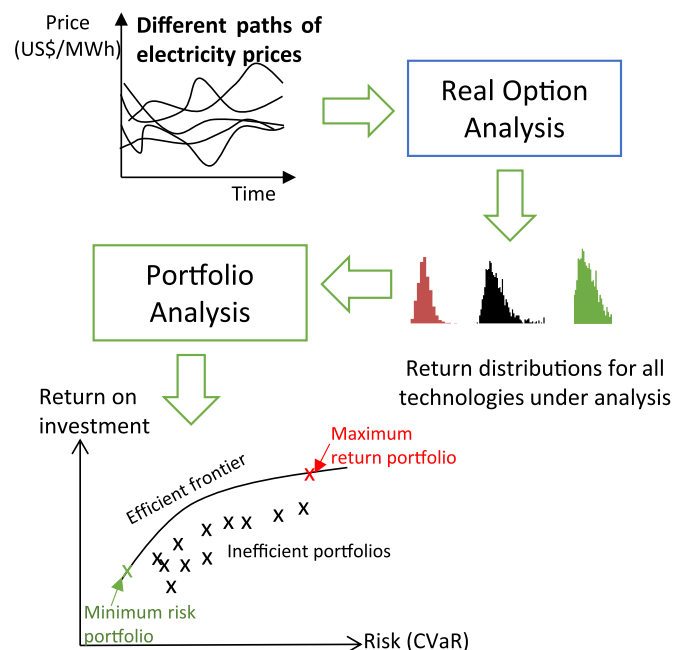


Fig. 5. General methodology used in [43]: use of real option analysis and portfolio optimization.

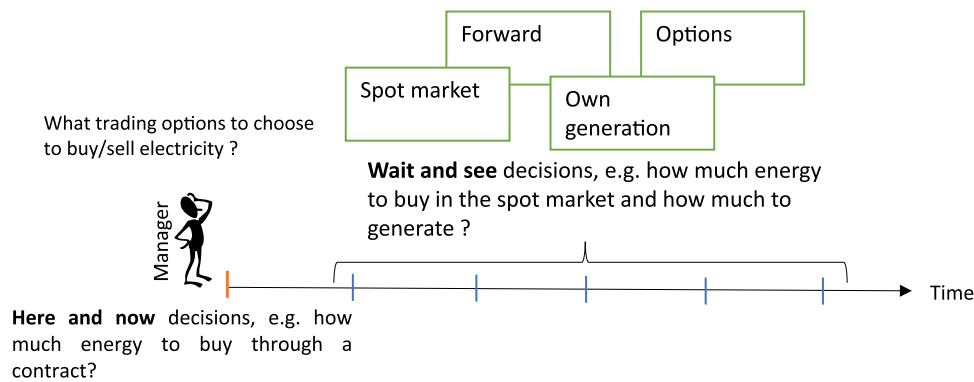


Fig. 6. Manager's Portfolio problem: defining efficient trading choices to maximize profit.

Exploring the combination of these tools—real option analysis and portfolio optimization—in the investment decision-making process is a great research opportunity. All the publications mentioned above ignore sources of uncertainty such as fuel costs and their possible complementarities, e.g. biomass cost declining as carbon price increases [45] or renewable resource uncertainty (wind speed, solar radiation, hydrologies, etc.), among other sources of uncertainty that investors face in the real investment decision process.

### 3. Portfolio optimization as a management tool for electricity sellers and buyers

Energy managers, both managers of electricity production firms and of big energy consumers, seek to limit their price risks by using instruments to hedge against spot price fluctuations. As investors, managers seek to maximize the firm's expected return while limiting its risks. However, instead of allocating capital among different investment opportunities, managers allocate electricity among different instruments (day-ahead markets, real-time markets, bilateral contracts, forward, etc.) as is shown in Fig. 6. Financial instruments have different delivery periods and maturity dates. While the spot market is nearly instantaneous, bilateral contracts can last for years. These facts introduce difficulties to the optimization because decisions for some trading instruments can be deferred in time according to new information on prices (e.g. how much energy to buy/sell on the spot market), while other decisions must be made in a specific period (e.g. how much energy buy/sell through a long-term forward contract).

A big energy consumer can take advantage of portfolio optimization not only by choosing among the instruments, but also by choosing among generation technologies. For a big energy consumer, there is a difference between signing a bilateral contract with a conventional generation plant or signing it with a solar PV plant, a wind power plant, or a combination of any of these alternatives. A consumer's preference for one supplier over another depends upon factors such as the demand profile, carbon footprint, and willingness to pay, etc. For example, the subway in Santiago de Chile recently signed two bilateral contracts, one with a solar PV plant and one with a wind power plant, and the two suppliers will cover approximately 60% of its energy needs. Because the subway system has greater energy needs during the day, the solar PV plant option is a good opportunity, although its daily load curve has two peaks, one in the morning and one in the late evening, just when the electricity produced by a solar PV plant is low, so the subway's energy managers chose a complementary wind power plant to avoid having to buy energy on the spot market. Portfolio optimization is a formal and well-tested tool for tackling this kind of problem, both for determining the type of instruments to use and for dealing with different technologies and locations.

