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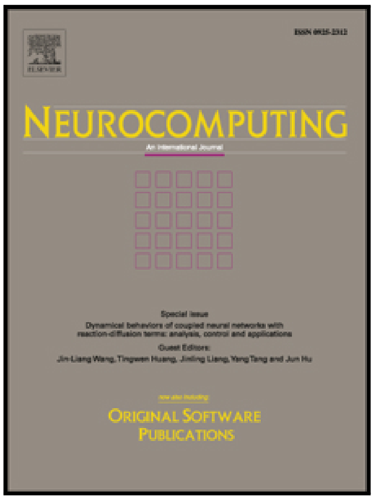

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Real-time pricing scheme based on Stackelberg game in smart grid with multiple power retailers

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Abstract As an essential characteristic of smart grid, demand response may reduce the power consumption of users and the operating expense of power suppliers. Real-time pricing is the key component of demand response which encourages power utilization in an efficient and economical way. In this paper, we study the real-time pricing scheme in smart grid with multiple retailers and multiple residential users using Stackelberg game. Additionally, the price competition among power retailers is formulated as a non-cooperative game, while the coordination among the residential users is formulated as an evolutionary game considering the private information of power retailers and residential users. The existence of Stackelberg equilibrium is proved. Moreover, two special algorithms are developed to solve the equilibrium. Numerical results show the convergence of algorithms, and also confirm the efficiency and effectiveness of proposed real-time pricing scheme.

Keywords smart grid, demand response, real-time pricing, Stackelberg game

1. Introduction

Advanced communication and information technologies [1] have made energy management more flexible in smart grid [2-4]. As a key component of smart grid technology, demand response (DR) may maintain the balance of power supply and demand by peak load shaving. Among demand response schemes, real-time pricing (RTP) is regarded as an efficient way to manage price-responsive loads [5-10]. In recent years, RTP has drawn more attention from policy makers, power companies and many academic researchers. Many methods and technologies, such as optimization theory and game theory [11], have been applied to study RTP.

A lot of works on RTP using game theory, mainly concentrate on the game relationship among the power generators or on the interaction among users [12-14]. In addition, the Stackelberg game approach is applied in [15-19] to study RTP problems. It is worth noting that the above works merely consider one power supplier for reducing the computational complexity. Multiple energy sources and the competition among them still receive very little attention [8]. In fact, considering the further opening of power market and development of new renewable energy sources, users, especially, the users who live in residential district, would find it easier to obtain power from different power suppliers than ever, which complicates the interaction behaviors between power suppliers and residential users [20]. Thus, some sophisticated hierarchical games have been leveraged to shed light on the multi-seller-multi-user RTP problems in the complex power market, see for [21-22].

Different from [21], the smart grid system considered by us in this paper is a power system in smart residential district like [22] where all the residential users have homogeneity in the power consumption process. The smart residential district grid system, the latest hot research area in smart grid, is also what our research focuses on. The power demand of users is accurately described by solving an optimization problem in [21], however, since the users are special residential users in this paper, solving an optimization problem like [21] will meet some obstacles: the power consumption behaviors of each residential user are embodied in choosing a power retailer to purchase power, but each residential user doesn't know the choice of other residential users, which is seen as a privacy issue. Then, the strategy of selecting power retailer to purchase power of each residential user is a mixed strategy process. At last, the evolutionary game is generated to describe the evolution process of power consumption behaviors among residential users. The above solution is what makes our paper innovative and different from [21].

The authors of [22] propose a two-level game where power utilities play a non-cooperative game and residential users play an evolutionary game, which is similar to our paper. But in real smart grid system, the retailers set the unit power prices based on available power and announce them to the residential users, thereby, residential users respond to the prices by an optimal power amount. Since the retailers act first and then the residential users make their decision based on the prices, it is a sequential action for the two participants, which is ignored in [22]. Then, though Nash equilibrium among power retailers and evolutionary equilibrium among residential users are reached, it does not guarantee that the strategy interactions between power retailers and residential users remain stable. That is to say, the equilibrium of Stackelberg game between power retailers and residential users does not necessarily keep existence in [22].

Based on the above reasons, both the power consumption characteristics of residential users and the sequential competition between power retailers and residential users are considered at the same time when demand response mechanism is designed in this paper. So the Stackelberg game model is adopted to study RTP in the power retailing market with multiple retailers and multiple residential users. The retailers procure power from the power wholesale market, and set the real-time electricity prices. Thus, the retailers play the role of the leaders and the residential users have to be the followers. The optimization problems are considered for each retailer and each residential user, respectively. The users who live in a neighborhood area are treated as a population. The evolution process which adjusts power consumption from the retailers is an optimal response to the real-time power prices. Therefore, we formulate an evolution game for the residential users. After the optimal power consumption of residential users is obtained by the evolution equilibrium, the power demand of users is transmitted to power retailers, and then the price competition among the power retailers is formulated as a non-cooperative game. Finally, each retailer sets the optimal real-time price according to the power demand of users. When the residential users and the retailers reach their equilibriums and the sequential competition could not change their equilibriums, the Stackelberg equilibrium (SE) is also achieved.

