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Sonam Norbu, Santanu Bandyopadhyay

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Power Pinch Analysis for Optimal Sizing of Renewablebased Isolated System with Uncertainties

Sonam Norbu

Department of Electrical Engineering, College of Science and Technology, Royal University of Bhutan, Rinchending, Phuentsholing-21101, Bhutan

and

Santanu Bandyopadhyay*

Department of Energy Science and Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400 076, India.

**Corresponding author: Tel:*+91-22-25767894; *Fax:*+91-22-25726875 *E-mail: santanub@iitb.ac.in*

HIGHLIGHTS

- Method to determine the minimum generator size and storage capacity is proposed.
- Limits of the minimum generator size are determined.
- Proposed method accounts for uncertainty associated with the renewable resources.
- Proposed method incorporates chance constrained programming in Power Pinch Analysis.
- Demonstration and validation of the proposed methodology with photovoltaic-battery example.

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Abstract

Isolated renewable energy system offers promising options to electrify communities located in remote areas where the utility grid is not available or extension of the grid is not economical. Proper sizing of renewable energy conversion system along with storage capacity is the key element to achieve the technical and economical feasibility of such renewable-based isolated system. In this paper, a methodology, based on the concept of Power Pinch Analysis, is proposed to determine the minimum renewable generator area, its extreme limits, and the corresponding storage capacity. The proposed methodology accounts for the uncertainties associated with the renewable resource to size the overall system with a predefined reliability. The concept of the chance constrained programming is applied within the Pinch Analysis framework to incorporate stochastic nature of the renewable energy resources. The applicability of the methodology is demonstrated with an illustrative example of photovoltaic-battery system and verified using sequential Monte-Carlo simulation approach as well as through annual simulation of the system with actual isolation data.

Keywords

Power Pinch analysis; Renewable energy system; Isolated energy system; Chance constrained programming; Photovoltaic battery system

Nomenclature

A	renewable generator area	(m^2)	
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- A_{min} minimum renewable generator area (m²)
- A^* minimum limit of A_{min} (m²)
- A_{α} renewable generator area considering the uncertainty of renewable source (m²)
- C_p power coefficient
- *D* demand (W)

DoD depth of discharge

- *f* net charging/discharging efficiency
- I_T solar radiation intensity (W/m²)
- *P* renewable generator output power (W)

 Q_s storage energy (Wh)

- *R* renewable sources power density (W/m^2)
- S_C storage capacity (Wh)

v wind velocity (m/s)

 Z_{α} standard normal variate with a cumulative probability of α

Greek Letter

- α system reliability level
- λ_{mc} reliability level from Monte-Carlo simulation
- η cyclic efficiency of the storage system
- η_c charging efficiency
- η_d discharging efficiency
- η_{pv} photovoltaic system efficiency
- ρ air density (kg/m³)

 μ_{P_T} mean of the renewable generator output power

 δ_{P_T} standard deviation of the renewable generator output power

Subscript

С	charging
С	storage capacity
d	discharging
min	minimum
S	storage

Abbreviations

ESCA	Electric	System	Cascade	Analysis
LBCA	LICCUIC	System	Cascauc	Anarysis

- GCC Grand Composite Curves
- PCC Power Composite Curves
- PoPA Power Pinch Analysis
- SAHPPA Stand-Alone Hybrid System Power Pinch Analysis

1. Introduction

Renewable energy resources are considered as an important means for overcoming the dependency of the power generation from fossil fuels and the subsequent climate change as well as adverse environmental impacts. In particular, the ability of renewable energy resources to generate electrical energy on small to large scales in remote areas where utility grid is not available or extension of the grid is not economical has drawn interest to form renewable energy based isolated systems [1]. The combination of different renewable energy sources with suitable energy storage system reduces the problem of matching the intermittence of the power supplied by renewable energy sources. Recently, Power Pinch Analysis (PoPA) has been developed to design isolated renewable energy system.

Techniques of Pinch Analysis have been widely used for the conservation of various resources such as heat [2], water [3], mass [4], gas [5], carbon [6], and so on. These techniques are primarily used for process design (both grassroots and retrofits) with special emphasis on the efficient utilization of resources and the reduction of environment pollution [2, 7]. Principles of Pinch Analysis are extended to design renewable-based isolated energy system [8]. Pinch Analysis helps in establishing the minimum resource targets prior to the detail design of the overall system. Application of Pinch Analysis for power targeting in isolated energy systems has been proposed recently. Insight-based graphical and numerical PoPA tools used for optimal hybrid power system supply planning and demand management have gained more attention over conventional mathematical programming approaches.

