Electrical Power and Energy Systems 94 (2018) 179–187

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Dynamic energy management in smart grid: A fast randomized first-order optimization algorithm

Dong Han [a,*], Weiqing Sun [a], Xiang Fan [b]

[a] Department of Electrical Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China
[b] Power Dispatch and Control Center, Guizhou Power Grid Corporation Limited, Guiyang 550000, China

ARTICLE INFO

Article history:
Received 19 March 2017
Received in revised form 26 June 2017
Accepted 5 July 2017

Keywords:
Dynamic energy management
Distributed energy resources
First-order optimization method
Augmented Lagrangian function
Low-rank matrix approximation

ABSTRACT

A crucial issue in the smart grid is how to manage the controllable load resources of end-users, in order to reduce the economic costs of system operation and facilitate to utilize renewable energies. This paper investigates a fast randomized first-order optimization method to explore the solution of dynamic energy management (DEM) for the smart grid integrated large-scale distributed energy resources. A complicated time-coupling and multi-variable optimal problem is presented to determine the load scheduling for the electricity customers. The main challenge of the proposed problem is to enable the efficient processing of the large data volumes and optimization of aggregated data involved in DEM. The first-order method as one of big data optimization algorithms is able to exhibit significant performance for computing globally optimal solutions based on randomization techniques. Using such solution approach, we can reformulate the original problem into an unconstrained augmented Lagrangian function. The optimal results can be obtained from computing the gradient based on the information of the first-order derivative. To speed up the calculations of obtaining the feasible solutions, the optimization variable matrix used to update the Lagrangian multiplier can be replaced with the corresponding low-rank representation in the iteration process. Both theoretical analysis and simulation results suggest that the proposed approach may effectively solve the optimal scheduling problem of DEM considering users' participation.

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1. Introduction

The dynamic energy management (DEM) enables electricity customers to change their energy consumptions by means of price-driven demand response mechanisms, to facilitate the users to actively take participation into the process of power system operation. In the conventional power grids, DEM is a well-investigated topic but not the case for the smart grid [1]. A smart grid is the modern power grid integrated with a large amount of distributed renewable energy, controllable electricity appliances, energy storage devices from the energy supply and consumption sides. This will make the DEM become much more complicated, which brings the challenge to achieve the real-time decision-making process for the control center of the power system operation. The infrastructure shows the advanced technical characteristics of the smart grid can be beneficial to enhance the economic, reliability and sustainability of the power system, but it also will pose a significant problem that how to manage the electricity load and controllable distributed energy resources in the context of massive data information. In more detail, DEM in the power grid can be described as an optimal problem for the scheduling of all of the controllable appliances of the users [2]. However, the large volumes of aggregated data generated by millions of end-users in the optimization process become a simple optimal control problem into a time-coupling, multi-variable, high-dimensional optimization issue in the smart grid environment. Thus, in order to achieve more efficient analyzing performance of DEM in the load scheduling, the solution algorithm to tackle such complicated optimization problem, which is associated with massive load data, needs to be investigated and studied.

Some researchers have focused on the DEM problem in the context of the smart grid. In [3], an optimal DEM scheme based on Lyapunov optimization theory was presented to perform the load scheduling with consideration of unpredictable load demands and distributed energy resources. An approach based on the mixed-integer linear programming paradigm was developed to determine power consumption and management of renewable resources in [4]. The work in [5] showed a rule-based energy management strategy was able to improve the fuel economy of plug-in hybrid electric vehicles using dynamic programming. Different

* Corresponding author.
E-mail address: d.han1984@hotmail.com (D. Han).

http://dx.doi.org/10.1016/j.ijepes.2017.07.003
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Nomenclature

Symbols

$g_{DG,i}(t)$	output of distributed generation resource of user i in time period t
$d_{IDR,i}(t)$	energy provided for satisfying interruptible load demand of user i in time period t
$d_{DR,i}(t)$	energy provided for satisfying load demand of demand response resources of user i in time period t
$d_{G,i}(t)$	energy sold to power grids in time period t
$c_{DG,i}(t)$	energy saved in time period t
$g^{\max}_{\mathrm{DG,i}}$	maximum output of the distributed generation resource of user i
$L_{IDR,i}(t)$	interruptible load demand of user i in time period t
$L^{\max}_{\mathrm{IDR,i}}$	maximum interruptible load demand of user i
$g_{IDR,i}(t)$	energy drawn from the power grid to meet user i's interruptible load demand in time period t
$r_{IDR,i}(t)$	energy provided from energy storage device of user i to satisfy interruptible load demand in time period t
$L_{DR,i}(t)$	load demand of user i's demand response programs in time period t
$L^{\max}_{\mathrm{DR,i}}$	maximum load demand of user i's demand response programs
U_{DR}	aggregate demand of user i's all elastic load resources
T_D	set of times that the demand response appliances can work
$g_{DR,i}(t)$	energy drawn from the power grid to meet user i's elastic demand in time period t
$r_{DR,i}(t)$	energy provided from energy storage device of user i to satisfy elastic load demand in time period t
$E_i(t)$	energy level of user i's energy storage device in time period t
$E_i^{\max}$	maximum capacity of user i's energy storage device
$C_i(t)$	energy charging the energy storage device in time period t
$R_i(t)$	energy discharged from energy storage device in time period t
$C_i^{\max}$	maximum charging capacity of user i's energy storage device
$R_i^{\max}$	maximum discharging capacity of user i's energy storage device
$c_{G,i}(t)$	energy drawn from the power grid for user i's energy storage device in time period t
$r_{G,i}(t)$	energy sold to the grid from user i's energy storage device in time period t
$I_{C_i(t)>0}$	state of charging the energy storage device
$I_{R_i(t)>0}$	state of discharging the energy storage device
$P(t)$	energy supplied from the load-serving entity in time period t
E_i^T	energy level of user i's energy storage device in time period T
λ	Lagrangian multiplier associated with equality constraints
μ	Lagrangian multiplier associated with inequality constraints
σ	scalar parameter
v	iteration count
α_v	iteration step-size
r	rank of matrix
N	number of end-users