The contributions of our paper are summarized as follows.

• We formulate the RTP between multiple power retailers and multiple residential users as a Stackelberg game. At the same time, an evolutionary game is generated for the residential users while a non-cooperative game is proposed for the power retailers.

• We design an algorithm to achieve the equilibrium of generated evolutionary game. The existence of Nash equilibrium (NE) is proved for the non-cooperative game among the power retailers. Therefore, after the evolutionary equilibrium is achieved, we also design a distributed algorithm for the power retailers to obtain NE, and then the SE is also reached.

The rest of this paper is organized as follows. We give the system model in Section 2. In Section 3, we formulate the evolutionary game among the residential users. An iterative algorithm is proposed to achieve evolution equilibrium. In Section 4, a non-cooperative game is proposed for the price competition behaviors among the retailers. Section 5 gives the Stackelberg game, and existence of its equilibrium is proved. We provide numerical results and discuss the performance of the proposed pricing scheme in Section 6. The last Section concludes this paper.

2. System model

Now we consider a smart power system with multiple retailers and multiple residential users consisting of a set $\mathbf{M} = \{1, 2, ..., m\}$ of power retailers and a set $\mathbf{N} = \{1, 2, ..., n\}$ of residential users. The smart meters are equipped for the residential users to make the residential users schedule energy consumption. A distribution power station supplies power for the residential users in a certain area.

We take one day as a period and the period is divided into K time slots. $\mathbf{K}$ denotes the set of time slots, and k denotes each time slot, where $k \in \mathbf{K}$. The power retailers accept electricity demand of all users and send real-time unit prices to the users in each time slot. Let p_j^k be the price of retailer j in the k th time slot, and $\mathbf{p}^k = (p_1^k, ..., p_j^k, ..., p_m^k)$ be the price strategy vector. All retailers purchase power and set price to maximize their profits according to the real-time power demand of all users. The retailers compete with each other to maximize their profits by adjusting the real-time electricity unit prices.

2.1. *Utility Function of Residential User*

User i selects a power retailer to serve himself in time slot k, its real-time power demand is denoted by x_i^k, $x_{i,\min}^k \le x_i^k \le x_{i,\max}^k$, where $x_{i,\min}^k$ and $x_{i,\max}^k$ represent minimum and maximum of electricity consumption of user i, respectively. $\mathbf{x}^k = (x_1^k, ..., x_i^k, ..., x_n^k)$ is the real-time power demand vector of the residential users in time slot k.

The power demand of each residential user varies from time to time. The different behaviors of users are depicted by different utility functions. In recent studies about RTP ([5], [11], [13]), the power demand is predicted by exploring the history dates of power consumption. In this paper, we still adopt the quadratic utility function

$$u_i^k(x_i^k, \omega_i^k) = \begin{cases} \omega_i^k x_i^k - \dfrac{\alpha_i^k}{2}(x_i^k)^2, & 0 \le x_i^k \le \dfrac{\omega_i^k}{\alpha_i^k}, \\ \dfrac{(\omega_i^k)^2}{2\alpha_i^k}, & x_i^k > \dfrac{\omega_i^k}{\alpha_i^k}, \end{cases} \tag{1}$$

where ω_i^k and α_i^k are user-specific parameters. Same to [22], we take

$$u_i^k(x_i^k, \omega_i^k) = \omega_i^k x_i^k - \frac{\alpha_i^k}{2}(x_i^k)^2, \quad x_{i,\min}^k \le x_i^k \le x_{i,\max}^k \tag{2}$$

After the m retailers announce their price vector $\mathbf{p}^k = (p_1^k, ..., p_j^k, ..., p_m^k)$ at the k th time slot, user i pays $p_j^k x_i^k$ when consuming x_i^k amount of power if he selects retailer j as the power supplier. Therefore, the welfare function of user i is described as follows

$$\begin{gathered} U_i^k(x_i^k) = u_i^k(x_i^k) - p_j^k x_i^k = \omega_i^k x_i^k - \frac{\alpha_i^k}{2}(x_i^k)^2 - p_j^k x_i^k, \\ x_{i,\min}^k \le x_i^k \le x_{i,\max}^k. \end{gathered} \tag{3}$$

2.2. *Cost and Revenue Function of Power Retailer*

The cost function C_j^k of power retailer j is defined as the cost for procuring the power amount of real-time demand from the power wholesale market at time slot k, and it is denoted as

$$C_j^k = pL_j^k, \tag{4}$$

where p is the electricity price procured by retailer j in power wholesale market, and is set as a constant parameter, L_j^k denotes the amount of power procured by retailer j from power wholesale market at slot time k. Hence, the revenue function of retailer j is given by

$$R_j^k(p_j^k, s_j^k) = p_j^k s_j^k - pL_j^k, \tag{5}$$

where s_j^k denotes the amount of power sold by retailer j in retail power market at slot time k, $s_j^k = \min(L_j^k, D_j^k)$, D_j^k is the power demand of the users from retailer j to be defined later in (9).