Due to non-storability, inelastic demand, and a steep supply curve, electricity spot prices suffer from high variability. That is why most agents usually use contracts and other financial/physical instruments

to hedge against these fluctuations. These instruments play a very important role in some electricity markets for future price discovery and price certainty. In fact, there are some electricity markets that rely entirely on bilateral contracts, such as the Chilean electricity markets. The most basic instruments that offer future price discovery and price certainty to electricity sellers and purchasers are forwards, futures, and swaps. All of these instruments may have different delivery periods and maturity dates. In fact, the maturity periods of forwards contracts range from hours to years [46].

The task of energy managers is to choose from among these instruments to maximize return and at the same time limit its risks. A correct strategy allows firms to avoid losses due to price fluctuations, reduce the volatility of earning, and meet regulatory requirements [47]. Portfolio optimization has been used in the literature as a tool to efficiently choose from among these instruments as well as from among real-time markets (real-time and day-ahead markets). It should be noted that managers have two types of decisions, “here and now” or “wait and see.” While “here and now” decisions are those that the manager has to make in the present, such as about how much energy to sell/buy using a long-term contract, “wait and see” decisions can be delayed to expect future developments, such as how much energy should be bought or sold using the real-time market, which is a decision that can be postponed until the need becomes urgent.

#### 3.1. Static approaches: traditional portfolio optimization applied to the manager problem

A traditional static portfolio optimization approach is formulated by Liu and Wu [48], who consider the problem of energy allocation for a power producer allowing three types of trading approaches: risk-free (local) contracts, riskier contracts (non-local), and the spot market. In this formulation, the planning period may be one day, one week, one year, or several years, etc. Non-local bilateral contracts are subject to risk because generation companies may face congestion transmission charges that depend on the difference between nodal prices. The uncertainty is then only present in electricity locational spot prices because fuel prices are assumed to be fixed in their work. Liu and Wu [48] present a static approach in which spot prices are characterized only by mean, variance, and spatial correlations, and they assume that nodal prices follow a multivariate normal distribution.

Treating the spot market as an individual asset has the disadvantage of some loss of information, because hourly spot prices reflect seasonal behavior, which is usually given by the behavior of the demand. When spot prices are treated as an asset and represented by a price distribution, the known seasonality is wrongly translated as an additional variability. By contrast, treating each period as a different asset gives more degrees of freedom to include this seasonality as new information (see Fig. 7). For example, Gokgoz and Atmaca [49] use mean-variance portfolio optimization by taking spot market hourly prices as separate assets in addition to bilateral contracts in the Turkish

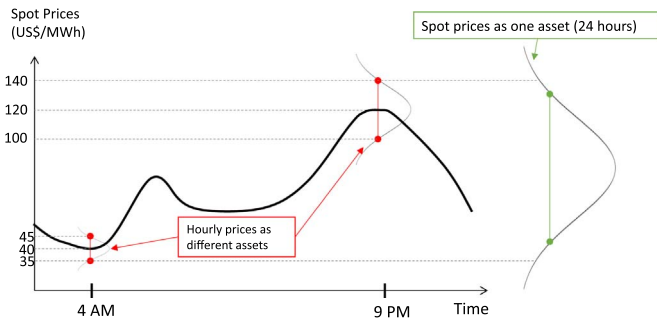


Fig. 7. Example of normal distributions capturing daily and hourly spot prices. Spot prices of long periods lose seasonal information that is translated into a greater variance.

electricity market. Turkey has no local, zonal, or nodal pricing system, so spot pricing is used as a signal for the entire system, and therefore there are no congestion charges. The assumption of 24 selling alternatives is new in this kind of study and allows sellers to choose according to their risk-return preferences to sell different hours, either on the spot market or via bilateral contracts.

Unlike dynamic models, which require large computational capacities because uncertainties (prices, costs, resources, etc.) are modeled in time, static models are simpler and therefore other sources, in addition to electricity prices, can be considered. For example, fossil fuel prices (oil, gas, and coal) present high variability, are highly correlated [50], and introduce uncertainty into generation costs. Mathuria et al. [51] consider spot market and bilateral contracts as trading options for a generation company in Sweden that faces risks from electricity prices, fuel prices, and from emission prices. The authors find a strong correlation between electricity spot prices and emission prices (see Fig. 8). This enables risks to be hedged by changing the allocation on the spot market, since a price change in the emission market (cost side) is compensated by a corresponding price change on the spot market (income side).

Fig. 8 shows estimated correlations by Mathuria et al. [51] between electricity prices, coal prices, gas prices, and the European Union Allowance (EUA), which are climate credits that represent the right to emit one ton of CO<sub>2</sub> into the atmosphere.