Abbreviations

DEM	dynamic energy management
RTP	real-time pricing
SVD	singular value decomposition
RAM	random access memory
CPU	central processing unit
GA	genetic algorithm

from centralized optimization approaches, [6] established a decentralized model with minimal information exchange and communications between users to determine optimal energy trading amounts. Another modeling technique was proposed in [7–10], where the study of DEM with residential energy system was conducted. The authors addressed the model with individual end-user behavior constraints, whereby the optimal load scheduling could be obtained. Additionally, DEM can be effectively implemented in the manner of demand response programs or transactive energy [11]. The setting of the power prices will have a profound impact on encouraging consumers' participation in energy savings and cooperation. A load scheduling problem with price uncertainty and temporally-coupled constraints in the smart grid was presented in [12], where the real-time pricing (RTP) model was proposed to incentivize energy resources scheduling. The incentive mechanism is also used to perform household energy management. Ref. [13] designed a policy scheme to regulate household energy consumption behavior in a dynamic active energy demand management system. According to [14], the proposed real-time optimal demand response management for residential appliances was designed via stochastic optimization and robust optimization approached considering deferrable/non-deferrable and interruptible/non-interruptible load models. On optimization in residential energy management, [15] presented a mixed integer multi-time scale stochastic optimization to formulate the load scheduling considering different types of load classes.

It is highlighted that DEM can be described as an optimal control problem for the scheduling of all of the controllable appliances in the smart grid. Considering the complexity of modeling and solving such problem, some previous research associated with its modeling techniques and solution algorithms has been somewhat carried out. The study of [16] deals with load control in a multiple-residence setup, from which the optimal amount of electricity production and consumption schedule can be obtained using a distributed subgradient method. In addition, the authors of [17] proposed a joint scheduling scheme for the electric supply and demand of home energy management system in term of the sequential procedure of prediction. Instead of using an optimization formulation, [18] employed a simulation testing method for conducting the pre-cooling strategies of thermal appliance scheduling. Under the environment of the smart grid, mass energy appliances in the electricity demand side will participate into DEM and interact with the smart grid. The main challenge is how to analyze the emerging control problem for such DEM integrated with aggregated big data from energy customers.

In this paper, we proposed an optimization method from the area of big data analytics for the DEM considering different demand response programs. Our method makes decisions on the load scheduling of both elastic load and inelastic load, as well as the operations of distributed renewable energy resources and energy storage devices. We aim to obtain an optimal energy management scheduling associated with the usage of all the energy resources in the smart grid network based on the proposed randomized first-order optimization method. Specifically, our contributions in this paper include:

(1) An optimization model about comprehensive load scheduling is developed in the framework of DEM. We formulate the optimal scheduling problem by minimizing the generation cost of the associated power utility, as the large number of users' controllable devices is accommodated into the system. The model demonstrates that the available energy resources from the customers are capable of providing the benefits for the power grid economic operation.
(2) The proposed problem consists of a large number of variables and constraints, which can be formulated into a large-scale optimization problem. As such, we present a fast randomized first-order optimization algorithm to efficiently solve the problem.
(3) Using the proposed methodology, the case studies show the solution to determine the optimal load scheduling can be effectively obtained in the environment of diversified and complex data. Moreover, it also illustrates that the centralized computational method in this paper has better performance of solving the optimization problem than others.

The remainder of this paper is organized as follows: The optimal load scheduling problem with consideration of different end-use energy resources is presented in Section 2. Section 3 provides a computational framework using the proposed randomized first-order algorithm. The simulation results in Section 4 shows the application of the proposed methodology. Finally, Section 5 summarizes the main conclusions and contributions of this study.

2. Problem formulation

In this section, we describe the model of DEM, and formulate the optimal load scheduling problem of integrated end-use controllable devices. The presented optimization model is to focus on the operation characteristics of end-use appliances and power utility from the component and system perspectives.

2.1. System model

2.1.1. Distributed generation resource

We assume that each end-user has a distributed generation resource, whose output energy can be used to serve the load demand, charge energy storage device, and sell to the grid. Hence, we have

$$g_{DG,i}(t) = d_{IDR,i}(t) + d_{DR,i}(t) + d_{G,i}(t) + c_{DG,i}(t) \tag{1}$$

$$0 \leqslant g_{DG,i}(t) \leqslant g_{DG,i}^{\max} \tag{2}$$

where $g_{DG,i}(t)$ is the output of the distributed generation resource of user i in time period t, $d_{IDR,i}(t)$ is the energy provided for satisfying interruptible load demand of user i in time period t, $d_{DR,i}(t)$ is the energy provided for satisfying load demand of demand response resources of user i in time period t, $d_{G,i}(t)$ is the energy sold to power grids in time period t, $c_{DG,i}(t)$ is the energy saved in time period t, and $g_{\mathrm{DG,i}}^{\max}$ is the maximum output of the distributed generation resource of user i.

