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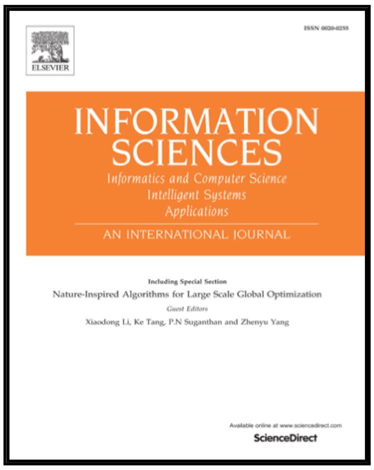

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Using Fuzzy Transform in Multi-Agent based Monitoring of Smart Grids

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Abstract

In this paper, we discuss the main scientific aspects of a Multi-Agent System (MAS), which was designed for monitoring Smart Grids (SGs) with assessment of optimal settings obtained through approximate Optimal Power Flow (OPF) solutions. The consideration behind the approach is that large historical operation data-sets are usually available in SGs and employed to extract useful information; besides, such datasets are also expected to grow over and over because of the pervasive deployment of SGs sensors. So we use Fuzzy transform in order to respond to two issues, that is first to reduce the storage need, by compressing the historical datasets, and second to provide agents with fast and reliable actions to get accurate OPF solutions, by a similarity search throughout the compressed historical dataset. A formal discussion on properties involved by the application of the method is afforded. Numerical results, obtained both on small and large-scale power systems, support the theoretical achievements, by showing the effectiveness of the proposed

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methodology in the task of solving realistic smart grid operation problems.

Keywords: Optimization problems, Power generation dispatch,
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1. Introduction

Smart grids (SGs) deal with information, communication infrastructures and computing technologies in order to transmit and distribute electric energy more efficiently, involving different problems ranging from network optimization to security issues.

In particular, issues such as grid efficiency improvement, flexible load supply, demand side management, emission control, and optimal network regulation can be addressed by a smart management system, which aims at acquiring and processing the available set of information describing the actual smart grid operation state. It is clear that this computing process is complex and time-consuming, since it requires the periodic estimation of the power system state, the analysis of the massive data streams generated by the grid sensors and the repetitive solution of large–scale optimization problems, which are complex, non-linear, and NP-hard problems. Moreover, in order to provide the grid operators with updated information to better understand and reduce the impact of system uncertainties associated with load and generation variations (e.g. in solar and wind power sources), the required computation times should be fast enough [14].

This may also apply to Optimal Power Flow (OPF) analysis, which is widely used for solving many complex power system operation problems (e.g. network reconfiguration, optimal power dispatch, voltage control), since the primary goal of a generic OPF problem is to minimize the total production costs of the entire system to serve the load demand while maintaining the security of the system operation.

Recently, the multiagent system (MAS) approach has been recognized as a promising new paradigm for power grid planning, design and operation [20, 21]. In particular, the MAS technology can meet the SGs requirements [42], with a good employment for control issus in SGs [19].

A MAS involves different kind of intelligent agents which interact both with each other and their environment to achieve some goals. Agents communicate with neighbors and with centralized controller if necessary, gather data from environment and may be able to perform some computations [18].

It is well-known that MASs were used in several contexts, e.g. for intelligent manufacturing systems [10], for the implementation of virtual enterprises [41], for supply chains [17].

In particular, a MAS can be used to solve OPF problems in different ways [33]–[16], as it will be detailed in the next section.

Now, it should be pointed out that in the SG context, the large volume of the datasets makes data collection, storage and processing a very complex and demanding task. Hence, effective tools aimed at reducing the size and the cardinality of SGs data may be very beneficial. Besides, the solution of OPF problems in SG often requires the compliance with strictly time constraints. Hence an approximated solution, if quickly computed, is often more useful than a high quality one, which requires more computation times.

In this paper, in order to respond efficiently to some SGs management functions, such as state estimation and network optimization, we propose an approach which combines the MAS technology with the approximation properties of Fuzzy transform (F-Transform).

In particular, the use of F-Transform is aimed at reducing the cardinality of OPF problems, allowing approximate solutions with a certain accuracy.

F-transform is a fuzzy approximation technique proposed by Perfilieva [28], by stating a functional dependency through a linear combination of basic functions. F–transform was mainly used for image compression/processing (e.g. [3]–[12]) or data compression (e.g. [6, 7]). Other interesting applications deal with data analysis and time series analysis [5]–[27].

The MAS we refer to herein is structured in different types of agents managing the characterizing elements of the grid (e.g. load demand, power generation, active power) and the OPF problem. In particular, the agent in charge of solving the OPF problem uses F–transform in order to get a solution in a reduced domain with a low computational cost.

In the perspective of a two–stage computational paradigm, this MAS approach represents the online stage.

In the offline stage, the F-transform is first used for reducing the cardinality of a knowledge-base, which includes the relevant matrices of the historical power system states and the corresponding OPF solutions. This dataset is subdivided in a proper number of clusters and, for each cluster, a local regression model is determined, providing a nonlinear mapping between the input and the output vectors.

In the online stage, suitable agents, first apply the F-transform to the vector of the measured smart grid state and determine the reference cluster through a similarity search in the compressed dataset, after applies the corresponding local regression model, providing the approximate OPF solution. Finally, the effectiveness of this approximate solution is assessed by checking the problem constraints satisfaction and, if the constraints are not satisfied, the OPF problem is rigorously solved by calling an external suitable function and the approximate solution used as an initial estimation. The obtained solution, jointly with the corresponding given vector of measurements, is then used to adjourn the knowledge-base.

What we have outlined above is supported by formal proofs showing that F–transform preserves similarity. More precisely, since under certain conditions, the minimum Euclidean distance in the transformed domain corresponds to the minimum Euclidean distance in the original domain, then search operations in the transformed domain provide not only fast but also reliable results.

This represents the main scientific contribution of the present paper.

The effectiveness of the proposed approach is demonstrated by means of numerical results obtained for both small and large power networks, namely the IEEE 30-bus test system and the 2383-bus Polish power system. The running times involved by the computation of the approximate OPF solutions are compared with the ones of the rigorous solution obtained by a state-of-the-art approach.

The paper is sectioned as follows. Section 2 briefly introduces OPF problems and a relevant literature review. In Section 3, the theoretical foundations and the main features of the proposed framework are presented. In Section 4, detailed numerical results are discussed. Finally, Section 5 gives some conclusions.

2. Optimal Power Flow problems: an overview

An Optimal Power Flow (OPF) problem can be formulated as a non-linear and non-convex optimization problem. It aims at identifying the value of decision variables, including the control and the state variables, that minimizes one or more objective functions, by satisfying both equality and inequality constraints.

Consider an n_b-bus power system. Let $\mathbf{u}$ be the vector of the control variables, $\mathbf{x}$ the vector of state variables, $\mathbf{f}(.)$ a q-dimensional objective function vector, $\mathbf{g}(.)$ and $\mathbf{h}(.)$ the p-dimensional and r-dimensional vectors representing the problem constraints, respectively. Then the problem can be formalized in a compact form as follows:

$$\begin{aligned} \min_{(\mathbf{x},\mathbf{u})} \quad & \mathbf{f}(\mathbf{x},\mathbf{u}), \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{x},\mathbf{u}) = 0, \\ & \mathbf{h}(\mathbf{x},\mathbf{u}) < 0. \end{aligned} \tag{1}$$

The objective functions $\mathbf{f}(.)$ in (1) can take several forms according to the specific application domain. For example, it can be referred to the minimization of the production costs, the minimization of the transmission line losses, the minimization of the voltage deviations and so on.

The vector of state variables $\mathbf{x}$ in it may be represented by some quantities, such as the voltage magnitude at load buses and the reactive power output of the generators. Similarly, the vector of control variable $\mathbf{u}$ may be mainly written in terms of some quantities, such as the active power output of the generators and the voltage magnitude at the generator buses.