On the electricity purchaser side, Huisman et al. [52] propose the use of a static mean-variance framework to optimally allocate positions in the day-ahead energy market as well as peak and off-peak forward contracts. Peak-forward contracts involve the delivery of power capacity during certain hours of high demand; off-peak contracts involve the delivery of a base capacity at all hours. Uncertainty is introduced through prices of the day-ahead energy market and consumption

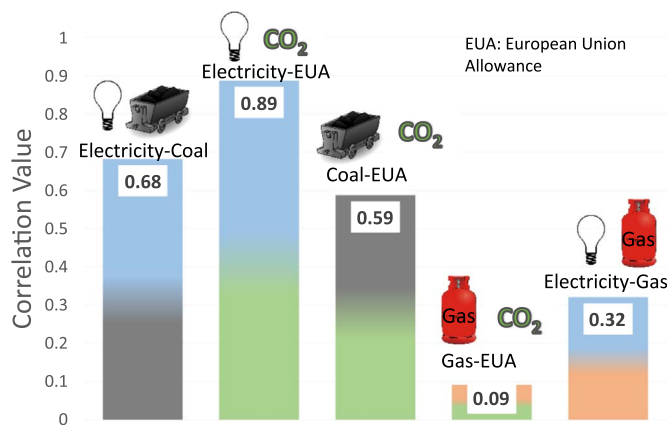


Fig. 8. Correlations of electricity spot prices with coal, EUA, and gas prices and correlations of coal and gas prices with EUA prices. Source: Mathuria et al. [51].

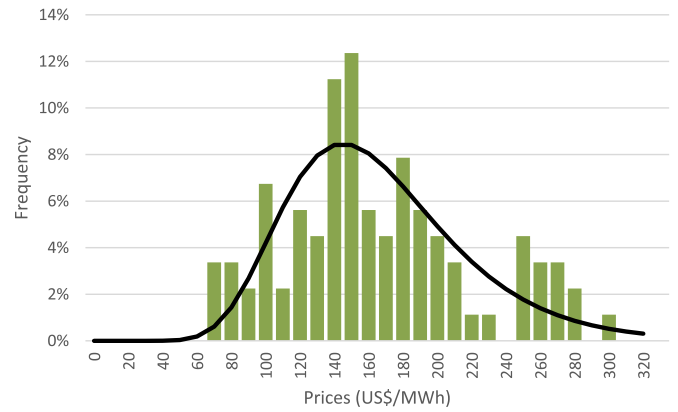


Fig. 9. Histogram of the spot prices in the Alto Jahuel 220 kV node in Chile from 2008 to 2015.

volumes. Day-ahead prices and hourly demand are assumed to be entirely characterized by their historical mean and variance. The problem is then to minimize the total electricity cost subject to a maximum level of risk, where the total cost is given by the sum of the cost of off-peak forward contracts, peak forward contracts and day-ahead energy market purchases. The authors assume a price-taker purchaser, i.e., the trading of electricity does not affect prices, and they show that the optimal allocation to peak contracts relative to off-peak contracts is the same for all purchasers. The differences in the exact allocation, including positions in the day-ahead market, are determined by their risk attitude.

Several studies have argued that electricity prices and fossil fuel prices show a positive level of skewness and leptokurtosis [53–55], so it does not seem enough to characterize them solely by mean and variance. Skewness is the extent to which a statistical distribution is not symmetrical, and leptokurtosis occurs when the distribution is more peaked than normal. See, for example, the asymmetry and fat tails of the histogram of monthly average spot prices from January 2008 to January 2016 for the Alto Jahuel 220 kV, a key transmission node in central Chile, presented in Fig. 9.

Pindoriya et al. [56] include skewness in their portfolio optimization analysis. They propose a mean-variance-skewness (MVS) model to set the energy allocation of generation companies among the spot energy market and bilateral contracts with clients located in different zones. A positive skewness means that the density function has a right-handed tail and therefore maximizing skewness in a context in which the distribution reflects profitability, implies the minimization of possibilities of low profits. Accordingly, an MVS model maximizes the return and the skewness (first and third moments of the distribution) and minimizes the variance (second moment), transforming the problem into a multi-objective optimization problem.

Suksonghong et al. [57] proposed a similar problem, but also added maximizing **diversification** as another objective to the optimization. This was implemented by minimizing the difference between the highest and lowest allocations. According to the authors, including the fourth objective of diversification effectively caused a more uniform allocation among all the instruments. The inclusion of skewness and other conflicting objectives makes the optimization problem very difficult to solve, so different optimization tools are used for these types of problems. A multi-objective optimization problem can be tackled by different methods [58], such as scalarization techniques, e-constraints methods, goal programming, among others [59].