Some of the available graphical and numerical PoPA tools for setting design targets are: Grand Composite Curves (GCC) [8], Power Composite Curves (PCC) [9], outsourced and storage electricity curves [10], power cascade table [11], etc. The concept of GCC has been applied to size the renewable generators and corresponding storage [8]. The PoPA tools have been extended further by considering the energy losses that occurs during conversion, storage and transfer processes to reflect the actual performance of hybrid power systems. A numerical tool, called the Electric System Cascade Analysis (ESCA), considers the inverter and battery charging/discharging efficiencies in Pinch Analysis framework [12]. ESCA has been applied to optimally design hybrid power system with non-intermittent [12] and intermittent [13] power generators. An enhanced graphical tool, called the Stand-Alone Hybrid System Power Pinch Analysis (SAHPPA) has also been proposed [14]. Storage cascade table has been further modified to include energy losses and to consider AC/DC topology [15]. This modified

algorithm has been applied to optimally design the hybrid power systems with multiple generators (intermittent and non-intermittent) [16].

The PoPA tools have been applied for demand-side management of the hybrid power system also, apart from applications in supply planning. Graphical tools with outsourced and storage electricity curves, have been applied to reduce the maximum storage capacity and the maximum power demand via load shifting [10]. ESCA has been applied to optimize load profiles, storage capacities, and charging/discharging schedule via load shifting [17]. Set of strategies for cost-effective load shifting by manipulating the peak and off-peaks load in hybrid power system using PoPA has also been proposed [18].

PoPA tools are used to explore various energy storage options for integration with conventional battery storage schemes. PoPA tool with AC/DC modified storage cascade table utilizes a framework that applies PoPA to screen various energy storages in hybrid power systems considering the efficiencies, power trends, and economics [19]. Extended PoPA tools are developed to design hybrid power system with battery and hydrogen storage system [20], the minimum required external AC and DC electricity sources and the hydrogen tank capacity [21], profitability of hybrid power system with different feed-in tariffs [22], etc. Along with the development of PoPA concepts, a probability theory as a tool has also been proposed [23]. The summary of available PoPA tools with various applications is shown in Table 1.

Although the insight-based graphical and numerical PoPA tools have been developed, progress has recently been made on the development of mathematical based approaches also. Mathematical PoPA tools includes condensed transshipment model for the analysis and design of off-grid hybrid power system [24], and superstructure-based model for hybrid power system design with energy loss considerations [25].

Identifications of the generator size and storage capacity are the key factors to achieve technical and economical feasibility of an isolated renewable energy system. Available graphical and numerical PoPA tools has been mainly focused on determining the minimum target for outsourced electricity and the amount of excess electricity for storage during start up and normal operations. Incorporation of the renewable resource uncertainty is important for sizing the overall system with given reliability. A methodology, combining the merits of both deterministic and probabilistic approaches through the concept of design space, incorporating the stochastic nature of renewable energy resources and the system reliability, has been proposed [26]. Different generator capacities with their corresponding storage sizes are constructed to establish a feasible design options or the design space. Identification of the design space is done by constructing the sizing curve, which represents the minimum storage capacity for a given generator rating.

In this paper, a methodology to determine the minimum renewable generator size and the corresponding storage capacity is proposed. The proposed methodology accounts for the renewable resource uncertainty through incorporating the chance constrained programming in Power Pinch Analysis framework and helps in sizing an isolated system with predefined system reliability. Accounting for the uncertainty of the renewable source into the framework of Pinch Analysis is one of the major contribution of this paper. The applicability of the proposed methodology is determined through a photovoltaic-battery renewable system for rural application in Bhutan. Results are verified using a sequential Monte-Carlo simulation approach as well as through simulation based on annual insolation data.

2. Renewable energy system modeling

Schematic of an isolated renewable energy system is shown in Fig. 1. The load dispatch is such that, the excess power available after meeting the load is stored in the storage system and when generation of power from renewable generator is insufficient to meet the load, energy is drawn from the storage system, provided the storage system has not reached the depth of the discharge (*DoD*). When the storage system is fully charged, the available excess power is either dumped or not generated.