Equality constraints express the non-linear power flow equations, considering as state variables the voltage magnitude and phase angle at load buses, the voltage phase angle and the reactive power generated at the generation buses, and the active and reactive power generated at the slack bus. Instead, the inequality constraints describe the network operating constraints, which include the maximum allowable power flows for the power lines, the minimum and maximum allowable limits for most control variables, such as generator voltages, and for some dependent variables, such as bus voltage limits.

Many important power systems operation problems can be formalized by an OPF problem. In this context, the optimal active power dispatch analysis is recognized as one of the most fundamental tool, since it aims at assessing the optimal output of a number of power generators which meets the system load, at the lowest possible cost, and assures a secure a reliable power system operation.

Let $N_b = \{1, \ldots, n_b\}$ be the set of all buses and N_G the set of generator buses.

The overall problem can be formalized by the following constrained non-linear optimization programming problem [9]:

$$
\begin{aligned}
&\min_{(V_i,\delta_i\ \forall i\in[1,N_b],P_{Gj},Q_{Gj}\forall j\in[1,N_G])} \sum_{i=1}^{N_G} (\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2), \\
\text{s.t. } & P_{Gi} - P_{Di} - V_i \sum_{j=1}^{N} V_j Y_{ij} cos\,(\omega_{ij}) = 0, \quad \forall i \in N_b \\
& Q_{Gj} - Q_{Dj} - V_j \sum_{k=1}^{N} V_k Y_{jk} sin\,(\omega_{jk}) = 0, \quad \forall j \in N_b \\
& V_{i,min} \le V_i \le V_{i,max}, \quad \forall i \in N_b \\
& Q_{Gi,min} \le Q_{Gi} \le Q_{Gi,max}, \quad \forall i \in N_G \\
& P_{Gi,min} \le P_{Gi} \le P_{Gi,max}, \quad \forall i \in N_G
\end{aligned}
\tag{2}
$$

by having assumed $\omega_{ij} = \delta_i - \delta_j - \theta_{ij}$ and where:

- δ_i is the voltage angle at node i;
- θ_{ij} is the phase angle of the ij^{th} element of the bus admittance matrix;
- α_i, β_i, and γ_i are the cost coefficients of the i^{th} generator;
- P_{G_i} and Q_{G_j} are the real and reactive power generated at i^{th} and j^{th} bus;
- P_{D_i} and Q_{D_j} are the real and reactive power demanded at i^{th} and j^{th} bus;
- V_i is the i^{th} bus voltage magnitude;
- Y_{ij} is the ij^{th} element of the bus admittance matrix;
- $V_{i,min}$ and $V_{i,max}$ are the minimum and maximum allowable limits for the voltage magnitude at i^{th} bus;
- $P_{Gi,min}$ and $P_{Gi,max}$ are the minimum and maximum active power limits for the i^{th} generator;
- $Q_{Gi,min}$ and $Q_{Gi,max}$ are the minimum and maximum reactive power limits for the i^{th} generator.

This mathematical formalization can easily be extended to solve the power flow analysis, which is another relevant problem in modern power system operation.

Let N_{PV} be the set of voltage controlled buses, N_P the set of buses where the injected active power is fixed, N_Q the set of buses where the injected reactive power is fixed.

The power flow problem can be stated as a particular instance of the OPF problem (see [31] for details).

The OPF problem can be solved by means of several traditional approaches such as linear programming, non-linear programming, quadratic programming, Newton-based techniques and interior point methods (IPMs) [23, 24].

The IPM seems to be one of the most efficient algorithms for solving the OPF problem [22, 45].

In contrast to traditional methods, several population-based techniques, such as genetic algorithm [2], particle swarm optimization [20], differential evolution [39], imperialist competitive algorithms (ICAs) [8], appeared in the last decades. Such techniques require multiple trials as well as the tuning of some parameters.

Recently, some MAS-based approaches were proposed to solve OPF problems, in order to benefit from a distributed intelligence.

In [33] a MAS was integrated with Differential Evolution (DE). In the resulting algorithm, each agent represents an individual in DE, that is a solution vector of the OPF with its fitness value. The agent with the minimum fitness value is the winner. Simulations were performed on a 6-bus system and the IEEE 30-bus system, by achieving the best values, after 30 different runs.

In [25] a MAS was used to solve the PF problem in an unbalanced distribution system. The radial distribution system was splitted in shunt components and series components; consequently, there was two classes of agents. The agents use the backward/forward sweep technique to iteratively solve the power flow in a completely distributed way. Three test cases were considered, exhibiting a computing time increasing with the dimension of the network. The largest test case, with 369 nodes, showed a computing time of 81.96 s.

The approach proposed in [25] was specialized in [44] to solve another optimization problem in distribution systems, i.e. the Volt/Var Control. In such kind of problem, the optimization objectives include maintaining the system voltage profile within a specified range, minimizing system loss and reducing the switching of shunt capacitors. The performance of the algorithm was validated through the modified IEEE 34 node test feeder.

In [26], the PF problem was solved by means of a distributed version of the shortest path and cost-scaling push-relabel algorithms; 5-bus systems were used as test cases.

Again the PF problem was solved in [38], by rewriting in a distributed way the iterative approach presented in [35]. As a test case was considered a radial system that is the IEEE-18 bus system.

In [16] the OPF problem was solved through a MAS, by establishing six agent types; four of them interact with the grid elements (generators, transformer, etc), one calls a Matlab function for computing the OPF solution and another one manages all the agents. The standard IEEE 6-bus test power system was employed.

Similarly to what proposed in [16], we used several types of agents referred to the characterizing elements of the grid and to the OPF problem, but differently from it, we used F–transform to benefit from a fast and reliable computation of an approximate solution of the OPF problem, as discussed in the next section.

As mentioned above, several MAS–based distributed approaches are appearing in order to avoid the shortcomings of traditional approaches based on centralized computing paradigms, which usually require a central fusion centre acquiring and processing all the grids measurements. Since in the next years, the current data acquisition for SGs are expected to be quadruplicated it is easy to desume the unsuitability of centralized control architecture. It is also true that distributed approaches may have a certain computational cost (e.g. see [25]). In this context, our computing scheme can be regarded as a decentralized approach, where several local devices perform observations, without communicating to each other, but providing necessary information to a supervisor to take a global decision.

It is well–known that decentralized approaches can be collocated at an intermediate level between the centralized and distributed ones, but in our opinion a suitable computing scheme should ensure an acceptable trade-off between a cost-effective solution and better management, higher performance and reliability. So in this paper we explore the potentialities of a fuzzy technique in a decentralized MAS-based scheme for solving efficiently OPF problems.

3. The proposed approach

It is usual in the SGs context mentioning big data, because of the continously increasing historical data [11]. Such data can contribute to a better energy planning, efficient energy generation and distribution. Herein, historical data are used to solve OPF problems in n_b-bus power systems. In this case, each row of the historical database is composed at least by $4n_b$ variables, that is at least $2n_b$ measured variables, such as the active and reactive power injected at each bus, and the $2n_b$ dependent variables describing the OPF solution (bus voltage magnitude and angle at each bus). Since a realistic number of buses n_b may be of the order of several thousand and the number of rows of the historical database increases in time, the cardinality of the problem might be prohibitive.

To address the need for robust and fast OPF solutions in the context of modern SGs, we propose a computational paradigm based on the approximation properties of the F-transform. The underlying principle is that, in practical applications, optimization algorithms are often invoked to solve OPF problems in power system configurations which are not too far different from previously encountered ones. Thus, the idea is to extract useful information from historical OPF solutions, allowing fast and accurate approximate solutions, without unnecessary calculations for similar smart grid states. The use of F-transform ensures a fast and reliable solution process. The main features of the proposed methodology are summarized in the following subsections.