### 3.2. Dynamic and multi-stage approaches

New information might require the consideration of the allocation problem at multiple stages, requiring a transition from static to

dynamic analysis. Only a few dynamic portfolio optimization approaches have been developed. Indeed, the application of multi-stage optimization models is relatively new in the literature on portfolio optimization in electricity markets for electricity sellers and purchasers. Multistage portfolios enable the modeling to optimize the rebalancing of the portfolio at multiple points in the future based on the information available at that time. The most common problem formulation in multi-stage stochastic optimization formulations is the equivalent deterministic form, which can be very large and require excessive computational capacities [60]. Thus, the most common multistage optimization application focuses on just two stages.

In stochastic problems with two stages, the first stage is when the decision maker takes action before random variables are revealed (“here-and-now-decisions”), and the second stage decisions are made after the random effect occurs (“wait-and-see decisions”). García-González et al. [61] present an example of a two-stage stochastic optimization problem in an electricity market in which a generation company that owns a wind farm and pumped-storage facility optimizes its bidding policy in the first stage and the decision on the operation of the pumped-storage for each possible realization of the random variables in the second stage. In that case, random variables are wind production and market prices as presented in Fig. 10.

Lorca and Prina [62] tackle the problem for a **power producer** holding thermal generation units and considering locational electricity prices. They use a stochastic optimization model to optimize the trading of electricity from a power producer in two locations through forwards contracts, a contract for differences, and the spot market. Their model obtains a set of contractual decisions at the beginning of the time horizon (“here-and-now decisions”) and a set of own-generation and spot market trading decisions in future time (“wait-and-see decisions”). They use a time series model to capture temporal and spatial correlations of locational electricity prices. The authors use CVaR as risk measurement, including it into the objective function multiplied by a risk aversion parameter. The main drawback to this formulation is the dimensionality problem. Modeling more than two buses make the problem too large to solve in reasonable time. Accordingly, the methodology is very useful to theoretically assess

how changes in price parameters cause changes in contractual and trading decisions, but it cannot be used in real-case scenarios in which the producer faces multiple locational electricity prices. Indeed, Lorca and Prina [62] found that changing the correlation parameter  $\rho_{ij}$  for locational electricity prices significantly affected the relationship between expected profit and risk. For fixed values of expected profit, as the correlation parameter between locational electricity prices decrease, the risk is also decreased.

On the electricity purchaser side, Rocha and Kuhn [63] present a multistage mean-variance model for the management of electricity derivatives from the point of view of an electricity purchaser who is a price-taker and the need to satisfy its clients’ demand. Electricity purchasers have three alternatives for acquiring electricity in time—spot market, forwards contracts, and call options—and **stochasticity appears in the form of uncertain electricity demand, spot prices, and derivative prices**, which are revealed sequentially over time. They present a stochastic optimization problem with aggregation of decision stages and Linear Decision Rules (LDR) approximation, avoiding the use of a large decision trees and limiting the computational burden. Spot prices are modeled by an **Ornstein-Uhlenbeck process** with seasonality, which is a **mean-reversion** stochastic process traditionally used to simulate electricity prices [64,65]. Electricity demand is also modeled as a stochastic mean-reversion process with seasonality. Rocha and Kuhn [63] found that incorporating **adaptivity** in portfolio optimization models is beneficial, especially in the presence of high spot-price volatility. The authors show that adapting to different market conditions provides a flexibility that makes it possible to obtain a better mean-risk profile, particularly when the decision maker is risk averse.

3.3. Diversification beyond energy markets: ancillary services, capacity market, and demand response

In addition to energy markets, in some countries power producers have other markets that could allow them to diversify risks. For example, capacity markets, ancillary services markets, and regulation services markets are options in which some generators can participate to mitigate risks of electricity markets. Similarly, load-serving entities have other resources beyond bilateral contracts to manage risks, such as demand response programs. Few publications have included these markets as part of portfolio optimization models, although one exception is a paper by Yu [66], which presents a model that can be used for multiple commodity electricity products that may include electricity, spinning reserve, or regulation, etc. The objective function is the minimization of risk, defined as the portfolio cost variance subject to the exceedance of the desired net profit. The author presents a case study involving two power pools, NYPP and PJM, each with two available markets, day-ahead energy and spinning reserve. The model includes constraints such as transaction costs and wheeling contracting, leading to a mixed integer formulation.

On the other side, an electricity buyer such as a retailer may be able to hedge risk using demand response programs. High demand usually implies high electricity prices (because of electricity’s steep supply function), and therefore there is a positive correlation between electricity spot price and customer demand [67]. Demand response (DR) programs help mitigate this correlation, lending an additional extent of flexibility to decrease exposure to risk. Deng and Xu [67] show that including DR programs, specifically in the form of interruptible contracts, significantly improve the profit-risk profile of portfolios for an electricity buyer considering the following instruments: spot market buyer, forwards contracts and DR programs. The authors used both variance and VaR as risk measures and found that the role played by DR program is dependent upon the choice of risk measure. Given a fixed expected profit, a 95%-VAR minimization problem holds all available interruptible programs, suggesting that DR programs may be especially useful in the worst-case scenarios.