The power generated by a renewable generator at any given time *t* is given by:

$$P(t) = A R(t) \tag{1}$$

where A is the total area of the renewable power generator (m²) and R(t) is the renewable power density (W/m²). For photovoltaic generator $R(t) = \eta_{pv}I_T(t)$, where η_{pv} is the photovoltaic system efficiency and $I_T(t)$ is the total instantaneous radiation of the array (W/m²) as a function of time. For wind generator $R(t) = \frac{1}{2}\rho v(t)^3 C_p \eta_0$, where v(t) is the time dependent wind velocity, ρ is the air density, C_p is the power coefficient of the wind turbine and η_0 is the overall efficiency of the electro-mechanical subsystems. Similarly, the power density for other alternate renewable energy sources can be suitably defined.

Based on the energy balance of the overall system, the net power flow across the storage is accounted for by considering the efficiencies of the power conversion during charging and discharging processes. The rate of change of energy stored (dQ_{s}/dt) in the storage system is proportional to the net power (difference between the power generated by the renewable generator, *P* and the power supplied to the load, *D*) [27].

$$\frac{dQ_S}{dt} = (P(t) - D(t))f(t) \tag{2}$$

where f(t) represents the efficiencies associated with the charging and discharging process at the time step.

$$f(t) = \begin{cases} \eta_c & \text{whenever } P(t) \ge D(t) \\ \frac{1}{\eta_d} & \text{whenever } P(t) < D(t) \end{cases}$$
(3)

These equations relate the rate of change of energy of the storage with the input power, demand power and the power conversion efficiencies during charging/discharging. The inverter efficiency is accounted for in the estimation of the demand. Change in the stored energy over a time period of Δt , may be expressed as follows:

$$Q_{S}(t + \Delta t) = Q_{S}(t) + \int_{t}^{t + \Delta t} (P(t) - D(t)) f(t) dt$$
(4)

For a relatively small time period Eq. (4) may be approximated as:

$$Q_S(t + \Delta t) = Q_S(t) + (P(t) - D(t))f(t)\Delta t$$
(5)

During the operation of the renewable energy system over the time period Δt , whenever the P(t) > D(t), the excess or energy surplus is stored in the storage system and the charging takes place until the maximum storage capacity is reached, after that the controller stops the charging process and the excess energy is dumped. Whenever P(t) < D(t), the energy deficit is covered by storage, the storage system discharges until it reaches the maximum allowable depth of discharge (*DoD*) at which the controller disconnects the load from the storage system. The self-discharge loss for the storage is assumed to be negligible. However, the proposed methodology is generic and the self-discharging losses can easily be incorporated.

The minimum renewable power generator area required $(A = A_{min})$ and the corresponding storage capacity for meeting the specified load may be obtained by solving Eq. (5) over the entire time horizon. A sustainable storage state of energy may be obtained by determining the minimum renewable generator area such that it satisfies the energy balance Eq. (5) with the following additional constraints [27]:

$$Q_S(t) \ge 0 \quad \forall t \tag{6}$$

$$Q_{S}(t=0) = Q_{S}(t=T)$$
(7)

Eq. (6) ensures that the storage system energy level is always non-negative, while Eq. (7) represents the repeatability of the storage state of energy over the time period. The repeatability condition implies that there is no net energy supplied to or drawn from the storage system over the time horizon, T. It is assumed that the load is occurring in the same pattern after time T. The required storage capacity (S_c) is obtained as:

(8)

$$S_C = \frac{max\{Q_S(t)\}}{DoD}$$

3. Minimum renewable generator area (A_{min}) and its limits

Determination of the minimum renewable power generator area is discussed in this section. The energy balance equation is used to obtain the minimum renewable power generator area, by integration of Eq. (2) over the entire time period we get:

$$\int_{0}^{T} \frac{dQ_{S}(t)}{dt} = \int_{0}^{T} [P(t) - D(t)] f(t) dt$$
(9)

At minimum renewable power generator area $(A = A_{min})$ applying the condition stated by Eq. (7), Eq. (9) may be expressed as:

$$\int_{0}^{T} [P(t) - D(t)] f(t) dt = [Q_{S}(t)]_{0}^{T} = 0$$
(10)

The repeatability constraint of the storage state of energy over the entire time period as expressed by Eq. (7) can be satisfied at the minimum renewable generator area ($A = A_{min}$) only. Considering the charging and discharging processes, as represented by Eq. (3), Eq. (10) may be expressed as:

$$\eta \int_0^T [P(t) - D(t)]^+ dt + \int_0^T [P(t) - D(t)]^- dt = 0$$
(11)

where, $\eta (= \eta_c \eta_d)$ is the cyclic efficiency of the storage system.