3.1. F–transform: basic notions and new properties

Before introducing the F-transform, the notion of fuzzy partition has to be recalled. Let $I = [a, b]$ be a closed interval and $\{z_1, z_2, \ldots, z_n\}$, with $n \geq 3$, be points of I, called nodes, such that $a = z_1 < z_2 < \ldots < z_n = b$. A fuzzy partition of I is defined as a sequence $\{A_1, A_2, \ldots, A_n\}$ of fuzzy sets $A_i : I \rightarrow [0, 1]$, with $i = 1, \ldots, n$ such that

- $A_i(z) = 0$ if $z \notin (z_{i-1}, z_{i+1})$,
- A_i is continuous and has its unique maximum at z_i, where $A_i(z_i) = 1$,
- $\sum_{i=1}^{n} A_i(z) = 1, \quad \forall z \epsilon I$.

The fuzzy sets $\{A_1, A_2, \ldots, A_n\}$ are called basic functions. They form an uniform fuzzy partition, if the nodes are equidistant, and they can be triangular shaped or not. The norm of a uniform fuzzy partition is $h = (b-a)/(n-1)$.

Usually the sinusoidal shaped basic functions are used:

$$A_j(z) = \begin{cases} \frac{1}{2}\left(\cos\frac{\pi(z-z_j)}{(z_{j+1}-z_j)}+1\right), & z \in [z_j, z_{j+1}] \\ \frac{1}{2}\left(\cos\frac{\pi(z-z_j)}{(z_j-z_{j-1})}+1\right), & z \in [z_{j-1}, z_j] \\ 0, & otherwise. \end{cases} \quad (3)$$

Let n and m be two integers so that $n < N$ and $m < M$. The discrete F–transform gives a linear mapping from $\mathbb{R}^M$ to $\mathbb{R}^m$, in the one–dimensional case, or from $\mathbb{R}^{N\times M}$ to $\mathbb{R}^{n\times m}$, in the two–dimensional case, so that for any $\alpha, \gamma \in \mathbb{R}$

$$\mathbf{F}^{[m]}[\alpha\mathbf{v} + \gamma\mathbf{w}] = \alpha\mathbf{F}^{[m]}[\mathbf{v}] + \gamma\mathbf{F}^{[m]}[\mathbf{w}], \quad (4)$$

with the vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^M$ or

$$\mathbf{F}^{[nm]}[\alpha\mathbf{D} + \gamma\overline{\mathbf{D}}] = \alpha\mathbf{F}^{[nm]}[\mathbf{D}] + \gamma\mathbf{F}^{[nm]}[\overline{\mathbf{D}}], \quad (5)$$

with the matrices $\mathbf{D}, \overline{\mathbf{D}} \in \mathbb{R}^{N\times M}$.

More precisely, let $\{A_1, \ldots, A_n\}$ and $\{B_1, \ldots, B_m\}$ be two fuzzy partitions. The discrete F–transform of the $N \times M$ data matrix $\mathbf{D}$ with respect to $\{A_1, \ldots, A_n\}$ and $\{B_1, \ldots, B_m\}$ is the $n \times m$ matrix $\mathbf{F}^{[nm]}$, of which elements are

$$F_{kl} = \frac{R_{kl}}{T_{kl}}, \qquad k = 1\ldots, n \quad l = 1, \ldots, m \quad (6)$$

being

$$\mathbf{R} = \mathbf{A}^T\mathbf{D}\mathbf{B}, \quad (7)$$

$$\mathbf{T} = \mathbf{A}^T\mathbf{J}\mathbf{B}, \quad (8)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the matrices with elements A_{rk} and B_{sl} respectively, with $r = 1, \ldots, N$, $s = 1, \ldots, M$, $\mathbf{J}$ is a $N \times M$ matrix with all unit elements.

The case of F–transform in one-dimensional domains can be regarded as a singular case where $N = 1$. So, for an M–sized row data vector $\mathbf{v}^T$, one has

$$F_i = \frac{P_i}{S_i}, \qquad i = 1, \ldots, m \quad (9)$$

where P_i and S_i are respectively the elements of the vectors $\mathbf{P} = \mathbf{v}^T\mathbf{B}$ and $\mathbf{S} = \mathbf{1}^T\mathbf{B}$, being $\mathbf{1}$ the M–sized vector with all unit elements.

Eq. (9) can be conveniently written in compact form as follows

$$\mathbf{F}^{[m]} = \mathbf{v}^T\mathbf{B}\overline{\mathbf{S}}, \tag{10}$$

where $\overline{\mathbf{S}}$ is the inverse of the diagonal matrix of order m, of which non-null entries are the elements of $\mathbf{S} = \mathbf{1}^T\mathbf{B}$.

The discrete inverse F–transform allows to get the approximate solution, that is

$$\mathbf{v}^F = \mathbf{B}^T\mathbf{F}^{[m]}, \tag{11}$$

for the one–dimensional case, being $\mathbf{F}^{[m]}$ the vector obtained through Eq. (10) or

$$\mathbf{D}^F = \mathbf{A}\mathbf{F}^{[nm]}\mathbf{B}^T, \tag{12}$$

for the two–dimensional case, being $\mathbf{F}^{[nm]}$ the matrix obtained through Eq. (6).

Fig. 1 summarizes how the discrete F–transform works in one dimensional domains.

Remark 1. *As one can desume from [28], one can write $F_i = v_k \pm \overline{\epsilon}$, for any $k \in [x_i, x_{i+1})$ and arbitrarily small $\overline{\epsilon} > 0$ and $i = 1, \ldots, m$. Similar results can be deduced for the two–dimensional case.*

In order to support the discussion in the next sections, we state the following properties. For the reminder of this work, $\|(.)\|_p$ will denote the p–norm and $\mathbf{I}_N$ will denote the identity matrix of order N.

Lemma 2. *Let $\mathbf{v}$ and $\mathbf{c}$ be two vectors of $\mathbb{R}^M$. Then the following inequality holds*

$$\|\mathbf{F}^{[m]}[\mathbf{v}] - \mathbf{F}^{[m]}[\mathbf{c}]\|_2 \leq \|\mathbf{v} - \mathbf{c}\|_2 \max_j(\overline{S}_{jj})\sigma_{max}(\mathbf{B}), \tag{13}$$

where $\sigma_{max}(\mathbf{B})$ is the maximum singular value of the matrix $\mathbf{B}$.

Proof. The conclusion readily holds by considering that

$$\|\mathbf{F}^{[m]}[\mathbf{v}] - \mathbf{F}^{[m]}[\mathbf{c}]\|_2 = \|\mathbf{F}^{[m]}[\mathbf{v} - \mathbf{c}]\|_2 \leq \|\mathbf{v} - \mathbf{c}\|_2\|\mathbf{B}\|_2\|\overline{\mathbf{S}}\|_2. \tag{14}$$

□

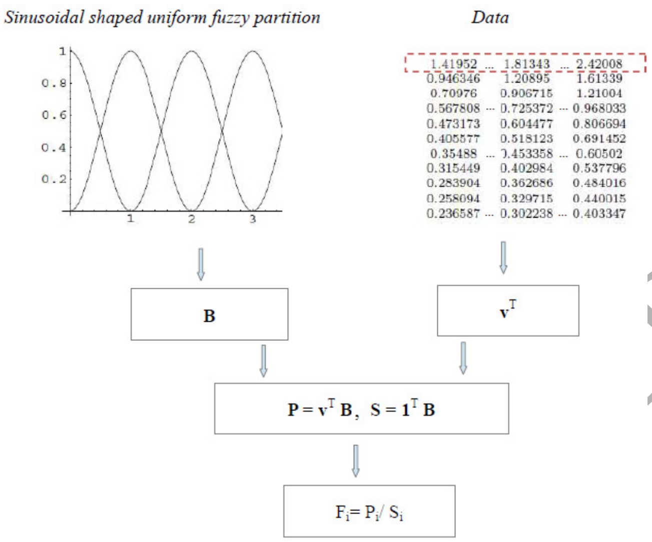

Figure 1: The discrete F-transform

Theorem 3. *Let* $\mathbf{v}$, $\mathbf{c}_1$, $\mathbf{c}_2$ *be three vectors of* $\mathbb{R}^M$. *If* $\max\limits_j(\overline{S}_{jj}) \leq \frac{1}{m}$, *with* $m < M$, *and* $\|\mathbf{F}^{[m]}[\mathbf{v}-\mathbf{c}_1]\|_1 = \|\mathbf{v}-\mathbf{c}_1\|_1+\epsilon$, *for any* $\epsilon > 0$, *then the inequality*

$$\|\mathbf{F}^{[m]}[\mathbf{v}] - \mathbf{F}^{[m]}[\mathbf{c}_1]\|_2 \leq \|\mathbf{F}^{[m]}[\mathbf{v}] - \mathbf{F}^{[m]}[\mathbf{c}_2]\|_2 \tag{15}$$

implies

$$\|\mathbf{v} - \mathbf{c}_1\|_2 \leq \|\mathbf{v} - \mathbf{c}_2\|_2. \tag{16}$$