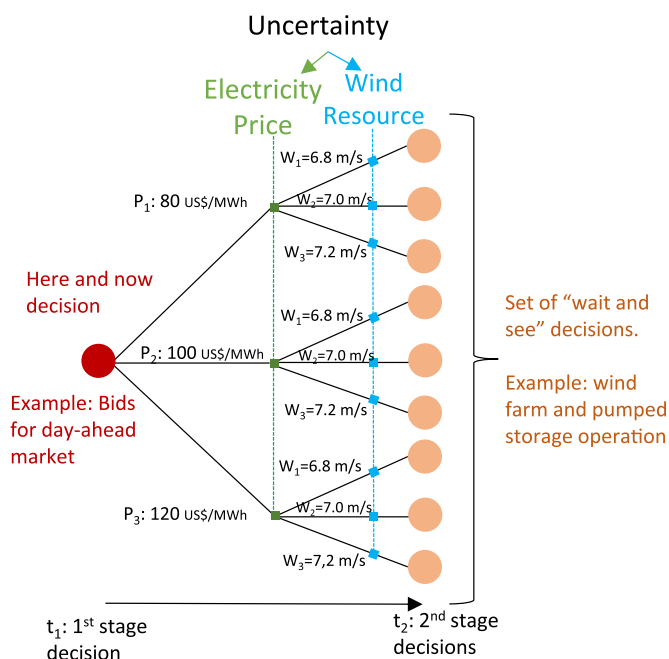


Fig. 10. Uncertainty representation in a two-stage model. Adapted from [61].



**Table 2**  
Frequently used Price processes in portfolio optimization literature.

Price Processes	Description	References
<b>Distribution fitting</b>	Fit a probability distribution to a series of historical data of prices. Examples of the distributions used are Normal distribution, Lognormal, Beta, and Pearson, among others.	[27–30]
<b>Time series models</b>	Time series are widely used for multiple applications, and price modeling is no exception. Markets with locational prices require a multivariate time series model. Examples of time series models are ARMA models, ARIMA model, GARCH models, etc.	[64,66,76]
<b>Continuous-time stochastic process</b>	Geometric Brownian Motion (GBM) and Ornstein-Uhlenbeck processes are examples of continuous-time stochastic processes. These models are widely used in mathematical finance to model price evolution. While GBM has a constant drift over time, the Ornstein-Uhlenbeck process tends to drift toward a long-term mean (mean-reverting). Both processes satisfy a stochastic differential equation.	[31,65,77,78]

**4. Cross-cutting issues in portfolio optimization for investors and managers: price process modeling and renewable complementarity**

There are cross-cutting issues to be found in the literature on portfolio optimization from the investor and the manager perspectives. First, most literature ignores the fundamentals of power system structures in price modeling, while in turn there is an excessive support on technical approaches that attempt to model stochastic behavior by using statistical analysis and historical data. Second, renewable complementarity is ignored, although there is strong evidence that the geographic diversification of solar and wind power plants in different locations may present complementarity generation profiles [68–72], and this complementarity has not yet been included in portfolio models.

*4.1. Modeling price process in portfolio optimization models*

Modeling electricity prices is critical for evaluating risks for both investors and managers. Electricity prices directly affect incomes, so it is crucial to model them in properly to account for the corresponding risk. There are mainly two families of approaches to model electricity price processes [46,73], structural or fundamental approaches that rely on simulation of the operation of the electricity system, and technical approaches which rely on historical data and statistical analysis to model the future behavior of prices. Fundamental approaches are more realistic since they allow for simulating new scenarios that cannot be considered with technical approaches, although they do require extensive computational effort. Most publications on portfolio optimization rely on technical approaches. Moreover, most publications on portfolio analysis simply use price processes such as those presented in Table 2, without using more complex price forecasting models like those reviewed by Weron [37].