2.3. *Interaction between Electricity Retailers and Residential Users*

The retailers provide their power to the residential users in order to obtain larger revenues at lower cost, whereas, the users decide the power consumption to maximize their satisfaction and welfare with a lower payment. According to the behaviors of the power retailers and the residential users, we design a new study scheme which is proposed to assure the profit maximization of the two sides. Therefore, we develop appropriate strategies to maintain the supply-demand balance of power between power retailers and residential users.

In view of the sequence of actions of the retailers and the users, the RTP problem is formulated as a Stackelberg game which is divided into two sides: the retailer side and the user side. Meanwhile, besides the game between the retailers and the users, there is also other competition in each side. One is the competition among the residential users so as to reach welfare maximization, and the other one is the evolutionary process among the residential users. Each residential user selects one retailer to buy power at each time slot in this paper.

3. Evolutionary game among residential users

Evolutionary game [23] has general applications in the engineering field with multiple buyers and multiple sellers [24-25]. An evolutionary game with limited rationality is formulated in our RTP problem, where the player is residential user, the population is set of the residential users, and the strategy is the selection of the retailers. Therefore, the evolutionary equilibrium is achieved by designing a suitable replicator. Similar to [22], we design a replicator to ensure that the evolutionary equilibrium among the residential users is achieved, an iterative algorithm is developed to carry out the replicator dynamics.

3.1. *Formulation of Evolutionary Game*

We consider one population scenario through using bi-directional communication structure in this model. The strategy of each residential user in the population is identical. Each user selects a retailer to buy power when residential users receive the power prices announced by power retailers, then each user gradually adjusts its behaviors and acts independently in the selection process. If the probability of a user choosing retailer j is expressed by $y_j^k (0 \le y_j^k \le 1, \sum_{j=1}^{m} y_j^k = 1)$ at time slot k, the population state is denoted as $\mathbf{y}^k = (y_1^k, y_2^k, ..., y_j^k, ..., y_m^k)$.

3.2. *Replicator Dynamics*

According to (3), when purchasing $x_{i,j}^k$ amount of power from retailer j, the welfare function of residential user i is denoted as

$$\begin{gathered} U_{i,j}^k(x_{i,j}^k) = u_{i,j}^k(x_{i,j}^k) - p_j^k x_{i,j}^k = \omega_i^k x_{i,j}^k - \frac{\alpha_i^k}{2}(x_{i,j}^k)^2 - p_j^k x_{i,j}^k, \\ x_{i,j,\min}^k \le x_{i,j}^k \le x_{i,j,\max}^k. \end{gathered} \tag{6}$$

where $x_{i,j,\min}^k$ and $x_{i,j,\max}^k$ represent minimum and maximum of power consumption of user i from retailer j, respectively. Thus, if $x_{i,j,\min}^k \le x_{i,j}^k \le x_{i,j,\max}^k$, the optimal power demand of user i is achieved at

$$x_{i,j}^k = \frac{\omega_i^k - p_j^k}{\alpha_i^k}. \tag{7}$$

We next redefine the optimal power consumption of user i when purchasing electricity from retailer j at time slot k as follows

$$(x_{i,j}^k)^* = \begin{cases} x_{i,\min}^k, & \dfrac{\omega_i^k - p_j^k}{\alpha_i^k} < x_{i,\min}^k, \\ \dfrac{\omega_i^k - p_j^k}{\alpha_i^k}, & x_{i,\min}^k \le \dfrac{\omega_i^k - p_j^k}{\alpha_i^k} < x_{i,\max}^k, \\ x_{i,\max}^k, & x_{i,\max}^k < \dfrac{\omega_i^k - p_j^k}{\alpha_i^k}. \end{cases} \tag{8}$$