The term $\int_0^T [P(t) - D(t)]^+ dt$ represents the excess energy available for charging the storage whenever the power generated by the renewable generator is greater than the demand (represented by Area-1 in Fig. 2). On the other hand, the term $\int_0^T [P(t) - D(t)]^- dt$ represents

the total amount of energy deficit to be covered by storage whenever the power generated by renewable generator is not enough to meet the demand (represented by Area-2 and Area-3 in Fig. 2).

During the operation of the renewable energy system over the entire time period, the following conditions may prevail based on the selected renewable power generator area:

1.
$$\eta \int_{0}^{T} [P(t) - D(t)]^{+} dt = -\int_{0}^{T} [P(t) - D(t)]^{-} dt$$
, whenever $A = A_{min}$ (12)

2.
$$\eta \int_0^T [P(t) - D(t)]^+ dt > - \int_0^T [P(t) - D(t)]^- dt$$
, whenever $A > A_{min}$ (13)

3.
$$\eta \int_{0}^{T} [P(t) - D(t)]^{+} dt < -\int_{0}^{T} [P(t) - D(t)]^{-} dt$$
, whenever $A < A_{min}$ (14)

The excess energy available for charging the storage increases as the renewable generator area is increased above the minimum generator area, beyond the charging capacity of the storage this excess power needs to be dumped. On the other hand, the storage requirement increases due to increase in the requirement of deficit energy to be recovered from the storage as the renewable generator is decreased beyond the minimum generator area. Whenever, the renewable generator area is equal to the minimum generator area ($A = A_{min}$) the positive area exactly matches with the negative area such that the excess energy generated is enough to charge the storage and deficit energy is made available from the storage. The minimum storage capacity is obtained by reducing both the excess energy generated and the deficit energy requirement from storage whenever the renewable generator area is equal to the minimum generator area is equal to the renewable generator area is equal to the minimum generator area is equal to the renewable generator area is equal to the minimum generator area is equal to the renewable generator area is equal to the minimum

In terms of Pinch Analysis, the variation of stored energy level is equivalent to the Grand Composite Curve in heat exchange networks and the point of the minimum stored energy is equivalent to the Pinch Point [2, 26]. The minimum storage capacity is obtained for the minimum renewable generator area, by targeting the excess energy generated by renewable generator and the amount of deficit energy available from the storage.

3.1. Determination of limits for A_{min}

As the cyclic efficiency of the storage system is always less than one ($\eta \le 1$), the excess energy generation can be expressed as:

$$\int_{0}^{T} [P(t) - D(t)]^{+} dt \ge \eta \int_{0}^{T} [P(t) - D(t)]^{+} dt$$
(15)

Adding $\int_0^T [P(t) - D(t)]^- dt$ to both sides of Eq. (15), we get:

$$\int_{0}^{T} [P(t) - D(t)]^{+} dt + \int_{0}^{T} [P(t) - D(t)]^{-} dt \ge \eta \int_{0}^{T} [P(t) - D(t)]^{+} dt + \int_{0}^{T} [P(t) - D(t)]^{-} dt$$
(16)

At the minimum generator area ($A = A_{min}$), Eq (17) may be simplified as:

$$\int_{0}^{T} [P(t)]dt \ge \int_{0}^{T} D(t)dt \tag{17}$$

Using Eq. (1), Eq. (17) can be further simplified to:

$$A_{min} \ge \frac{\int_0^T D(t)dt}{\int_0^T R(t)dt} = A^*$$
(18)

As expressed by Eq. (18), A^* is the minimum limit for the renewable generator area to meet the specified demand. A^* represents the minimum renewable generator area when the load perfectly follows the renewable resource pattern. In such a case, power generation is exactly equals to the demand (i.e., $\int_0^T P(t)dt = \int_0^T D(t)dt$). As the generated power is immediately utilized, there is no requirement of storage and hence, this condition is independent of the cyclic efficiency of the storage.

Similarly, the inequality for the cyclic efficiency of the storage system leads to the following expression:

$$\int_{0}^{T} [\eta P(t) - D(t)]^{-} dt \leq \int_{0}^{T} [P(t) - D(t)]^{-} dt$$
(19)

After adding $\int_0^T [P(t) - D(t)]^+ dt$ to both sides of Eq. (19) and with the help of some algebraic rearrangement, we get the following inequality at the minimum generator area ($A = A_{min}$).

$$A_{min} \le \frac{\int_0^T D(t)dt}{\eta \int_0^T R(t)dt} = \frac{A^*}{\eta}$$
(20)

As expressed by Eq. (20), A^*/η is the maximum limit for the renewable generator area to meet the specified demand. A^*/η represents the renewable generator area when the load is perfectly mismatched with the renewable power generation. In such a case, all the power generated has to be stored in the storage system and the demand is met by draining the storage.