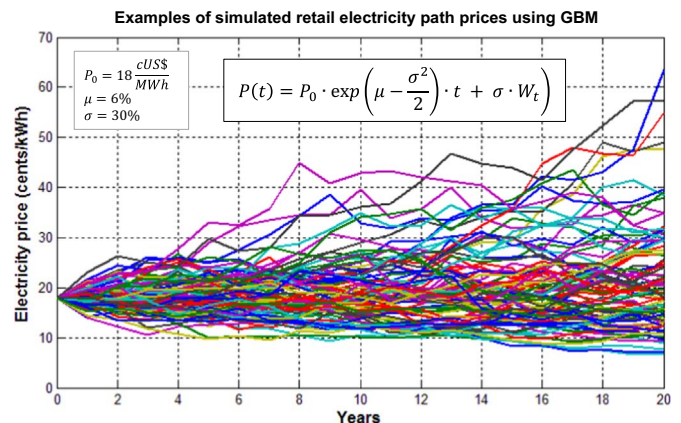
Among the most common techniques used by publications on portfolio optimization to model long-run electricity and fuel prices are the Geometric Brownian Motion (GBM) process [25,74] and distribution fitting. GBM processes are governed by a stochastic differential equation that describes a process in which the relative change of price is a **combination of deterministic proportional growth plus a normally distributed random change**.<sup>2</sup> The choice for GBM is often driven by the simplicity of its closed-form solution. However, real price statistics and patterns often don't match such process, presenting cycles driven by demand patterns and price spikes driven by supply and demand shocks. Examples using GBM to generate different simulations of annual electricity prices are presented in Fig. 11.<sup>3</sup>

<sup>2</sup> Geometric Brownian Motion (GBM) process stochastic differential equation:  $\frac{dP(t)}{P(t)} = \mu \cdot dt + \sigma \cdot dW_t$ , where  $W_t$  is a Wiener process and its solution (for any value of t) is a log-normally distributed random variable.

<sup>3</sup> The price function has an initial value of  $P_0=18$  US\$/MWh, an annual trend of  $\mu=6\%$ , and a standard deviation of  $\sigma=30\%$ .

Eydeland and Wolyniec [53] describe the main pros and cons of using GBM model the spot prices of energy commodities. On one side, GBM is an industry standard, its properties are well known and can be easily implemented in efficient computer implementations, and it is very useful for modeling cross-commodity correlations. But on the other side, the downside of using GBM as described in reference [53] includes the difficulty of calibrating because it offers few degrees of freedom (just two parameters) to match historical data. Furthermore, if it is used for pricing power products, the problem of non-storability of power makes it impossible to use the standard no-arbitrage argument to validate the common pricing formulas. Finally, the GBM price process does not allow for modeling the fat tails of price distributions or price spikes with the magnitude of real energy markets. In summary, GBM processes may be appropriate for some applications based on the criteria of normality and independence, but not for other applications, depending on the characteristics of the process and time frame, etc. For example, a process in which the drift is dependent upon time is not appropriate for GBM because GBM has a constant drift and variance over time. More examples of using GBM in different applications can be found in reference [75].

Other publications on portfolio optimization often assume some well-known probability distributions and estimate their parameters from time-series data and performing Monte Carlo simulation later to generate price trajectories. For example, such as Roques et al. [27], assume a normal distribution for fuel, electricity, and CO2 prices, and the parameters of these distributions (mean and variance) are estimated from historical time series. Similarly, Muñoz et al. [28] fitted a **Pearson distribution** to historical electricity pool prices in the Spanish market and assumed three scenarios with different degrees of growth per year. Madlener and Wenk [76] fitted a **log-normal distribution** to time-series price data derived from the European Electricity Exchange (EEX), and Glensk and Madlener [77] fitted a **beta distribution** to their electricity price data. After deciding how uncertainty factors are to be modeled and estimating parameters, these



**Fig. 11.** Examples of simulated paths of retail electricity prices using GBM over 20 years.

papers run simulations to compute cash flows and their main measures (NPV, IRR, etc.) and their distributions. These distributions and their correlations are then used in portfolio models.

The main issue with technical approaches is that past price values only do a good job representing the behavior of future prices while the system (transmission system, demand, and supply) remains static. But, if the system changes, prices can also change dramatically and therefore the risk analysis is no longer useful. In contrast, structural analyses allow for the production of price series that are consistent with the system and its possible changes in the future. This is a topic of active research as the greater penetration of renewable generation and operational and transmission constraints is becoming more important to defining prices.

Additionally, on the demand side, most consumers today are protected against price fluctuations by regulations and therefore do not directly see major risks, although they pay for them in the form of risk premiums embedded into their tariffs. This is rapidly changing, however, as the smart grid and distributed generation is becoming massive and makes consumers more proactive. These changes on the distribution network will have an impact on the wholesale market too affecting prices, expansion times, power flows, etc. The system is changing on all fronts, which reinforces the need for a structural analysis that considers different technological scenarios that cannot be captured by statistical analyses.

The failure to considering transmission capacity constraints could lead to an incorrect measure of income for some generation projects. Transmission constraints isolate different areas of electricity markets and sometimes create the possibility of exercising market power [78], such that local transmission constraints may lead to price risks (significant reduction of local marginal prices) as well as to volumetric risks (less electricity production caused by a capacity constraint). As an example, the mismatch in China between the wind installed capacity and wind generation is mainly explained by the inadequacy of the power transmission grid [79]. Transmission constraints affecting specific generation projects should be included in the modeling. The capacity of a transmission line determines the degree to which generators in different locations can compete with each other [80], and therefore ignoring transmission constraints may lead to significant errors in estimating the revenue for a generator firm.