So the total demand for electricity coming from retailer j at time slot k is

$$D_j^k = y_j^k \sum_{i=1}^{n} (x_{i,j}^k)^*. \tag{9}$$

It is worth noticing that p_j^k and L_j^k keep unchanged in the evolutionary process of residential users once they are announced by retailer j. $(x_{i,j}^k)^*$ in (8) is also a constant. We still use a so-called *net utility* [22] to describe the accumulated welfare of users obtained from retailer j. Therefore, the *net utility* becomes

$$\pi_j^k = \begin{cases} \dfrac{1}{2}\sum_{i=1}^{n} \alpha_i^k (x_{i,j}^k)^{*2}, & L_j^k \ge D_j^k, \\ \left[\dfrac{L_j^k}{D_j^k} - \dfrac{(\frac{L_j^k}{D_j^k})^2}{2} \right] \sum_{i=1}^{n} \alpha_i^k (x_{i,j}^k)^{*2}, & L_j^k < D_j^k, \end{cases} \tag{10}$$

where $\sum_{i=1}^{n} \alpha_i^k (x_{i,j}^k)^{*2}$ is regarded as a constant in the evolution process.

In the following, the replicator dynamics is designed to depict the selection dynamics of the population

$$\frac{\partial y_j^k}{\partial t} = y_j^k (\pi_j^k - \overline{\pi}^k), \tag{11}$$

where $\overline{\pi}^k = \sum_{j=1}^{m} y_j^k \pi_j^k$ is the average value of *net utility*.

3.3. *Evolutionary Equilibrium*

The evolutionary equilibrium is achieved when the population keeps its selection unchanged. We determine the selection according to the difference between the *net utility* and its average value [22], thus the evolutionary equilibrium is achieved when the difference is tiny enough, i.e.,

$$\frac{\partial y_j^k}{\partial t} = 0. \tag{12}$$

Hence,

$$\frac{\partial \sum_{j=1}^{m} y_j^k}{\partial t} = \sum_{j=1}^{m} y_j^k (\pi_j^k - \bar{\pi}^k) = \bar{\pi}^k - \bar{\pi}^k \sum_{j=1}^{m} y_j^k = 0. \tag{13}$$

Similar to [22], Lyapunov method [26] is adopted to achieve the convergence of the evolutionary equilibrium (12) with the replicator dynamics (11). The evolutionary equilibrium is denoted by

$$\mathbf{y}^k * = (y_1^k *, y_2^k *, ..., y_j^k *, ..., y_m^k *).$$

3.4. *Iterative Algorithm*

For describing the replicator dynamics, we use the discrete replicator to give an iterative algorithm as follows.

$$y_{j,s+1}^k = y_{j,s}^k + \sigma_1 y_{j,s}^k (\pi_{j,s}^k - \bar{\pi}_s^k), \tag{14}$$

where s is the iteration number, and σ_1 denotes the step size. The terminal criterion is expressed as

$$\left| \pi_{j,s}^k - \bar{\pi}_s^k \right| < \varepsilon, \tag{15}$$

where ε is an arbitrary small positive constant. The detailed algorithm is listed as follows

Table 1

Algorithm 1
1: User i arbitrarily chooses one retailer j to buy power, $\forall i \in \mathbf{N}, j \in \mathbf{M}$;
2: $s = 1$;
3: Repeat.
4: Compute $\pi_{j,s}^k$ by (10);
5: Compute the average value of *net utility* after obtaining all $\pi_{j,s}^k$;
6: Replace retailer to purchase power with the probability $y_{j,s}^k$ by (14);
7: $s = s + 1$;
8: End when (15) is satisfied.

4. Non-cooperative game among retailers

Each retailer aims at maximizing its own revenue through selling the power to the users since it is selfish and rational, then the non-cooperative game is proposed to model the price competition among the retailers.

4.1. *Analysis of the Non-Cooperative Game*

Definition 1. ([27]) A non-cooperative game is a triple $G = \{\mathbf{N}, (S_i)_{i \in N}, (U_i(l)_{i \in N}\}$, where $\mathbf{N} = \{1, 2, ..., N\}$ is the set of players participating in the game. $S_i = \left\{ l_i \middle| l_i \in \left[l_i^{\min}, l_i^{\max} \right] \right\}$ is the set of possible strategies that player i

takes, and $U_i(l)$ is the payoff function.

Definition 2. ([27]) For a non-cooperative game $G=\{\mathbf{N},(S_i)_{i\in N},(U_i(l)_{i\in N}\}$, a vector of strategies $l^*=(l_1^*,l_2^*,...,l_N^*)$ is said to be a Nash equilibrium if and only if $U_i(l_i^*,l_{-i}^*)\geq U_i(l_i^{'},l_{-i}^*)$ for all $i\in\mathbf{N}$ and any other $l_i^{'}\in S_i$, where $l_{-i}=(l_1,l_2,...,l_{i-1},l_{i+1},...,l_N)$ is the set of strategies selected by all the consumers except for consumer i, $(l_i,l_{-i})=(l_1,l_2,...,l_{i-1},l_i,l_{i+1},...,l_N)$ is the strategy profile, and $U_i(l_i,l_{-i})$ is player i's resulting payoff given the strategies of other players.