Combining Eqs. (18) and (20), the extreme limits for A_{min} can be expressed.

$$A^* \le A_{min} \le \frac{A^*}{\eta} \tag{21}$$

It should be noted that the limits of the inequality are the tightest, i.e., these limits cannot be improved further. Therefore, the minimum generator size lies within these two extreme limits.

3.2. Determination of A_{min}

Combining Eqs. (10) and (11), the difference between the final and the initial state of storage may be expressed as:

$$\eta_d [Q_S(t=T) - Q_S(t=0)] = \eta \int_0^T [P(t) - D(t)]^+ dt + \int_0^T [P(t) - D(t)]^- dt$$
(22)

As discussed earlier, the initial energy state of the storage is equal to the final energy state of the storage, i.e., $Q_S(t = 0) = Q_S(t = T)$ at the minimum renewable generator area $(A = A_{min})$. If the final stored energy level at the end of the time horizon is lower that the initial storage, i.e., $Q_S(t = 0) > Q_S(t = T)$, then the renewable power source is insufficient to meet the load. Similarly, if the final stored energy level at the end of the time horizon is higher that the initial storage, i.e., $Q_S(t = 0) < Q_S(t = T)$, then the renewable power source is more than the initial storage, i.e., $Q_S(t = 0) < Q_S(t = T)$, then the renewable power source is more than the initial storage, i.e., $Q_S(t = 0) < Q_S(t = T)$, then the renewable power source is more than the minimum and consequently, excess energy is generated.

Using Eq. (1), Eq. (22) may be expressed as:

$$\eta_d [Q_S(t=T) - Q_S(t=0)] = \eta \int_0^T [AR(t) - D(t)]^+ dt + \int_0^T [AR(t) - D(t)]^- dt$$
(23)

Equation (23) implies that the difference between the final and the initial state of storage, Q_S $(t = T) - Q_S(t = 0)$, is a linear function of the renewable generator area (A). It should be noted that the slope of the line depends on the excess energy available as well as energy deficit in different time intervals. In most practical cases, both the renewable resources and the power demand are expressed as step-wise function. In such a case, the slope of the linear relation between the final and the initial state of storage and the renewable generator area depends on the number of intervals with excess energy and the number of intervals with overall energy deficits. Therefore, in general the relation between the difference between the final and the renewable generator area is represented by a piece-wise linear function.

When the renewable generator area is equal to A^* , the difference between the final and the initial state of storage is always non-positive. On the other hand, when the renewable generator area is equal to A^*/η , the difference between the final and the initial state of storage is always non-negative. If either of these values is zero, the minimum generator area can be directly

calculated. Typically, this is not the case. Possible values of the generator area where the slope of the piece-wise linear curve may change, can be calculated by calculating the renewable area required for any time interval. This is calculated by equating the demand and the generation of the renewable energy for each time interval. For interval *i*, the interval renewable area may be calculated as:

$$A_i = \frac{D(i)}{R(i)} \tag{24}$$

Note that the continuous time t is replaced by the time interval index i in equation (24). The interval renewable areas that satisfy the extreme inequality are selected and the difference between the final and the initial state of storage for these areas are determined. As soon as the difference between the final and the initial state of storage changes sign (either positive to negative or negative to positive), the minimum renewable power generator area (A_{min}) can be calculated by linear interpolation. Corresponding storage capacity may be calculated using Eq. (8). This discussion is summarized as a flow chart in Fig. 3.

4. Accounting for resource uncertainties

Determination of the minimum renewable generator area with a given reliability for uncertain renewable resources is discussed in this section. The renewable generator output power P(t), as expressed in Eq. (1), is stochastic due to uncertainty associated with the availability of renewable source. The hourly renewable energy input is assumed as stochastic variable that follows certain probability distribution with known probability parameters. Considering the demand load at time step to be deterministic, the load can be met if the power supplied by the overall renewable energy system ($P_s(t)$) is at least equal to or greater than the demand. To account for the uncertainty of the power output from the renewable generator, the event of P_s (t) being greater than the demand D(t) may be said to have a certain probability of occurrence (α) as expressed by the following chance constraint:

$$\operatorname{Prob}[P_s(t) \ge D(t)] \ge \alpha \qquad 0 \le \alpha \le 1 \tag{25}$$

The chance constraint, expressed in Eq. (25), suggests that the probability of the load being met has a certain confidence level, which is denoted by α and thus, it defines the system reliability. Chance constrained programming is a useful tool applicable for studying mathematical models with random variables, as introduced by Charnes and Cooper [28].