Proof. Beforehand note that $\sigma_{max}(\mathbf{B}) \leq 1$. So in force of Lemma 1, and by considering that
$\frac{1}{m}\|\mathbf{v} - \mathbf{c}_1\|_2 \leq \frac{1}{m}\|\mathbf{v} - \mathbf{c}_1\|_1 \leq \frac{1}{m}\|\mathbf{F}^{[m]}[\mathbf{v}] - \mathbf{F}^{[m]}[\mathbf{c}_1]\|_1 \leq \frac{1}{\sqrt{m}}\|\mathbf{F}^{[m]}[\mathbf{v}] - \mathbf{F}^{[m]}[\mathbf{c}_1]\|_1 \leq \|\mathbf{F}^{[m]}[\mathbf{v}]-\mathbf{F}^{[m]}[\mathbf{c}_1]\|_2 \leq \|\mathbf{F}^{[m]}[\mathbf{v}]-\mathbf{F}^{[m]}[\mathbf{c}_2]\|_2 \leq \|\mathbf{v}-\mathbf{c}_2\|_2 \max_j(\overline{S}_{jj})\sigma_{max}(\mathbf{B})$
the conclusion readily holds. □

Let E be the finite set of the Euclidean distances of the vector $\mathbf{v}$ from the vectors $\mathbf{c}_1, \ldots, \mathbf{c}_{N_c}$ and let $\overline{E}$ be the finite set of the Euclidean distances of the transformed vector $\mathbf{F}^{[m]}[\mathbf{v}]$ from the transformed vectors $\mathbf{F}^{[m]}[\mathbf{c}_1], \ldots, \mathbf{F}^{[m]}[\mathbf{c}_{N_c}]$. By means of Theorem 3 it is easy to prove the following Corollary.

Corollary 4. *Suppose the condition of Theorem 3 satisfied. Let* $\|\mathbf{F}^{[m]}[\mathbf{v}] - \mathbf{F}^{[m]}[\mathbf{c}_k]\|_2$ *be the minimum distance in* $\overline{E}$*, with* $k \in \{1, \ldots, N_c\}$*. Then* $\|\mathbf{v} - \mathbf{c}_k\|_2$ *is the minimum distance in* E*.*

3.2. The offline stage

The offline stage is aimed firstly at compressing the historical data in order to respond to storage saving issues and secondly at finding the approximating relations between some measurements and the optimal settings of the problem defined in Section 2.

More formally, we have a dataset given by an $N \times M$ matrix $\mathbf{X}$ and an $N \times P$ matrix $\mathbf{Y}$.

The rows of $\mathbf{X}$ are the power system state vectors $\mathbf{v}_j^T = (x_{j1}, \ldots, x_{jM})$, here included the active and reactive powers measured at each network bus, while the rows of $\mathbf{Y}$ are the vectors of the corresponding OPF solutions.

Let $\mathbf{C}$ be an $N_c \times M$ matrix, of which rows are the vectors $\mathbf{C}_k$ representing substantially the centers of the N_c clusters, and let $\mathbf{Y}^C$ be the $N_c \times P$ matrix, of which rows are the vectors $\mathbf{Y}_k^C$ of the rigorous OPF solution related to the input $\mathbf{C}_k$.

Note that $N_c << N$ is the total number of clusters grouping similar state vectors and there is no overlapping between clusters.

The off-line computational scheme can be summarized by the following algorithm:

1. compute the discrete F-transform (see Eq. ((6))) of the matrices $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{C}$, i.e. in the order the $n \times m$ matrix $\mathbf{F}^{[nm]}$, the $n \times P$ matrix $\mathbf{F}^{[nP]}$ (obtained by $\mathbf{B} = \mathbf{I}_P$ in Eqs. (7), (8)) and the $N_c \times m$ matrix $\mathbf{F}^{[N_c m]}$ (obtained by $\mathbf{A} = \mathbf{I}_{N_c}$ in Eqs. (7), (8)), with $n < N$ and $m < M$;
2. for each value $k = 1, \ldots, N_c$
   2.1) compute the Euclidean norm
   $$d(\mathbf{F}_i^X, \mathbf{F}_k^C) = \|\mathbf{F}_i^X - \mathbf{F}_k^C\|_2, \tag{17}$$
   for $i = 1, \ldots, n$;
   2.2) construct the set of r vectors, with $r \leq n/N_c$
   $$Y_F^{(k)} = \left\{\mathbf{F}_i^Y : 1 \leq i \leq n, d(\mathbf{F}_i^X, \mathbf{F}_k^C) \leq \epsilon\right\}, \tag{18}$$
   being ϵ a fixed though arbitrary small real number;

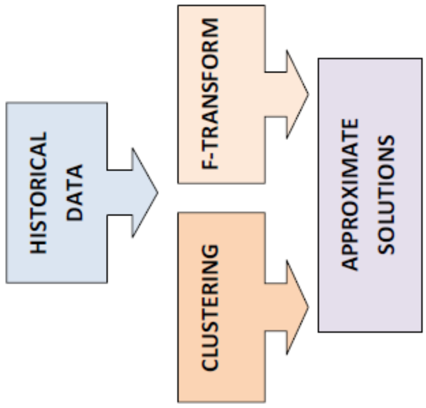

Figure 2: The offline stage

3. for $\mathbf{F}_j^{Y,(k)} \in Y_F^{(k)}$, $j = 1, \ldots, r$, find the mapping

$$\hat{\mathbf{Y}}_k = \beta(\mathbf{F}_1^{Y,(k)}, \ldots, \mathbf{F}_r^{Y,(k)}), \qquad k = 1, \ldots, N_c. \tag{19}$$

being β an unknown functional form to be determined by a regression analysis, i.e. by minimizing the deviations between $\hat{\mathbf{Y}}_k$ and the vector $\mathbf{Y}_k^C$ of the rigorous OPF solution related to the input $\mathbf{C}_k$.

Notice that the set $Y_F^{(k)}$ is substantially the cluster with center $\mathbf{F}_k^C$.
If a linear regression model is adopted, then for any $k = 1, \ldots, N_c$

$$\hat{\mathbf{Y}}_k = \sum_{j=1}^{r} a_{kj} \mathbf{F}_j^{Y,(k)}, \qquad k = 1, \ldots, N_c. \tag{20}$$

i.e. in compact form the regression problem can be formulated as follows

$$\min_{\mathbf{a}} \|\mathbf{a}_k \mathbf{F}^{Y,(k)} - \mathbf{Y}_k^C\|^2, \tag{21}$$

where $\mathbf{a}_k$ is the vector of the unknown real parameters a_{kj}.
The offline stage is outlined in Fig. 2.
The computational cost of the off–line stage is substantially due to the compression of the matrices $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{C}$ by F-transform, that is $O(nM(N+m) + nNP + N_cMm)$ and to the clustering method adopted. For large

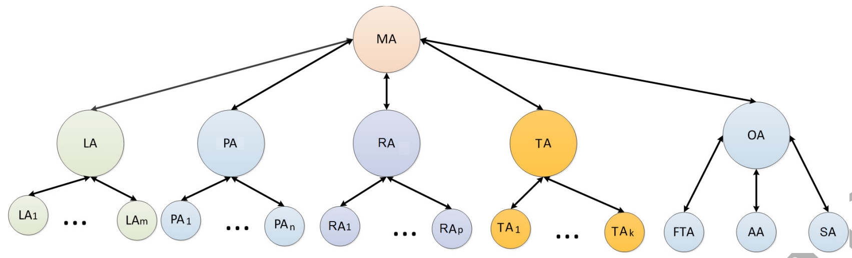

Figure 3: The proposed MAS architecture

datasets, k-means and k-medoids clustering seem more suitable than hierarchical clustering. However, k-medoids clustering is more robust than k-means in presence of noise and outliers. Its cost for each iteration is $O(\overline{k}(N_d - \overline{k})^2)$, where $\overline{k}$ is the number of clusters, N_d the number of elements in the dataset [13].