In the proposed non-cooperative game among the retailers, the player is retailer, the price p_j^k is strategy of retailer j at time slot k, and retailer j 's revenue function given in (5) is written as

$$R_j^k(p_j^k,s_j^k)=\begin{cases} p_j^k D_j^k - pL_j^k, & L_j^k > D_j^k, \\ p_j^k L_j^k - pL_j^k, & L_j^k \leq D_j^k. \end{cases} \tag{16}$$

Lemma 1. ([27]) Nash equilibrium exists in the game if

1) The player set is finite;

2) The strategy sets are closed, convex, and bounded;

3) The utility functions are continuous and quasi-concave in the strategy space.

Based on lemma 1, we obtain the following theorem.

Theorem 1. Nash equilibrium exists in the non-cooperative game among the retailers.

Proof. There are tighter limits on such that $p_j^k\in\left[p_{j,\min}^k,p_{j,\max}^k\right]$ in the non-cooperative game among retailers. The lower limit is the generation costs and associated operating expenses. The retailers keep their price above $p_{j,\min}^k$. The upper bound $p_{j,\max}^k$ is fixed by the policies from government. The retailers must consider these price limits. Therefore, for the m retailers, the strategy sets are nonempty, closed, bounded, compact, and convex subset of $\Re^m$. Substituting (8) into (9), we have

$$D_j^k = y_j^k \sum_{i=1}^{n} \frac{\omega_i^k - p_j^k}{\alpha_i^k}, \quad x_{i,\min}^k \leq \frac{\omega_i^k - p_j^k}{\alpha_i^k} < x_{i,\max}^k. \tag{17}$$

Thus, (16) is rewritten as

$$R_j^k(p_j^k,s_j^k)=\begin{cases} p_j^k y_j^k \sum_{i=1}^{n} \frac{\omega_i^k - p_j^k}{\alpha_i^k} - pL_j^k, & L_j^k > D_j^k, \\ p_j^k L_j^k - pL_j^k, & L_j^k \leq D_j^k. \end{cases} \tag{18}$$

It is obvious that R_j^k is continuous with respect to p_j^k in (18). We next consider the quasi-concave of R_j^k. Since the revenue R_j^k of retailer j is increasing about the amount of power for a given p_j^k, each retailer aims to announce its power price and sell out all the procured power when the available power L_j^k procured from the power whole market is given. i.e., $L_j^k\leq D_j^k$. Actually, the non-cooperative game ends when $L_j^k=D_j^k$, which ensures the power supply-demand balance.

We consider two cases.

1) $L_j^k \le D_j^k$. we have

$$\frac{d^2 R_j^k}{d(p_j^k)^2} = 0 \tag{19}$$

2) $L_j^k > D_j^k$. The second order derivative of R_j^k with respect to p_j^k is

$$\frac{d^2 R_j^k}{d(p_j^k)^2} = -2y_j^k \sum_{i=1}^{n} \frac{1}{\alpha_i^k}. \tag{20}$$

Combing (19) with (20) leads to $\frac{d^2 R_j^k}{d(p_j^k)^2} \le 0$. In summary, R_j^k is always quasi-concave in p_j^k for the two cases. By virtue of Lemma 1, we conclude that Nash equilibrium exists in the non-cooperative game.

5. Stackelberg game between retailers and residential users

5.1. *Stackelberg Game*

From Section 4, we know that the power retailers compete with each other to set the power real-time prices. After the power prices are announced to the residential users by the retailers, the residential users are involved in the evolutionary game and finally their optimal power demand reachs the evolutionary equilibrium. After that, the power demand of residential users is transferred into the retailers, then the interaction behaviors between the retailers and the users are formulated as a Stackelberg game considering the sequential competition action of the retailers and the users, where the retailers involved in the price competition are the multiple leaders and the residential users are the multiple followers. Their objectives are to obtain the Stackelberg equilibrium. The equilibrium strategy for the users in the Stackelberg game is to constitute an optimal response for the announced p_j^k by the leaders.