2.1.2. Inelastic load demand

Each user operates a set of inelastic load appliances which are interruptible at their working time. The energy demand of the inelastic load appliances can be satisfied from user's distributed generation resource, the power grid, or energy storage.

$$0 \leqslant L_{IDR,i}(t) \leqslant L_{IDR,i}^{\max} \tag{3}$$

$$L_{IDR,i}(t) = d_{IDR,i}(t) + g_{IDR,i}(t) + r_{IDR,i}(t) \tag{4}$$

where $L_{IDR,i}(t)$ is the interruptible load demand of user i in time period t, $L_{\mathrm{IDR,i}}^{\max}$ is the maximum interruptible load demand of user i, $g_{IDR,i}(t)$ is the energy drawn from the power grid to meet user i's interruptible load demand in time period t, and $r_{IDR,i}(t)$ is the energy provided from energy storage device of user i to satisfy interruptible load demand in time period t.

2.1.3. Demand response

Still, user i can optimize the energy consumption of flexible load appliances across the setting time T_D.

$$0 \leqslant L_{DR,i}(t) \leqslant L_{DR,i}^{\max} \tag{5}$$

$$\sum_{t}^{T_D} L_{DR,i}(t) \geqslant U_{DR} \tag{6}$$

$$L_{DR,i}(t) = d_{DR,i}(t) + g_{DR,i}(t) + r_{DR,i}(t) \tag{7}$$

where $L_{DR,i}(t)$ is the load demand of user i's demand response programs in time period t, $L_{\mathrm{DR,i}}^{\max}$ is the maximum load demand of user i's demand response programs, U_{DR} is the aggregate demand of user i's all elastic load resources during the time T_D, T_D is the set of times that the demand response appliances can work, $g_{DR,i}(t)$ is the energy drawn from the power grid to meet user i's elastic demand in time period t, and $r_{DR,i}(t)$ is the energy provided from energy storage device of user i to satisfy elastic load demand in time period t.

2.1.4. Energy storage

A customer i can store the energy to implement the flexible load scheduling using the battery. We model the dynamics of the energy storage device as follows.

$$E_i(t+1) = E_i(t) + C_i(t) - R_i(t) \tag{8}$$

$$C_i(t) = c_{DG,i}(t) + c_{G,i}(t) \tag{9}$$

$$R_i(t) = r_{IDR,i}(t) + r_{DR,i}(t) + r_{G,i}(t) \tag{10}$$

$$\mathrm{I}_{C_i(t)>0} + \mathrm{I}_{R_i(t)>0} \leqslant 1 \tag{11}$$

$$0 \leqslant E_i(t) \leqslant E_i^{\max} \tag{12}$$

$$0 \leqslant C_i(t) \leqslant \min\left[C_i^{\max}, E_i^{\max} - E_i(t)\right] \tag{13}$$

$$0 \leqslant R_i(t) \leqslant \min\left[R_i^{\max}, E_i(t)\right] \tag{14}$$

where $E_i(t)$ is the energy level of user i's energy storage device in time period t, $E_i^{\max}$ is the maximum capacity of user i's energy storage device, $C_i(t)$ is the energy charging the energy storage device in time period t, $R_i(t)$ is the energy discharged from energy storage device in time period t, $C_i^{\max}$ and $R_i^{\max}$ are the maximum charging and discharging capacity of user i's energy storage device, $c_{G,i}(t)$ is the energy drawn from the power grid for user i's energy storage device in time period t, and $r_{G,i}(t)$ is the energy sold to the grid from user i's energy storage device in time period t. $I_{C_i(t)>0}$ and $I_{R_i(t)>0}$ indicate the states of charging and discharging the energy storage device respectively.

2.1.5. System-Level load serving

From the system-level perspective, we consider the load scheduling can be conducted by a load-serving entity. The load-serving entity may represent a regulated monopoly like most utility companies which serve to satisfy the energy demand of a set users. Considering some end-use controllable resources integrated into the grid, the model of energy generation for the load-serving entity is developed as follows.

$$P(t) = \sum_i \begin{pmatrix} L_{DR,i}(t) + L_{IDR,i}(t) + c_{G,i}(t) \\ -d_{IDR,i}(t) - g_{IDR,i}(t) - d_{DR,i}(t) - g_{DR,i}(t) \\ -r_{IDR,i}(t) - r_{DR,i}(t) - r_{G,i}(t) \end{pmatrix} \tag{15}$$

where $P(t)$ is the energy supplied from the load-serving entity.

2.2. Modeling for dynamic energy management

We can now state precisely the objective of the DEM model as the constrained minimization of the expected generation cost for a load-serving entity.

$$F = \min \frac{1}{T}\sum_{t=1}^{T-1} \mathrm{E}\{f[P(t)]\} \tag{16}$$

$$f(P(t)) = aP^2(t) + bP(t) + c \tag{17}$$

Each day is divided into T periods of equal duration, indexed by $t \in \{1, 2, \ldots, T\}$. $f(P(t))$ is the utility company's energy generation cost function [19].