3.3. The online MAS-based stage

The online stage is referred to the monitoring of the network, by computing for the online measurements the optimal setting through the OPF solution.

At this end, we designed a MAS (Fig. 3) with the following types of agents (the index j is referred to the jth element at the lowest level):

- Main Agent (MA), which communicates with all the agents at the lower level and coordinates their actions;
- Load Agent (LA), which receives the load demand from its load measurement devices LA_j;
- Power Agent (PA), which manages the communications between generators (PA_j) and MA; as soon as the OPF solution is computed, then the new power settings are sent to each generator agent PA_j;
- Reactive power Agent (RA), which handles messages between the MA and each reactive power compensator agent (RA_j);
- Transformer Agent (TA), which manages messages between the MA and each transformer device agent (TA_j);

- Optimization Agent (OA), which receives the measured data from MA and solve the OPF problem, by communicating the solution to MA; it manages three agents:

- F-Transform Agent (FTA), which computes the F-transform vector $\mathbf{F}^{[m]}$ of the state vector $\mathbf{v}^T$ and the Euclidean distance $d(\mathbf{F}^{[m]}, \mathbf{F}_k^{[N_c m]})$ between $\mathbf{F}^{[m]}$ and the center of the kth cluster $\mathbf{F}_k^{[N_c m]}$, for $k = 1, \ldots, N_c$; this distance is sent to OA, which, if $d(\mathbf{F}^{[m]}, \mathbf{F}_k^{[N_c m]}) < \overline{d}$, for a fixed though arbitrary $\overline{d}$, communicates to AA the corresponding not null value of k, otherwise (for a null value of k, that is no cluster is individuated) SA is invoked;

- Approximate solution Agent (AA), which calls the approximating function related to the cluster k and executes a check on the constraints satisfaction; if the check fails, an error message is sent to OA, which calls SA;

- rigorous Solution Agent (SA), which calls an external function for the classical OPF solution; this solution so obtained, jointly with the vector $\mathbf{v}$, is then used off–line for adjourning the knowledge-base by updating the cluster centers and refining the local models.

In Fig. 4, a sequence diagram is shown: it illustrates the case when a cluster k is individuated and a suitable approximate solution (AASOL) is computed by AA and communicated by MA to PA, RA, TA which adjust the settings of their devices.

Remark 5. *OA, through FTA, finds the cluster k which allows the minimum Euclidean distance in the transformed domain. Under the condition of Corollary 3, this minimum corresponds to the one in the original domain, by meaning that the values range is different in the two domains, but the minimum is achieved with regard to the same cluster k.*

It should be pointed out that due to the increasing complexity of control systems in the SG context, it is important embedding knowledge-based and intelligent components in real-time systems.

The proposed computing framework has been designed to be deployed by a hybrid control environment such as DICE [15] which allows interfacing with

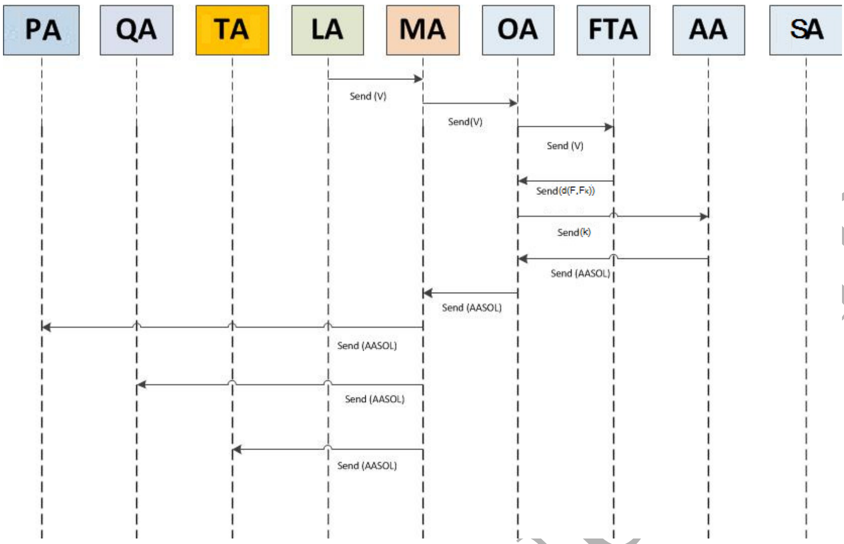

Figure 4: A sequence diagram

external software. The usual requirements regarding scalability, reliability, resource optimization and soft-real-time modelling are pursued by allowing for the dynamics of agents on a network of computers, requiring as unique constraint the presence of the plug-in DICE component on the machine. The agents infrastructure conceived in this way supports the communication of control-based components of a larger systems, considered as unified Java-based environments. Such facility allows an effective monitoring of the agent network.

4. Simulation results

This section presents the results obtained by applying the proposed methodology in the task of solving OPF problems for both small and large-scale power systems.

For each example, we give some comments about the data range and the covariance matrix of data.

In order to fix the number of clusters, we used the Gap statistics, which usually outperforms other methods [36]. However, we also tried an assessment through the well-known Silhouette criterion [32]. It is the case to recall that the Silhouette index is based on the pairwise difference of between and within-cluster distances; the optimal cluster number is obatained by means of the maximum value of such index.

Instead, the Gap test is based on a comparison between the dispersion of clusters generated from the data and the one derived from a sample of null hypothesis sets, by setting a certain sensitivity or tolerance. For a given number of null hypothesis sets, the higher the tolerance the fewer the number of clusters. According to the Gap criterion, the optimal number of clusters corresponds to the solution with the largest local or global gap value within a tolerance range.

With regard to the running times, they were compared to the ones obtained by means of Matpower [43]. Matpower is based on IPM, which is one of the most efficient methods for solving OPF problems, as mentioned in Section 2. In [34] several IPM based solvers have been compared and Matpower turned out to be the best solver for large networks, that is for a number of nodes higher than 100; for a smaller number of nodes, its computation time is competitive in any case. All this motivated us to consider Matpower a good reference for a comparison.

We used the percentage rate $d_r = \frac{t_{rig}-t_{app}}{t_{rig}} \times 100$ as a measure of the relative distance between the running time t_{rig} for the rigorous solution and the running time t_{app} for the approximate solution.

4.1. First example

The first example application deals with the solution of the OPF problem for the IEEE 30-bus test system. The dataset is composed by a 1133×45 matrix (**X** matrix) of input state variable values (that is the values of 45 state variables, in terms of active and reactive power in 1133 instants) and a 1133×60 matrix (**Y** matrix), of which rows represent the OPF solutions (that is bus voltage magnitudes and angles) for the corresponding 45 state variables vector.

The maximum and minimum values of matrices **X** and **Y** are tabled in Table 1.

The covariance matrix of the whole dataset shows a certain degree of dependence, since its entries vary between -9.775 and 27.749.