Definition 3. ([27]) Let $\Gamma_{R,j}$ and $\Gamma_{U,i}$ be the strategy sets for retailer j and residential user i , respectively. The strategy sets of the retailers and the residential users are $\Gamma_R = \Gamma_{R,1} \times \Gamma_{R,2} \times ...... \times \Gamma_{R,m}$ and $\Gamma_U = \Gamma_{U,1} \times \Gamma_{U,2} \times ...... \times \Gamma_{U,N}$. Then, $p_j^k * \in \Gamma_R$ is a Stackelberg equilibrium strategy of retailer j if it satisfies

$$R_j^k(\mathbf{p}^k\mathbf{*}, \mathbf{y}(\mathbf{p}^k\mathbf{*})) \ge R_j^k(p_j^k, \mathbf{p}_{-j}^k*, \mathbf{y}(p_j^k, \mathbf{p}_{-j}^k*)), \forall j \in M, \tag{21}$$

where $\mathbf{p}^k\mathbf{*} = \left\{p_j^k*\right\}$, $\mathbf{y} := \left(\mathbf{y_1^k;y_2^k;...;y_n^k}\right)$ is the strategy of the users, $\mathbf{y}(\mathbf{p}^k\mathbf{*})$ is the optimal response of the users, which is obtained by Algorithm 1**.** The above process cycle until $\mathbf{p}^k\mathbf{*}$ and $\mathbf{y}^k\mathbf{*}$ remain unchanged, and then vector $(\mathbf{p}^k\mathbf{*}, \mathbf{y}^k\mathbf{*})$ is a Stackelberg equilibrium.

5.2. *Existence of Stackelberg Equilibrium*

In the Stackelberg game, when the retailers announce the power price vector, all residential users receive the announced price information, and participate in the evolutionary game. Finally, the evolutionary equilibrium is achieved. As a result, if the retailers adjust their power price to converge to a NE in the non-cooperative game among the retailers, the Stackelberg game owns a Stackelberg equilibrium.

Theorem 2. The Stackelberg equilibrium exists between the power retailers and the residential users.

Proof. The existence of NE is proved in Theorem 1. Based on the equilibrium price of retailers, the convergence to the evolutionary equilibrium with the replicator dynamics (11) is guaranteed, i.e., the optimal response of the users is obtained. Then, the equilibrium of Stackelberg game also exists.

5.3. *Distributed algorithm for SE*

In the game model, the residential users achieve optimal power demand based on the power prices and procured power amount offered by the retailers. A distributed algorithm is developed for the retailers to obtain the NE when each retailer does not know the information of other retailers, thus the Stackelberg equilibrium is reached. The price updating strategy of retailer j is designed by using

$$p_{j,s+1}^{k} = p_{j,s}^{k} + \sigma_2 (D_{j,s}^{k} - L_j^k), \tag{22}$$

where σ_2 denotes a speed adjustment parameter, and s is the iteration number. The terminal criterion of the distributed algorithm is

$$p_{j,s+1}^{k} = p_j^k, \ \left| D_{j,s}^{k} - L_j^k \right| < \varepsilon, \tag{23}$$

where $\varepsilon > 0$. After the power price is adjusted, the residential users evolve to obtain a new equilibrium. Then, the prices of retailers are adjusted again. This process is operated by the following algorithm.

Table 2

Algorithm 2 Distributed algorithm

1: For $\forall\ k \in K$ do.
2: For $s = 1$, arbitrarily choose $p_{j,1}^k$, $\forall j \in \mathrm{M}$;
3: Repeat for $s = s + 1$;
4: User $i = 1, 2, ..., n$;
5: Operating **Algorithm 1**;
6: Compute power demand $D_{j,s}^k$ of the residential users according to (9);
7: Transmit $D_{j,s}^k$ to each power retailer j;
8: Compute $p_{j,s+1}^k$ using (22);
9: Until (23) is satisfied.

6. Numerical results

In this section, we provide some numerical results to discuss the performance of proposed algorithm and validate the above model analysis, then examine how the residential users buy their optimal electricity based on the unit price vector of the retailers and how the retailers optimize their unit price vector based on available power constraints. In the following simulation results, we consider the scenario consisting of two retailers and five residential users. The operating time is divided into 24 time slots. ω_i is randomly selected from [4,10], and $\alpha_i = 0.5$. In the retailer side, and the power wholesale price $p = 0.3$, the available power constraints of all retailers are $L_1 = 22$, $L_2 = 11$ in a time slot.

6.1. *Evolutionary Game*

To evaluate the convergence of Algorithm 1, we simulate for verifying the users to reach the equilibrium by conducting Algorithm 1. We see the residential users converge to the equilibrium quickly with a view to the probability of buying power from two retailers in Fig.1. Fig.2 shows the dynamic change process of the average net utility. Clearly, the residential users have better welfare.