In summary, combined with user model and supply model before mentioned, the mathematical model of load scheduling for the DEM can be obtained according to (1)–(17). We can see the presented model is a time-coupling optimization problem due to constraint (8)–(10), (13) and (14). The decision variables in the model include $d_{IDR,i}(t)$, $d_{DR,i}(t)$, $d_{G,i}(t)$, $c_{DG,i}(t)$, $c_{G,i}(t)$, $g_{IDR,i}(t)$, $g_{DR,i}(t)$, $r_{IDR,i}(t)$, $r_{DR,i}(t)$, and $r_{G,i}(t)$. Generally, the DEM problem is formulated as the mixed integer linear programming model, where the binary variable as the states of charging and discharging the energy storage device. In the specific case, when the states of all the energy storage are given, all entries in the control decision will be equal to 1, and in this case the DEM problem can be described as a linear programming problem. Considering the computational complexity of such problem due to multiple numbers of users in the smart grid, this paper focuses on this specific case.

For dynamic energy management (DEM), we consider an electric power distribution network consisting of a set of energy users. Each user has a renewable distributed generation unit, an energy storage device, and a connection to the power grid, which collaboratively meet its elastic and inelastic load demand. Load scheduling decisions are made dynamically by the utility company in each time slot. In comparison, the DEM problem can be formulated as the load scheduling model. The conventional economic dispatch problem considering the participation of distributed generations and demand response is not load scheduling problem, but generation scheduling problem. In mathematics, the decision variables of the proposed DEM model are used to represent the behaviors of load appliances, energy storage devices, and distributed generation sources in the user sector. Generally, the control variables of the conventional economic dispatch model denote the output of the generation units in the generation sector. Hence, it exits the difference between the DEM model and the conventional economic dispatch problem, regardless of concepts or models.

3. Solution methodology

In order to obtain the optimal scheduling of the DEM, a fast randomized first-order optimization algorithm is presented, which is used to solve the DEM model based on the Lagrangian multiplier framework.

3.1. The Lagrangean multiplier framework

For the complicated constraint (8) which has the time-coupling characteristic, we can relax the energy level dynamics by using another set of multipliers [20]. Thus, the energy level can be determined based on the initial energy level, and power discharged or charged as:

$$E_i(t) = E_i(0) - \sum_{n=1}^{t}(C_i(n) - R_i(n)), t = 1, 2, \ldots, T-1 \tag{18}$$

By submitting (18) into (12), the formulation of the energy level dynamics for an energy storage can be described as

$$E_i(0) - E_i^{\max} \leqslant \sum_{n=1}^{t}(C_i(n) - R_i(n)) \leqslant E_i(0) \tag{19}$$

and

$$\sum_{n=1}^{T}(C_i(n) - R_i(n)) = E_i(0) - E_i^T \tag{20}$$

where E_i^T indicates the energy level of user i's energy storage device in time period T.

The original problem for the DEM can be reformulated as follows.

$$\begin{aligned}
\min\quad & \frac{1}{T}\sum_{t=1}^{T-1}\mathrm{E}\{f[P(t)]\} \\
s.t.\quad & E_i(0) - E_i^{\max} \leqslant \sum_{n=1}^{t}(C_i(n) - R_i(n)) \leqslant E_i(0) \\
& \sum_{n=1}^{T}(C_i(n) - R_i(n)) = E_i(0) - E_i^T \\
& L_{IDR,i}(t) = d_{IDR,i}(t) + g_{IDR,i}(t) + r_{IDR,i}(t) \\
& L_{DR,i}(t) = d_{DR,i}(t) + g_{DR,i}(t) + r_{DR,i}(t) \\
& g_{DG,i}(t) = d_{IDR,i}(t) + d_{DR,i}(t) + d_{G,i}(t) + c_{DG,i}(t) \\
& \sum_{t}^{T_D} L_{DR,i}(t) \geqslant U_{DR} \\
& 0 \leqslant C_i(t) \leqslant \min\left[C_i^{\max}, E_i^{\max} - E_i(t)\right] \\
& 0 \leqslant R_i(t) \leqslant \min\left[R_i^{\max}, E_i(t)\right] \\
& \mathrm{I}_{C_i(t)>0} + \mathrm{I}_{R_i(t)>0} \leqslant 1
\end{aligned} \tag{21}$$

For simplification of the modeling and simulation, (21) can be abstracted as

$$\begin{aligned}
\min\quad & f(x) \\
s.t.\quad & h(x) = 0 \\
& g(x) \leqslant 0
\end{aligned} \tag{22}$$

Due to recent advances in convex optimization algorithms for big data, first-order methods play a profound impact on solving large-scale optimization problems. First-order methods can obtain low- or medium-accuracy solutions by utilizing only first-order oracle information from gradient calculations for an optimization model. A classical first-order technique is the gradient method, which uses the gradient vector and iteratively performs the following update.

$$x^{v+1} = x^{v} - \alpha_v \nabla f(x^{v}) \tag{23}$$

where v is the iteration count, α_v is a scalar parameter indicates an iteration step-size, and $\nabla f(\cdot)$ represents the gradient variable.

In order to employ the first-order method to solve problem (22), we can transform such problem into an unconstrained optimization problem. Then, the augmented Lagrangian function for the abstracted original problem is structured below

$$\phi(x,\lambda,\mu,\sigma) = f(x) + \lambda^T h(x) + \frac{1}{2}\sigma\|h(x)\|^2 + \frac{1}{2\sigma}\{[\max(0,\mu+\sigma g(x))]^2 - \mu^2\} \quad (24)$$

where λ and μ are Lagrangian multipliers associated with equality and inequality constraints, as well as σ is a large enough scalar.