According to the Gap criterion, by fixing the usual settings, that is the dimension of null hypothesis sets equal to 5 and the tolerance equal to 1, the optimal number of clusters is 12, whereas the Silhouette index provides an optimal value equal to 3 (Figure 10a). It should be pointed out that a similar

Table 1: Minimum and maximum values of the data for the first and second example

dataset	min	max
1st Example (**X**)	0.0211865	41.0147
1st Example (**Y**)	-0.0586714	1
2nd Example (**X**)	-7.2489	653.63
2nd Example (**Y**)	-0.6087	1.0633

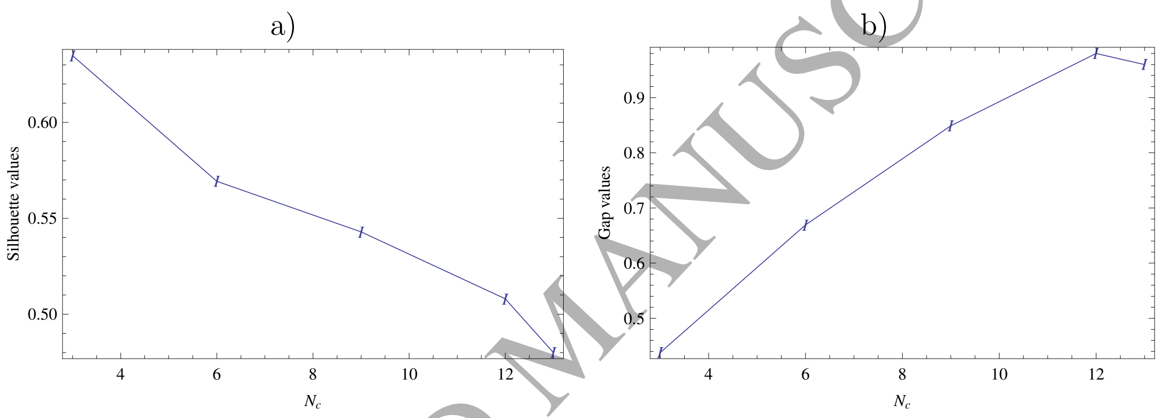

Figure 5: First example, cluster validity indices: a) Silhouette, b) Gap

value can be obtained by the Gap criterion by fixing the tolerance equal to 18.

So the dataset is organized in 12 different sized clusters, from which total 210 sampling cases are randomly extracted, in order to be used in the online stage simulation. In this way, the matrices above turn out to have 923 rows.

In the offline stage, the discrete F–transform is applied to:

- the matrices **X** and **Y**, reducing their cardinality to [300,30] and [300,60];
- to the matrix of centroids **C**, by reducing its cardinality from [12,45] to [12,30].

In Fig. 6 the errors for the validation of the offline stage are depicted. The errors are to be intended as the difference between the computed approximate solution for each cluster and the reference OPF solution related to

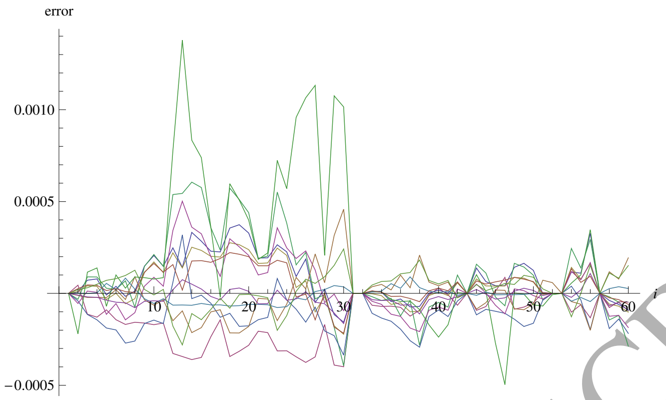

Figure 6: First example: validation errors for the offline stage

the centroid. By using a linear approximation, the maximum error is equal to $1.38E - 03$.

With regard to the online stage, that is a MAS running cycle:

- for the generic input vector $\mathbf{v}^T$, with size $M = 45$, the F-transform action was invoked in order to get a reduced size $m = 30$;
- the distance between the transformed vector and the transformed centroid of each cluster was computed in order to find the reference cluster;
- the approximating function for the detected cluster was retrieved.

In Fig. 7a and 7b the maximum absolute error and the mean error are depicted. These errors vary according to the chosen admissible Euclidean distance (Fig. 7c). For instance, if such value is 29 then the maximum error and the mean error are respectively 1.03E-02 and 2.98E-03; instead, for an admissible value of the Euclidean distance equal to 41.5, the maximum and mean error are 3.38E-02 and 9.94E-03 respectively. The lower the Euclidean distance, the lower the maximum and mean errors.

In Fig. 8 the computed approximate solutions are depicted.

In order to check further our results, we considered the control variables, i.e. the active power generated by 6 generators (in this example). The mean error $\overline{e}_c$ and the standard deviation $\overline{\sigma}_c$ affecting the ith control variable (in p.u.) are tabled in Table 2. These results are good enough, by considering that the maximum values of mean error and standard deviation for the pre-

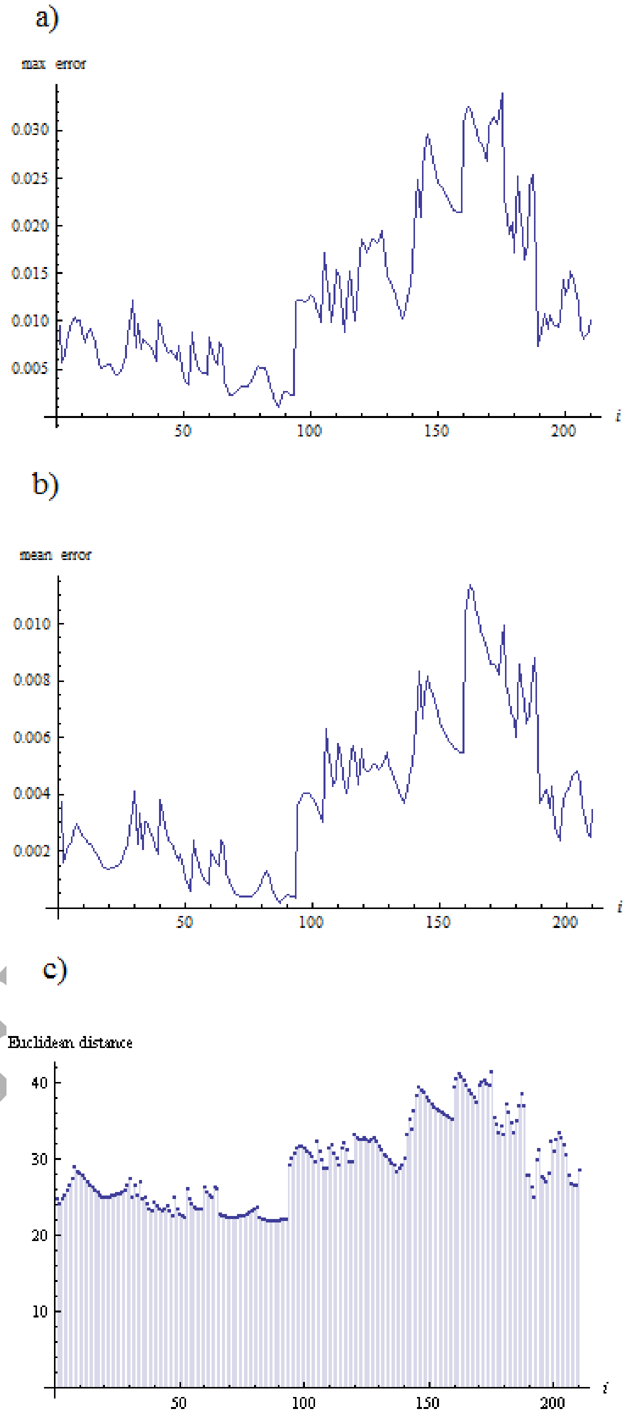

Figure 7: First example: a) maximum errors, b) mean errors, c) Euclidean distances in the online stage