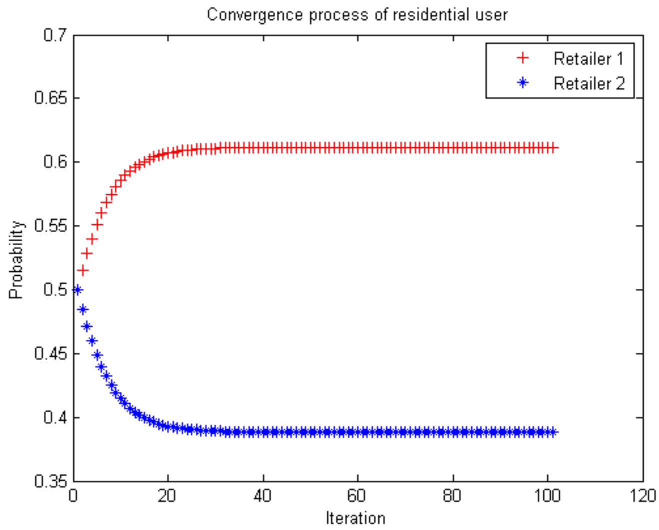

Fig1. Convergence process of residential users

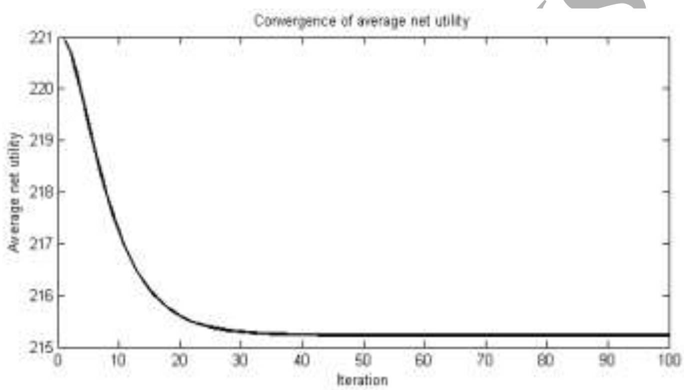

Fig2. Convergence process of the average net utility

The scalability of Algorithm 1 is well reflected in Fig. 3. Algorithm 1 has good scalability with the increasing user number. It is seen that the iteration number always keep in low range.

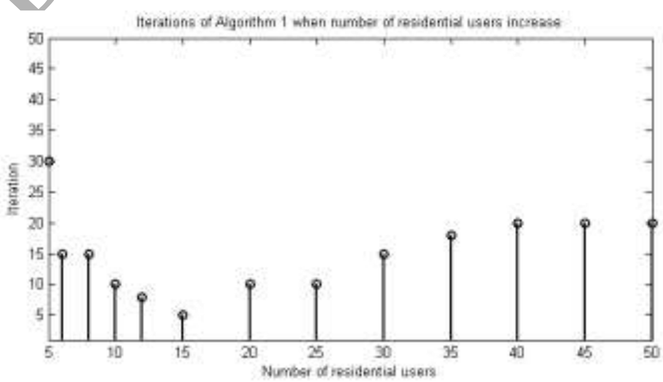

Fig3. Iterations of Algorithm 1 when the number of residential users increases

6.2. *Non-cooperative Game among Retailers and Stackelberg Equilibrium*

For evaluating the performance of Algorithm 2 and obtaining the Stackelberg Equilibrium between the retailers and the residential users, we consider the competition and convergence of Nash equilibrium among the retailers in the non-cooperative game. Fig.4(a) shows that the welfare of the retailers is significantly improved.

Finally, Nash equilibrium is reached and the welfare functions of the retailers are maximized. Fig.4 (b, c) show the convergence process of the retailers according to the amount of power demand and power price. It is obvious that