The abovementioned mathematical programming problem can be further relaxed in the following representation.

$$\phi^*(x,\lambda,\mu,\sigma) = f(x) + \lambda^T h(x) + \frac{1}{2}\sigma\|h(x)\|^2 + \frac{1}{2\sigma}\{[(\mu+\sigma g(x))]^2 - \mu^2\} \quad (25)$$

The basic idea of augmented Lagrangian technique is to relax the complicated constraints by using Lagrangian multiplier. Furthermore, the proposed original problem can be transformed into a simple and unconstrained optimization problem.

3.2. Constructing low-rank matrix approximation

The proposed DEM problem contains a large number of variables and constrains and becomes a large-scale optimization problem in this paper. For such optimization problem, it is difficult to solve the model (25) using general optimization tools. Ref. [21] demonstrates first-order methods are well-positioned to address such large-scale problem. First-order methods can obtain numerical solutions by using only first-order oracle information from the objective of the optimization problem. The key link of applying first-order methods to solving optimization problems is making use of proximal mapping principle to handle a great deal of complicated variables [22]. Generally, randomization is the approximation technique that is used to implement and enhance the scalability of first-order methods. The basic idea of randomization techniques is to randomly update optimization variables, and replace the deterministic gradient with the simple linear algebra operation.

The low-rank matrix approximation technique as the randomized linear algebra plays a significant role in data analysis and scientific computing [23]. When the matrix objects have low-rank representations, the efficiency of seeking the solution for the optimization problem will improve. The work of computing a low-rank approximation to a given matrix can be divided into two computational stages. The first stage is to establish a low-dimensional subspace that captures the property of the matrix. The second stage is to restrict the matrix to the subspace and implement the computation of a standard factorization, such as QR factorization, and singular value decomposition (SVD), of the reduced matrix. We can summarize the implementation of the applications on the low-rank matrix approximation technique as follows.

Stage (I) Randomized range finder

Given an $m \times n$ matrix $\boldsymbol{M}$ and an integer r, this scheme computes an $m \times r$ orthonormal matrix $\boldsymbol{Q}$ whose range approximates the range of $\boldsymbol{M}$.

(1) Generate an $n \times r$ Gaussian random matrix $\boldsymbol{\Omega}$.
(2) Form the $m \times r$ matrix $\boldsymbol{W} = \boldsymbol{M}\,\boldsymbol{\Omega}$.
(3) Construct an $m \times r$ matrix $\boldsymbol{Q}$ whose columns form an orthonormal base for the range of $\boldsymbol{W}$ using the QR factorization.

Stage (II) Direct SVD

Given matrices $\boldsymbol{M}$ and $\boldsymbol{Q}$ holds, this procedure computes an approximate factorization $\boldsymbol{M} \approx \boldsymbol{U}\sum\boldsymbol{V}^*$, where $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthonormal, and $\sum$ is a nonnegative diagonal matrix.

(1) Form the matrix $\boldsymbol{B} = \boldsymbol{Q}^T\boldsymbol{M}$.
(2) Compute an SVD of the small matrix: $\boldsymbol{B} \approx \tilde{U}\sum\boldsymbol{V}^T$.
(3) Form the orthonormal matrix $\boldsymbol{U} = \boldsymbol{Q}\tilde{U}$.

3.3. Algorithm for obtaining feasible solutions

The whole procedure for solving the problem in (25) using the randomized first-order optimization method in the Lagrangian framework is described as follows.

Step 0: Initialization. Initialize multipliers λ and μ as well as penalty parameter. Set the initial iteration $v = 0$.
Step 1: Solution of the relaxed primal problem. Solve the relaxed prime problem at the iteration v. We can obtain the optimal solution by using the following representation.

$$\nabla_x \phi^*_{(v)}(z,\lambda,\mu,\sigma) = 0 \quad (26)$$

Step 2: Randomization. At the vth iteration, constructing low-rank matrix approximation $\boldsymbol{U}^{(v)}$.
Step 3: Multiplier updating. The augmented Lagrangian algorithm is used to update the multipliers in the whole iteration process.

$$\begin{cases} \lambda^{(v+1)} = \lambda^{(v)} + \sigma\{h(x^{(v)})_{[U^{(v)}]}\} \\ \mu^{(v+1)} = \max\{0, \mu^{(v)} + \sigma\{g(x^{(v)})_{[U^{(v)}]}\}\} \end{cases} \quad (27)$$

Step 4: Convergence checking. If multipliers do not change significantly in two consecutive iterations, stop, the solution has been reached; otherwise, $v = v + 1$ and continue with Step 1.

4. Simulation results

In this section, we provide numerical examples to complement the analysis in the previous sections. To evaluate the performance of our proposed algorithm, Matlab Optimization Toolbox is used to implement the simulations based on the assumptions and the data. All tests are conducted in the environment of a 32-bit computer with 12 GB of RAM and an Intel core 2 duo CPU. Hence, the numerical results from the scenarios simulated and compare will be obtained.