As the load is allowed to be met by the combined sources of storage system and renewable power generator, for a small time step Δt , Eq. (25) may be modified as follows:

$$\operatorname{Prob}\left[P(t) + \frac{Q_{S}(t)}{f(t)\Delta t} - \frac{Q_{S}(t+\Delta t)}{f(t)\Delta t} \ge D(t)\right] \ge \alpha$$
(26)

By rearranging the Eq. (26), the random variable P(t) can be isolated to the left hand side of the inequality sign within the square brackets:

$$\operatorname{Prob}\left[P(t) \le D(t) + \frac{Q_{S}(t+\Delta t)}{f(t)\Delta t} - \frac{Q_{S}(t)}{f(t)\Delta t}\right] \le 1 - \alpha$$
(27)

It may be noted that Eq. (27) represents the cumulative distribution function (CDF) of P(t), which is described to have certain probability distribution [27]. Considering the probability distribution function of P(t), Eq. (27) may be solved analytically to express the deterministic equivalent of the energy balance equation:

$$F_{P_t}\left[\frac{Q_s(t+\Delta t)}{f(t)\Delta t} - \frac{Q_s(t)}{f(t)\Delta t} + D(t)\right] \le 1 - \alpha$$
(28)

$$Q_{S}(t + \Delta t) = Q_{S}(t) + \left[F_{P_{t}}^{-1}(1 - \alpha) - D(t)\right]f(t)\Delta t$$
(29)

The term $F_{P_t}^{-1}(1 - \alpha)$ is the inverse of the cumulative distribution function of renewable energy source evaluated at the particular value of $1 - \alpha$. It represents the power which should be generated by the renewable power generator so as to meet the system reliability requirement of α .

Assuming that P(t) is described in terms of normal probability distribution with known mean and standard deviation, the term $F_{P_t}^{-1}(1 - \alpha)$ can be obtained as follows:

$$F_{P_{t}}^{-1}(1-\alpha) = P(t) = \mu_{P_{t}}(t) - \sigma_{P_{t}}(t)Z_{\alpha}$$
(30)

Where $\mu_{P_t}(t)$ is mean renewable generator power, $\sigma_{P_t}(t)$ is standard deviation of renewable generator power and Z_{α} is the inverse of the cumulative standard normal probability distribution (with zero mean and unity standard deviation) corresponding to the required confidence level α . Equation (30) may be interpreted as the effective renewable power generation for a given system reliability.

$$P(t) = R_{(effective)}(t)A = (\mu_{R_{Effec}}(t) - \delta_{R_{Effec}}(t)Z_{\alpha})A$$
(31)

Equation (21) may be suitably modified to determine the extremes limits on the minimum renewable generator area.

$$A_{\alpha}^{*} \leq A_{\min} \leq \frac{A_{\alpha}^{*}}{\eta}$$
(32)

Where A_{α}^{*} is expressed as follows:

$$A_{\alpha}^{*} = \frac{\int_{0}^{T} D(t)dt}{\int_{0}^{T} R_{Effec}(t)dt} = \frac{\int_{0}^{T} D(t)dt}{\int_{0}^{T} [\mu_{R_{Effec}}(t) - \delta_{R_{Effec}}(t)Z_{\alpha}]dt}$$
(33)

After determining the effective renewable power generation for a given system reliability, methodology described in the previous section may be followed to determine the minimum renewable generator size and the corresponding storage capacity.

5. Illustrative Examples

Applicability of proposed methodology for determining the minimum renewable power generator area (A_{min}) , as well as the maximum generator area $(\frac{A^*}{\eta})$ and the minimum generator area (A^*) limits are demonstrated by considering a photovoltaic battery system as an example of renewable energy system for a remote location in Bhutan.