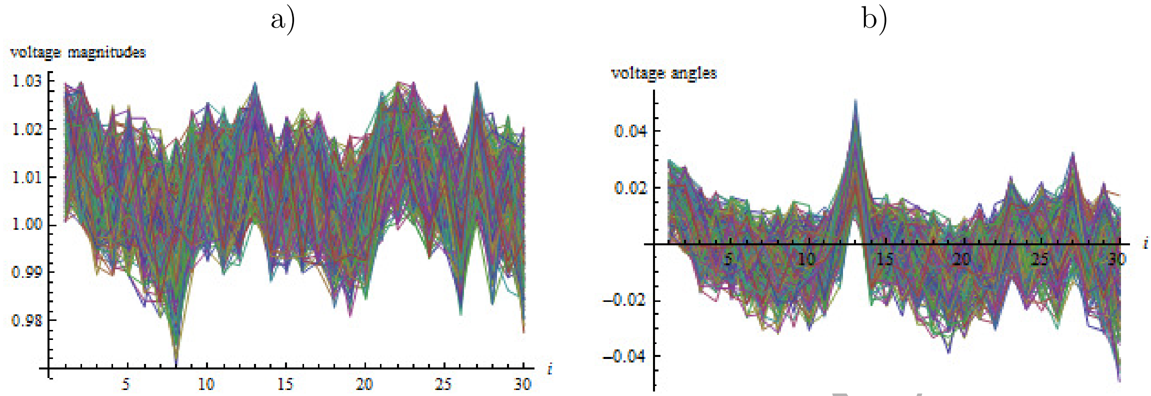

Figure 8: First example: approximate solutions, a) voltage magnitudes b) voltage angles

dicted bus voltage magnitude are $\overline{e}_p = 1.63E-02$, $\overline{\sigma}_p = 9.3E-03$, whereas one has $\overline{e}_p = 1.62E-02$, $\overline{\sigma}_p = 9.2E-03$ for the predicted bus voltage angles.

Finally, the number of samples n_s with respect to a certain percentage rate d_r occurs is shown in Fig. 9. For 1/3 about of the sampling cases, the running time for computing the rigorous solution is $50-60\%$ higher than the one for computing the approximate solution; for one half about of the sampling cases, the running time for computing the rigorous solution is $30-50\%$ higher than the one for computing the approximate solution. In any case, the running time for the rigorous solution is higher.

4.2. Second example

In order to further test the performance of the proposed methodology, the constrained power flow analysis of the 2383-bus Polish power system, which represents the Polish 400, 220 and 110 kV networks composed by 2383 buses, 327 generators, 2056 loads and 2896 lines, was considered. The 3970 state variables of this optimization problem are the voltage magnitude at the load buses, and the voltage angle at all buses except the slack bus. The problem objective is to minimize the power mismatch between the computed and the fixed active and reactive bus powers, satisfying the power flow equations, and assuring that the voltage magnitude at each bus and the reactive power generated at the generation buses are within the allowable ranges.

The dataset is composed by a 1300×3970 matrix (**X** matrix) of input state variable values at 1300 instants and a 1300×4766 matrix (**Y** matrix), of which rows represent the OPF solutions for the corresponding 3970 state

Table 2: First example: mean error $\overline{e}_c$ and standard deviation $overline\sigma_c$ for the control variables

i	$\overline{e}_c$	$\overline{\sigma}_c$
1	1.76E-01	1.22E-01
2	2.39E-01	1.73E-01
3	7.27E-02	5.00E-02
4	5.35E-01	3.66E-01
5	7.14E-02	5.22E-02
6	8.04E-02	5.65E-02 height

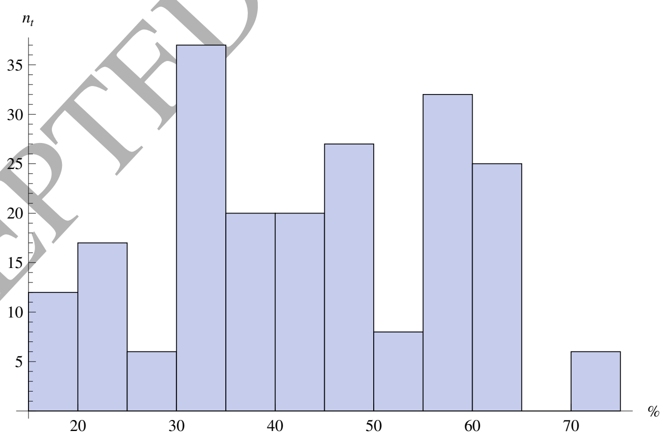

Figure 9: First example: n_s versus d_r

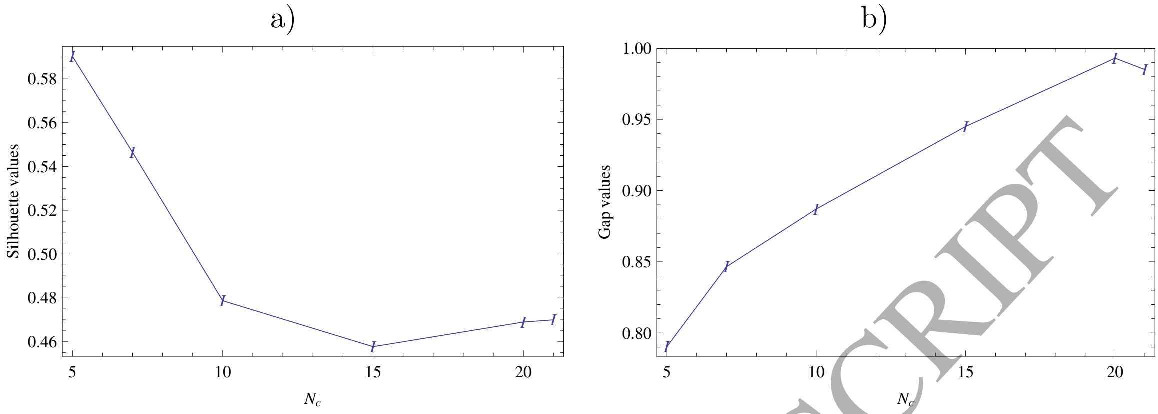

Figure 10: Second example, cluster validity indices: a) Silhouette, b) Gap

variables vectors.

The maximum and minimum values of matrices **X** and **Y** for this example are tabled in Table 1, as well as for the first example.

The covariance matrix entries vary between -878.767 and 6401.46, which means a degree of dependence between data.

The dataset is organized in 20 different sized clusters, from which total 330 cases are randomly extracted and used during the online stage simulation. In this way, the matrices above have 970 rows. As in the first example, $N_c = 20$ is fixed by means of the Gap criterion. In Fig. 10, the Silhoutte and Gap values are shown.

In the offline stage, the discrete F–transform is applied to:

- the matrices **X** and **Y**, reducing their cardinality to [300,600] and [300,700];
- to the matrix of centroids **C**, by reducing its cardinality from [20,3970] to [20,600].

Fig. 11 shows the validation errors for this second example. During the online stage,the F-transform was applied to the generic input vector $\mathbf{v}^T$ with size $M = 3970$, in order to get a reduced size $m = 600$.

For this second example application, the highest values of the maximum absolute error and the mean error throughout the 310 cases are respectively 4.82E-02 and 7.75E-03, with an Euclidean distance equal to 33.

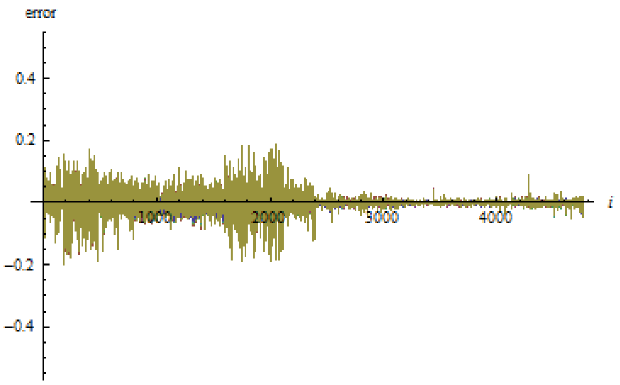

Figure 11: Second example: validation errors for the offline stage

In Fig. 12a, 12b, 12c the maximum absolute errors, the mean errors and the Euclidean distances respectively for 110 test cases are depicted.