4.1. Description of the data and assumptions

It is assumed that a 24 h operation horizon is taken into account in the simulation procedure. In the example system, there are 50 users with renewable distributed generation sources, energy storage devices, elastic and inelastic load appliances in basic case. We assume that the maximum capacity of each user's renewable distributed generation source is 0.2 kW. Considering the stochastic characteristics of distributed generation resource, we consider the output of the distributed generation resource is the random variable, which satisfies the normal distribution with the mean value is 0.1 and the standard deviation is 0.05. The maximum charging and discharging capacities of user's energy storage devices are assumed to be 1.5 kW h. The maximum energy level of the energy storage equipment is set to be 0.24 kW h. Both elastic and inelastic load demands are considered as uniform random variables over the intervals [1,4] and [1,8] kW h. We set all elastic load demand deadlines to 12 h in one day, i.e., $T_D = 12$. We select $a = 0.75$, $b = 0.1$, and $c = 0$ in all simulations, unless otherwise stated.

4.2. Numerical results and discussions

To demonstrate the advantage of the proposed algorithm, we show a numerical comparison of the randomized first-order optimization method and the general Lagrangian multiplier method in Fig. 1. For the Lagrangian multiplier method with low-rank matrix approximation, the fitness value of the objective converges to 4.5×10^9 in about 10 iterations, while the other method needs 40 iterations to yield the same fitness value. This is due to the fact that the optimization variable matrix can be described using a low-rank representation, and consequently improves the computational speed. Further, notice that the randomized first-order method exhibits its significant acceleration since it can rapidly obtain an optimal solution with randomized linear algebra operations.

It is an important task to find an approximate value for the rank r of optimization variable matrix. Generally, higher values of r will increase the column or row subset selection for the optimization variable matrix, and the resultant optimal solution can be achieved more rapidly. To investigate the impact of the selection of r on computational performance, Fig. 2 plots the correlation between the iterations and fitness values of the objective function in problem (21). Fig. 2 shows an example of a randomized low-rank approximation, which is performed based on the classical QR factorization and SVD technique, using a random initial value. The result in Fig. 2 displays that we can significantly accelerate the computation efficiency by setting a higher value r in the optimization procedure. Thus, the approximation can be very efficiency if the rank of the matrix is chosen properly.

In order to demonstrate the impact of number of dimensions on convergence performance of the proposed method, four scenarios associated with different amounts of end-users are designed. Assuming that N is the number of end-users, Fig. 3 shows effect of number of end-users on the total cost of DEM during the overall scheduling period. In each case that each user has both an elastic load and an inelastic load demands, we can compare the total energy generation cost of the DEM scheme for four designed cases. The computational complexity of the DEM problem depends on number of decision variables, time slots in the simulation process, and number of end-users.

Particularly, we notice that the number of dimensions of decision variables is $500 \times 24 \times 10$ when the number of end-users is 500, which produces an exceptionally large-scale optimization problem. To evaluate the impact of number of end-users on the computational performance, we add some scenarios in the case study when number of end-users is set different values $N = 100$, 200, 500. The number of dimensions of decision variables is 24,000, 48,000, and 120,000 in the scenarios where $N = 100$, 200, 500. The result shown in Fig. 3 suggests that the numerical benefits of such randomized first-order method, since the randomization step in the optimization process makes the sparsity of the variable matrix much better.

The preferences assigned by the aggregated end-users to the time slots for the inelastic and elastic loads are displayed in Fig. 4. The simulation results in Fig. 4 demonstrate that the power to satisfy the different loads is more provided from end-users than grids. Fig. 4 illustrates the benefits of dynamic energy management in the user sector over the traditional economic dispatching. Since more energy users can interact with the actual operation of power grids, the sharing energy benefits significantly for each participant in the energy market.

To further study the effectiveness of our proposed methodology in solving the large-scale optimization problem, we report the results associated with computational performance indices in Table 1, based on the comparison of the computational performance of different methods. All that includes the proposed algorithm, Genetic Algorithm (GA), and gradient descent method, is run under the same computer environment. The optimal solutions of the proposed optimization problem using randomized first-order algorithm and other methods are very close, which

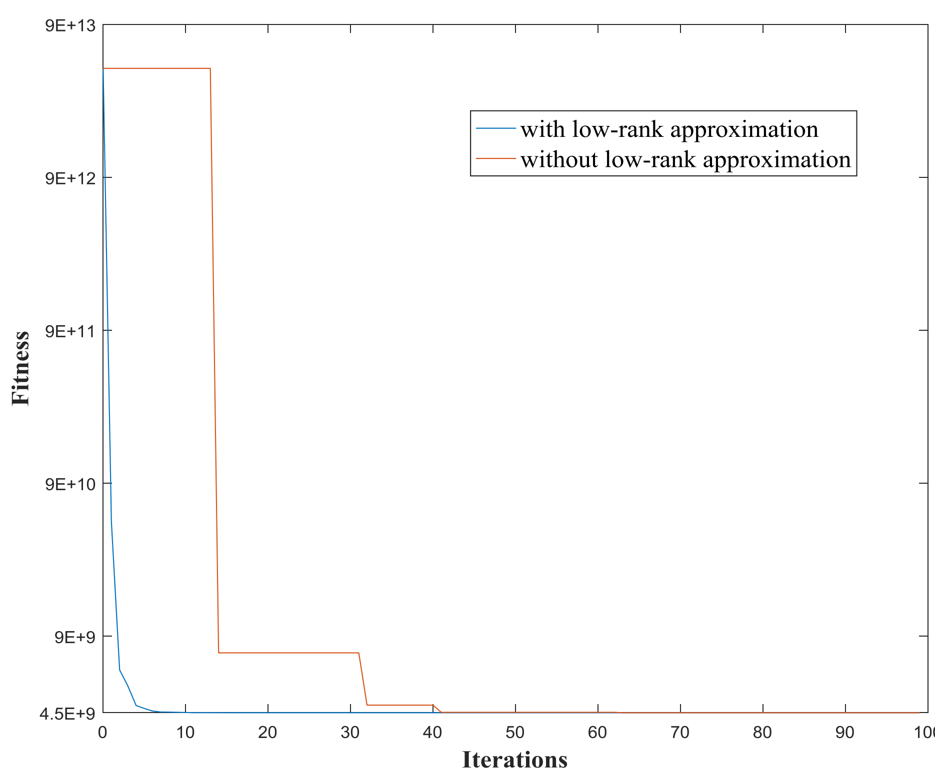

Fig. 1. A numerical comparison of Lagrangian multiplier method with low-rank approximation versus without low-rank approximation.