Bhutan as country in the Himalayan region is rich of natural resources. Hydropower continues to remain as a major source of energy in Bhutan. Given signs of drying water, increase in energy demand, the reliance on a single electricity source, increasing fossil fuel imports, and low hydropower production in winter months; it is evident that there is an urgent need to develop alternative renewable energy systems to diversify the energy policy [30] reports outline an urgent need to optimize and conserve the usage of grid-based power through promotion of dispersed energy generation options such as solar thermal, solar photovoltaic and other stand-alone systems. These reports clearly highlight the need for community-based initiatives in the form of decentralized and distributed generations in remote areas of the country, which have been largely dependent on firewood and kerosene for cooking, heating, and lighting purposes. Integrated master plan study for district-wise electrification in Bhutan [31] identifies that at present only 30% of rural population in Bhutan has access to electricity. It may be noted that 43,673 households in 1716 villages of 20 districts of Bhutan still need to

be electrified [32]. The extension of the national electricity grid into rural areas is technoeconomically unviable, as rural areas are isolated and scattered in the mountains and rugged terrain of the country. Renewable-based isolated energy system has been strongly proposed as an alternative approach to rural electrification [32, 33].

At present, no irradiation measurement stations are available in Bhutan. Based on available processed satellite data, it is estimated that there is an enormous potential of solar energy utilization. Photovoltaic battery system offers a promising option for isolated power generation in these remote locations. One-year solar radiation data for the isolated location of Rinchending (26°51'N, 89°23'E) in Chukhha district of Bhutan is considered for the illustration. Rinchending is located at an elevation of 396m above the mean sea level. Fig. 4 shows the hourly variation of the solar insolation in terms of hourly mean and standard deviation over an average day of the year. Various parameters used for illustration of the proposed methodology are given in Table 2. The daily load curve of the Rinchending region is shown in Fig. 5. The load data is normalized to daily demand of 1 kW according to the pattern of daily electricity use for Rinchending region provided in Power Data Book [34]. The peak load is 54.64W, the average load of 41.87W and the minimum load is 27.23W.

For illustration, the case with system reliability ($\alpha = 0.5$) is considered first. The minimum array area limit (A_{α}^{*}) is found to be 1.63 m² and the maximum array area limit $\left(\frac{A_{\alpha}^{*}}{\eta}\right)$ is found to be 2.26 m². Array area is calculated at each time period Δt and the array area of 1.69 m² is found to be lying between these extreme limits. Corresponding differences between the final and the initial state of storage are shown in Table 3 and Fig. 6. The minimum array area (A_{min}) obtained for the system reliability of 50% (i.e., $\alpha = 0.5$) is 1.95m² with a battery capacity of 844.36 Wh.

Similarly, the minimum array area and the corresponding battery capacity for different system reliabilities are tabulated in Table 4. These results are verified using sequential Monte-Carlo simulation of the system as well as long-term simulation using annual radiation data (see Table 4). It should be noted that the design values are conservative. It may be concluded that application of the chance constraint programming method provides a conservative solution in sizing the overall renewable system.

It may be observed that both the area of the renewable generator and the battery capacity increase with increasing system reliability. For low system reliability the rate of increase of the generator area and the corresponding battery capacity is low. For example, increasing the

system reliability from 60% to 70%, the generator area increases by 16.3% and the corresponding battery capacity increases by 1.8%. On the other hand, the rate of increase of the generator are and the corresponding battery capacity is significant for high reliability. For example, increasing the system reliability from 95% to 99%, the generator area increases 8-fold (by 795.5%) and the corresponding battery capacity increases by 23.7%. Typically, increasing the system reliability beyond 99% is not advised unless the tail distribution is known very precisely. Determination of the system reliability can be determined based on the overall economic affordability. Optimum system reliability can be determined based on the overall economic evaluation of the entire system.

6. Conclusion

A methodology to determine the minimum renewable power generator area, its extreme limits, and the corresponding storage capacity is proposed in this study. The uncertainties associated with the renewable resources to size the overall system with a predefined reliability is considered by integrating the concept of the chance constrained programming with the framework of Power Pinch Analysis. With the known renewable resources, the load profile and system specifications, a methodology to determine the extreme limits of the minimum renewable generator area is derived rigorously. The relationship between the renewable generator area and the difference between the final and the initial state of the storage is explored to determine the minimum renewable generator size. The proposed methodology is illustrated using the photovoltaic-battery system for a remote location in Bhutan for various system reliability. The methodology is verified using a sequential Monte-Carlo simulation approach as well as through simulation based on annual insolation data. Results from Monte-Carlo simulation shows that the proposed methodology provides a conservative estimate in determining the minimum renewable generator area and storage capacity. The proposed methodology is generic in nature and can be easily extended to incorporate other renewable energy sources. The proposed methodology may be suitably combined with available graphical and numerical PoPA tools for optimal design of the isolated energy system, in particular for the application of PoPA tools to generate sizing methods for optimal hybrid power systems.