Theoretically, spot prices vary spatially according to their contribution to marginal losses and the marginal congestion component. These prices show the instant value of the energy for the system, and it can differ greatly from one zone to another, and therefore ignoring the location of injections despite these components and assigning the same value to an MWh injected anywhere in the system is sometimes quite wrong. This inefficiency is ignored in a market of a global spot prices (mainly in Europe), but it is an issue for investors in the market of nodal prices (mainly in America). However, papers on portfolio optimization from the investor's perspective tend to ignore this effect.

#### 4.2. Renewable profiles and complementarities

Renewable profiles, especially wind power profiles, depend on local meteorological features and atmospheric and geographic phenomena that are very volatile and difficult to predict. Therefore, two wind power plants with the same model and number of units will produce very different power profiles when placed in locations with different meteorological and relief conditions. Using two complementary profiles helps reduce the need for storage and produce a smoother combined output profile, which may be a much more appealing "product" for a buyer.

Spatial diversification of solar PV production can be achieved by distributing PV plants across different locations, taking advantage of differences on sunrise/sunset times (and therefore solar PV peak production times), cloud regimes, ground albedos, among other geographical features which allow smoothing out changes in PV

production. The extent of the smoothing effect depends mainly on the number of PV plants, the composition of the ensemble, longitudinal differences between sites, area of dispersion and irradiation variability [81]. The impressive curve of declining costs of PV technology, its fast deployment in recent years across the globe and its expected growing participation in the energy markets have pushed for new research to address problems associated with short-term variability of PV production. This issue was initially seen as a potential limiting factor for PV integration into the grid [82]. Mills and Wiser [82] were one of the first to account for geographic diversity to reduce volatility of PV production. In fact, they concluded that the need for additional reserves to manage variability of PV plants is considerably reduced by geographic diversity in a wide area. More recently, David et al. [83] have shown that solar PV geographical diversification can also be achieved in small territories with different microclimates. Two different regimes of cloudiness appears to be enough to greatly improve the diversification effect in their study. Finally, several publications have pointed out that the smoothing effect could lead to lower forecasting errors, since plant spacing decreases the correlation value of their forecasting errors [84,85].

For investors, the most relevant profile complementarity is in the annual and monthly time scales, because they help to reduce financial risks to the portfolio. Investing in two different renewable power plants that have complementary generation profiles is less risky than investing all the capital in a single project that is twice the size. For managers, the complementarity of renewable profiles help to increase profits (energy sellers), reduce costs (energy buyers) and mitigate risks overall. Complementarity allows energy sellers to offer output that is much less volatile, and that can result in a more valuable product for energy buyers who may be willing to pay more. On the other side, those who buy energy from different complementary renewable generators may reduce their exposure to the real spot market and their footprint at the same time. Moreover, technological and spatial diversification of renewable energies can reduce vulnerability of the entire power system [86].

Although there is active research underway on the quantification of the geographic complementarity of solar and wind power plants [68–72], there are currently no publications that integrate and analyze their effect on the technological portfolio. How a smart choice of solar and/or wind power plants across a determined territory improve the return/cost - risk profile? This represents a significant opportunity for further research because everything suggest that renewable energy will continue its aggressive entrance into the market in the future.

## 5. Conclusions

The electric power industry is very dynamic and in constant search of efficiency, especially in a world full of uncertainties. Today more than ever, the electric power industry faces a high degree of uncertainty in every dimension, from operations to investments. Part of this uncertainty is caused by renewable energy technologies that have been experiencing a rapid progress and wide deployment, producing a sharp drop in its investment and deployment costs and reducing the energy price at which they can supply electricity competitively. Indeed, more fundamental changes in the industry are expected over the next 10 years. A boost in electricity demand due to electric vehicles, as well as an increase in distributed generation, massive storage, and the deployment of smart grids are some of the sector's upcoming challenges. Large investments will be required to address these new challenges, and that is why agents need protection and are constantly looking for risk management tools. To contribute to this task, this paper presents a selection of applications, issues, and opportunities for further research on portfolio optimization from the perspectives of both investors and portfolio managers.

Better use of infrastructure, avoiding unnecessary investment, and optimal resource management are essential skills for today's energy

companies. For investors, developing a generation project involves an enormous amount of capital along with very large risk, so investors usually adopt different diversification strategies to mitigate those risks, such as investing in power plants of different sizes and places, with different technologies and more importantly, with different types of fuel or resources. These strategies reveal that the return on a portfolio of projects is not simply the sum of the returns from all of the individual projects. The “correct” return is the result of the return on individual projects plus the “interaction” among them. This “interaction” is key in portfolio optimization; interaction among projects allows for diversification and the cancellation of risks. Most papers determine the proxy of projects’ profit and their interaction to be the **Net Present Value** or **Internal Rate of Return** of the projects and their respective correlations. This correlation between cash flows quantifies interaction gains or losses. In addition to diversification strategies, investors can wait and defer the investment if there is excessive uncertainty (due to a regulatory change for example). However, the literature places little emphasis on the **value of waiting** or deferring a project or a set of projects within the context of a portfolio. Decision makers who are unwilling to take risks in the face of insufficient information might be well advised to consider the option of waiting.