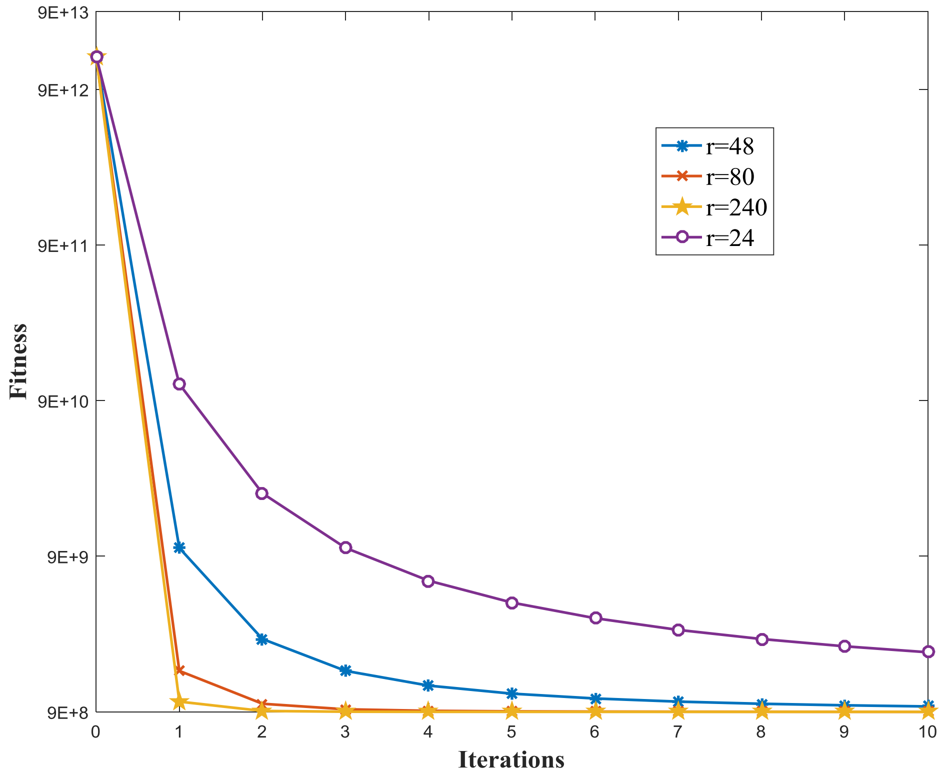

Fig. 2. The impact of randomization on the convergence performance of the proposed algorithm.

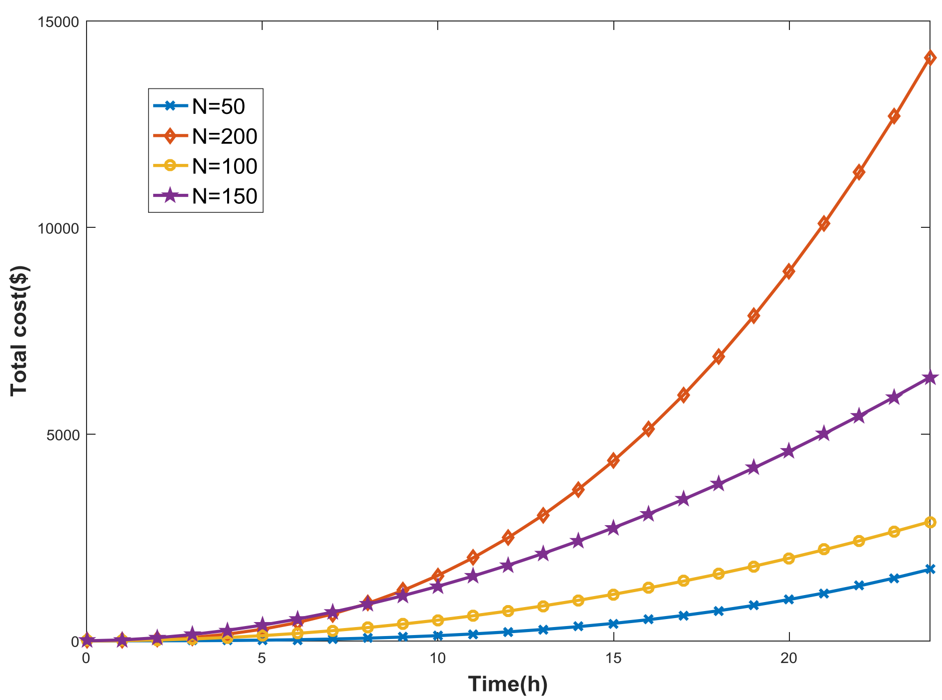

Fig. 3. The effect of number of end-users on the total cost of DEM during the overall scheduling period.