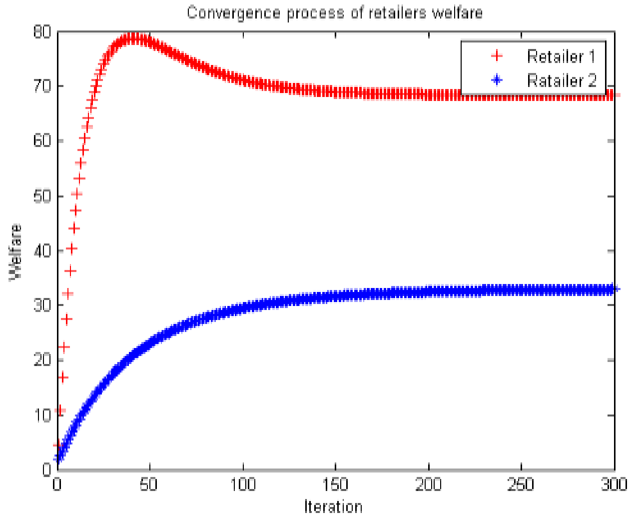

Fig4 (a). Convergence process of retailers' welfare

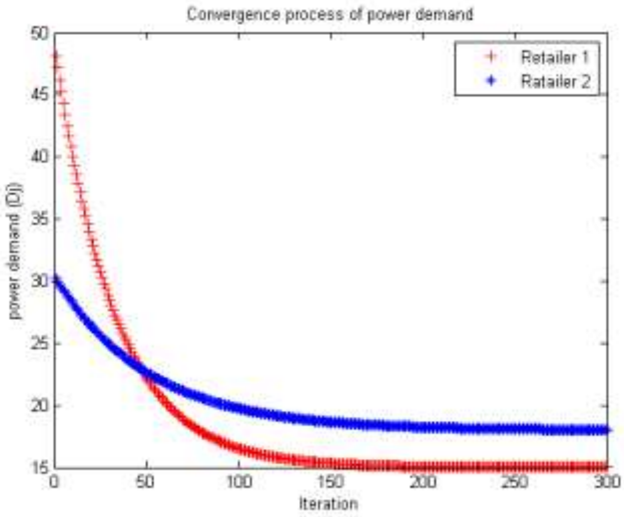

Fig4 (b). Convergence process of retailers' power demand

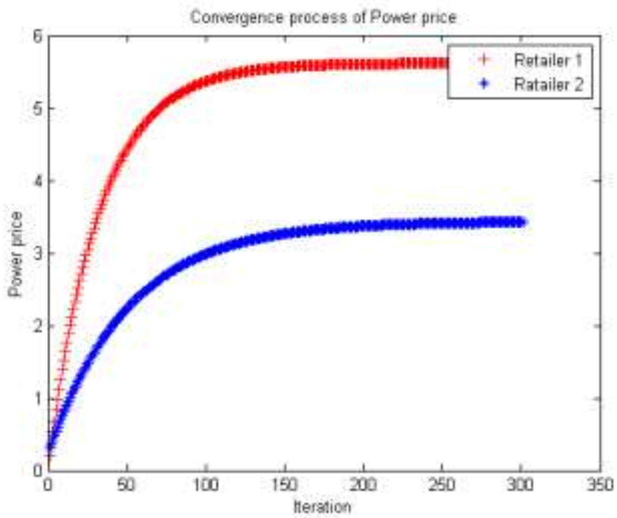

Fig4 (c). Convergence process of retailers' power price both the amount of power demand and the power price converge to a contant which ensures power supply and demand balance. From the above results, we obtain the NE of the non-cooperative game, i.e., equilibrium price of the retailers. We next achieve the evolutionary equilibrium by virtue of section 6.1, then, the Stackelberg equilibrium is reached between the power retailers and the residential users.

6.3. *Comparison with Fixed Pricing Scheme*

To evaluate the performance of proposed real-time pricing scheme, we consider the fixed pricing scheme proposed in [5] for making a comparison with the new pricing scheme in this paper. In this fixed pricing scheme, the retailer keeps a fixed value in the entire process of each time slot. Therefore, the residential users have not taken demand response because of the lack of enthusiasm. Fig.5 shows the power equilibrium price of retailer 1 under two different schemes in 24 hours. It is obvious that the proposed real-time pricing scheme is very remarkable in cutting down the real-time power price. Therefore, the residential users are able to benefit from the proposed pricing scheme in saving payment.

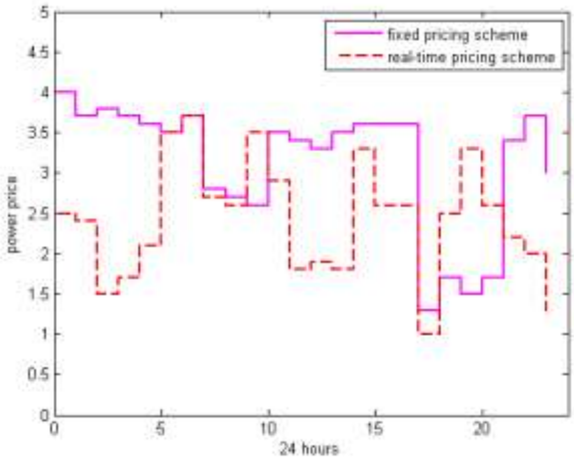

Fig5. Power equilibrium price of retailer 1 under different pricing schemes

7. Conclusion

In this paper, we have proposed a real-time pricing scheme with multiple retailers and multiple residential users. We model the decision process of RTP as a Stackelberg game framework. The price competition among the power retailers are formulated as a non-cooperative game, while the coordination among residential users is modeled as an evolutionary game. We show that all the games converge to the corresponding equilibriums on the basis of proposed scheme.

In addition, we design two iterative algorithms to achieve the equilibrium strategies. Simulation results confirm that the power price is reduced significantly with the proposed real-time pricing scheme, while the power consumption payment of residential users is decreased. Correspondingly, the residential users are able to benefit from the proposed pricing scheme.

However, since smart grid is a complex power system, many uncertainty factors and nonlinear stochastic characteristics exist in such system [28, 29]. It is interesting for us to extend and explore the uncertainty about the power loads of residential users and consider the effect of price prediction [30, 31].

Acknowledgments

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