On the other side, portfolio managers of large electricity sellers/buyers must deal with electricity spot prices that are very volatile due to the special properties of electricity, such as non-storability and a non-linear, steeply rising supply curve. Unrestrained exposure to **price risks** may produce overwhelming consequences for agents. Take for example the price spikes presented in [46] in which high spot prices led different agents to bankruptcy, with devastating consequences for the economy. The California electricity crisis of 2000–2001 is one example in which prices persistently reached US\$500/MWh, and retailers had not hedged against price risk through other financial instruments, leading to a major crisis in the sector. In the case of volumetric risk, retailers who are forced to serve their entire load must also be concerned with uncertainty on the load, since there are no simple financial instruments to deal with changes in the demanded volume, especially because mass electricity storage is still not an economically viable option. [46]. Exploiting the high correlation between demand and prices using **trading mechanisms** such as bilateral contracts, forwards, futures, call/put options, among others instruments are alternatives to mitigate part of the volumetric risks. In the portfolio literature, most applications rely on **static models**. Conversely, **dynamic and multi-stage** applications are more limited, mainly because of their need for great computational power, which restricts the design of real-world applications. Active research is required, therefore, in the development of new methodologies to deal with the current computational limits. Rocha et al. [63] have made notable advances in that direction by using linear decision rules to approximate the solution of a stochastic optimization.

One cross-cutting issue in the literature on portfolio optimization that affects both investors and portfolio managers is the frequent use of statistical rather than structural models. In fact, most of the available literature trusts statistical approaches to model future price behavior, although history will not repeat itself and the past is now a poor predictor of future behavior. Given the radical changes and uncertainties we are facing, structural-based methods are required to model future behavior of prices. The next challenges in portfolio analysis are in modeling a much more realistic power system, where the future equilibrium prices are not simply estimated from historical values, but justified by structural models of supply and demand. The rapid progress of generation technologies, distributed generation, and communications that can radically alter the electricity system as we know it today limit the usefulness of currently used models based on statistical analyses. As the penetration of intermittent renewable energies be-

comes higher, it is no longer acceptable to refrain from modeling operational constraints and using stylized temporal representations. Urgent attention is required in the development of methodologies and algorithms that can handle higher levels of temporal, spatial, and technical details. This is not only important for the planner, but also for investors and managers who are in constant need of future price projections to make investment and commercial decisions respectively. Electricity spot prices depend heavily upon the physical infrastructure of the power system, and a new transmission line or generator could completely change all of the price statistics, since its impacts power plant dispatch, marginal losses, and congestion, the key components of locational spot prices.

In addition to intermittency, renewable resources such as solar, wind and perhaps future tidal power plants bring other special features, and this is complementarity in different time scales. Unlike conventional generators, which are fully controllable, these renewable generators depend on the availability of natural resources that are beyond our control. However, numerous publications have demonstrated that geographical diversification can significantly decrease variability in different time frames, especially of wind power production [68,69,87–90]. Spatial diversification of solar PV plants is very useful to smooth out the production in small time frames, ranging from seconds and minutes to hours. A smoother PV production decreases the cost of system integration allowing better forecasting and requiring less primary/secondary reserves. Depending on the market rules, this cost reduction could affect in more or less extent the income/cost of the project participant (project investors and portfolio managers). Nevertheless, renewable complementarity is currently entirely absent in planning portfolio literature. The potential gains in efficiency (return-risk) from geographical diversification are currently neglected, which minimizes the relevance of transmission capacity constraints and cross-border interconnections. Well distributed energy resources may offer investors and managers a good alternative for diversifying risks, but some of their diversification benefits are actually being overlooked.

Finally, the primary gap in the portfolio literature is the lack of the consideration for the general public or small electricity users. Consumers are slowly taking a more active role in the electricity market through residential generation, smart metering infrastructure, demand response, smart grid deployments, and other areas associated with the raising figure of the “prosumer.” Thus, an excellent opportunity for research lies in analyzing the impact of the new small and distributed energy systems with the active participation of the demand side of the portfolio, changing its composition, or becoming a component of the optimal portfolio as an energy resource.

Power systems are changing in several areas, including technologies, regulations, relationships with consumers, resources, and others described in this paper. Private agents trading over the system can no longer manage their decision making through the use of traditional methodologies because they have limited capabilities to simultaneously represent different sources of risks. Using tools to explicitly include different risk measures and their interactions, such as portfolio optimization, in making investment and management decisions is crucial to surviving in a fast-changing world.

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