illustrates the effectiveness of randomized first-order approach as a good choice for the big data optimization problem. We can see that the running time for one iteration, which is defined as the time for reading, processing and writing data, shows our presented method can save more computational time than other methods. Moreover, the index of total time indicates the time for obtaining an optimal DEM strategy using the corresponding method. The numerical result shows the proposed algorithm in this paper has less running time than others in whole optimization procedure, which demonstrates the computational benefit of our method. The optimal results of total energy generation cost via GA and gradient descent algorithm are nearly similar, which are different from the one of the randomized first-order method. Specifically, the optimal value of total cost obtained from the proposed algorithm has worse optimality than the other algorithms. This demonstrates that even through the randomized first-order method leads

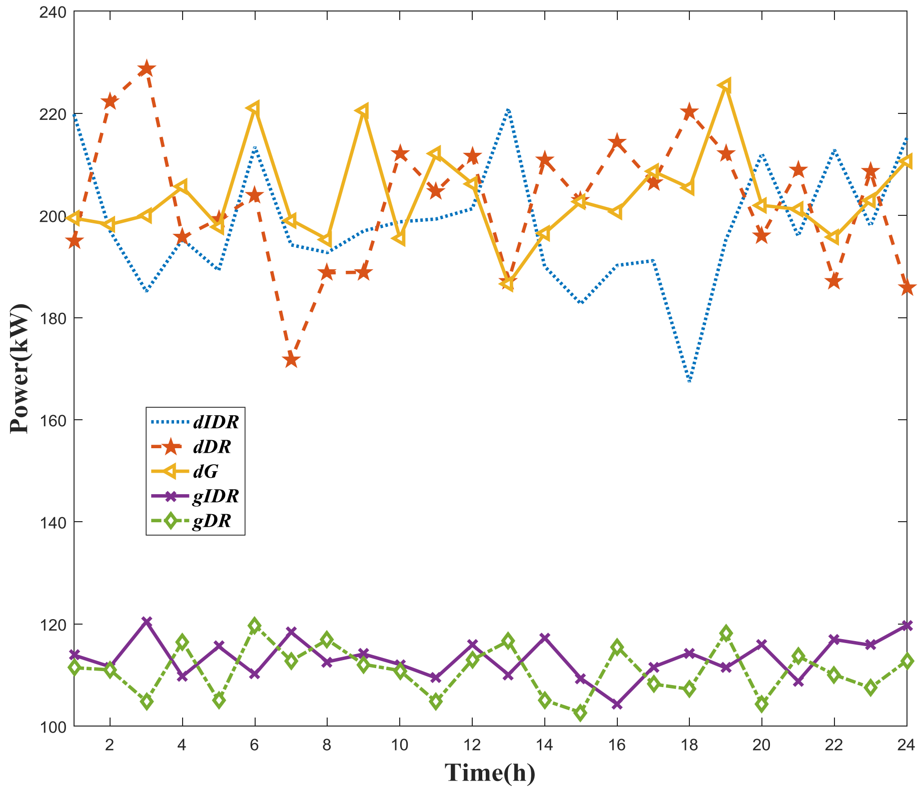

Fig. 4. Preferences assigned by the aggregated end-users to the time slots in the basic case.

Table 1
Comparison of the computational performance of different methods. ($N = 500$, $r = 24$).

	The proposed algorithm	Genetic algorithm	Gradient descent algorithm
One iteration (sec)	3.8	230.5	515.2
Optimal value of total cost ($)	4.604×10^4	4.207×10^4	4.182×10^4
Total time (min)	6.2	481.1	846.5

to rapidly achieve a feasible solution, it may incur the solution is not optimal. Hence, the case study in this paper holds the idea that the randomized first-order algorithm no longer seeks to high-accuracy solutions since big data models are necessarily inexact or simple, which is similar to one in [24]. As for real-world problems, increasingly large data sets may affect the optimality and feasibility of the solution at the optimization phase. Often, it is difficult to deal with optimization problems efficiently, of which the data and parameter sizes are too large. The randomized first-order algorithms can exhibit significant acceleration over their deterministic counterparts since they can generate a good quality solution with high probability by inspecting only a negligibly small fraction of the data. Although traditional optimization tools can be used to address the issue of large data optimization, it is computationally intractable when the problem dimensions grow. In a big data optimization approach, first-order methods with randomization for scalability are assumed to base on simple linear algebra principles and the aiming is to gain surprisingly accelerations even on classical optimization problems. Therefore, the randomized first-order method is more computationally tractable as compared to a traditional optimization problem in the context of big data.

5. Conclusions

In this paper, we investigate how to conduct the DEM strategy when considering the integration of a large deal of end-users who have controllable energy resources in the smart grid. A randomized first-order methodology is presented and used to approach the large-scale optimization problem. The proposed algorithm can be implemented using the Lagrangian multiplier framework with randomization. The low-rank approximation technique as a randomized linear algebra is employed to conduct a sparse matrix whose entries represent all decision variables. As such, a feasible scheduling solution for the DEM in the smart grid can be obtained in high-efficiency. Numerical results demonstrate the computational performance and convergence of our proposed algorithm. Furthermore, the results also display our model can provide a feasible load scheduling for all end-users to achieve the minimal energy generation cost. Finally, we also notice that even though the proposed algorithm has the computational benefit for large-scale optimization problem, it may incur a medium-accuracy solution based on the idea of big data analytics.

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