The approximate solutions are depicted in Fig. 13.

In Fig. 14 the average prediction error on the control variable (again the generated active power) is shown; the maximum value of such error is 2.7E-02.

The number of samples n_s with respect to a certain percentage rate d_r occurs is shown in Fig. 15. For 2/3 about of the sampling cases, the running time for computing the rigorous solution is $35-55\%$ higher than the one for computing the approximate solution; for 1/6 about of the sampling cases, the running time for computing the rigorous solution is $60-80\%$ higher than the one for computing the approximate solution. In any case, the running time for the rigorous solution is higher. We wish to point out that in [8], it was reported that for solving the simpler OPF case in an IEEE 57-bus test system by means of different versions of ICAs, the mean CPU time varies between 53 s and 63 s, by using MATLAB 7.6 and a CPU clocking in at 2.50 GHz. By means of a CPU with similar performances (i.e. 2.40 GHz), the mean CPU time for our approximate OPF solutions in the 2383-bus Polish power system is 1.50 s.

5. Conclusions

In this paper, a computational framework based on MAS and F-transform is proposed for reducing the complexity of Smart Grids optimization. More

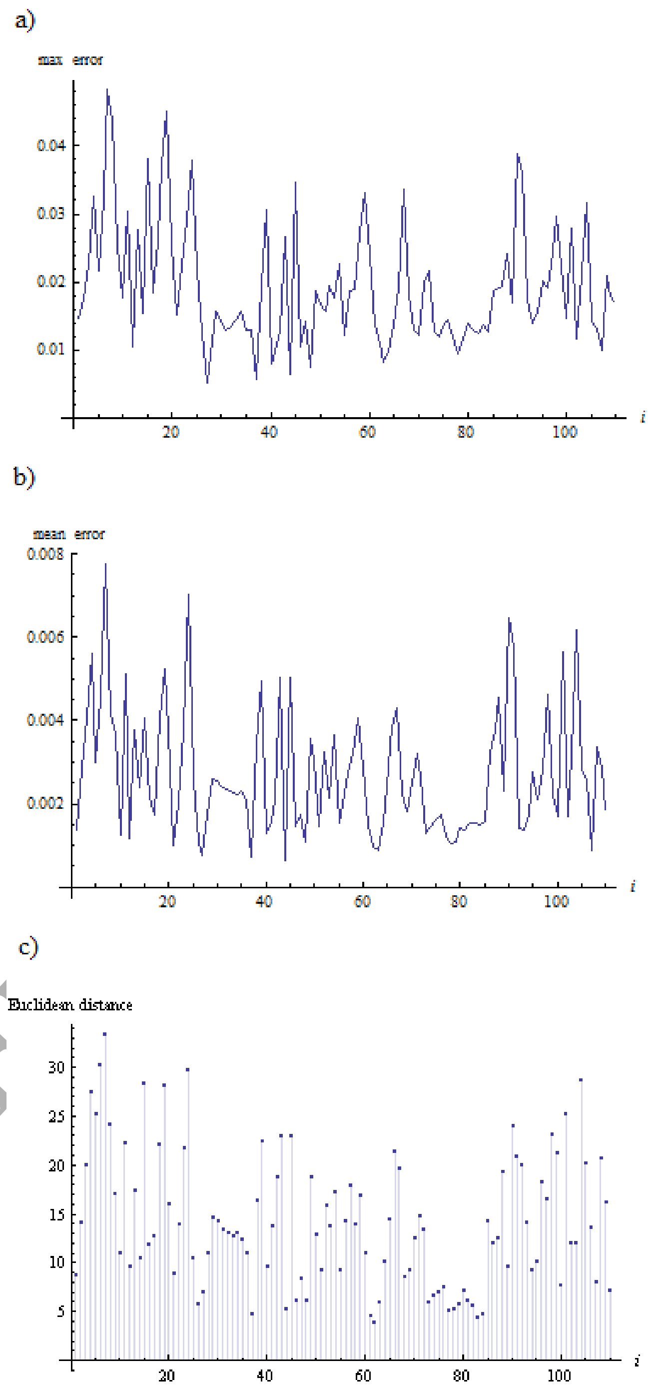

Figure 12: Second example: a) maximum errors, b) mean errors, c) Euclidean distances in the online stage

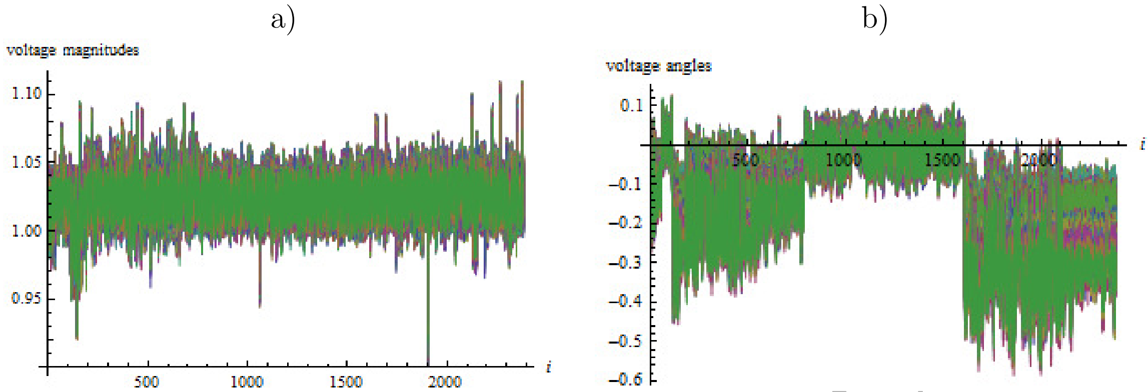

Figure 13: Second example: approximate solutions, a) voltage magnitudes b) voltage angles

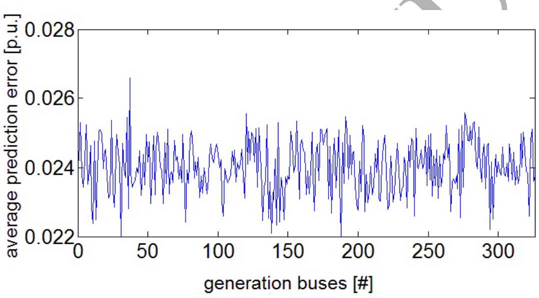

Figure 14: Second example: average prediction error on the control variable

precisely, in view of the huge quantity of historical data available in the modern SGs, we discussed a two-stage scheme which exploits F-transform in two complementary ways:

- reducing the storage occupancy of a clustered historical database in the offline stage, aimed at constructing approximate OPF solutions for each cluster;
- reducing the computational burden of the OPF problem, retrieving the approximate OPF solution by means of a similarity search in the transformed (reduced) domain, allowing fast agent actions in the online stage for the monitoring of the grid.

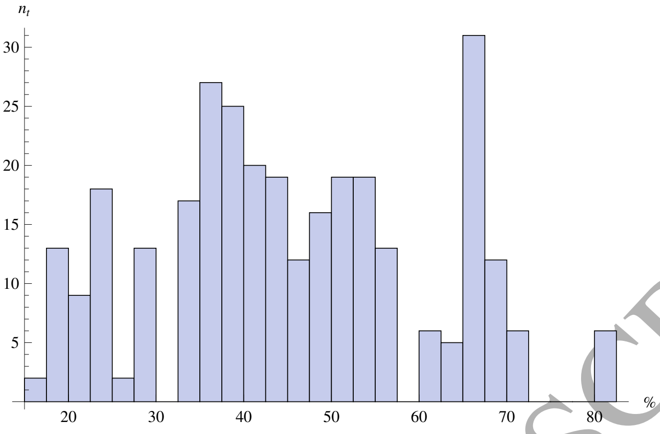

Figure 15: Second example: n_s versus d_r

The numerical results, supported by some theoretical achievements, were obtained for both small and large power systems and they showed the effectiveness of the method.

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