In the proposed methodology, the uncertainty in the renewable resources is only considered. To device, a methodology to determine the minimum renewable generator and storage capacity by taking into account the uncertainty of both renewable resources and the uncertainty in demand side is identified as one of our future work.

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PoPA Tools	Type of Methods	Remarks
GCC with design space	Graphical	-Targeting storage requirement [8].
PCC and continuous PCC	Graphical	-Targeting electricity at start-up and normal 24-hour operations [9].
Outsourced and Storage Electricity Curves	Graphical	-Allocations of storage and outsourced electricity [10].
		-Demand side management via load shifting [10].
Power Cascade Table and the Storage Cascade Table	Numerical	-Targeting electricity and allocations at start- up and normal 24-hour operations [11].
Electricity system cascading analysis (ESCA)	Numerical	-Isolated renewable energy system design with energy loss considerations [12].
		-Optimal design of isolated energy system with intermittent [13] and non-intermittent [12] power generators.
		-Demand side management via load shifting [17].
Stand-alone hybrid system power pinch analysis (SAHPPA)	Graphical and Numerical	-Enhance graphical PoPA with correction factors for isolated renewable energy system design with energy loss considerations [14].
		-To size intermittent and non-intermittent generators [14].
Modified Storage Cascade Table	Numerical	-Optimally design hybrid power system with multiple generators (intermittent and non- intermittent) [16].
		-Isolated renewable energy system design considering energy losses and AC/DC topology [15].
Extended-PoPA	Graphical and Numerical	-Design of hybrid power system with combined battery and hydrogen storage [20].
Modified Extended PoPA	Graphical and numerical	-To determine the minimum required external AC and DC electricity sources and hydrogen tank electricity capacity [21].
On-Grid Problem Table	Numerical	-To test sensitivity and profitability of a hybrid power system with respect to renewable energy systems (Feed-in-Tariff) failure or shutdown [22].
Probability-PoPA	Numerical	-Eliminates the need to manually match the AC/DC resources and demands [23].

Table 1. Various Power Pinch Analysis (PoPA) tools

Table 2. Parameters	used in	illustrative	example
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Photovoltaic system efficiency (%)	15
Net charging efficiency (%)	85
Net discharging efficiency (%)	85
Battery depth of discharge (%)	70

Table 3 Difference between the final and the initial state of storage for different photovoltaic array area (at $\alpha = 0.5$)

A_{α}^{*}	1.63	-167.9
A _i	1.69	-135.3
A_{α}^{*}	2.26	+165.8
η		

Table 4. Minimum photovoltaic array area and corresponding battery storage capacity for

 different system reliabilities along with Monte-Carlo simulations and long-term simulations

Minimum array	Battery size	Calculated system reliability	
area, $A_{min} (m^2)$	(Wh)	Monte Carlo simulation	Long-term simulation
1.95	844.4	0.89	0.80
2.21	855.5	0.95	0.84
2.57	871.2	0.98	0.89
3.18	900.5	1.00	0.94
4.64	950.9	1.00	0.98
7.36	1023.2	1.00	0.99
65.91	1266.1	1.00	1.00
	Minimum array area, A _{min} (m ²) 1.95 2.21 2.57 3.18 4.64 7.36 65.91	Minimum array area, A_{min} (m^2)Battery size (Wh)1.95844.42.21855.52.57871.23.18900.54.64950.97.361023.265.911266.1	Minimum array area, A_{min} (m^2)Battery size (Wh)Calculated syste Monte Carlo simulation1.95844.40.892.21855.50.952.57871.20.983.18900.51.004.64950.91.007.361023.21.0065.911266.11.00

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Fig. 2: Storage state of energy at minimum renewable power generator area ($A = A_{min}$)



Fig. 3: Procedure to determine minimum renewable generator area and the storage capacity



Fig. 4: Variation in the mean (solid line) and standard deviation (dashed line) of the hourly solar insolation on the array surface for an average day.



Fig. 5: Demand variations for the average day for the location



Fig. 6: Variation of the difference between the final and the initial state of storage with different photovoltaic array area (at $\alpha = 0.5$)

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